Effects of Erlang Call Holding Times on PCS Call Completion

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***Abstract*—This paper studies personal communications services (PCS's) channel allocation assuming the Erlang call holding time distribution (a generalization of the exponential distribution) to investigate the effect of the variance of the call holding times on the call completion probability. Our analysis indicates that the call completion probability decreases as the variance of the call holding times decreases. This effect becomes more pronounced as the variance of the cell residence times decreases.**

***Index Terms*— Channel allocation, Erlang distribution, handover, personal communication services.**

I. Introduction

A *PERSONAL communications services* (PCS's) network [1], [2] allows users to communicate with the network while on the move. During a communication session, a radio link is established between the portable (the *mobile phone* or *mobile computer*) and a *base station* of the PCS network, if the portable is in the *cell* or the coverage area of the base station. If, during the conversation, the portable moves from one cell to another, the radio link between the old base station and portable is removed, and in order to continue the call the portable must obtain a new radio link in the new cell. If no radio link is available, the call is *forced terminated* [3].

Performance modeling of a PCS system can be conducted at two levels. The first-level modeling uses the number of radio channels in cells as an input parameter to determine the new call blocking probability and the forced termination probability. Second-level modeling uses the given new call blocking and the given forced termination probabilities to study the call completion probability (or the probability that a call is successfully completed).

This paper focuses on the second-level modeling which derives call completion probability with the given new/handover call blocking probability. Since existing cellular systems are typically engineered at 1%–2% new call blocking and forced termination, these default values may be used as the reference input parameters for the second-level modeling. However, the call completion probability cannot be derived directly from these two probabilities. Both the cell residence times and the call holding times of portables are required to investigate the call completion probability. To provide an accurate analysis, the cell residence time and the call holding time distributions must be carefully chosen to reflect the real system. In our model, a general cell residence time distribution is considered, which can be used to accommodate any real PCS system. The selection of the call holding time distribution is subtle. In the early wireline telephone network modeling, the call holding times were typically assumed to be exponentially distributed. A previous study [4] indicated that the exponential assumption may not be valid for modern telephone services. In recent telephone network engineering [4], lognormal distributions [5] have been used to approximate the wireline call holding times.

In this paper, we consider the Erlang call holding time distribution. The Erlang distribution is a special case of the gamma distribution. Statistically, both the lognormal and gamma distributions have the same capability to approximate measured data [6]. An important advantage of the Erlang/gamma distribution over the lognormal distribution is that the Erlang/gamma distribution has a simple Laplace transform format, a desirable property in our modeling. Although the Erlang distribution is a specific case of a gamma distribution, our results (see the Conclusion) also apply to the cellular systems with gamma call holding time distribution.

In a followup work, we have generalized the Erlang call holding time distribution to a general distribution by using complex inverse Laplace technique [7]. While the result of the work is more general, it involves nonintuitive mathematics. Furthermore, the technique only applies to the second-level modeling. On the other hand, the derivations of this paper are easy to understand, and following these derivations, the results can be extended from the second-level modeling to the first-level modeling [8].

II. Assumptions

In this paper, we derive the call completion probability with the following three assumptions. The first two assumptions follow those given in our previous work [9]. The third assumption generalizes the results presented in [9].

- The call arrivals form a Poisson process with arrival rate λ.
- The *cell residence times* (the intervals that a portable stays in the cells) have identical, but arbitrary nonlattice distribution with mean $1/\eta$. The call completion probability p_c will be derived under the general cell residence time

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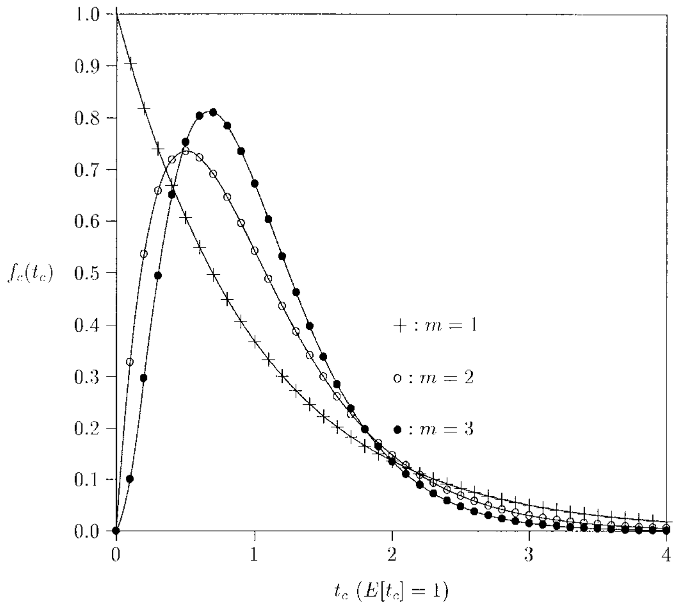


Fig. 1. The Erlang/gamma distribution.

distribution. Then gamma cell residence time distribution is used in the numerical examples. For a gamma cell residence time distribution with the shape parameter γ and the scale parameter $\beta = \gamma\eta$ [and the variance of the distribution is $V = (1/\gamma\eta^2)$]; the density function $f(t)$ is expressed as

$$f(t) = \frac{\beta^\gamma t^{\gamma-1} e^{-\beta t}}{\Gamma(\gamma)}, \qquad \text{where} \quad \gamma > 0. \tag{1}$$

Note that the γ value is a positive real number. As pointed out in [9], the gamma distribution is selected because the distribution is representing a good second approximation for the data measured from the PCS field trials.

- The call holding times have an Erlang distribution with the shape parameter m and the scale parameter $\alpha = m\mu$ where $1/\mu$ is the expected call holding time, and the variance is $V = 1/(m\mu^2)$. The density function f_c for the call holding time t_c is expressed as

$$f_c(t_c) = \frac{(\alpha t_c)^{m-1}}{(m-1)!} \alpha e^{-\alpha t_c}, \quad \text{where } m = 1, 2, 3, \cdots. \tag{2}$$

Note that an Erlang distribution is a special case of the gamma distribution where the shape parameter m is a positive integer. When $m = 1$, the call holding time has an exponential distribution. Similar to the gamma distribution, the Erlang distribution can be used to approximate the measured data for some special cases. It can also be used to illustrate the effects of the variance and the skewness of the call holding time distribution on the call completion probability. Fig. 1 plots the Erlang density functions with mean 1 and $m = 1$, 2, and 3 (or the gamma density function with mean 1 and $\gamma = 1$, 2, and 3).

III. The Derivation for the Call Completion Probability

This section derives the call completion probability p_c. We note that the derivation of (4) is exactly the same as that in [9]. The results of [9] based on the exponential call holding time distribution have been generalized in this paper by using Erlang call holding time distribution. We note that this distinction is nontrivial. First, the derivation of p_c based on the Erlang call holding time distribution is much more difficult than that based on the exponential distribution. Second, with the Erlang distribution, we are able to observe the effects of the variance of the call holding time distribution on the call completion probability.

Consider the timing diagram in Fig. 2 where the events occur at times $x_0, x_1, x_2, \cdots, x_{k+2}$. At time x_0, the portable enters the first cell. At time x_1, a new call arrives (i.e., a call connection request for the portable occurs). The call is complete at time x_{k+2} if it is not forced terminated. In other words, the call holding time is $t_c = x_{k+2} - x_1$ [which has the density function $f_c(t_c)$ as defined in (2)]. The portable moves from cell $i-1$ to cell i at time x_i $(i \geq 2)$. If the call is forced terminated when the portable enters cell k, then the effective call holding time for this incomplete call is $t_e = x_{k+1} - x_1$. In Fig. 2, $t_1 = x_2 - x_0$ is the time that the portable resides at cell 1, and $t_i = x_{i+1} - x_i$ (where $i \geq 2$) is the cell residence time at cell i. We assume that the cell residence times $t_1, t_2, t_3, \cdots, t_k$ are independent and identically distributed random variables[1] with an arbitrary nonlattice density function $f(\cdot)$ and the mean $1/\eta$. Let $f^*(s)$ be the Laplace transform of the cell residence time distribution. Then

$$f^*(s) = \int_{\tau=0}^{\infty} f(\tau) e^{-s\tau} \, d\tau \, .$$

Suppose that a call for the portable occurs when the portable is in cell 1. In Fig. 2, T_1 is the interval between when the call arrives and when the portable moves out of cell 1. As illustrated in Fig. 2, $t_e = T_1 + t_2 + \cdots + t_k$. Thus, the density function $f_k(t_e)$ for t_e is expressed as the convolution of the density functions for $T_1, t_2, \cdots, t_k$

$$f_k(t) = \int_{T_1=0}^{t} \int_{t_2=0}^{t-T_1} \cdots \int_{t_{k-1}=0}^{t-T_1-\cdots-t_{k-2}} \gamma(T_1) f(t_2) \cdots \times f(t_{k-1}) f(t - T_1 - \cdots - t_{k-1}) \, dT_1 \, dt_2 \cdots dt_{k-1}.$$

Let $f_k^*(s)$ be the Laplace Transform of the T_e distribution. From [8], we have

$$f_k^*(s) = \frac{\eta}{s}[1 - f^*(s)][f^*(s)]^{k-1}. \tag{3}$$

Let p_o be the probability that a new call attempt is blocked (i.e., the call is never connected), p_f be the probability that a handover call is forced terminated, and p_c be the probability that a call is completed (i.e., the call is connected and completed). By using $p_o, p_f, f_c(t_c)$, and $f_k(t_e)$, we derive p_c as follows. From the definitions of p_o and p_c, the probability of an incomplete call (i.e., the call is connected but is eventually

[1] As pointed out by an anonymous reviewer of this paper, this assumption may fit to certain conditions. For example, consider a hexagon-cell structure. If we draw an arbitrary line across the six hexagon cells, it can be found that on average, four out of six segments crossing the cells are of the same length.

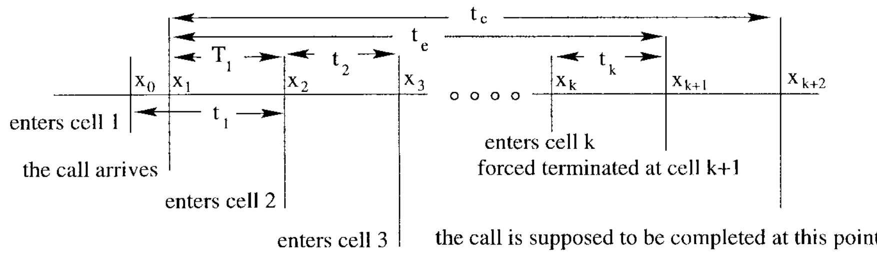


Fig. 2. The timing diagram for a forced terminated call.

forced terminated) is $1-p_o-p_c$ and is expressed as

$$\begin{aligned}1-p_o-p_c &= \sum_{k=1}^{\infty}\left[\int_{t=0}^{\infty}\int_{t_c=t_e}^{\infty}(1-p_o)f_k(t_e)\right.\\&\qquad \left.\times\,(1-p_f)^{k-1}p_f f_c(t_c)\,dt_c\,dt_e\right]\\&=\sum_{k=1}^{\infty}\left\{\int_{t_e=0}^{\infty}(1-p_o)f_k(t_e)(1-p_f)^{k-1}p_f\right.\\&\qquad \left.\times\left[\int_{t_c=t_e}^{\infty}f_c(t_c)\,dt_c\right]dt_e\right\}. \quad (4)\end{aligned}$$

In (4), the term in $\{\cdot\}$ is the probability that a call is forced terminated at the kth handover [note that the call is connected with the probability $1-p_o$, then makes $k-1$ successful handovers with the probability $(1-p_f)^{k-1}$, and is forced terminated at the kth handover with the probability p_f]. The call incompletion probability is the summation of the probabilities for $1\le k<\infty$. From (2), we have

$$\begin{aligned}\int_{t_c=t_e}^{\infty}f_c(t_c)\,dt_c &= \int_{t_c=t_e}^{\infty}\frac{(\alpha t_c)^{m-1}}{(m-1)!}\alpha e^{-\alpha t_c}\,dt_c\\&=\sum_{i=0}^{m-1}\frac{(\alpha t_e)^i}{i!}e^{-\alpha t_e}\end{aligned}$$

and (4) is rewritten as

$$\begin{aligned}1-p_o-p_c &= \sum_{k=1}^{\infty}\left\{\int_{t_e=0}^{\infty}(1-p_o)f_k(t_e)(1-p_f)^{k-1}p_f\right.\\&\qquad\left.\times\left[\sum_{i=0}^{m-1}\frac{(\alpha t_e)^i}{i!}e^{-\alpha t_e}\right]dt_e\right\}\\&=\sum_{k=1}^{\infty}\left[\sum_{i=0}^{m-1}(1-p_o)(1-p_f)^{k-1}p_f\left(\frac{\alpha^i}{i!}\right)\right.\\&\qquad\left.\times\int_{t_e=0}^{\infty}t_e^i f_k(t_e)e^{-\alpha t_e}\,dt_e\right] \quad (5)\\&=\sum_{i=0}^{m-1}\left\{\sum_{k=1}^{\infty}(1-p_o)(1-p_f)^{k-1}p_f\left(\frac{\alpha^i}{i!}\right)\right.\\&\qquad\left.\times\left[\left.\frac{(-1)^i d^i f_k^*(s)}{d^i s}\right|_{s=\alpha}\right]\right\} \quad (6)\end{aligned}$$

where (6) is derived from (5) by using the fact [10, Rule P.3.1.1] that

$$g(t)=tf(t)\Rightarrow g^*(s)=-\frac{df^*(s)}{ds}.$$

Let $p_c(m)$ be the p_c value when the Erlang call holding time distribution has the shape parameter m. From (6), we have

$$p_c(m)=1-p_o-X(m) \quad (7)$$

where

$$\begin{aligned}X(m)=\sum_{j=0}^{m-1}&\left\{\sum_{k=1}^{\infty}(1-p_o)(1-p_f)^{k-1}p_f\left(\frac{\alpha^j}{j!}\right)\right.\\&\left.\times\left[\left.\frac{(-1)^j\,d^j f_k^*(s)}{d^j s}\right|_{s=\alpha}\right]\right\}. \quad (8)\end{aligned}$$

Appendix A derives $X(m)$ for $m=1, 2$, and 3. These results will be used in the next section.

IV. Discussion

In this paper, we assume a PCS system such as AMPS or global system for mobile communications (GSM), where the channel assignments for handover calls and the new calls are identical. That is, $p_f=p_o$. Based on (7), we plot p_c as the function of $p_o\ (=p_f), \eta, \gamma, \mu$, and m. Note that the variance of the Erlang call holding times is $1/(m\mu^2)$, and a large m implies a small variance. Similarly, the variance of the gamma cell residence times is $1/(\gamma\eta^2)$, and a large γ implies a small variance. In a PCS system, a typical value for $1/\mu$ is between 1–3 min. In Figs. 3–8(a), we plot p_c as functions of various input parameters. Figs. 3–8(b) plot the proportional changes of p_c between $m=2, 3$, and $m=1$. In other words, we plot

$$\frac{p_c(1)-p_c(m)}{p_c(m)}, \qquad m=2 \text{ and } 3$$

as functions of various input parameters. We observe the following results.

1) General Effect of m on p_c: Figs. 3–8(a) indicate that p_c decreases as m increases. In other words, when the cell is engineered at a fixed blocking probability (p_o), as the variance of call holding time decreases, the call completion performance degrades. Figs. 3–8(b) indicate that the effect of m on p_c can be ignored for $m\le 2$, and the effect becomes significant when $m\ge 3$.

2) General Effect of the Interaction Between m and γ: Figs. 3 and 6(b) indicate that when the variance of the cell residence time is small (i.e., γ is large), the effect of changing m becomes significant. On the other hand, Figs. 5 and 8(b) indicate that when the variance of the cell residence time is large (i.e., γ is small), the effect of changing m can be ignored.

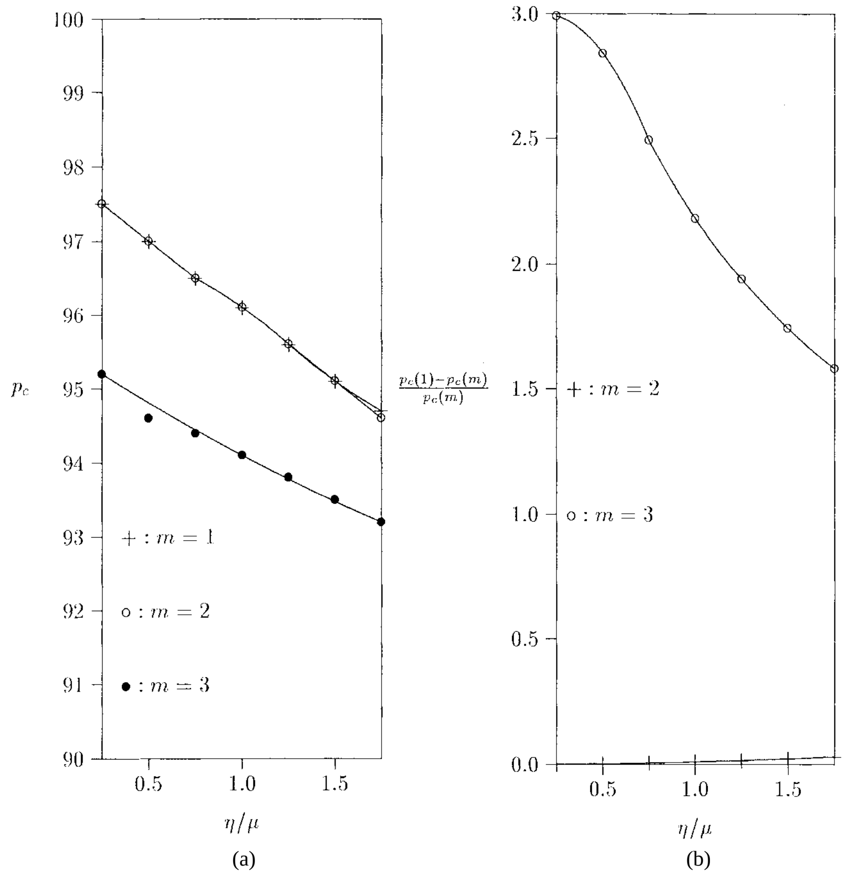


Fig. 3. The effect of η ($\gamma = 10$).

3) Interaction Between η and m: Suppose that a cell is engineered at 2% blocking probability (i.e., $p_o = p_f = 2\%$). Fig. 3 indicates intuitive results that p_c decreases as mobility η increases. The effect of η on p_c becomes more significant as m increases. Similar results are observed in Figs. 4 and 5. Furthermore, Figs. 4 and 5(b) indicate that if the variance of the cell residence time is large (i.e., $\gamma \leq 1$), the effect of m on p_c becomes more significant as η increases. On the other hand, if the variance of the cell residence time is small [i.e., $\gamma \geq 10$; see Fig. 3(b)], the effect of m on p_c becomes less significant as η increases.

4) Interaction Between $p_o(p_f)$ and m: As shown in Figs. 6–8, it is intuitive that p_c decreases as p_o increases. These figures also indicate that the effect of m on p_c becomes more significant as p_o increases. Figs. 6–8(b) indicate that if the variance of the cell residence time is large (i.e., γ is small), the effect of m on p_c is significantly affected by the change of p_o. On the other hand, if the variance of the cell residence time is small (i.e., γ is large), the effect of m on p_c is not affected by the change of p_o.

The above discussion leads to an important observation: the call completion probability decreases as the variance of the call holding times decreases. This effect becomes more significant as the variance of the cell residence times decreases.

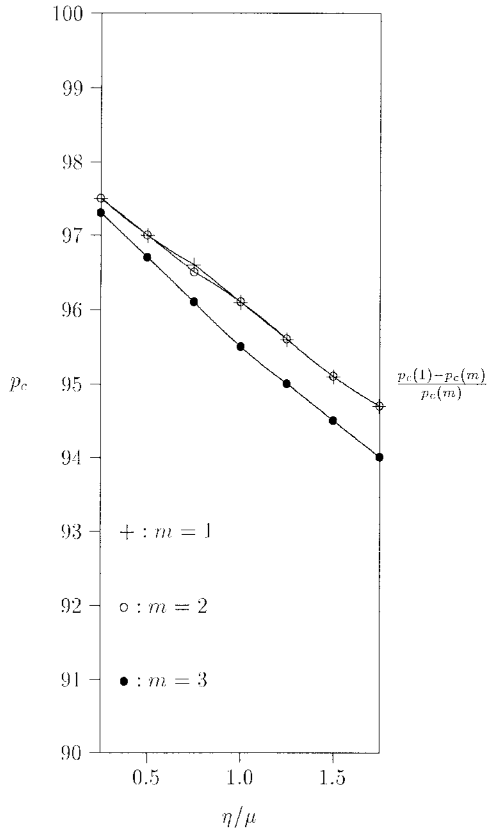
V. Conclusions

This paper introduced a model for studying emerging PCS cellular systems where a general distribution for modeling the call holding times is necessary. By characterizing the call holding times via the Erlang distribution (a generalization of the exponential distribution) we were able to investigate the effect of the variance of the call holding times on the call completion probability. Using this model it was possible to make the significant observation that in these systems the call completion probability decreases as the variance of the call holding times decreases. This effect becomes more significant as the variance of the cell residence times decreases. In other words, when the variances of the call holding times and the cell residence times are small, a cell should be engineered at a smaller blocking probability (i.e., p_o should be small) to achieve the same call completion performance as that for a cell where the variances of the call holding times and the cell residence times are large.

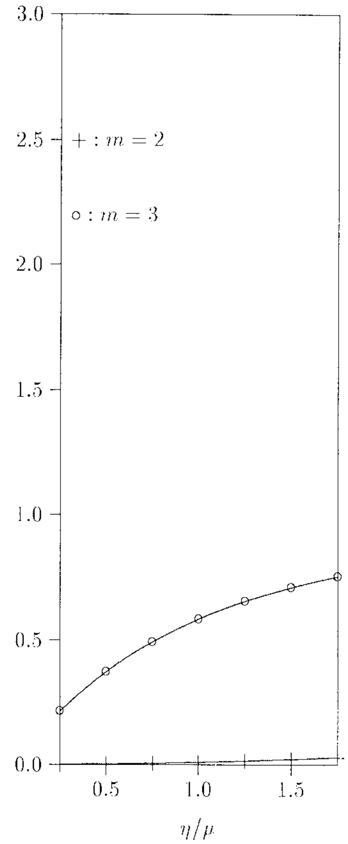
Appendix I
The Derivations for $X(m)$

Denote $p_c(m)$ as the p_c value when the Erlang call holding time distribution has the shape parameter m. For $m = 1$, $p_c(1)$ can be derived as follows. From (3), we have [9]

$$\begin{aligned} X(1) &= \sum_{k=1}^{\infty}(1-p_o)(1-p_f)^{k-1}p_f \\ &\quad\times\left(\frac{\eta}{s}\right)[1-f^*(s)][f^*(s)]^{k-1} \\ &= \frac{\eta(1-p_o)[1-f^*(\alpha)]p_f}{\alpha[1-(1-p_f)f^*(s)]}. \end{aligned} \quad (9)$$



(a)



(b)

Fig. 4. The effect of η ($\gamma = 1$).

From (7) and (9), we have

$$p_c(1) = (1 - p_o)\left\{1 - \frac{\eta[1 - f^*(\alpha)]p_f}{\alpha[1 - (1 - p_f)f^*(\alpha)]}\right\}. \tag{10}$$

For $m > 1$, in order to derive $X(i)$, we need to compute the derivatives of $f_k^*(s)$ and $f^*(s)$. Let

$$f_k^{(i)}(s) = \frac{d^i f_k^*(s)}{ds^i} \quad \text{and} \quad f^{(i)}(s) = \frac{d^i f^*(s)}{ds^i}.$$

Then from (3)

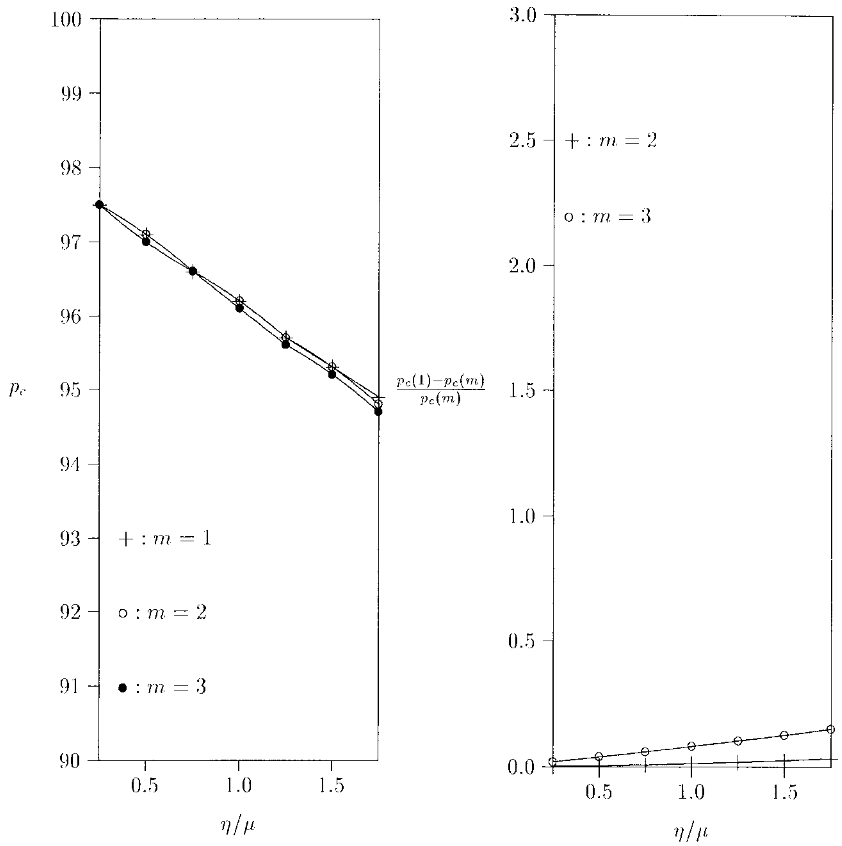
$$\begin{aligned} f_k^{(1)}(s) &= -\frac{\eta}{s^2}[1 - f^*(s)][f^*(s)]^{k-1} - \frac{\eta}{s}\left[\frac{df^*(s)}{ds}\right] \\ &\quad \cdot [f^*(s)]^{k-1} + \frac{\eta}{s}[1 - f^*(s)](k-1)[f^*(s)]^{k-2} \\ &\quad \cdot \left[\frac{df^*(s)}{ds}\right] \\ &= -f_k^*(s)\left[\frac{1}{s} + \frac{f^{(1)}(s)}{1 - f^*(s)} + \frac{f^{(1)}(s)}{f^*(s)} - \frac{kf^{(1)}(s)}{f^*(s)}\right] \quad (11) \\ &= -A(s)[f^*(s)]^{k-1} + B(s)k[f^*(s)]^{k-1} \quad (12) \end{aligned}$$

where

$$\begin{aligned} A(s) &= \frac{\eta[1 - f^*(s)]}{s}\left[\frac{1}{s} + \frac{f^{(1)}(s)}{1 - f^*(s)} + \frac{f^{(1)}(s)}{f^*(s)}\right] \\ B(s) &= \frac{\eta f^{(1)}(s)[1 - f^*(s)]}{sf^*(s)}. \end{aligned} \tag{13}$$

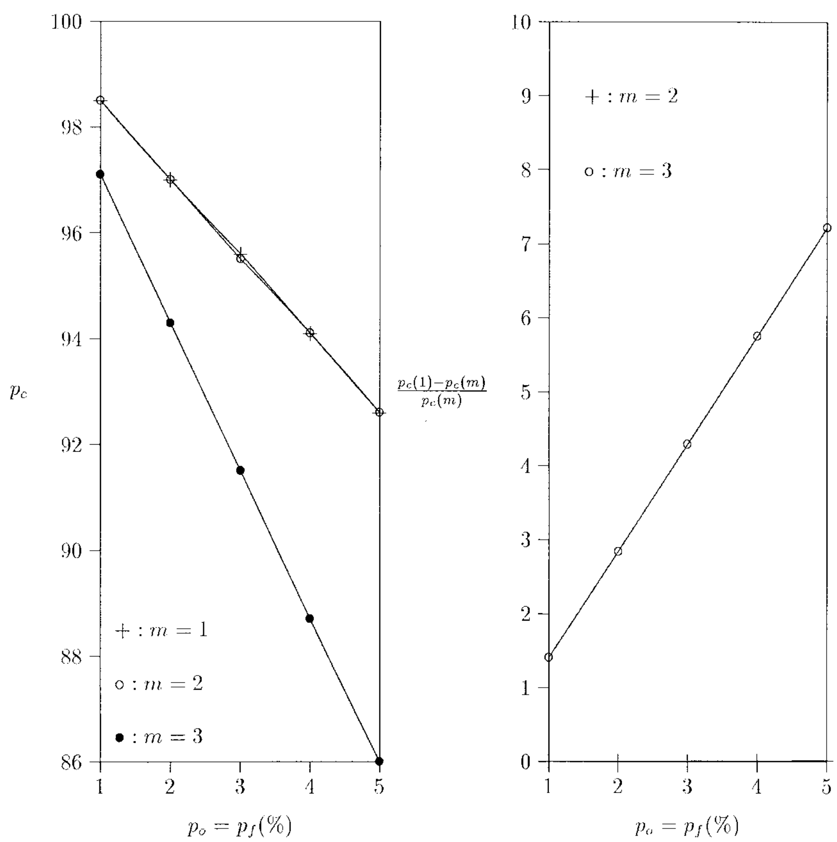
Consider $m = 2$. From (8) and (12), we have

$$\begin{aligned} X(2) &= X(1) + \sum_{k=1}^{\infty}(1 - p_o)(1 - p_f)^{k-1}p_f \\ &\quad \cdot \left[-\alpha\frac{df_k^*(s)}{ds}\right]\Bigg|_{s=\alpha} \\ &= X(1) + \left[\frac{\alpha(1 - p_o)p_f}{f^*(\alpha)(1 - p_f)}\right] \\ &\quad \times \left\{A(\alpha)\left\{\sum_{k=1}^{\infty}[(1 - p_f)f^*(\alpha)]^k\right\}\right. \\ &\quad \left. - B(\alpha)\left\{\sum_{k=1}^{\infty}k[(1 - p_f)f^*(\alpha)]^k\right\}\right\} \\ &= X(1) + \alpha(1 - p_o)p_f \\ &\quad \times \left\{\frac{A(\alpha)}{1 - (1 - p_f)f^*(\alpha)} - \frac{B(\alpha)}{[1 - (1 - p_f)f^*(\alpha)]^2}\right\}. \end{aligned}$$



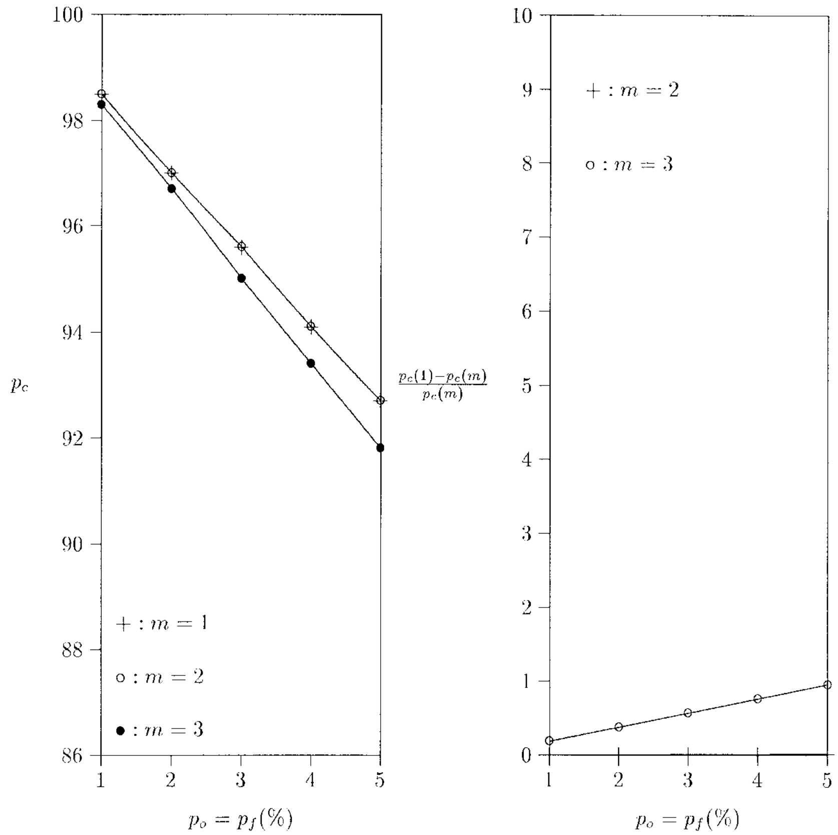
(a) (b)

Fig. 5. The effect of η ($\gamma = 0.1$).



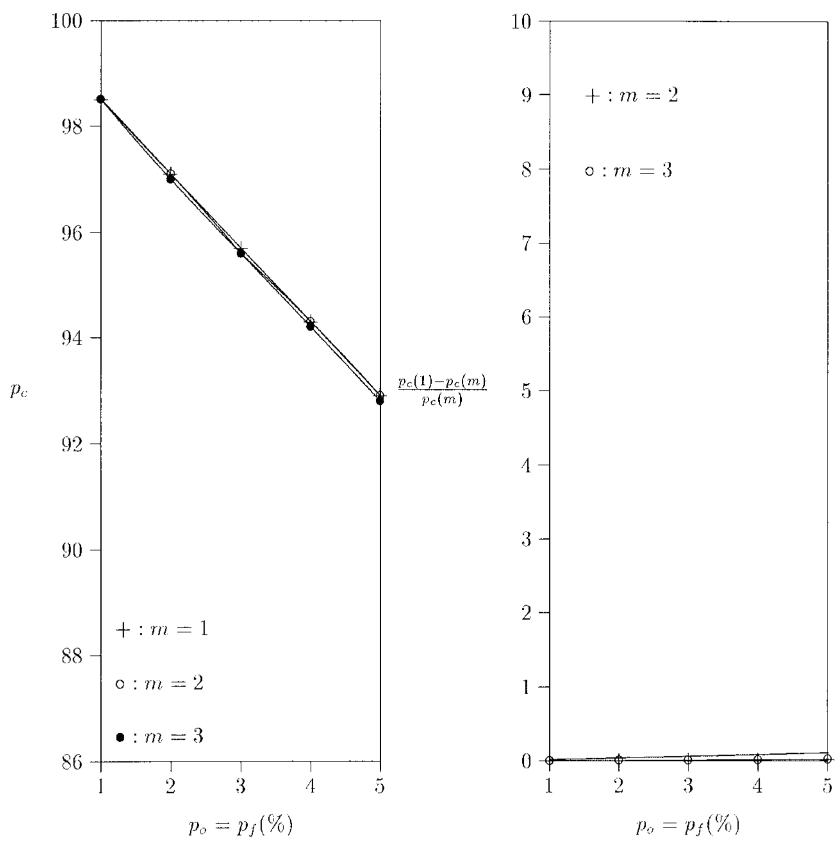
(a) (b)

Fig. 6. The effect of p_o ($= p_f$) ($\gamma = 10$).



(a) (b)

Fig. 7. The effect of p_o $(= p_f)$ $(\gamma = 1)$.



(a) (b)

Fig. 8. The effect of p_o $(= p_f)$ $(\gamma = 0.1)$.

Consider $m = 3$. From (11), we have

$$
\begin{aligned}
f_k^{(2)}(s) &= -f_k^{(1)}(s)\left[\frac{1}{s} + \frac{f^{(1)}(s)}{1-f^*(s)} - \frac{(k-1)f^{(1)}(s)}{f^*(s)}\right] \\
&\quad - f_k^*(s)\left\{-\frac{1}{s^2} + f^{(2)}(s)\left[\frac{1}{1-f^*(s)} - \frac{(k-1)}{f^*(s)}\right]\right. \\
&\quad \left. + \left\{\frac{f^{(1)}(s)}{[1-f^*(s)]^2} + \frac{(k-1)f^{(1)}(s)}{[f^*(s)]^2}\right\} f^{(1)}(s)\right\} \\
&= A(s)\left[\frac{1}{s} + \frac{f^{(1)}(s)}{1-f^*(s)} + \frac{f^{(1)}(s)}{f^*(s)}\right][f^*(s)]^{k-1} \\
&\quad - A(s)\left[\frac{f^{(1)}(s)}{f^*(s)}\right]k[f^*(s)]^{k-1} - B(s) \\
&\quad \cdot \left[\frac{1}{s} + \frac{f^{(1)}(s)}{1-f^*(s)} + \frac{f^{(1)}(s)}{f^*(s)}\right]k[f^*(s)]^{k-1} \\
&\quad + B(s)\left[\frac{f^{(1)}(s)}{f^*(s)}\right]k^2[f^*(s)]^{k-1} + \frac{\eta[1-f^*(s)]}{s} \\
&\quad \cdot \left\{\frac{1}{s^2} - \frac{f^{(2)}(s)}{1-f^*(s)} + \frac{f^{(2)}(s)}{f^*(s)} - \left[\frac{f^{(1)}(s)}{1-f^*(s)}\right]^2\right. \\
&\quad \left. - \left[\frac{f^{(1)}(s)}{f^*(s)}\right]^2\right\}[f^*(s)]^{k-1} + \frac{\eta[1-f^*(s)]}{s} \\
&\quad \cdot \left\{\frac{f^{(2)}(s)}{f^*(s)} - \left[\frac{f^{(1)}(s)}{f^*(s)}\right]^2\right\}k[f^*(s)]^{k-1} \\
&= C(s)[f^*(s)]^{k-1} + D(s)\times k[f^*(s)]^{k-1} + E(s) \\
&\quad \cdot k^2[f^*(s)]^{k-1}
\end{aligned}
\tag{14}
$$

where

$$
\begin{aligned}
C(s) &= A(s)\left[\frac{1}{s} + \frac{f^{(1)}(s)}{1-f^*(s)} + \frac{f^{(1)}(s)}{f^*(s)}\right] + \frac{\eta[1-f^*(s)]}{s} \\
&\quad \cdot \left\{\frac{1}{s^2} - \frac{f^{(2)}(s)}{1-f^*(s)} + \frac{f^{(2)}(s)}{f^*(s)}\right. \\
&\quad \left. - \left[\frac{f^{(1)}(s)}{1-f^*(s)}\right]^2 - \left[\frac{f^{(1)}(s)}{f^*(s)}\right]^2\right\} \\
D(s) &= \frac{\eta[1-f^*(s)]}{s}\left\{\frac{f^{(2)}(s)}{f^*(s)} - \left[\frac{f^{(1)}(s)}{f^*(s)}\right]^2\right\} \\
&\quad - A(s)\left[\frac{f^{(1)}(s)}{f^*(s)}\right] \\
&\quad - B(s)\left[\frac{1}{s} + \frac{f^{(1)}(s)}{1-f^*(s)} + \frac{f^{(1)}(s)}{f^*(s)}\right] \\
E(s) &= B(s)\left[\frac{f^{(1)}(s)}{f^*(s)}\right].
\end{aligned}
\tag{15}
$$

From (8) and (14), we have

$$
\begin{aligned}
X(3) &= X(2) + \sum_{k=1}^{\infty}(1-p_o)(1-p_f)^{k-1}p_f \\
&\quad \times \left[\left(\frac{\alpha^2}{2}\right)\frac{d^2 f_k^*(\alpha)}{ds^2}\right]\Bigg|_{s=\alpha} \\
&= X(2) + \left[\frac{\alpha^2(1-p_o)p_f}{2f^*(\alpha)(1-p_f)}\right] \\
&\quad \times \left\{C(\alpha)\sum_{k=1}^{\infty}[(1-p_f)f^*(\alpha)]^k\right. \\
&\quad + D(\alpha)\sum_{k=1}^{\infty}k[(1-p_f)f^*(\alpha)]^k \\
&\quad \left. + E(\alpha)\sum_{k=1}^{\infty}k^2[(1-p_f)f^*(\alpha)]^k\right\} \\
&= X(2) + \left[\frac{\alpha^2(1-p_o)p_f}{2}\right] \\
&\quad \cdot \left\{\frac{C(\alpha)+E(\alpha)}{1-(1-p_f)f^*(\alpha)}\right. \\
&\quad \left. + \frac{D(\alpha)-E(\alpha)}{[1-(1-p_f)f^*(\alpha)]^2} + \frac{2E(\alpha)}{[1-(1-p_f)f^*(\alpha)]^3}\right\}.
\end{aligned}
$$

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