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Computer-aided optimization of multi-pass turning operations for continuous forms on CNC lathes

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The Computer Numerical Control (CNC) machine is one of the most effective production facilities used in manufacturing industry. Determining the optimal machining parameters is essential in the machining process planning since the machining parameters significantly affect production cost and quality of machined parts. Previous studies involving machining optimization of turning operations concentrated primarily on developing machining models for bar components. Machined parts on the CNC lathes, however, typically have continuous forms. In this study, we formulate an optimization model for turned parts with continuous forms. Also, a stochastic optimization method based on the simulated annealing algorithm and the pattern search is applied to solve this machining optimization problem. Finally, the applications of the developed machining model and the proposed optimization algorithm are established through the numerical examples.

1. Introduction

Optimizing machining conditions has become increasingly important owing to the extensive applications of Computer Numerical Control (CNC) machines. In the process planning of CNC machining, determining optimal machining parameters is necessary to satisfy requirements involving machining economics, machining quality and machining safety. The machining parameters in multi-pass turning operations consist of cutting speed, feed, depth of cut and number of passes. The machining parameters significantly affect the cost, productivity and quality of machined parts. Determining machining parameters is generally based on optimizing certain economic criteria which are subjected to a set of machining constraints. A number of economic criteria such as minimum unit production cost, minimum unit production time and maximum profit rate are used to measure the performance of machining operations. The machining constraints usually consider the CNC machine specifications, CNC machine dynamics, cutting tool dynamics and machined part design specifications. Therefore, the machining optimization problems are generally constrained nonlinear programming problems with a high computational complexity.

Although previous studies involving optimizing turning operations are remarkable, they concentrated primarily on developing cutting models and solution approaches for bar components [1–15]. The machining models formulated in these works only considered bar components requiring straight turning. This process involves cutting a workpiece in the longitudinal (z) direction to produce a constant stock diameter. In the real-world turning process, the parts turned on CNC lathes frequently have continuous forms. Figure 1 presents a turned part with a continuous form. These usually consist of bar components, taper components and circular components. Such turned parts additionally require taper turning, face turning and circular turning (see Fig. 1). A taper is a uniform reduction in diameter measured along the axis of the workpiece; in addition, linear interpolation is used to cut the taper. Face turning involves material removal as the tool moves in the x direction that is perpendicular to the z axis. Circular turning is typically used to machine concave or convex circular shapes. In this study, we formulate a machining model for turned parts with continuous forms to extend machining optimization applications.

Many solution approaches have been used to optimize turning operations, e.g., calculus differential approach [12], geometric programming [5,11,12], Lagrangian optimization method [3,9], goal programming [12,14], Sequential Unconstrained Minimization Technique (SUMT) [8], linear approximation method [15], simplex search [1,2], direct search combining random search and Hooke–Jeeves pattern search [10], a combination of dynamic programming and Fibonacci search [13], and a combination of geometric and linear programming [6,7].

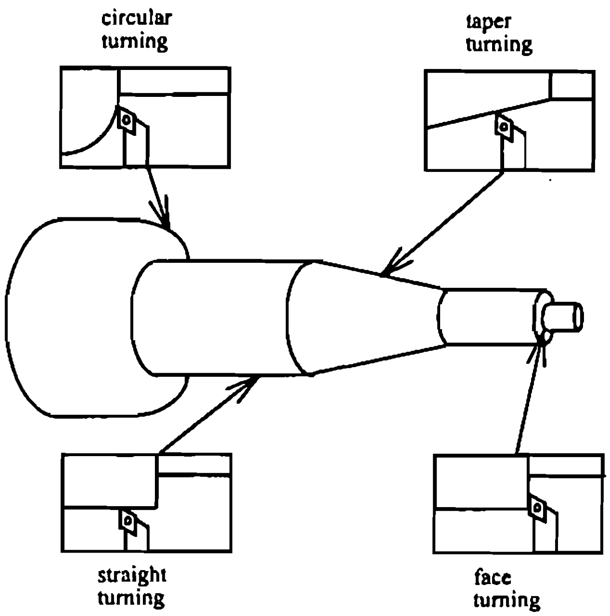

Fig. 1. A turned part with the continuous form.

Among the existing methods, no generalized solution method is available for all machining optimization problems [16]. The existing methods differ in their reliability, efficiency and sensitivity to initial solution. Furthermore, they are only useful for a specific problem or are inclined to obtain a local optimal solution.

This study aims to formulate a machining model for turned parts with continuous forms, and to develop a generalized solution method for determining the optimal machining parameters. The turned components considered herein can be characterized in terms of straight, taper and circular profiles. The turning operations for machined parts with continuous forms studied here have multiple rough cuts and a single finish cut. The optimal machining parameters are determined with respect to minimum unit production cost criterion and a set of practical constraints. The proposed optimization algorithm is a hybrid approach that combines Simulated Annealing (SA) [17,18] and Hooke–Jeeves Pattern Search (PS) [19]. This algorithm can solve large turning optimization problems for actual manufacturing applications, and also deliver a solution approximating to the global optimum. It has been reported that the future of SA depends on its application with other well-tuned heuristics [20]. Whilst the SA technique is well-known for its capability to escape from local optima, it is not efficient with respect to the number of iterations [21]. PS has been widely used and is considered to be efficient in the area of nonlinear programming [22]; however, it is more inclined to terminate on a local optima. In this study, a more efficient stochastic optimization approach is developed by incorporating PS into SA.

The rest of this paper is organized as follows. Section 2 discusses the expressions for cutting time for various turning operations. Section 3 formulates the machining model for turned parts with continuous forms. Section 4 presents the proposed optimization algorithm. In Section 5, two numerical examples are described and their computational results are summarized. Concluding remarks are finally made in Section 6.

2. Computation of cutting time

For turned parts with continuous forms, turning processes include circular interpolation and/or linear interpolation. The linear interpolation can be classified into three types of operations: straight turning, face turning and taper turning. Previous studies have developed turning models for bar components, in which the cutting time is only calculated for straight turning. Therefore these previous models cannot be employed to optimize turning operations for parts with continuous forms since the computation of cutting time for each type of turning process is different. The expressions of cutting time for various turning processes are presented as follows.

The cutting time of straight turning T (min) can be calculated by [23]

$$T = \frac{\pi DL}{1000Vf}, \tag{1}$$

where D and L are the diameter and length of the workpiece (mm); V is the cutting speed (m/min); and f is the feed (mm/rev). From Fig. 2, the formula for the cutting time (Equation (1)) for straight turning between any two points $P_1(z_1, x_1)$ and $P_2(z_2, x_2)$ can be rewritten as

$$T = \frac{\pi|x(z_2 - z_1)|}{500Vf}, \tag{2}$$

where $x = x_1 = x_2$; $2x = D$; and $|z_2 - z_1| = L$.

In turning operations, the spindle speed N (rpm) can be represented as the cutting speed V divided by the circumference length of the given diameter D [24], thus

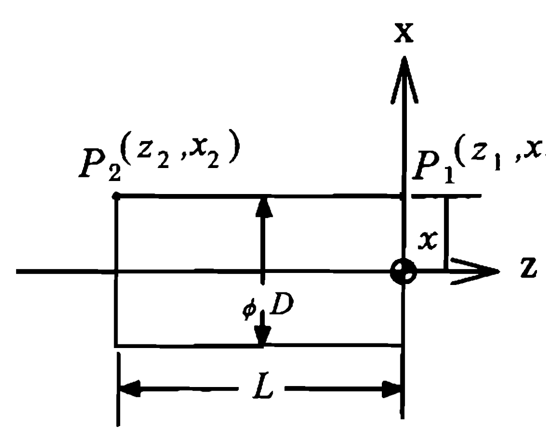

Fig. 2. Straight turning.

$$N = \frac{1000V}{\pi D}. \tag{3}$$

In straight turning the distance between the tool and the workpiece center is a constant. Thus, a constant cutting speed V can be obtained if the spindle speed N does not vary. However, the diameter D in the cases of face turning, taper turning and circular turning is not a constant. Therefore, the cutting speed V cannot be constant due to the instantaneously changing diameter D (refer to Equation (3)). It is desirable to maintain a constant cutting speed even though the effective diameter of the cutting changes during machining [25]. To provide a constant cutting speed in such cases, the CNC controller monitors the changing distance between the tool and the workpiece center, and adjusts the spindle speed. Therefore, Equation (1) cannot be used to calculate the cutting times for face turning, taper turning and circular turning. For such turning operations, the cutting times can be calculated by the formulas derived by Lee [26]. These formulas are briefly described as follows. Lee presents a more detailed discussion.

Figure 3 illustrates the geometric representation of the linear turning. For the linear turning between any two points $P_1(z_1, x_1)$ and $P_2(z_2, x_2)$, the cutting time can be calculated by

$$T = \int_{x_1}^{x_2} \frac{\pi D}{1000Vf \sin\theta}\,\mathrm{d}x = \int_{x_1}^{x_2} \frac{2\pi x}{1000Vf \sin\theta}\,\mathrm{d}x, = \frac{\pi}{1000Vf}\left|\frac{x_2^2 - x_1^2}{\sin\theta}\right|, \tag{4}$$

where $\theta = \tan^{-1}((x_2 - x_1)/(z_2 - z_1)); 0 < \theta < \pi$ or $\pi < \theta < 2\pi$. If θ is not equal to 0, $\pi/2$, π, or $3\pi/2$, the linear turning can be classified as taper turning.

In the case of straight turning, where θ in Equation (4) is 0 or π, Equation (4) is found to be equivalent to Equation (1). In the case of face turning, where θ in Equation (4) is $\pi/2$ or $3\pi/2$ ($\sin\theta = \pm 1$), the cutting time T becomes

$$T = \frac{\pi}{1000Vf}\left|x_2^2 - x_1^2\right|. \tag{5}$$

Figure 4 presents a geometric representation of circular turning. For circular turning between any two points $P_1(z_1, x_1)$ and $P_2(z_2, x_2)$, the cutting time can be calculated by

$$T = \int_{\theta_1}^{\theta_2} \frac{2\pi r_a x}{1000Vf}\,\mathrm{d}\theta = \int_{\theta_1}^{\theta_2} \frac{2\pi r_a}{1000Vf}(x_c + r_a \sin\theta)\,\mathrm{d}\theta, = \frac{\pi r_a}{500Vf}\left|x_c(\theta_2 - \theta_1) - r_a(\cos\theta_2 - \cos\theta_1)\right|, \tag{6}$$

where $\theta_1 = \tan^{-1}((x_1 - x_c)/(z_1 - z_c)); \theta_2 = \tan^{-1}((x_2 - x_c)/(z_2 - z_c)); 0 \le \theta_1 \le 2\pi; 0 \le \theta_2 \le 2\pi$; r_a represents the radius of the circular arc at a point $P(x, z)$ between P_1 and P_2; $P_c(z_c, x_c)$ represents the center of the circular arc.

Next, we derive the analytical formulas for cutting time of the multi-pass machining models, in which the turned parts have continuous forms.

3. Development of machining model

In the cutting of forged and cast workpieces, the cutter path pattern, which moves along the part contour, is repeated as many times as required [25]. The turning operations generally include a roughing stage which has multiple rough cuts, and a finishing stage which has a single finish cut. For such multi-pass turning operations, the number of rough cuts, as well as cutting speeds, feeds, and depths of cut for roughing and finishing are considered as decision variables. In this study, the optimal machining parameters (decision variables) are determined with respect to the minimum unit production cost criterion. The unit production cost is minimized while satisfying a set of machining constraints.

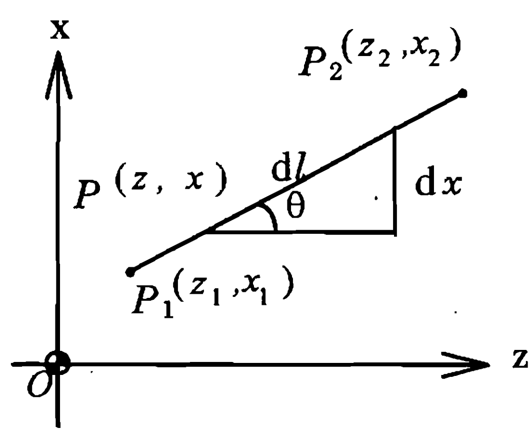

Fig. 3. Linear turning (Lee [26]).

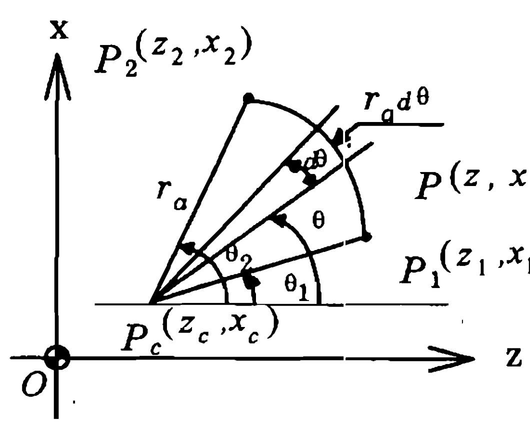

Fig. 4. Circular turning (Lee [26]).

3.1. *Cutting time expressions for multi-pass turning operations*

3.1.1. *Cutting time of multi-pass straight turning*

A turned part may consist of several segments which require straight turning. For the i-th straight turning segment between two points $P_{1(i)}(z_{1(i)}, x_{1(i)})$ and $P_{2(i)}(z_{2(i)}, x_{2(i)})$ (refer to Fig. 2), the cutting time of the g-th rough cut $T_{hr(i)(g)}$ (min) can be calculated by using Equation (2), and is expressed as

$$T_{hr(i)(g)} = \frac{\pi|x_{(g)(i)}(z_{2(i)} - z_{1(i)})|}{500V_r f_r}, \quad (7)$$

where $x_{(g)(i)}$ represents the x-coordinate of the g-th rough cut and takes the form

$$x_{(g)(i)} = \frac{D_{(i)}}{2} + d_t - gd_r. \quad (8)$$

In Equation (7), $z_{(1)(i)}$ and $z_{(2)(i)}$ are the z-coordinates of $P_{1(i)}$ and $P_{2(i)}$; V_r, f_r and d_r are the cutting speed (m/min), feed (mm/rev) and depth of each cut (mm) for roughing, respectively; $D_{(i)}$ is the diameter of the finished part for the i-th straight turning segment; and d_t is the total depth of metal to be removed. By substituting $x_{(g)(i)}$ in Equation (7) for Equation (8), $T_{hr(i)(g)}$ can be rewritten as

$$T_{hr(i)(g)} = \frac{\pi|((D_{(i)}/2) + d_t - gd_r)(z_{2(i)} - z_{1(i)})|}{500V_r f_r} \quad (9)$$

For the i-th straight turning segment, the cutting time of multi-pass roughing $S_{hr(i)}$ can be expressed as

$$S_{hr(i)} = \sum_{g=1}^{n} T_{hr(i)(g)} = \sum_{g=1}^{n} \frac{\pi|((D_{(i)}/2) + d_t - gd_r)(z_{2(i)} - z_{1(i)})|}{500V_r f_r}, \quad (10)$$

where n is the number of rough passes.

For single-pass finishing, the cutting time of the i-th straight turning segment $S_{hs(i)}$ can be expressed as

$$S_{hs(i)} = \frac{\pi|x_{s(i)}(z_{2(i)} - z_{1(i)})|}{500V_s f_s} = \frac{\pi|D_{(i)}(z_{2(i)} - z_{1(i)})|}{1000V_s f_s}, \quad (11)$$

where $x_{s(i)} = D_{(i)}/2$ represents the x-coordinate of the i-th straight turning segment for the finish cut; and V_s and f_s are the cutting speed and feed for finishing, respectively.

The cutting time of the i-th straight turning segment is the sum of the cutting time of multi-pass roughing $S_{hr(i)}$ and the cutting time of single-pass finishing $S_{hs(i)}$. If there are n_h segments of straight turning, the total cutting time of all such segments S_h becomes

$$S_h = \sum_{i=1}^{n_h} (S_{hr(i)} + S_{hs(i)}). \quad (12)$$

3.1.2. *Cutting time of multi-pass taper turning*

For the j-th taper between two points $P_{1(j)}(z_{1(j)}, x_{1(j)})$ and $P_{2(j)}(z_{2(j)}, x_{2(j)})$ (refer to Fig. 3), the x-coordinates of $P_{1(j)(g)}$ and $P_{2(j)(g)}$ of the g-th rough cut can be represented as

$$x_{1(j)(g)} = \frac{D_{1(j)}}{2} + d_t - gd_r, \quad x_{2(j)(g)} = \frac{D_{2(j)}}{2} + d_t - gd_r, \quad (13)$$

where $D_{1(j)}$ and $D_{2(j)}$ are the diameters of the finished part at points $P_{1(j)}(z_{1(j)}, x_{1(j)})$ and $P_{2(j)}(z_{2(j)}, x_{2(j)})$, respectively. By substituting x_1 and x_2 in Equation (4) for $x_{1(j)(g)}$ and $x_{2(j)(g)}$ using Equation (13), the cutting time of the g-th rough cut for taper turning $T_{tr(j)(g)}$ can be obtained as

$$T_{tr(j)(g)} = \frac{\pi}{1000V_r f_r}\left|\frac{((D_{2(j)}/2) + d_t - gd_r)^2 - ((D_{1(j)}/2) + d_t - gd_r)^2}{\sin\theta_{(j)}}\right|, = \frac{\pi}{1000V_r f_r}\left|\frac{0.25(D_{2(j)}^2 - D_{1(j)}^2) + (D_{2(j)} - D_{1(j)})(d_t - gd_r)}{\sin\theta_{(j)}}\right|, \quad (14)$$

where $\theta_{(j)} = \tan^{-1}((x_{2(j)} - x_{1(j)})/(z_{2(j)} - z_{1(j)})) = \tan^{-1}((D_{2(j)} - D_{1(j)})/2(z_{2(j)} - z_{1(j)}))$; $0 < \theta_{(j)} < \pi$; $\pi < \theta_{(j)} < 2\pi$.

For the multi-pass roughing, the cutting time of the j-th taper $S_{tr(j)}$ can be defined as

$$S_{tr(j)} = \sum_{g=1}^{n} T_{tr(j)(g)} = \sum_{g=1}^{n} \frac{\pi}{1000V_r f_r}\left|\frac{0.25(D_{2(j)}^2 - D_{1(j)}^2) + (D_{2(j)} - D_{1(j)})(d_t - gd_r)}{\sin\theta_{(j)}}\right|. \quad (15)$$

For the single-pass finishing of the j-th taper, the x-coordinates of $P_{1(j)}$ and $P_{2(j)}$ of the finish cut, $x_{1s(j)}$ and $x_{2s(j)}$, can be represented as

$$x_{1s(j)} = \frac{D_{1(j)}}{2}, \quad x_{2s(j)} = \frac{D_{2(j)}}{2}. \quad (16)$$

By substituting x_1 and x_2 into Equation (4) for $x_{1s(j)}$ and $x_{2s(j)}$ using Equation (16), the cutting time of the finish cut for the j-th taper $S_{ts(j)}$ can be expressed as

$$S_{ts(j)} = \frac{\pi}{1000V_s f_s}\left|\frac{(D_{2(j)}/2)^2 - (D_{1(j)}/2)^2}{\sin\theta_{(j)}}\right| = \frac{\pi}{1000V_s f_s}\left|\frac{(D_{2(j)}^2 - D_{1(j)}^2)}{4\sin\theta_{(j)}}\right|. \quad (17)$$

The cutting time of the j-th taper is the sum of the cutting time of multi-pass roughing $S_{tr(j)}$ and the cutting time of single-pass finishing $S_{ts(j)}$. The total cutting time of all taper turning segments S_t can be expressed as

$$S_t = \sum_{j=1}^{n_t} \left(S_{tr(j)} + S_{ts(j)}\right), \tag{18}$$

where n_t is the number of tapers.

3.1.3. *Cutting time of multi-pass face turning*

For face turning, the formulas for cutting time can be obtained by setting θ in Equation (4) to $\pi/2$ or $3\pi/2$ ($\sin\theta = \pm 1$). The cutting time of the multi-pass roughing for the k-th facing segment $S_{vr(k)}$ can be obtained from Equation (15) and expressed as

$$S_{vr(k)} = \sum_{g=1}^{n} \frac{\pi}{1000 V_r f_r} \left| \frac{1}{4}\left(D_{2(k)}^2 - D_{1(k)}^2\right) + \left(D_{2(k)} - D_{1(k)}\right)\left(d_t - g d_r\right) \right|, \tag{19}$$

where $D_{1(k)}$ and $D_{2(k)}$ are the diameters of the finished part for the k-th facing segment at points $P_{1(k)}\left(z_{1(k)}, x_{1(k)}\right)$ and $P_{2(k)}\left(z_{2(k)}, x_{2(k)}\right)$, respectively.

Similarly, the cutting time of the finish cut for the k-th face turning segment $S_{us(j)}$ can be obtained from Equation (17) and is expressed as

$$S_{vs(k)} = \frac{\pi}{1000 V_s f_s} \left| \frac{1}{4}\left(D_{2(k)}^2 - D_{1(k)}^2\right) \right|. \tag{20}$$

The cutting time of the k-th facing segment is the sum of the cutting time of multi-pass roughing $S_{vr(k)}$ and the cutting time of single-pass finishing $S_{vs(k)}$. The total cutting time of all facing segments S_v can be defined as

$$S_v = \sum_{k=1}^{n_v} \left(S_{vr(k)} + S_{vs(k)}\right), \tag{21}$$

where n_v is the number of facing segments.

3.1.4. *Cutting time of multi-pass circular turning*

For the l-th circular arc between two points $P_{1(l)}\left(z_{1(l)}, x_{1(l)}\right)$ and $P_{2(l)}\left(z_{2(l)}, x_{2(l)}\right)$(refer to Fig. 4), the arc radius in the g-th rough cut $r_{a(l)(g)}$ can be defined as

$$r_{a(l)(g)} = R_{(l)} + d_t - g d_r, \tag{22}$$

where $R_{(l)}$ is the radius of the finished part for the l-th circular arc. By replacing r_a in Equation (6) by $r_{a(l)(g)}$ using Equation (22), the cutting time of the g-th rough cut for the l-th circular arc $T_{cr(l)(g)}$ can be obtained as

$$T_{cr(l)(g)} = \frac{\pi\left(R_{(l)} + d_t - g d_r\right)}{500 V_r f_r} \left| x_{c(l)}\left(\theta_{2(l)} - \theta_{1(l)}\right) - \left(R_{(l)} + d_t - g d_r\right)\left(\cos\theta_{2(l)} - \cos\theta_{1(l)}\right) \right| \tag{23}$$

where

$$\theta_{1(l)} = \tan^{-1}\left(\frac{x_{1(l)} - x_{c(l)}}{z_{1(l)} - z_{c(l)}}\right) = \tan^{-1}\left(\frac{\left(D_{1(l)}/2\right) - x_{c(l)}}{z_{1(l)} - z_{c(l)}}\right);$$

$$\theta_{2(l)} = \tan^{-1}\left(\frac{x_{2(l)} - x_{c(l)}}{z_{2(l)} - z_{c(l)}}\right) = \tan^{-1}\left(\frac{\left(D_{2(l)}/2\right) - x_{c(l)}}{z_{2(l)} - z_{c(l)}}\right),$$

$D_{1(l)}$ and $D_{2(l)}$ are the diameters of the finished part for the l-th circular arc at points $P_{1(l)}$ and $P_{2(l)}$, respectively; and $P_{c(l)}\left(z_{c(l)}, x_{c(l)}\right)$ is the center of the l-th circular arc.

For the multi-pass roughing, the cutting time of the l-th circular arc $S_{cr(l)}$ can be defined as

$$S_{cr(l)} = \sum_{g=1}^{n} T_{cr(l)(g)}, = \sum_{g=1}^{n} \frac{\pi\left(R_{(l)} + d_t - g d_r\right)}{500 V_r f_r} \left| x_{c(l)}\left(\theta_{2(l)} - \theta_{1(l)}\right) - \left(R_{(l)} + d_t - g d_r\right)\left(\cos\theta_{2(l)} - \cos\theta_{1(l)}\right) \right|. \tag{24}$$

For the single-pass finishing of the l-th circular arc, the cutting time of the finish cut $S_{cs(l)}$ can be expressed as

$$S_{cs(l)} = \frac{\pi R_{(l)}}{500 V_s f_s} \left| x_{c(l)}\left(\theta_{2(l)} - \theta_{1(l)}\right) - R_{(l)}\left(\cos\theta_{2(l)} - \cos\theta_{1(l)}\right) \right|. \tag{25}$$

The cutting time of the l-th circular arc is the sum of the cutting time of multi-pass roughing $S_{cr(l)}$ and the cutting time of single-pass finishing $S_{cs(l)}$. The total cutting time of all circular arcs S_c can be expressed as

$$S_c = \sum_{l=1}^{n_c} \left(S_{cr(l)} + S_{cs(l)}\right), \tag{26}$$

where n_c is the number of circular arcs.

Finally, the total cutting time T_M can be obtained by summing up the cutting times of: multi-pass straight turning S_h, of multi-pass taper turning S_t, multi-pass face turning S_v and multi-pass circular turning S_c. Hence, the total cutting time T_M becomes

$$T_M = S_h + S_t + S_v + S_c. \tag{27}$$

3.2. *Formulation of the objective function*

The economic criterion considered here is the minimum unit production cost which includes the cutting cost by actual time, the machine idling cost due to loading and unloading operations and idling tool motion, the tool replacement cost and the tool cost.

The cutting cost C_M (\$/piece) can be expressed as

$$C_M = k_o T_M, \tag{28}$$

where k_o is the sum of direct labor cost and overhead (\$/min); T_M is the actual cutting time which can be calculated by Equation (27).

The machine idling cost C_I ($/piece) can be expressed as

$$C_I = k_o T_I \tag{29}$$

where T_I is the machine idling time (min). It is divided into a constant term (t_c) due to loading and unloading operations and a variable term due to idling tool motion. The variable term, idling tool motion time t_v (min), can be represented as the distance of tool rapid traverse l_a (mm) divided by the rapid speed V_a (mm/min), thus

$$t_v = \frac{l_a}{V_a}. \tag{30}$$

The distance of rapid traverse l_a can be defined as

$$l_a = 2l_1 + (n+1)(l_2 + l_3), \tag{31}$$

where l_1 is the distance between the reference point and the cycle start point; l_2 is the distance between the cycle start point and the cutting start point; and l_3 is the distance between the cutting end point and the cycle start point [25]. Consequently, the machine idling time T_I can be expressed as

$$T_I = t_c + t_v = t_c + \frac{2l_1 + (n+1)(l_2 + l_3)}{V_a}. \tag{32}$$

Hence, the machine idling cost C_I becomes

$$C_I = k_o T_I = k_o \left[t_c + \frac{2l_1 + (n+1)(l_2 + l_3)}{V_a} \right]. \tag{33}$$

The tool replacement cost C_R ($/piece) can be expressed as

$$C_R = k_o T_R, \tag{34}$$

where T_R is the tool replacement time (min). The tool replacement time can be written in terms of tool life t_l (min), time required to exchange a tool t_e (min) and cutting time T_M. It is given by

$$T_R = t_e \frac{T_M}{t_l}. \tag{35}$$

The Taylor equation for tool-life is given by [23]

$$t = \frac{C_0}{V^\alpha f^\beta d^\gamma}, \tag{36}$$

where α, β, γ and C_0 are constants. The same tool is assumed to be used for the entire machining process, i.e., both roughing and finishing. The wear rate of tools usually differs between roughing and finishing because the machining condition is different. The tool life t_l in such a situation can be expressed as [13]

$$t_l = w t_r + (1-w) t_s, \tag{37}$$

where $t_r = C_0 / V_r^\alpha f_r^\beta d_r^\gamma$ represents tool life for rough machining; $t_s = C_0 / V_s^\alpha f_s^\beta d_s^\gamma$ represents tool life for finish machining; w is a weight for the tool-life equation; and $0 \le w \le 1$.

Hence, the tool replacement cost C_R can be expressed as

$$C_R = k_o T_R = k_o t_e \frac{T_M}{t_l}. \tag{38}$$

The tool cost C_T ($/piece) can be obtained by

$$C_T = k_t \frac{T_M}{t_l}, \tag{39}$$

where k_t is the cutting edge cost ($/edge).

Finally, by using the above mathematical manipulations, the unit production cost UC ($/piece) can be obtained as

$$\begin{aligned} UC &= C_M + C_I + C_R + C_T, \\ &= k_o T_M + k_o T_I + k_o \left(t_e \frac{T_M}{t_l} \right) + k_t \left(\frac{T_M}{t_l} \right). \end{aligned} \tag{40}$$

3.3. *Cutting constraints*

The practical constraints imposed during the roughing and finishing operations include: (1) parameter bounds; (2) tool-life constraint; (3) operating constraints consisting of surface finish constraint (only for finish machining), cutting force constraint and power constraint; (4) stable cutting region constraint; (5) chip-tool interface temperature constraint and (6) parameter relationship constraints consisting of relations between roughing and finishing parameters, and total depth of cut constraint. They are discussed as follows.

3.3.1. *Rough machining*

Parameter bounds: Owing to the limited capacity of CNC machines, the safety of the operator, the type of cutting tool and the material of the workpiece, the machining parameters are restricted to be within the lower and upper bounds.

Bounds on the cutting speed:

$$V_{rL} \le V_r \le V_{rU}, \tag{41}$$

where V_{rL} and V_{rU} are the lower and upper bounds of the cutting speed in roughing, respectively.

Bounds on the feed:

$$f_{rL} \le f_r \le f_{rU}, \tag{42}$$

where f_{rL} and f_{rU} are the lower and upper bounds of the feed in roughing, respectively.

Bounds on the depth of cut:

$$d_{rL} \le d_r \le d_{rU}, \tag{43}$$

where d_{rL} and d_{rU} are the lower and upper bounds of the depth of cut in roughing, respectively.

Tool-life constraint: Considering production economics and the required quality of the machined parts, the tool life should be within an acceptable range. The constraint on the tool life is taken as

$$T_L \le t_r \le T_U, \tag{44}$$

where T_L and T_U are the lower and upper bounds of the tool life, respectively.

Cutting force constraint: A constraint must be placed on the cutting force to limit any deflection of either the workpiece or the cutting tool that would cause dimensional errors, and to reduce the power required for the cutting process. An expression for the cutting force constraint is [18]

$$F_r = k_f f_r^{\mu} d_r^{\upsilon} \leq F_U, \tag{45}$$

where F_r is the cutting force during rough machining (kgf); k_f, μ and υ are constants pertaining to specific tool-workpiece combination; and F_U is the maximum allowable cutting force (kgf).

Power constraint: The power required during the cutting operation should not exceed the available power of the machine tool. The power constraint is given by [13]

$$P_r = \frac{k_f f_r^{\mu} d_r^{\upsilon} V_r}{6120\eta} \leq P_U, \tag{46}$$

where P_r is the cutting power during rough machining (kW); η is the power efficiency; and P_U is the maximum allowable cutting power (kW).

Stable cutting region constraint: To prevent chatter vibration, adhesion and the formation of a built-up edge, a constraint on the stable cutting region has been imposed. This constraint is expressed as [11]

$$V_r^{\lambda} f_r d_r^{\upsilon} \geq S_L, \tag{47}$$

where λ and υ are constants pertaining to specific tool-workpiece combinations; and S_L is the limit of the stable cutting region.

Chip-tool interface temperature constraint: The hardness and sharpness of a tool decrease if the temperature generated at the chip-tool interface exceeds the available limit and the tool can no longer be used for cutting. This constraint can be expressed as [8]

$$Q_r = k_q V_r^{\tau} f_r^{\phi} d_r^{\delta} \leq Q_U, \tag{48}$$

where Q_r is the temperature during roughing (°C); k_q, τ, ϕ and δ are constants, and Q_U is the maximum allowable temperature (°C).

3.3.2. *Finish Machining*

All the constraints other than the surface finish constraint are similar for rough and finish machining.

Parameter bounds:

Bounds on the cutting speed:

$$V_{sL} \leq V_s \leq V_{sU}, \tag{49}$$

where V_{sL} and V_{sU} are the lower and upper bounds on the cutting speed in finishing, respectively.

Bounds on the feed:

$$f_{sL} \leq f_s \leq f_{sU}, \tag{50}$$

where f_{sL} and f_{sU} are the lower and upper bounds on the feed in finishing, respectively.

Bounds on the depth of cut:

$$d_{sL} \leq d_s \leq d_{sU}, \tag{51}$$

where d_{sL} and d_{sU} are the lower and upper bounds on the depth of cut in finishing, respectively.

Tool-life constraint:

$$T_L \leq t_s \leq T_U. \tag{52}$$

Cutting force constraint:

$$F_s = k_f f_s^{\mu} d_s^{\upsilon} \leq F_U, \tag{53}$$

where F_s is the cutting force during finish machining (kgf).

Power constraint:

$$P_s = \frac{k_f f_s^{\mu} d_s^{\upsilon} V_s}{6120\eta} \leq P_U, \tag{54}$$

where P_s is the cutting power during finish machining (kW).

Stable cutting region constraint:

$$V_s^{\lambda} f_s d_s^{\upsilon} \geq S_L. \tag{55}$$

Chip-tool interface temperature constraint:

$$Q_s = k_q V_s^{\tau} f_s^{\phi} d_s^{\delta} \leq Q_U, \tag{56}$$

where Q_s is the temperature during finishing (°C).

Surface finish constraint: The surface finish dominates the quality of a machined part, and is generally influenced by factors such as speed, feed, depth of cut, tool geometry and material of the tool. Furthermore, some undesirable machining conditions such as excessive tool wear, built-up edge and chatter, deteriorate the surface finish. However, only feed and the nose radius of the cutting tool R_n (mm) are considered here since they have the most significant effect on the surface finish [11]. This constraint takes the form

$$\frac{f_s^2}{8R_n} \leq R_a, \tag{57}$$

where R_a is the maximum allowable surface roughness (μm).

Relations between roughing and finishing parameters: In addition to the constraints mentioned previously, the relations between the speed, feed and depth of cut in both roughing and finishing must be defined [16]. During roughing, the values of the depth of cut and feed are usually greater than that for finishing. Nevertheless, the speed for roughing is usually less than that for finishing. These relations are important in determining machining parameters, and can be expressed as

$$V_s \geq k_1 V_r, \tag{58}$$

$$f_r \geq k_2 f_s, \tag{59}$$

$$d_r \geq k_3 d_s, \tag{60}$$

where k_1, k_2, k_3 are relationship coefficients and k_1, k_2, $k_3 \geq 1$.

Total depth of cut constraint: The depth of the finish cut (d_s) should be equal to the total depth of cut (d_t) minus the total depth of the rough cut (nd_r). Therefore, this equality equation can be defined as

$$d_s = d_t - nd_r. \tag{61}$$

Bounds on the number of rough cuts: The bounds on the number of rough cuts can be expressed as follows:

$$N_L \leq n \leq N_U, \tag{62}$$

where $N_L = (d_t - d_{sU})/d_{rU}$, $N_U = (d_t - d_{sL})/d_{rL}$ are the lower and upper bounds on the number of rough cuts, respectively.

For each possible depth of a finish cut (d_s), the corresponding depth of the rough cut (d_r) can be computed. Equation (61) can be rewritten as

$$d_r = \frac{d_t - d_s}{n}. \tag{63}$$

By using the above mathematical manipulation, d_r and the equality constraint, Equation (61), can be eliminated in the optimization algorithm.

3.4. *Summary of the cutting model*

Based on the above discussion, this study has formulated a machining model to optimize multi-pass turning operations for parts with continuous forms. The turning model formulated above is a constrained nonlinear optimization problem with multiple continuous/integral variables (machining parameters). The unit production cost, as given by Equation (40) is the objective function of the developed machining optimization model. The machining constraints include constraints (41)–(62). The aforementioned optimization techniques are not suitable for solving this cutting model owing to the complexity arising from the nonlinear objective function and the numerous nonlinear constraints. The following section describes the proposed optimization algorithm.

4. Proposed optimization algorithm

In this section, we propose an optimization algorithm to effectively deal with the complex multi-pass turning problems developed in Section 3. The proposed optimization algorithm is an approach combining SA and PS. To reduce the computational requirement of SA and increase the chance of optimality of PS, the proposed optimization algorithm embeds the PS into SA as the move generation mechanism.

4.1. *Simulated annealing*

The Simulated Annealing (SA) algorithm [17,18] is a stochastic search technique designed to guide a search procedure away from the trap of local optimality. Koulamas *et al.* [27] have surveyed SA applications in production/operations management and operations research problems. From their review, SA has developed into a powerful optimization technique for difficult combinatorial optimization problems. By defining a specific move generation mechanism, Corana *et al.* [28] have developed an SA approach to solve optimization problems in which the variables are continuous.

Analogous to statistical mechanics, the search procedure in SA performs with respect to a transition probability. The transition probability is determined by the control temperature and the change in objective function. SA is a variation of neighborhood search, which will move "uphill" with respect to cost, replacing the current solution with a higher cost. By allowing a move to a worse solution in a controlled situation, SA can escape from a local optimum and potentially find a more promising "downhill" path. Furthermore, the "uphill" moves are carefully controlled by the temperature parameter. When the temperature is high, the probability of the "uphill" move is high. With a gradual decrease in temperature, the probability of an "uphill" move becomes small. Although SA is typically considered to be a heuristic optimization method, it allows the global optima to be found, provided that certain conditions are satisfied [29].

4.2. *Pattern search*

The Hooke–Jeeves Pattern Search (PS) [19] is incorporated into the SA algorithm as the move generation mechanism. PS proceeds according to a series of either exploratory or pattern moves. The exploratory moves examine the local behavior of a function and seek to locate the direction of any sloping valleys that might be present. The pattern moves utilize the information generated in the exploration to step rapidly along the valleys.

The step sizes of the variables are adjusted if the vector of the pattern direction is equal to zero, i.e., no move is accepted in the completed exploratory move with the current step sizes. The step sizes will be increased first until the user-specified limit is reached. If this fails, the step sizes will be decreased, and the exploratory moves are repeated.

4.3. *The proposed optimization algorithm*

By considering the enormous computational requirements, the SA approach for optimization problems with continuous variables as developed by Corana *et al.* [28], is not suitable for solving complex machining optimization problems. To reduce the computational cost of the method developed by Corana *et al.*, we propose an optimization algorithm which incorporates PS into SA as the move generation mechanism. Applying the SA approach initially requires defining four basic components of the algorithm. The four basic components in the proposed optimization algorithm are

(1) *Configuration*: A legal configuration is one combination of variables, i.e., a vector of machining parameters (S) related to the turning model.
(2) *Move set*: All S's obtained from the PS are elements of the move set. The moves in the optimization algorithm are determined by PS. It also adjusts the step sizes and determines the search directions in which the optimization procedure will proceed.
(3) *Cost function*: The objective function which defines the unit production cost with any given combination of variables (see Equation (40)) is the cost function for the machining optimization problem.
(4) *Cooling schedule*: We carried out a simple one, the geometric cooling schedule [30]. Following completion of a specified number of moves (M), the temperature (Γ) is replaced by the old temperature multiplied by a constant (c). The constant, called the cooling ratio, is less than 1 and greater than zero. The process is judged to be frozen when the procedure has made K consecutive loops of M moves with no change in the current best solution. van Laarhoven and Aarts [21] have discussed a number of cooling schedules. The geometric cooling schedule has been implemented successfully for many optimization problems, e.g., graph partitioning [31], resource-constrained scheduling [32] and manufacturing systems layout design [33]. This cooling schedule cools rapidly and performs quite well [32]. Since the machining optimization model formulated above is large and complex, the geometric cooling schedule is proposed here to reduce the computational requirements.

Next, the proposed optimization algorithm is formally presented. Table 1 lists the notations and user-specified data of the proposed optimization algorithm

Step 1. (*Initialize the search procedure*)
- (a) Obtain an initial solution S^0, an initial control temperature Γ^0 and initial step sizes U^0.
- (b) Set $S = S^0$, $U = U^0$, $\Gamma = \Gamma^0$, $n_K = 0$, $n_M = 0$, $n_I = 0$, $n_D = 0$. Evaluate $F(S)$.

Step 2. (*Exploratory move*) Set $\Delta S = 0$, $S' = S$.
For $j = 1$ to m
- (a) Set $s'_j = s_j + u_j$, $\Delta E = F(S') - F(S)$.
  If $\Delta E < 0$ (*downhill move*),
  set $S = S'$, $\Delta s_j = u_j$, $imp = 1$.
- (b) If $\Delta E \geq 0$ (*uphill move*), set $S = S'$, $\Delta s_j = u_j$, $imp = 1$ with probability $e^{-\Delta E/\Gamma}$.
- (c) Perform sub-procedure **CHECK**.
  If $frozen = 1$, go to Step 5.
- (d) If $\Delta s'_j = 0$, set $s'_j = s_j - u_j$, $\Delta E = F(S') - F(S)$.
  Otherwise, return to (a).
- (e) If $\Delta E < 0$ (*downhill move*),
  set $S = S'$, $\Delta s_j = -u_j$, $imp = 1$.
- (f) If $\Delta E \geq 0$ (*uphill move*), set $S = S'$, $\Delta s_j = -u_j$, $imp = 1$ with probability $e^{-\Delta E/\Gamma}$.
- (g) Perform sub-procedure **CHECK**.
  If $frozen = 1$, go to Step 5.

Step 3. (*Check pattern direction and adjust step sizes*)
- (a) If $\Delta S \neq 0$, go to Step 4.
- (b) If $n_I \leq I$, set $n_I = n_I + 1$ (*increase step sizes*).
  For $i = 1$ to m
  If s_i is continuous, set $u_i = (r_I)^{n_I} \times u_i^0$.
  Otherwise, if s_i is integral, set $u_i = (n_I + 1) \times u_i^0$.
  If $u_i > u^M$, set $u_i = u^M$.
  Go to Step 2.
- (c) Otherwise, set $n_D = n_D + 1$ (*decrease step sizes*).
  For $i = 1$ to m
  If s_i is continuous, set $u_i = (r_D)^{n_D} \times u_i^0$.
  Otherwise, if s_i is integral, set $u_i = u_i^0 - n_D$.
  If $u_i < u^m$, set $u_i = u^m$.
  Go to Step 2.

Step 4. (*Pattern move*)
- (a) Set $S' = S + \Delta S$, $\Delta E = F(S') - F(S)$.
- (b) If $\Delta E < 0$ (*downhill move*), set $S = S'$, $imp = 1$ and go to (d).
- (c) If $\Delta E \geq 0$ (*uphill move*), set $S = S'$, $imp = 1$ with probability $e^{-\Delta E/\Gamma}$.
- (d) Perform sub-procedure **CHECK**.
  If $frozen = 1$, go to Step 5.
- (e) If $S = S'$, return to (a) (*continue pattern move*).
- (f) Otherwise, return to Step 2 with S.

Step 5. (*Termination*)
Return S^* and terminate search.

Sub-Procedure CHECK (*Check improvement in current best solution during M moves and lower control temperature*)

Step 1. Set $n_M = n_M + 1$.
If $n_M = M$, go to Step 2.
Otherwise, go to Step 4.

Step 2. (*Check improvement during M moves*) Set $n_M = 0$.
- 2.1 If $imp = 1$, (*current best solution improved*)
  Set $n_K = 0$.
- 2.2 Otherwise, set $n_K = n_K + 1$.
  - 2.2.1 If $n_K = K$, (*frozen state achieved*)
    set $frozen = 1$.
  - 2.2.2 Otherwise, set $frozen = 0$.

Step 3. (*Lower control temperature*)
Set $\Gamma = c \times \Gamma^0$.

Step 4. Return.

In the proposed optimization algorithm, the search procedure begins at an initial solution. The initial solution can be either selected in an arbitrarily manner or by using a present solution (if an existing cutting condition is of concern). The initial solution can be generated randomly, if the manufacturing engineer does not have sufficient knowledge of how to set the initial values of parameters.

Table 1. Notations and specified data of the proposed optimization algorithm

m	number of decision variables, $m = 2$ (Example 1), $m = 6$ (Example2)
S	vector of decision variables (machining parameters), $\mathbf{S} = \{s_1, s_2, \ldots, s_m\}$
S′	a neighboring solution of S
$\mathbf{S}^0$	vector of initial solution, $\mathbf{S}^0 = \{s_1^0, s_2^0, \ldots, s_m^0\}$
$\mathbf{S}^*$	vector of final solution, $\mathbf{S}^* = \{s_1^*, s_2^*, \ldots, s_m^*\}$
U	vector of step sizes, $\mathbf{U} = \{u_1, u_2, \ldots, u_m\}$,
$\mathbf{U}^0$	vector of initial step sizes, $\mathbf{U}^0 = \{u_1^0, u_2^0, \ldots, u_m^0\}$, $\mathrm{U}_0 = \{0.01, 0.5\}$ (Example 1), $\mathbf{U}^0 = \{1, 0.01, 0.5, 0.01, 0.01, 0.5\}$ (Example 2)
$\mathbf{U}^M$	vector of maximal step sizes, $\mathbf{U}^M = \{u_1^M, u_2^M, \ldots, u_m^M\}$, $\mathbf{U}^M = \{1.0, 15.0\}$ (Example 1), $\mathbf{U}^M = \{1, 0.5, 15.0, 1.0, 0.5, 15.0\}$ (Example 2)
$\mathbf{U}^m$	vector of minimal step sizes, $\mathbf{U}^m = \{u_1^m, u_2^m, \ldots, u_m^m\}$, $\mathbf{U}^m = \{0, 0\}$ (Example 1), $\mathbf{U}^m = \{1, 0, 0, 0, 0, 0\}$ (Example 2)
$\Delta\mathbf{S}$	vector of pattern direction, $\Delta\mathbf{S} = \{\Delta s_1, \Delta s_2, \ldots, \Delta s_m\}$
ΔE	change in objective function, $\Delta E = F(\mathbf{S}') - F(\mathbf{S})$
n_I	a counter for number of step size increment
I	maximum number of step size increment allowed, $I = 10$
n_D	a counter for number of step size decrement
r_I	increasing rate of step sizes, $r_I > 1$, $r_I = 1.47$
r_D	decreasing rate of step sizes, $0 < r_D < 1$, $r_D = 0.77$
n_M	a counter for number of search points at a temperature level
M	specified number of search points at a temperature level, $M = 45$
n_K	a counter for checking frozen state achieved of the proposed optimization algorithm
K	specified maximum number of n_K, $K = 25$
N	maximum number of iterations for considering $\Delta E = 0$ as improvement, $N = 3000$
c	cooling ratio, a constant, $c = 0.95$
Γ	control temperature of the optimization algorithm
Γ^0	initial control temperature of the optimization algorithm, $\Gamma^0 = 1000$

The fact that the proposed optimization algorithm is a stochastic optimization approach may cause the cost of the current "best" solution to fluctuate. The solution procedure saves the lowest cost solution reached at any perturbation, thereby ensuring that the lowest cost solution can always be recovered. Johnson *et al.* [31] performed an empirical study, concluding that, at some point, it is more profitable to perform, e.g., two annealings of length H than one annealing of length $2H$. Therefore, we recommend the optimization algorithm be run twice. To reduce the number of iterations, no change in objective function (i.e., $\Delta E = 0$) will be consider as no improvement for this perturbation after a relatively large number of points have been reached (N).

The proposed optimization algorithm has several desirable characteristics, including the capability of escaping from the local optima, ease of implementation algorithmically, robustness in dealing with large machining problems, and no restrictive assumptions regarding objective function, parameter set and constraint set.

5. Numerical examples and discussions

5.1. *Examples*

Two different test examples are used in this study to evaluate the effectiveness of the proposed optimization algorithm. Example 1 is adopted from Philipson and Ravindran [12]. It is a relatively simple turning problem whose true optimum can be found by the differential calculus approach. This example is used to measure the absolute quality of obtained solution using the proposed optimization algorithm. Example 2 is a part with a continuous form to be turned in multiple passes. It is used to illustrate the complexity of the turning model developed in Section 3, and justify the viability of the proposed optimization algorithm in highly complex turning optimization problems. The proposed optimization algorithm has been tested by making 50 runs with different initial solutions for each test example. The initial solutions are randomly selected within the parameter bounds. The examples have been run on an IBM PC 486 compatible computer using C.

Example 1: A bar with a single diameter is to be turned in one pass using the optimal cutting speed and feed which will minimize the unit production cost. The machining constraints considered in this example include parameter bounds, cutting force constraint and stable cutting region constraint. A detailed formulation of this turning model can be found in the literature [12]. Owing to the simplicity of this example, its true optimum can be obtained from the calculus differential approach. The optimal values of cutting speed and feed are 153.37 ft/min and 0.035 in/rev [12], respectively. By inserting these values into the objective function, the unit production cost is $ 0.561/piece. Comparing our results of Example 1

Table 2. Computational results of Example 1

Final solution $S^* = \{f^*, V^*\}$		*Final cost* ($) UC^*($)	*Frequency*
{0.035, 153.432}		0.561	16/50
{0.035, 153.256}		0.561	25/50
{0.035, 153.204}		0.561	2/50
{0.035, 153.391}		0.561	5/50
{0.035, 153.500}		0.561	2/50
Average final cost ($)	0.561		
Standard deviation of the final cost	0.000		
Average search points	12565.2		
Average CPU time (seconds/run)*	2.0		

* The CPU time is based on an IBM PC 486 compatible computer.

(see Table 2) with the optimal solution found by Philipson and Ravindran [12] reveals that the final values of machining parameters are slightly different. However, the final values of objective function are $ 0.561/piece, and the standard deviation of the final values of objective function is 0.000. Results obtained from this example demonstrate that the proposed optimization algorithm performs quite well in terms of absolute solution quality.

Example 2: A turned part with a continuous form shown in Fig. 5 is taken as the second example. This turned part has five straight turning segments, one face turning segment, one taper and two circular arcs. The machining optimization model is established with respect to the formulation developed in Section 3. The set of decision variables in the optimization algorithm is $\{n, f_r, V_r, d_s, f_s, V_s\}$ (d_r can be obtained by Equation (63)). Table 3 summarizes the data for the objective function (unit production cost) and machining constraints. The machining data (refer to Table 3) can be read from existing database files in the CAPP (Computer-Aided Process Planning) system, or the data can be provided from the keyboard, with the option of saving the data into the appropriate databases. This example is a constrained nonlinear optimization problem with a high computational complexity. The computational results shown in Table 4 reveal that the proposed optimization algorithm is a viable alternative for complex turning problems for parts with continuous forms.

5.2. *Discussions*

The final solution of each run may vary due to the stochastic nature of the SA approach. A previous experiment of the SA approach performed by Johnson *et al.* [31] indicates that even with a large number of iterations in each run, there can still be a large variation in the quality of solutions found by different runs. In contrast, Tables 2 and 4 show that the variations in final solutions are only slight. The proposed optimization algorithm is reasonably consistent since the final solutions are insensitive to the initial solutions. Furthermore, the initial solution does not have to be a feasible one.

To further examine the performance of the proposed optimization algorithm for complex machining problems, an enormous set of 6.86×10^7 combinations of machining parameters in Example 2 are equally-spaced enumerated within the whole solution space. The lowest unit cost in these 6.86×10^7 combinations is 13.5094, associated with $\{n, d_r, f_r, V_r, d_s, f_s, V_s\} = \{2, 2.5, 0.7, 106.0, 1.0, 0.3, 180.0\}$. From the computational results shown in Table 4, the optimization algorithm commonly locates better final solutions except the 47-th and 50-th program runs (13.5769 and 13.5479). However, the number of iterations in the optimization algorithm is relatively small to that in the enumerative search (22 787.2 vs. 6.86×10^7, 0.033%). The computational results indicate that the proposed optimization algorithm can effectively generate high quality heuristic solutions.

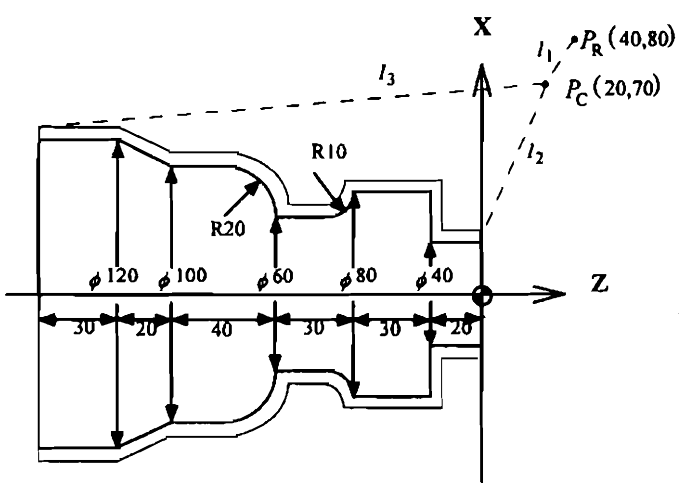

Fig. 5. Part drawing of Example 2.

Table 3. Data of Example 2

$V_{rU} = 500$ m/min	$V_{rL} = 50$ m/min	$f_{rU} = 0.9$ mm/rev
$f_{rL} = 0.2$ mm/rev	$d_{rU} = 3.5$ mm	$d_{rL} = 1.5$ mm
$V_{sU} = 500$ m/min	$V_{sL} = 50$ m/min	$f_{sU} = 0.9$ mm/rev
$f_{sL} = 0.2$ mm/rev	$d_{sU} = 2.8$ mm	$d_{sL} = 0.8$ mm
$\alpha = 5$	$\beta = 1.75$	$\gamma = 0.75$
$k_f = 108$	$\mu = 0.75$	$\upsilon = 0.95$
$\eta = 0.85$	$\lambda = 2$	$\nu = -1$
$k_q = 132$	$\tau = 0.4$	$\phi = 0.2$
$\delta = 0.105$	$R_n = 1.2$ mm	$k_o = 2.5$\$/min
$k_t = 15$\$/edge	$V_a = 5 \times 10^4$ mm/min	$t_e = 1.5$ min/edge
$C_0 = 6 \times 10^{11}$	$t_p = 2.5$	$F_U = 5.0$ kgf
$T_L = 25$ min	$T_U = 45$ min	$Q_U = 1000$°C
$P_U = 200$ kW	$R_a = 10\ \mu$m	$S_L = 140$
$k_1 = 1.2$	$k_2 = 1.5$	$k_3 = 2.0$
$d_t = 6$ mm		

Table 4. Computational results of Example 2

No	$S^*\{n^*, d_r^*, f_r^*, V_r^*, d_s^*, f_s^*, V_s^*\}$	UC^*($)	*No*	$S^*\{n^*, d_r^*, f_r^*, V_r^*, d_s^*, f_s^*, V_s^*\}$	UC^*($)
1	2, 2.400, 0.750, 102.732, 1.199, 0.300, 176.695	13.3275	26	2, 2.400, 0.750, 102.730, 1.199, 0.300, 176.700	13.3276
2	2, 2.400, 0.750, 102.747, 1.199, 0.300, 176.692	13.3274	27	2, 2.400, 0.750, 102.727, 1.199, 0.300, 176.696	13.3277
3	2, 2.400, 0.750, 102.731, 1.199, 0.300, 176.688	13.3272	28	2, 2.415, 0.740, 114.400, 1.170, 0.300, 176.640	13.4556
4	2, 2.400, 0.750, 102.724, 1.199, 0.300, 176.686	13.3272	29	2, 2.400, 0.750, 102.720, 1.199, 0.300, 176.688	13.3273
5	2, 2.400, 0.750, 102.740, 1.199, 0.300, 176.698	13.3276	30	2, 2.400, 0.750, 102.747, 1.199, 0.300, 176.693	13.3274
6	2, 2.400, 0.750, 102.725, 1.199, 0.300, 176.687	13.3273	31	2, 2.400, 0.750, 102.714, 1.200, 0.300, 176.677	13.3270
7	2, 2.400, 0.750, 102.724, 1.199, 0.300, 176.693	13.3274	32	2, 2.400, 0.750, 102.733, 1.199, 0.300, 176.692	13.3277
8	2, 2.400, 0.750, 102.726, 1.200, 0.300, 176.684	13.3272	33	2, 2.400, 0.750, 102.810, 1.199, 0.308, 175.108	13.2910[b]
9	2, 2.400, 0.750, 102.725, 1.199, 0.300, 176.692	13.3274	34	2, 2.426, 0.740, 103.043, 1.148, 0.280, 182.209	13.4609
10	2, 2.400, 0.750, 102.721, 1.200, 0.300, 176.684	13.3271	35	2, 2.400, 0.750, 102.729, 1.200, 0.300, 176.677	13.3270
11	2, 2.400, 0.750, 102.767, 1.199, 0.300, 176.696	13.3275	36	2, 2.400, 0.750, 102.724, 1.199, 0.300, 176.690	13.3274
12	2, 2.400, 0.750, 102.826, 1.200, 0.308, 175.104	13.2910	37	2, 2.400, 0.750, 102.746, 1.199, 0.300, 176.694	13.3274
13	2, 2.400, 0.750, 102.723, 1.199, 0.300, 176.692	13.3273	38	2, 2.400, 0.750, 102.723, 1.199, 0.300, 176.685	13.3273
14	2, 2.400, 0.750, 102.722, 1.199, 0.300, 176.686	13.3273	39	2, 2.400, 0.750, 102.724, 1.200, 0.300, 176.682	13.3272
15	2, 2.400, 0.750, 102.811, 1.199, 0.308, 175.040	13.2942	40	2, 2.400, 0.750, 102.720, 1.200, 0.300, 176.678	13.3272
16	2, 2.400, 0.750, 102.725, 1.199, 0.300, 176.693	13.3277	41	2, 2.400, 0.750, 102.729, 1.199, 0.300, 176.696	13.3277
17	2, 2.400, 0.750, 102.792, 1.200, 0.308, 175.105	13.2936	42	2, 2.400, 0.750, 102.734, 1.199, 0.300, 176.693	13.3277
18	2, 2.400, 0.750, 102.735, 1.199, 0.300, 176.699	13.3277	43	2, 2.400, 0.750, 102.724, 1.199, 0.300, 176.687	13.3273
19	2, 2.400, 0.750, 102.727, 1.199, 0.300, 176.688	13.3277	44	2, 2.400, 0.750, 102.725, 1.199, 0.300, 176.688	13.3273
20	2, 2.400, 0.750, 102.724, 1.199, 0.300, 176.698	13.3277	45	2, 2.400, 0.750, 102.731, 1.200, 0.300, 176.684	13.3271
21	2, 2.400, 0.750, 102.733, 1.199, 0.300, 176.698	13.3277	46	2, 2.400, 0.750, 102.721, 1.199, 0.300, 176.690	13.3274
22	2, 2.400, 0.750, 102.721, 1.199, 0.300, 176.688	13.3273	47	2, 2.426, 0.740, 103.043, 1.148, 0.260, 186.996	13.5769[a]
23	2, 2.400, 0.750, 102.719, 1.200, 0.300, 176.678	13.3270	48	2, 2.400, 0.750, 102.717, 1.200, 0.300, 176.678	13.3270
24	2, 2.400, 0.750, 102.885, 1.200, 0.308, 175.096	13.2937	49	2, 2.400, 0.750, 102.720, 1.199, 0.300, 176.691	13.3273
25	2, 2.400, 0.750, 102.727, 1.199, 0.300, 176.690	13.3274	50	2, 2.400, 0.750, 102.722, 1.199, 0.260, 185.774	13.5479

Average final cost ($)	13.3385	
The highest cost ($) in 50 runs	13.5769[a]	[a] represents Run No. 47
The lowest cost ($) in 50 runs	13.2910[b]	[b] represents Run No. 33
Standard deviation of the final cost	0.0538	
Average search points	22787.2	
Average CPU time (seconds/run)*	37.5	

*The CPU time is based on an IBM PC 486 compatible computer.

The initial solutions which are randomly selected within the parameter bounds are usually infeasible and far from the optimal solutions. To display the current solution changes with each iteration of the optimization algorithm, a run chart of the first annealing for Example 2 is plotted in Fig. 6. The run chart clearly demonstrates the cost improvement process. In this program run, most of the cost improvements are achieved within the first 500 iterations. For the first 400 iterations, the evaluated solutions are usually infeasible and far from the optimal solution. However, the optimization algorithm can efficiently approach to the vicinity of the optimal solution. For this program run, the lowest cost ($13.3277) of first annealing is obtained after 9000 iterations. Similar observations can be made from the other program runs.

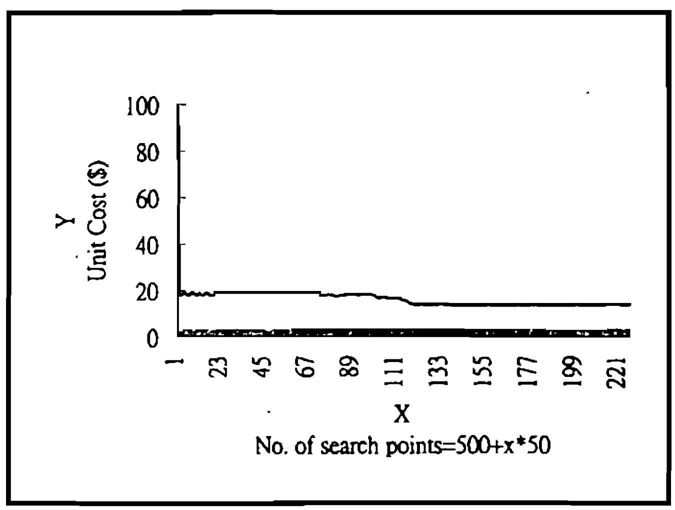

Fig. 6. Run chart of first annealing for Example 2.

Without exception, the proposed optimization algorithm requires a large number of iterations to locate a nearly-optimal solution, particularly, when solving large machining problems. However, it performs efficiently with respect to the required CPU time. On the average, the proposed optimization algorithm can generate a nearly-optimal solution within a reasonable computation time. In this experimental study, the average computation times of Examples 1 and 2 are 2.0 and 37.5 seconds per run, respectively. The computation time can certainly be reduced if a more powerful computer is used. Therefore, this approach can certainly be potentially extended as an on-line system for selecting optimal machining parameters.

6. Conclusions

In this study, we develop a machining model to optimize the multi-pass turning operations for turned parts with continuous forms. Owing to the high complexity of this machining optimization problem, a stochastic optimization method has been applied to solve this problem. The proposed optimization algorithm is a hybrid optimization method based on a simulated annealing algorithm and the Hooke–Jeeves pattern search. It can obtain a nearly optimal solution in an extremely huge solution space within a reasonable computation time. Computational results demonstrate that the proposed optimization algorithm can adequately solve complex machining economics problems.

This work has some initiative and encouraging features: (1) the machining optimization model for turned parts with continuous forms can be used to extend the application of machining economics for CNC lathe operations; (2) the proposed optimization algorithm is completely generalized and problem-independent so that it can be easily modified to optimize turning operations under various economic criteria and numerous practical constraints; (3) no special information regarding the solution surface, e.g., gradient and local curvature, need be identified; (4) since the proposed optimization algorithm can obtain a nearly-optimal solution within a reasonable execution time on a PC, it can potentially be extended to be an on-line adjustment system for machining parameters based on signals from sensors; and (5) experimental results demonstrate that the proposed optimization algorithm is consistent and effective since the final solutions are insensitive to the initial solutions (starting points).

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