ELSEVIER

Fuzzy Sets and Systems 104 (1999) 209–218

FUZZY sets and systems

New methods for students' evaluation using fuzzy sets

Shyi-Ming Chen[a,*], Chia-Hoang Lee[b]

[a]*Department of Electronic Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan, ROC*
[b]*Department of Computer and Information Science, National Chiao Tung University, Hsinchu, Taiwan, ROC*

Received April 1996; received in revised form May 1997

Abstract

In an earlier work, Biswas (1995) presented two methods for the application of fuzzy sets in students' answerscripts evaluation. In this paper, we extend his work to propose two new methods for evaluating students' answerscripts using fuzzy sets. The proposed methods can overcome the drawbacks in Biswas (1995) due to the fact that they do not need to perform the complicated matching operations and they can evaluate students' answerscripts in a more fair manner.

Keywords: Extended fuzzy grade sheet; Fuzzy set; Generalized extended fuzzy grade sheet; Letter-grade; Mid-grade-point

1. Introduction

In 1965, Zadeh proposed the theory of fuzzy sets [15]. In recent years, some research on the application of fuzzy set theory in education has begun [2, 4, 10]. In [10], Chiang et al. presented a method for the application of fuzzy set theory to teaching assessment. In [4], Chang et al. presented a method for fuzzy assessment of learning performance of junior school students. In [2], Biswas pointed out that the chief aim of education institutions should be to provide the students with the evaluation reports regarding their test/examination as sufficient as possible and with unavoidable error as small as possible. He also presented a fuzzy evaluation method (fem) for the application of fuzzy sets in students' answerscripts evaluation. The fem presented in [2] is a computer based fuzzy approach, where a vector valued marking is used. Furthermore, in [2], Biswas also generalized the fem to propose a generalized fuzzy evaluation method (gfem) in which a matrix-valued marking is adopted. However, the methods presented in [2] have the following drawbacks:

(1) Because they use a matching function S to measure the degrees of similarity between the standard fuzzy sets and the fuzzy marks of the questions, they will take a large amount of time to perform the matching operations.

(2) In Biswas's methods, two different fuzzy marks may be translated into the same awarded grade and this is unfair in students' evaluation.

* Corresponding author.

0165-0114/99/$ – see front matter
PII: S0165-0114(97)00208-X

Because Biswas's methods have the above two drawbacks in the task of students' answerscripts evaluation, it is necessary to develop new methods to overcome the above drawbacks.

In this paper, we present two new methods for the application of fuzzy sets in students' answerscripts evaluation. They can overcome the drawbacks of the ones presented in [2]. The proposed methods have the advantages of much faster execution and are more fair in the task of students' evaluation than the ones presented in [2].

The rest of this paper is organized as follows. In Section 2, we briefly review the theory of fuzzy sets from [5, 6, 12, 15, 16]. In Section 3, we briefly review Biswas's methods for students' answerscripts evaluation. In Section 4, we present a new method for students' answerscripts evaluation using fuzzy sets. In Section 5, we present a generalized fuzzy evaluation method for students' evaluation. The conclusions are discussed in Section 6.

2. Fuzzy set theory

The theory of fuzzy sets was proposed by Zadeh in 1965 [15]. Roughly speaking, a fuzzy set is a class with fuzzy boundaries. Let X be the universe of discourse, $X = \{x_1, x_2, \ldots, x_n\}$, and let A be a fuzzy set of X, then the fuzzy set A can be represented as

$$A = \{(x_1, f_A(x_1)), (x_2, f_A(x_2)), \ldots, (x_n, f_A(x_n))\}, \quad (1)$$

where f_A is the membership function of the fuzzy set A, $f_A: X \to [0, 1]$, $f_A(x_i)$ indicates the degree of membership of x_i in A. If the universe of discourse X is an infinite set, then the fuzzy set A can be expressed as

$$A = \int_X f_A(x_i)/x_i, \quad x_i \in X. \quad (2)$$

Example 2.1. Let X be the universe of discourse, $X = \{\text{red, black, yellow, blue, white, brown, green}\}$, and let "dark" be a fuzzy set of the universe of discourse X subjectively defined as follows:

$$\text{dark} = \{(\text{red}, 0.5), (\text{black}, 1.0), (\text{yellow}, 0.1), (\text{blue}, 0.6), (\text{white}, 0.0), (\text{brown}, 0.8), (\text{green}, 0.3)\}, \quad (3)$$

where "black" has the largest membership value (i.e., 1.0) in the fuzzy set "dark", and "white" has the smallest membership value (i.e., 0.0) in the fuzzy set "dark". Thus, "black" is most pertinent to the fuzzy set "dark", and "white" is impertinent to the fuzzy set "dark".

For convenience, if an element x_i has zero membership value in a fuzzy set A (i.e., $f_A(x_i) = 0$), then the ordered pair $(x_i, f_A(x_i))$ can be discarded from the representation of the fuzzy set A. Thus, in the above example, the fuzzy set "dark" also can be written as follows:

$$\text{dark} = \{(\text{red}, 0.5), (\text{black}, 1.0), (\text{yellow}, 0.1), (\text{blue}, 0.6), (\text{brown}, 0.8), (\text{green}, 0.3)\}. \quad (4)$$

Example 2.2. Let X be the universe of discourse, $X = [0, 100]$. Then, the fuzzy sets "young" and "old" may subjectively be defined as follows:

$$f_{\text{young}}(x) = \begin{cases} 1, & 0 < x \leqslant 20, \\ (1 + ((x - 20)/15)^2)^{-1}, & 20 < x \leqslant 100, \end{cases} \quad (5)$$

$$f_{\text{old}}(x) = \begin{cases} 0, & 0 < x \leqslant 40, \\ (1 + ((x - 40)/10)^{-2})^{-1}, & 40 < x \leqslant 100, \end{cases} \quad (6)$$

where f_{young} and f_{old} are the membership functions of the fuzzy sets "young" and "old", respectively.

3. Biswas's methods for students' answerscripts evaluation

In [2], Biswas used a matching function S to measure the degree of similarity between two fuzzy sets. Let A and B be two fuzzy sets of the universe of discourse X, where

$$A = \{(x_1, f_A(x_1)), (x_2, f_A(x_2)), \ldots, (x_n, f_A(x_n))\},$$

$$B = \{(x_1, f_B(x_1)), (x_2, f_B(x_2)), \ldots, (x_n, f_B(x_n))\},$$

$$X = \{x_1, x_2, \ldots, x_n\}.$$

By using the vector representation method, the fuzzy sets A and B can be represented by the vectors $\bar{A}$ and $\bar{B}$, respectively, where

$\bar{A} = \langle f_A(x_1), f_A(x_2), \ldots, f_A(x_n)\rangle$,

$\bar{B} = \langle f_B(x_1), f_B(x_2), \ldots, f_B(x_n)\rangle$.

Then, the degree of similarity $S(\bar{A}, \bar{B})$ between the fuzzy sets A and B can be defined by

$$S(\bar{A}, \bar{B}) = \frac{\bar{A} \cdot \bar{B}}{\mathrm{Max}(\bar{A} \cdot \bar{A}, \bar{B} \cdot \bar{B})}, \tag{7}$$

where $S(\bar{A}, \bar{B}) \in [0, 1]$. The larger the value of $S(\bar{A}, \bar{B})$, the more the similarity between the fuzzy sets A and B.

Based on the matching function S, Biswas introduced a "fuzzy evaluation method" (fem) for evaluating students' answerscripts. In the following, we briefly review Biswas's methods for students' answerscripts evaluation. In [2], Biswas used five fuzzy linguistic hedges (called Standard Fuzzy Sets (SFS)) for students' answerscripts evaluation, i.e., E (excellent), V (very good), G (good), S (satisfactory), and U (unsatisfactory), where

$X = \{0\%, 20\%, 40\%, 60\%, 80\%, 100\%\}$,

$E = \{(0\%, 0), (20\%, 0), (40\%, 0.8), (60\%, 0.9), (80\%, 1), (100\%, 1)\}$,

$V = \{(0\%, 0), (20\%, 0), (40\%, 0.8), (60\%, 0.9), (80\%, 0.9), (100\%, 0.8)\}$,

$G = \{(0\%, 0), (20\%, 0.1), (40\%, 0.8), (60\%, 0.9), (80\%, 0.4), (100\%, 0.2)\}$,

$S = \{(0\%, 0.4), (20\%, 0.4), (40\%, 0.9), (60\%, 0.6), (80\%, 0.2), (100\%, 0)\}$,

$U = \{(0\%, 1), (20\%, 1), (40\%, 0.4), (60\%, 0.2), (80\%, 0), (100\%, 0)\}$.

Based on the vector representation method, the fuzzy sets E, V, G, S, and U can be represented by the vectors $\bar{E}$, $\bar{V}$, $\bar{G}$, $\bar{S}$, and $\bar{U}$, respectively, where

$\bar{E} = \langle 0, 0, 0.8, 0.9, 1, 1\rangle$,

$\bar{V} = \langle 0, 0, 0.8, 0.9, 0.9, 0.8\rangle$,

$\bar{G} = \langle 0, 0.1, 0.8, 0.9, 0.4, 0.2\rangle$,

$\bar{S} = \langle 0.4, 0.4, 0.9, 0.6, 0.2, 0\rangle$,

$\bar{U} = \langle 1, 1, 0.4, 0.2, 0, 0\rangle$.

In [2], Biswas pointed out that "A", "B", "C", "D", and "E" are called letter grades, where

$0 \leqslant E < 30$, $30 \leqslant D < 50$, $50 \leqslant C < 70$, $70 \leqslant B < 90$, $90 \leqslant A \leqslant 100$.

Furthermore, he also introduced the concept of mid-grade-point, where the mid-grade-point of $A = 95$ is denoted by $P(A)$, $B = 80$ by $P(B)$, $C = 60$ by $P(C)$, $D = 40$ by $P(D)$, $E = 15$ by $P(E)$. Assume that an evaluator is to evaluate the ith question (i.e., Q.i) of an answerscript of a student using a fuzzy grade sheet shown in Table 1.

In the first row of Table 1, the fuzzy mark (fum) to the answer of question Q.1 shows the degrees of the evaluator's satisfication for that answer in 0%, 20%, 40%, 60%, 80%, and 100% are 0, 0.1, 0.2, 0.4, 0.4, and 0.6, respectively. Let the fuzzy mark of the answer of question Q.1 be denoted by F_1. Then, we can see that F_1 is a fuzzy set of the universe of discourse X, where

$X = \{0\%, 20\%, 40\%, 60\%, 80\%, 100\%\}$,

$F_1 = \{(0\%, 0), (20\%, 0.1), (40\%, 0.2), (60\%, 0.4), (80\%, 0.4), (100\%, 0.6)\}$.

Biswas's algorithm [2] for students' answerscript evaluation is summarized as follows.

Step 1: For each attempted question in the answerscript repeatedly perform the following steps:

Table 1
A fuzzy grade sheet

Question No.	Fuzzy mark						Grade
Q.1	0	0.1	0.2	0.4	0.4	0.6	
Q.2							
Q.3							
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
					Total mark =		

(1) The evaluator awards a fuzzy mark F_i to the question Q.i by his best possible judgement and fills up the cells of the ith row for the first seven columns. Let $\bar{F}_i$ be the vector representation of F_i.

(2) Calculate the following degrees of similarities: $S(\bar{E}, \bar{F}_i)$, $S(\bar{V}, \bar{F}_i)$, $S(\bar{G}, \bar{F}_i)$, $S(\bar{S}, \bar{F}_i)$, and $S(\bar{U}, \bar{F}_i)$, where $\bar{E}$, $\bar{V}$, $\bar{G}$, $\bar{S}$, and $\bar{U}$ are the vector representations of the standard fuzzy sets E (excellent), V (very good), G (good), S (satisfactory), and U (unsatisfactory), respectively.

(3) Find the maximum among the five values $S(\bar{E}, \bar{F}_i)$, $S(\bar{V}, \bar{F}_i)$, $S(\bar{G}, \bar{F}_i)$, $S(\bar{S}, \bar{F}_i)$, and $S(\bar{U}, \bar{F}_i)$. Assume that $S(\bar{V}, \bar{F}_i)$ is the maximum value among the values of $S(\bar{E}, \bar{F}_i)$, $S(\bar{V}, \bar{F}_i)$, $S(\bar{G}, \bar{F}_i)$, $S(\bar{S}, \bar{F}_i)$, and $S(\bar{U}, \bar{F}_i)$, then award grade "B" to the question Q.i due to the fact that grade "B" corresponds to V (very good) of the standard fuzzy set.

Step 2: Calculate the total score using the following formula:

$$\text{Total score} = \frac{1}{100}\sum [T(\text{Q}.i) \times P(g_i)], \tag{8}$$

where $T(\text{Q}.i)$ is the mark alloted to Q.i in the question paper, and g_i is the grade awarded to Q.i by Step 1 of the algorithm. Put this total score in the appropriate box at the bottom of the fuzzy grade sheet.

Furthermore, in [2], Biswas also presented a generalized fuzzy evaluation method (gfem), where a generalized fuzzy grade sheet shown in Table 2 is used to evaluate the students' answerscripts. In the grade sheet of Table 2, for all $j = 1, 2, 3, 4$, and for all i, g_{ij} is the calculated grade by fem for the awarded fum F_{ij}, and m_i is the calculated mark to be awarded to the attempted question Q.i using the formula:

$$m_i = \frac{1}{400} \cdot T(\text{Q}.i) \cdot \sum_{j=1}^{4} P(g_{ij}) \tag{9}$$

and Total mark $= \sum m_i$.

Table 2
A generalized fuzzy grade sheet

Question No.	gfum	Grade	Mark
Q.1	F_{11}	g_{11}	m_1
	F_{12}	g_{12}	
	F_{13}	g_{13}	
	F_{14}	g_{14}	
Q.2	F_{21}	g_{21}	m_2
	F_{22}	g_{22}	
	F_{23}	g_{23}	
	F_{24}	g_{24}	
…	…	…	…
…	…	…	…
…	…	…	…
		Total mark =	

However, the methods presented in [2] have the following drawbacks:

(1) Because they use a matching function S to measure the degrees of similarity between the standard fuzzy sets and the fuzzy marks of the questions, it will take a large amount of time to perform the matching operations. Especially, when the number of questions in the test/examination is very big.

(2) In Biswas's method, two different fuzzy marks may be translated into the same awarded grade and this is unfair in students' evaluation. For example, let F_i and F_j be two different fuzzy marks represented by fuzzy sets of the universe of discourse X, respectively, and let E (excellent), V (very good), G (good), S (satisfactory), and U (unsatisfactory) be standard fuzzy sets of the universe of discourse X, where $X = \{0\%, 20\%, 40\%, 60\%, 80\%, 100\%\}$ and the corresponding awarded grade of the standard fuzzy sets "E", "V", "G", "S", and "U" are "A", "B", "C", "D", and "E", respectively. Then, based on [2], we can calculate the following degrees of similarities:

Case 1: If $S(\bar{V}, \bar{F}_i)$ is the maximum value among the values of $S(\bar{E}, \bar{F}_i)$, $S(\bar{V}, \bar{F}_i)$, $S(\bar{G}, \bar{F}_i)$, $S(\bar{S}, \bar{F}_i)$, $S(\bar{U}, \bar{F}_i)$, then the fuzzy mark F_i is translated to the awarded grade "B" due to the fact that the grade "B" corresponds to V (very good).

Case 2: If $S(\bar{V}, \bar{F}_j)$ is the maximum value among the values of $S(\bar{E}, \bar{F}_j)$, $S(\bar{V}, \bar{F}_j)$, $S(\bar{G}, \bar{F}_j)$, $S(\bar{S}, \bar{F}_j)$, $S(\bar{U}, \bar{F}_j)$, then the fuzzy mark F_j is translated to the awarded grade "B" due to the fact that the grade "B" corresponds to V (very good).

From Cases 1 and 2, we can see that two different fuzzy marks F_i and F_j are translated to the same awarded grade "B", and this is unfair in the task of students' answerscripts evaluation.

Because Biswas's methods have the above two drawbacks in the task of students' answerscripts evaluation, a new method for students' answerscripts evaluation is required to overcome the above drawbacks.

4. A new method for student's evaluation using fuzzy sets

In this section, we present a new method for students' answerscripts evaluation. Assume that there are eleven satisfication levels to evaluate the students' answerscripts regarding a question of a test/examination, i.e., extremely good (EG), very very good (VVG), very good (VG), good (G), more or less good (MG), fair (F), more or less bad (MB), bad (B), very bad (VB), very very bad (VVB), and extremely bad (EB), where the degrees of satisfication of the eleven satisfication levels are shown in Table 3.

Table 3
Satisfication levels and their corresponding degrees of satisfication

Satisfication levels	Degrees of satisfication
Extremely good (EG)	100% (i.e., 1.00)
Very very good (VVG)	91%–99% (i.e., 0.91–0.99)
Very good (VG)	81%–90% (i.e., 0.81–0.90)
Good (G)	71%–80% (i.e., 0.71–0.80)
More or less good (MG)	61%–70% (i.e., 0.61–0.70)
Fair (F)	51%–60% (i.e., 0.51–0.60)
More or less bad (MB)	41%–50% (i.e., 0.41–0.50)
Bad (B)	25%–40% (i.e., 0.25–0.40)
Very bad (VB)	10%–24% (i.e., 0.10–0.24)
Very very bad (VVB)	1%–9% (i.e., 0.01–0.09)
Extremely bad (EB)	0% (i.e., 0)

Let X be a set of satisfication levels, $X =$ {extremely good (EG), very very good (VVG), very good (VG), good (G), more or less good (MG), fair (F), more or less bad (MB), bad (B), very bad (VB), very very bad (VVB), extremely bad (EB)}, and let T be a mapping function which maps a satisfication level to the maximum degree of satisfication of the corresponding satisfication level, where $T: X \rightarrow [0, 1]$. From Table 3, we can see that

$$T(\text{extremely good}) = 1.00 \text{ (i.e., } T(\text{EG}) = 1.00\text{)},$$
$$T(\text{very very good}) = 0.99 \text{ (i.e., } T(\text{VVG}) = 0.99\text{)},$$
$$T(\text{very good}) = 0.90 \text{ (i.e., } T(\text{VG}) = 0.90\text{)},$$
$$T(\text{good}) = 0.80 \text{ (i.e., } T(\text{G}) = 0.80\text{)},$$
$$T(\text{more or less good}) = 0.70 \text{ (i.e., } T(\text{MG}) = 0.70\text{)},$$
$$T(\text{fair}) = 0.60 \text{ (i.e., } T(\text{F}) = 0.60\text{)},$$
$$T(\text{more or less bad}) = 0.50 \text{ (i.e., } T(\text{MB}) = 0.50\text{)},$$
$$T(\text{bad}) = 0.40 \text{ (i.e., } T(\text{B}) = 0.40\text{)},$$
$$T(\text{very bad}) = 0.24 \text{ (i.e., } T(\text{VB}) = 0.24\text{)},$$
$$T(\text{very very bad}) = 0.09 \text{ (i.e., } T(\text{VVB}) = 0.09\text{)},$$
$$T(\text{extremely bad}) = 0 \text{ (i.e., } T(\text{EB}) = 0\text{)}. \quad (10)$$

Assume that an evaluator can evaluate the students' answerscripts using extended fuzzy grade sheets. The definition of the extended fuzzy grade sheets is presented as follows.

Definition 4.1. Extended fuzzy grade sheet: An extended fuzzy grade sheet is a matrix type structure containing thirteen columns and n rows, where n is the total number of questions in a test/examination. An example of an extended fuzzy grade sheet is shown in Table 4. At the bottom of the sheet there is a box which tells the total score. The first column reveals the serial numbers of the questions; in any row, the columns from the second to the twelfth shows the fuzzy mark awarded to the answer to the corresponding question in the first column, where the fuzzy mark is represented as a fuzzy set in the universe of discourse X, $X =$ {extremely good (EG), very very good (VVG), very good (VG), good (G), more or less good (MG), fair (F), more or less bad (MB), bad (B), very bad (VB), very very bad (VVB), extremely bad (EB)}, The last (i.e., the

Table 4
An extended fuzzy grade sheet

Question No.	Satisfication levels											Degree of satisfication
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
Q.1												
Q.2												
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Q.*n*												
												Total mark =

Table 5
An example of an extended fuzzy grade sheet

Question No.	Satisfication levels											Degree of satisfication
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
Q.1	0	0.9	0.8	0.5	0	0	0	0	0	0	0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
												Total mark =

thirteenth) column shows the degree of satisfication evaluated by the proposed method awarded to each question. The box at the bottom shows the total mark awarded to the student.

For example, assume that an evaluator is using an extended fuzzy grade sheet to evaluate the fuzzy mark of the first question (i.e., Q.1) of a test/examination of a student as shown in Table 5. From Table 5, we can see that the satisfication level regarding the first question of the student's answerscript is represented by a fuzzy set F(Q.1) of the universe of discourse X, where X = {EG, VVG, VG, G, MG, F, MB, B, VB, VVB, EB}, and

$$F(\text{Q.1}) = \{(\text{EG}, 0), (\text{VVG}, 0.9), (\text{VG}, 0.8), (\text{G}, 0.5), (\text{MG}, 0), (\text{F}, 0), (\text{MB}, 0), (\text{B}, 0), (\text{VB}, 0), (\text{VVB}, 0), (\text{EB}, 0)\}. \quad (11)$$

For convenience, the fuzzy set F(Q.1) can also be abbreviated into

$$F(\text{Q.1}) = \{(\text{VVG}, 0.9), (\text{VG}, 0.8), (\text{G}, 0.5)\}. \quad (12)$$

It indicates that the satisfication level of the student's answerscript with respect to the first question is described as 90% very very good, 80% very good, and 50% good.

The method for students' answerscripts evaluation is now presented as follows:

Step 1: Assume that the fuzzy mark of the question Q.i of a student's answerscript evaluated by an evaluator is shown in Table 6, where $y_i \in [0, 1]$ and $1 \leqslant i \leqslant 11$. From formula (10), we can see that T(EG) = 1, T(VVG) = 0.99, T(VG) = 0.90, T(G) = 0.80, T(MG) = 0.70, T(F) = 0.60, T(MB) = 0.50, T(B) = 0.40, T(VB) = 0.24, T(VVB) = 0.09, and T(EB) = 0. In this case, the degree of satisfication D(Q.i) of the question Q.i of the student's answerscript can be evaluated by the function D,

$$D(\text{Q}.i) = \frac{y_1 * T(\text{EG}) + y_2 * T(\text{VVG}) + \cdots + y_{11} * T(\text{EB})}{y_1 + y_2 + \cdots + y_{11}} \quad (13)$$

Table 6
Fuzzy mark of question Q.i in an extended fuzzy grade sheet

Question No.	Satisfication levels											Degree of satisfication
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Q.i	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
												Total mark =

where $D(\text{Q}.i) \in [0, 1]$. The larger the value of $D(\text{Q}.i)$, the more the degree of satisfication that the question Q.i of the student's answerscript satisfies the evaluator's opinion.

For example, let us consider the example shown in Table 5. From formula (10), we can see that $T(\text{VVG}) = 0.99$, $T(\text{VG}) = 0.90$, and $T(\text{G}) = 0.80$. By applying formula (13), the degree of satisfication $D(\text{Q}.1)$ of the student's answerscript regarding question Q.1 can be evaluated as follows:

$$D(\text{Q}.1) = \frac{0.9 * 0.99 + 0.8 * 0.90 + 0.5 * 0.80}{0.9 + 0.8 + 0.5} = 0.9141. \quad (14)$$

It indicates that the degree of satisfication of the question Q.1 of the student's answerscript evaluated by the evaluator is 0.9141 (i.e., 91.41%).

Step 2: Consider a candidate's answerscript to a paper of 100 marks. Assume that in total there were n questions to be answered:

TOTAL MARKS = 100

Q.1 carries s_1 marks

Q.2 carries s_2 marks

⋮

Q.n carries s_n marks,

where $\sum_{i=1}^{n} s_i = 100$, $0 \leqslant s_i \leqslant 100$, and $1 \leqslant i \leqslant n$. Assume that the evaluated degree of satisfication of the question Q.1, Q.2, …, and Q.n are $D(\text{Q}.1)$, $D(\text{Q}.2)$, …, and $D(\text{Q}.n)$, respectively, then the total score of the student can be evaluated as follows:

$$s_1 * D(\text{Q}.1) + s_2 * D(\text{Q}.2) + \cdots + s_n * D(\text{Q}.n). \quad (15)$$

Put this total score in the appropriate box at the bottom of the extended fuzzy grade sheet.

In the following, we use an example to illustrate the students' answerscript evaluation process.

Example 4.1. Consider a candidate's answerscript to a paper of 100 marks. Assume that in total there were four questions to be answered:

TOTAL MARKS = 100

Q.1 carries 20 marks

Q.2 carries 30 marks

Q.3 carries 25 marks

Q.4 carries 25 marks.

Assume that an evaluator awards the students' answerscript by an extended fuzzy grade sheet as shown in Table 7.

[*Step* 1] Based on formula (10) and by applying formula (13), we can see that

$$D(\text{Q}.1) = \frac{0.8 * T(\text{VVG}) + 0.9 * T(\text{VG})}{0.8 + 0.9} = \frac{0.8 * 0.99 + 0.9 * 0.90}{0.8 + 0.9} = 0.9424. \quad (16)$$

Table 7
Extended fuzzy grade sheet of Example 4.1

Question No.	Satisfication levels											Degree of satisfication
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
Q.1	0	0.8	0.9	0	0	0	0	0	0	0	0	0.9424
Q.2	0	0	0	0.6	0.9	0.5	0	0	0	0	0	0.7050
Q.3	0	0	0.8	0.7	0.5	0	0	0	0	0	0	0.8150
Q.4	0	0	0	0	0	0	0	0.5	0.9	0.2	0	0.2713
												Total mark = 67

$$D(\mathrm{Q}.2) = \frac{0.6 * T(\mathrm{G}) + 0.9 * T(\mathrm{MG}) + 0.5 * T(\mathrm{F})}{0.6 + 0.9 + 0.5}$$

$$= \frac{0.6 * 0.80 + 0.9 * 0.70 + 0.5 * 0.60}{0.6 + 0.9 + 0.5}$$

$$= 0.7050. \tag{17}$$

$$D(\mathrm{Q}.3) = \frac{0.8 * T(\mathrm{VG}) + 0.7 * T(\mathrm{G}) + 0.5 * T(\mathrm{MG})}{0.8 + 0.7 + 0.5}$$

$$= \frac{0.8 * 0.90 + 0.7 * 0.80 + 0.5 * 0.70}{0.8 + 0.7 + 0.5}$$

$$= 0.8150. \tag{18}$$

$$D(\mathrm{Q}.4) = \frac{0.5 * T(\mathrm{B}) + 0.9 * T(\mathrm{VB}) + 0.2 * T(\mathrm{VVB})}{0.5 + 0.9 + 0.2}$$

$$= \frac{0.5 * 0.40 + 0.9 * 0.24 + 0.2 * 0.09}{0.5 + 0.9 + 0.2}$$

$$= 0.2713. \tag{19}$$

[*Step* 2] By applying formula (15), the total mark of the student can be evaluated as follows:

$$20 * D(\mathrm{Q}.1) + 30 * D(\mathrm{Q}.2) + 25 * D(\mathrm{Q}.3) + 25 * D(\mathrm{Q}.4)$$

$$= 20 * 0.9424 + 30 * 0.7050 + 25 * 0.8150 + 25 * 0.2713$$

$$= 18.848 + 21.15 + 20.375 + 6.7825$$

$$= 67.155$$

$$\fallingdotseq 67 \text{ (assuming that no half mark is given in the total score).} \tag{20}$$

5. A generalized fuzzy evaluation method

In this section, we generalize the method presented in Section 4 to propose a weighted method for students' answerscripts evaluation using fuzzy sets. Consider a candidate's answerscript to a paper of 100 marks.

Step 1: Assume that in total there are n questions to be answered:

TOTAL MARKS = 100

Q.1 carries s_1 marks

Q.2 carries s_2 marks

⋮

Q.n carries s_n marks.

Assume that an evaluator evaluates the questions of students' answerscripts using the following four criteria [2]:

C1: Accuracy of information,
C2: Adequate converage,
C3: Conciseness,
C4: Clear expression,

Table 8
A generalized extended fuzzy grade sheet

Question No.	Criteria	Satisfication levels											Degree of satisfication for criteria	Degree of satisfication for questions
		EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB		
Q.1	C1												$D(C11)$	
	C2												$D(C12)$	$P(Q.1)$
	C3												$D(C13)$	
	C4												$D(C14)$	
Q.2	C1												$D(C21)$	
	C2												$D(C22)$	$P(Q.2)$
	C3												$D(C23)$	
	C4												$D(C24)$	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Q.n	C1												$D(Cn1)$	
	C2												$D(Cn2)$	$P(Q.n)$
	C3												$D(Cn3)$	
	C4												$D(Cn4)$	

Total mark $= s_1 * P(Q.1) + s_2 * P(Q.2) + \cdots + s_n * P(Q.n)$

and assume that the weights of the criteria C1, C2, C3, and C4 are w_1, w_2, w_3, and w_4, respectively, where $w_i \in [0, 1]$ and $1 \leqslant i \leqslant 4$. Furthermore, assume that the evaluator can evaluate each question of the students' answerscripts using the above four criteria based on the method described in Section 4. In this case, an evaluator can evaluate the student's answerscripts using a generalized extended fuzzy grade sheet as shown in Table 8, where the degrees of satisfication of the question Q.i of a students' answerscript regarding to the criteria C1, C2, C3, and C4 evaluated by the method presented in ection 4 are $D(Ci1)$, $D(Ci2)$, $D(Ci3)$, and $D(Ci4)$, respectively, where $0 \leqslant D(Ci1) \leqslant 1$, $0 \leqslant D(Ci2) \leqslant 1$, $0 \leqslant D(Ci3) \leqslant 1$, $0 \leqslant D(Ci4) \leqslant 1$, and $1 \leqslant i \leqslant n$.

Step 2: The degree of satisfication $P(Q.i)$ of the question Q.i of the student's answerscript can by evaluated as follows:

$$P(Q.i) = \frac{w_1 * D(Ci1) + w_2 * D(Ci2) + w_3 * D(Ci3) + w_4 * D(Ci4)}{w_1 + w_2 + w_3 + w_4}, \quad (21)$$

where $P(Q.i) \in [0, 1]$ and $1 \leqslant i \leqslant n$. The total score of the student can be evaluated and is equal to

$$s_1 * P(Q.1) + s_2 * P(Q.2) + \cdots + s_n * P(Q.n). \quad (22)$$

Put this total score in the appropriate box at the bottom of the extended fuzzy grade sheet.

6. Conclusions

In [2], Biswas has presented two fuzzy evaluation methods for students' answerscripts evaluation. In this paper, we extend the work of [2] to present two new methods for students' answerscripts evaluation. The proposed methods can be executed much faster than the ones presented in [2] due to the fact that they do not need to perform the complicated matching operations. Furthermore, they can make a more fair evaluation of students' answerscripts. The proposed methods can overcome the drawbacks of the ones presented in [2].

Acknowledgements

The authors would like to thank the referees for providing very helpful comments and suggestions. This work was supported in part by the National Science Council, Republic of China, under Grant NSC 86-2213-E-009-018.

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