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# $B \to \eta' X_s$ in the standard model

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**Abstract**

We study $B \to \eta' X_s$ within the framework of the Standard Model. Several mechanisms such as $b \to \eta' sg$ through the QCD anomaly, and $b \to \eta' s$ and $B \to \eta' s\bar{q}$ arising from four-quark operators are treated simultaneously. Using QCD equations of motion, we relate the effective Hamiltonian for the first mechanism to that for the latter two. By incorporating next-to-leading-logarithmic(NLL) contributions, the first mechanism is shown to give a significant branching ratio for $B \to \eta' X_s$, while the other two mechanisms account for about 15% of the experimental value. The Standard Model prediction for $B \to \eta' X_s$ is consistent with the CLEO data. © 1999 Elsevier Science B.V. All rights reserved.

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The recent observation of $B \to \eta' K$ [1] and $B \to \eta' X_s$ [2] decays with high momentum $\eta'$ mesons has stimulated many theoretical activities [3–10]. One of the mechanisms proposed to account for this decay is $b \to sg^* \to sg\eta'$ [3,4] where the $\eta'$ meson is produced via the anomalous $\eta' - g - g$ coupling. According to a previous analysis [4], this mechanism within the Standard Model (SM) can only account for $1/3$ of the measured branching ratio: $\mathcal{B}(B \to \eta' X_s) = \left[6.2 \pm 1.6(\text{stat}) \pm 1.3(\text{syst})^{+0.0}_{-1.5} \times (\text{bkg})\right] \times 10^{-4}$ [2] with $2.0 < p_{\eta'} < 2.7$ GeV. There are also other calculations of $B \to \eta' X_s$ based on four-quark operators of the effective weak-Hamiltonian [5,6]. These contributions to the branching ratio, typically $10^{-4}$, are also too small to account for $B \to \eta' X_s$, although the four-quark-operator contribution is capable of explaining the branching ratio for the exclusive $B \to \eta' K$ decays [8,9]. These results have inspired proposals for an enhanced $b \to sg$ and other mechanisms arising from physics beyond the Standard Model [4,6,7]. In order to see if new physics should play any role in $B \to \eta' X_s$, one has to have a better understanding on the SM prediction. In this letter, we carry out a careful analysis on $B \to \eta' X_s$ in the SM using next-to-leading effective Hamiltonian and consider several mechanisms simultaneously.

We have observed that all earlier calculations on $b \to sg\eta'$ were either based upon one-loop result [4]

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which neglects the running of QCD renormalization -scale from $M_W$ to $M_b$ or only taking into account part of the running effect [3]. Since the short-distance QCD effect is generally significant in weak decays, it is therefore crucial to compute $b \to sg\eta'$ using the effective Hamiltonian approach. As will be shown later, the process $b \to sg\eta'$ alone contribute significantly to $B \to \eta' X_s$ while contributions from $b \to \eta' s$ and $B \to \eta' s\bar{q}$ are suppressed.

The effective Hamiltonian [3] for the $B \to \eta' X_s$ decay is given by:

$$H_{\rm eff}(\Delta B=1) = \frac{G_F}{\sqrt{2}}\Bigg[\sum_{f=u,c} V_{fb}V_{fs}^{*}\left(C_1(\mu)O_1^f(\mu) + C_2(\mu)O_2^f(\mu)\right) - V_{ts}^{*}V_{tb}\left(\sum_{i=3}^{6} C_i(\mu)O_i(\mu) + C_8(\mu)O_8(\mu)\right)\Bigg], \quad (1)$$

with [4]

$$O_1^f = (\bar{s}_i f_j)_{V-A}(\bar{f}_j b_i)_{V-A},$$
$$O_2^f = (\bar{s}_i f_i)_{V-A}(\bar{f}_j b_j)_{V-A},$$
$$O_3 = (\bar{s}_i b_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V-A},$$
$$O_4 = (\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V-A},$$
$$O_5 = (\bar{s}_i b_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V+A},$$
$$O_6 = (\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V+A},$$
$$O_8 = -\frac{g_s}{4\pi^2}\bar{s}_i\sigma^{\mu\nu}(m_s P_{\rm L} + m_b P_{\rm R})T^a_{ij} b_j G^a_{\mu\nu}, \quad (2)$$

where $V \pm A \equiv 1 \pm \gamma_5$. In the above, we have dropped $O_7$ since its contribution is negligible. For numerical analyses, we use the scheme-independent Wilson coefficients discussed in Ref. [13,14]. For $m_t = 175$ GeV, $\alpha_s(m_Z^2) = 0.118$ and $\mu = m_b = 5$ GeV, we have [14]

$$C_1 = -0.313, \quad C_2 = 1.150, \quad C_3 = 0.017,$$
$$C_4 = -0.037, \quad C_5 = 0.010, \quad C_6 = -0.045. \quad (3)$$

At the NLL level, the effective Hamiltonian is modified by one-loop matrix elements which effectively change $C_i(\mu)(i=3,\cdots,6)$ into $C_i(\mu) + \overline{C}_i(q^2,\mu)$ with

$$\overline{C}_4(q^2,\mu) = \overline{C}_6(q^2,\mu) = -3\overline{C}_3(q^2,\mu) = -3\overline{C}_5(q^2,\mu) = -P_s(q^2,\mu), \quad (4)$$

where

$$P_s(q^2,\mu) = \frac{\alpha_s}{8\pi}C_2(\mu)\left(\tfrac{10}{9} + G(m_c^2,q^2,\mu)\right), \quad (5)$$

with

$$G(m_c^2,q^2,\mu) = 4\int x(1-x)\log\left(\frac{m_c^2 - x(1-x)q^2}{\mu^2}\right){\rm d}x. \quad (6)$$

The coefficient $C_8$ is equal to $-0.144$ at $\mu = 5\,\text{GeV}$ [5], and $m_c$ is taken to be 1.4 GeV.

Before we discuss the dominant $b \to sg\eta'$ process, let us first work out the four-quark-operator contribution to $B \to \eta' X_s$ using the above effective Hamiltonian. We follow the approach of Ref. [3,5,15] which uses factorization approximation to estimate various hadronic matrix elements. The four-quark operators can induce three types of processes represented by (1) $\langle \eta'|\bar{q}\Gamma_1 b|B\rangle\langle X_s|\bar{s}\Gamma_1' q|0\rangle$, (2) $\langle \eta'|\bar{q}\Gamma_2 q|0\rangle\langle X_s|\bar{s}\Gamma\, b|B\rangle$, and (3) $\langle \eta' X_s|\bar{s}\Gamma_3 q|0\rangle\langle 0|\bar{q}\Gamma_3' b|B\rangle$. Here $\Gamma_i^{(\prime)}$ denotes appropriate gamma matrices. The contribution from (1) gives a ‘‘three-body’’ type of decay, $B \to \eta' s\bar{q}$. The contribution from (2) gives a ‘‘two-body’’ type of decay $b \to s\eta'$. The contribution from (3) is the annihilation type which is relatively suppressed and will be neglected. Note that there are interferences

[3] For an extensive review on the subject of effective Hamiltonian, see Ref. [11], which contains a detailed list of original literatures.

[4] The sign of $O_8$ is consistent with the covariant derivative, $D_\mu = \partial_\mu - igT^a A^a_\mu$, in the QCD Lagrangian. See, [12].

[5] For an extensive review on the subject of effective Hamiltonian, see Ref. [11], which contains a detailed list of original literatures.

between (1) and (2), so they must be coherently added together [5].

Several decay constants and form factors needed in the calculations are listed below:

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u|\eta'\rangle = \langle 0|\bar{d}\gamma_\mu\gamma_5 d|\eta'\rangle = i f^u_{\eta'} p^{\eta'}_\mu ,$$

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 s|\eta'\rangle = i f^s_{\eta'} p^{\eta'}_\mu ,$$

$$\langle 0|\bar{s}\gamma_5 s|\eta'\rangle = i\left( f^u_{\eta'} - f^s_{\eta'}\right)\frac{m^2_{\eta'}}{2m_s} ,$$

$$f^u_{\eta'} = \frac{1}{\sqrt{3}}\left( f_1\cos\theta_1 + \frac{1}{\sqrt{2}} f_8\sin\theta_8\right),$$

$$f^s_{\eta'} = \frac{1}{\sqrt{3}}\left( f_1\cos\theta_1 - \sqrt{2} f_8\sin\theta_8\right),$$

$$\langle \eta'|\bar{u}\gamma_\mu b|B^-\rangle = \langle \eta'|\bar{d}\gamma_\mu b|\bar{B}^0\rangle$$

$$= F^{Bq}_1\left( p^B_\mu + p^{\eta'}_\mu\right)$$

$$+\left( F^{Bq}_0 - F^{Bq}_1\right)\frac{mB^2 - m^2_{\eta'}}{q^2} q_\mu ,$$

$$F^{Bq}_{1,0} = \frac{1}{\sqrt{3}}\left(\frac{1}{\sqrt{2}}\sin\theta F^{B\eta_8}_{1,0} + \cos\theta F^{B\eta_1}_{1,0}\right). \quad (7)$$

For the $\eta' - \eta$ mixing associated with decay constants above, we have used the two-angle -parametrization. The numerical values of various parameters are obtained from Ref. [16] with $f_1 = 157$ MeV, $f_8 = 168$ MeV, and the mixing angles $\theta_1 = -9.1°$, $\theta_8 = -22.1°$. For the mixing angle associated with form factors, we use the one-angle parametrization with $\theta = -15.4°$ [16], since these form factors were calculated in that formulation [5,15]. In the latter discussion of $b \to sg\eta'$, we shall use the same parametrization in order to compare our results with those of earlier works [3,4]. For form factors, we assume that $F^{B\eta_1} = F^{B\eta_8} = F^{B\pi}$ with dipole and monopole $q^2$ dependence for $F_1$ and $F_0$, respectively. We used the running mass $m_s \approx 120$ MeV at $\mu = 2.5$ GeV and $F^{B\pi} = 0.33$ following Ref. [9].

The branching ratios of the above processes also depend on two less well-determined KM matrix elements, $V_{ts}$ and $V_{ub}$. The dependences on $V_{ts}$ arise from the penguin-diagram contributions while the dependences on $V_{ub}$ and its phase $\gamma$ occur through the tree-diagram contributions. We will use $\gamma = 64°$ obtained from Ref. [17], $|V_{ts}| \approx |V_{cb}| = 0.038$ and $|V_{ub}|/|V_{cb}| = 0.08$ for an illustration. We find that, for $\mu = 5$ GeV, the branching ratio in the signal region $p_{\eta'} \geq 2.0$ GeV ($m_X \leq 2.35$ GeV) is

$$\mathscr{B}( b \to \eta' X_s) \approx 1.0 \times 10^{-4}. \quad (8)$$

The branching ratio can reach $2 \times 10^{-4}$ if all parameters take values in favour of $B \to \eta' X_s$. Clearly the mechanism by four-quark operator is not sufficient to explain the observed $B \to \eta' X_s$ branching ratio.

We now turn to the major mechanism for $B \to \eta' X_s$: $b \to \eta' sg$ through the QCD anomaly. To see how the effective Hamiltonian in Eq. (1) can be applied to calculate this process, we rearrange part of the effective Hamiltonian such that

$$\sum_{i=3}^{6} C_i O_i = \left( C_3 + \frac{C_4}{N_c}\right) O_3 + \left( C_5 + \frac{C_6}{N_c}\right) O_5$$
$$- 2( C_4 - C_6) O_A + 2( C_4 + C_6) O_V , \quad (9)$$

where

$$O_A = \bar{s}\gamma_\mu(1-\gamma_5) T^a b \sum_q \bar{q}\gamma^\mu\gamma_5 T^a q,$$

$$O_V = \bar{s}\gamma_\mu(1-\gamma_5) T^a b \sum_q \bar{q}\gamma^\mu T^a q. \quad (10)$$

Since the light-quark bilinear in $O_V$ carries the quantum number of a gluon, one expects [3] $O_V$ give contribution to the $b \to sg^*$ form factors. In fact, by applying the QCD equation of motion: $D_\nu G^{\mu\nu}_a = g_s \Sigma \bar{q}\gamma^\mu T^a q$, we have $O_V = (1/g_s)\bar{s}\gamma_\mu(1-\gamma_5)$-$T^a b D_\nu G^{\mu\nu}_a$ [6]. In this form, $O_V$ is easily seen to give

[6] By applying the QCD equation of motion or performing a direct calculation, it was shown that the operator basis of $O_3 - O_6$ are suitable to describe nonleptonic weak decays although effective vertices such as $s \to d +$ gluons are encountered. Here the operator basis on the r.h.s of Eq. (9) is more suitable for our purpose. For detail, see Ref. [18].

rise to $b \to sg^*$ vertex. Let us write the effective $b \to sg^*$ vertex as

$$\Gamma_\mu^{bsg} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \frac{g_s}{4\pi^2} \times \Big( \Delta F_1 \bar{s} \big( q^2 \gamma_\mu - q\!\!/ q_\mu \big) L T^a b - i F_2 m_b \bar{s} \sigma_{\mu\nu} q^\nu R T^a b \Big). \tag{11}$$

In the above, we define the form factors $\Delta F_1$ and $F_2$ according to the convention in Ref. [4]. Inferring from Eq. (9), we arrive at

$$\Delta F_1 = \frac{4\pi}{\alpha_s} \big( C_4(\mu) + C_6(\mu) \big), \quad F_2 = -2 C_8(\mu). \tag{12}$$

We note that our relative sign between $\Delta F_1$ and $F_2$ agree with those in Ref. [4,6], and shall result in a destructive interference for the rate of $b \to sg\eta'$. We stress that this relative sign is fixed by treating the sign of $O_8$ and the convention of QCD covariant derivative consistently. [7] To ensure the sign, we also check against the result by Simma and Wyler [19] on $b \to sg^*$ form factors. An agreement on sign is found. Finally, we remark that, at the NLL level, $\Delta F_1$ should be corrected by one-loop matrix elements. The dominant contribution arises from the operator $O_2$ where its charm-quark-pair meets to form a gluon. In fact, this contribution, denoted as $\Delta \bar{F}_1$ for convenience, has been shown in Eqs. (4)-(6), namely $\Delta \bar{F}_1 = \frac{4\pi}{\alpha_s} (\bar{C}_4(q^2,\mu) + \bar{C}_6(q^2,\mu))$.

To proceed further, we recall the distribution of the $b(p) \to s(p') + g(k) + \eta'(k')$ branching ratio [4]:

$$\frac{d^2 \mathscr{B}(b \to sg\eta')}{\mathrm{d}x\,\mathrm{d}y} \cong 0.2 \cos^2\theta \left( \frac{g_s(\mu)}{4\pi^2} \right)^2 \frac{a_g^2(\mu) m_b^2}{4} \times \left[ |\Delta F_1|^2 c_0 + \mathrm{Re}(\Delta F_1 F_2^*) \frac{c_1}{y} + |\Delta F_2|^2 \frac{c_2}{y^2} \right], \tag{13}$$

where $a_g(\mu) \equiv \sqrt{N_F}\, \alpha_s(\mu)/\pi f_{\eta'}$ is the strength of $\eta' - g - g$ vertex: $a_g \cos\theta \epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta$ with $q$ and $k$ the momenta of two gluons; $x \equiv (p'+k)^2/m_b^2$ and $y \equiv (k+k')^2/m_b^2$; $c_0$, $c_1$ and $c_2$ are functions of $x$ and $y$ as given by:

$$c_0 = \big[ -2x^2 y + (1-y)(y-x')(2x+y-x') \big]/2,$$
$$c_1 = (1-y)(y-x')^2,$$
$$c_2 = \big[ 2x^2y^2 - (1-y)(y-x')(2xy-y+x') \big]/2, \tag{14}$$

with $x' \equiv m_{\eta'}^2/m_b^2$; and the $\eta' - \eta$ mixing angle $\theta$ is taken to be $-15.4°$ as noted earlier. Finally, in obtaining the normalization factor: 0.2, we have taken into account the one-loop QCD correction [20] to the semi-leptonic $b \to c$ decay for consistency.

In previous one-loop calculations without QCD corrections, it was found $\Delta F_1 \approx -5$ and $F_2 \approx 0.2$ [3,4]. In our approach, we obtain $\Delta F_1 = -4.86$ and $F_2 = 0.288$ from Eqs. (3) and (12). However, $\Delta F_1$ is enhanced significantly by the matrix-element correction $\Delta \bar{F}_1(q^2,\mu)$. The latter quantity develops an imaginary part as $q^2$ passes the charm-pair threshold, and the magnitude of its real part also becomes maximal at this threshold. From Eqs. (3), (4) and (5), one finds $\mathrm{Re}(\Delta \bar{F}_1(4m_c^2,\mu)) = -2.58$ at $\mu = 5$ GeV. Including the contribution by $\Delta \bar{F}_1(q^2,\mu)$ with $\mu = 5$ GeV, and using Eq. (13), we find $\mathscr{B}(b \to sg\eta') = 5.6 \times 10^{-4}$ with the cut $m_X \equiv \sqrt{(k+p')^2} \le 2.35$ GeV imposed in the CLEO measurement [2]. This branching ratio is consistent with CLEO's measurement on the $B \to \eta' X_s$ branching ratio [2]. Without the kinematic cut, we obtain $\mathscr{B}(b \to sg\eta') = 1.0 \times 10^{-3}$, which is much larger than $4.3 \times 10^{-4}$ calculated previously [4]. We also obtain the spectrum $d\mathscr{B}(b \to sg\eta')/dm_X$ as depicted in Fig. 1. The peak of the spectrum corresponds to $m_X \approx 2.4$ GeV.

It is interesting to note that the CLEO analysis [2] indicates that, without the anomaly-induced contribution, the recoil-mass($m_X$) spectrum of $B \to \eta' X_s$ can not be well reproduced even if the four-quark operator contributions are normalized to fit the branching ratio of the process. On the other hand, if $b \to sg^* \to sg\eta'$ dominates the contributions to $B \to \eta' X_s$, as shown here, the $m_X$ spectrum can be fitted better as shown in Fig. 2 of Ref. [2]. It is also interesting to remark that although the four-quark operator contributions can not fit the branching ratio nor the spectrum, it does play a role in producing a small peak in the spectrum, which corresponds to the $B \to \eta' K$

[7] We thank A. Kagan for pointing out this to us, which helps us to detect a sign error in our earlier calculation.

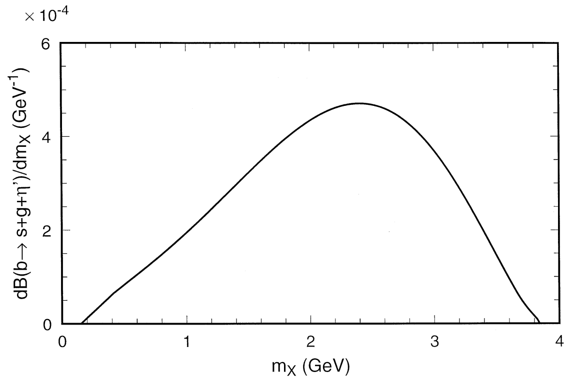


Fig. 1. The distribution of $\mathscr{B}(b \to s + g + \eta')$ as a function of the recoil mass $m_X$.

mode. Specifically, the $B \to \eta' K$ mode is accounted for by the $b \to s\eta'$ type of decays discussed earlier. Based on results obtained so far, one concludes that the Standard Model is not in conflict the experimental data on $B \to \eta' X_s$. It can produce not only the branching ratio for $B \to \eta' X_s$ but also the recoil-mass spectrum when contributions from the anomaly mechanism and the four-quark operators are properly treated.

Up to this point, $a_g(\mu)$ of the $\eta' - g - g$ vertex has been treated as a constant independent of invariant-masses of the gluons, and $\mu$ is set to be 5 GeV. In practice, $a_g(\mu)$ should behave like a form-factor which becomes suppressed as the gluons attached to it go farther off-shell [3,4,6]. However, it remains unclear how much the form-factor suppression might be. It is possible that the branching ratio we just obtained gets reduced significantly by the form-factor effect in $\eta' - g - g$ vertex. Should a large form-factor suppression occur, the additional contribution from $b \to \eta' s$ and $B \to \eta' s\bar{q}$ discussed earlier would become crucial. We however like to stress that our estimate of $b \to sg\eta'$ with $\alpha_s$ evaluated at $\mu = 5$ GeV is conservative. To illustrate this, let us compare branching ratios for $b \to sg\eta'$ obtained at $\mu = 5$ GeV and $\mu = 2.5$ GeV respectively. In NDR scheme [8], branching ratios at the above two scales with the cut $m_X \leq 2.35$ GeV are $4.9 \times 10^{-4}$ and $9.1 \times 10^{-4}$ respectively. One can clearly see the significant scale-dependence! With the enhancement resulting from lowering the renormalization scale, there seems to be some room for the form-factor suppression in the attempt of explaining $B \to \eta' X_s$ by $b \to sg\eta'$ [9].

It should be noted that the above scale-dependence is solely due to the coupling constant $\alpha_s(\mu)$ appearing in the $\eta' - g - g$ vertex. In fact, the $b \to sg^*$ vertex is rather insensitive to the renormalization scale. Indeed, from Eq. (11), we compute in the NDR scheme the scale-dependence of $g_s(\Delta F_1 + \Delta \bar{F}_1(q^2))$. We find that, as $\mu$ decreases from 5 GeV to 2.5 GeV, the peak value of the above quantity increases by only 10%. Therefore, to stabilize the scale-dependence, one should include corrections beyond those which simply renormalize the $b \to sg^*$ vertex. We shall leave this to a future investigation.

It is instructive to compare our results with those of Refs. [3,4]. With the kinematic cut, our numerical result for $\mathscr{B}(b \to sg\eta')$ is only slightly smaller than the branching ratio, $8.2 \times 10^{-4}$, reported in Ref. [3], where the $\alpha_s(\mu)$ coupling of $\eta' - g - g$ vertex is evaluated at $\mu \approx 1$ GeV, and $\Delta F_1$ receives only short-distance contributions from the Wilson coefficients $C_4$ and $C_6$. Although we have a much smaller $\alpha_s$, which is evaluated at $\mu = 5$ GeV, and the interference of $\Delta F_1$ and $F_2$ is destructive [4] rather than constructive [3], there exists a compensating enhancement in $\Delta F_1$ due to one-loop matrix elements. The branching ratio in Ref. [4] is $2 - 3$ times smaller than ours since it is given by a $\Delta F_1$ smaller than ours but comparable to that of Ref. [3]. Concerning the relative importance of $\Delta F_1$ and $F_2$, we find that $\Delta F_1$ alone gives $\mathscr{B}(b \to sg\eta') = 6.5 \times 10^{-4}$ with the kinematic cut $m_X \leq 2.35$ GeV. Hence the inclusion of $F_2$ lowers down the branching ratio by only 14%. Such a small interference effect is quite distinct from results of Refs. [3,4] where 20%–50% of inter-

[8] In NDR scheme, apart from a different set of Wilson coefficients compared to Eq. (3), the constant term: $\frac{10}{9}$ at the r.h.s. of Eq. (5) is replaced by $\frac{2}{3}$. For details, see, for example Ref. [21].

[9] We do notice that $B(b \to sg\eta')$ is suppressed by more than one order of magnitude if $a_g(\mu)$ in Eq. (13) is replaced by $a_g(m_{\eta'}) \cdot \frac{m_{\eta'}^2}{(m_{\eta'}^2 - q^2)}$ according to Ref. [6]. However, this prescription for $a_g$ stems from the assumption that $g^* \to g\eta'$ form factor behaves in the same way as the QED-anomaly form factor $\gamma^* \to \gamma\pi^0$. It remains unclear as raised in Refs. [3,4] that one could make such a connection between two distinct form factors.

ference effects are found. We attribute this to the enhancement of $\Delta F_1$ in our calculation.

Before closing we would like to comment on the branching ratio for $B \to \eta X_s$. It is interesting to note that the width of $b \to \eta sg$ is suppressed by $\tan^2\theta$ compared to that of $b \to \eta' sg$. Taking $\theta = -15.4°$, we obtain $\mathscr{B}(B \to \eta X_s) \approx 4 \times 10^{-5}$. The contribution from the four-quark operator can be larger. Depending on the choice of parameters, we find that $B(B \to \eta X_s)$ is in the range of $(6 \sim 10) \times 10^{-5}$.

In conclusion, we have calculated the branching ratio of $b \to sg\eta'$ by including the NLL correction to the $b \to sg^*$ vertex. By assuming a low-energy $\eta' - g - g$ vertex, and cutting the recoil-mass $m_X$ at 2.35 GeV, we obtained $\mathscr{B}(b \to sg\eta') = (5-9) \times 10^{-4}$ depending on the choice of the QCD renormalization-scale. Although the form-factor suppression in the $\eta' - g - g$ vertex is anticipated, it remains possible that the anomaly-induced process $b \to sg\eta'$ could account for the CLEO measurement on $\mathscr{B}(B \to \eta' X_s)$. For the four-quark operator contribution, we obtain $\mathscr{B}(B \to \eta' X_s) \approx 1 \times 10^{-4}$. This accounts for roughly 15% of the experimental central-value and can reach 30% if favourable parameters are used. Finally, combining contributions from the anomaly-mechanism and the four-quark operators, the entire range of $B \to \eta' X_s$ spectrum can be well reproduced.

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