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# Ferromagnetic behavior of a triplet superconductor

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Properties of a type II triplet superconductor with equal spin pairing are investigated using the phenomenological Ginzburg–Landau approach. It is shown that there exist two kinds of fluxons: vortices and magnetic skyrmions. Vector nature of the order parameter allows a direct coupling of the effective spin of the condensate to a magnetic field. This coupling, which is reminiscent to Zeeman interaction in the form, significantly modifies the structure of an isolated vortex, as compared to the usual Abrikosov vortex, and changes the energetics of a vortex lattice. If the coupling is sufficiently strong, then magnetization of the triplet superconductor becomes positive at high magnetic fields. Formation of spontaneous vortex state is also possible. © 1999 American Institute of Physics. [S0021-8979(99)55408-X]

In the majority of low  $T_c$  superconductors pairing occurs in  $s$  channel. In this conventional case the total spin of a Cooper pair is equal to zero. A magnetic field influences the superconducting condensate very strongly via coupling to orbital motion of the pairs. Zeeman coupling of the magnetic field to the spins of electrons is not effective in the first approximation and does not show up in most physical situations, although it contributes to the destruction of Cooper pairs and leads, for example, to corrections for the value of  $H_{c2}$ . The situation might be different in the case of triplet pairing when members of a Cooper pair have parallel spins.

The triplet pairing is suspected to occur in recently discovered new class of Ru based type II superconductors  $\text{Sr}_2\text{YRu}_{1-x}\text{Cu}_x\text{O}_6$ .<sup>1</sup> At the same temperature of about 60 K, at which superconductivity sets in, these materials begin to exhibit basic ferromagnetic properties like a hysteresis loop with a positive remanent magnetization. Exact overlap of superconductivity and ferromagnetism naturally suggests that in these particular materials Cooper pairs might in fact be magnetic moments and that they themselves are responsible, at least partially, for overcoming the usual diamagnetic response of a superconductor. In this paper we consider in some detail this fascinating scenario.

We employ a phenomenological Ginzburg–Landau (GL) theory. Within this framework, nonzero spin of the Cooper pair can be taken into account by introducing an order parameter of the vector type  $\psi_i(\mathbf{r})$ ,  $i=1,2,3$ .<sup>2</sup> The free energy density for such a model has the form:  $F=F_{\text{bulk}}+F_{\text{grad}}$ , where

$$F_{\text{bulk}} = -\alpha\psi_i\psi_i^* + \frac{\beta_1}{2}(\psi_i\psi_i^*)^2 + \frac{\beta_2}{2}|\psi_i\psi_i|^2, \quad (1)$$

$$F_{\text{grad}} = \frac{\hbar^2}{2m^*}(\mathcal{D}_j\psi_i)(\mathcal{D}_j\psi_i)^* + \frac{B_i^2}{8\pi} - \mu S_i B_i. \quad (2)$$

Magnetic field  $B$  is coupled to the order parameter in two ways [see Eq. (2)]. First, via covariant derivatives  $\mathcal{D}_i \equiv \partial_i$

$-i(e^*/c\hbar)A_i$ . This is the usual type of coupling leading to diamagnetism. Note also that it is the reason for the appearance of topological solitons in type II superconductors. The second type of coupling is a direct interaction with the spin  $S_i \equiv -i\epsilon_{ijk}\psi_j^*\psi_k$  carried by the order parameter field. It is only possible if the order parameter has a vector nature. It can lead, as shown below, to ferromagnetism in a sense that the system can show positive response on an external magnetic field and can have spontaneously magnetized ground state. The direct coupling  $-\mu S_i B_i$  looks exactly like Zeeman interaction (ZI), and below we adopt this name for clarity. Note, however, that in our theory  $S_i$  is actually an effective spin and  $\mu$  is a phenomenological parameter.

Our analysis consists of the following steps. First, we find which homogeneous superconducting phase ZI is effective and, thus, ferromagnetism could be possible. Second, we classify topologically stable line defects which would determine behavior of a type II superconductor in external magnetic fields. Third, we gain some knowledge about the topological objects by numerical solution of appropriate GL equations. Finally, we develop a simple analytical model of the vortex lattice.

Minimization of Eq. (1) yields two different homogeneous superconducting phases. Only one of them has  $S \neq 0$  and we will subsequently concentrate on it. The order parameter has the form  $\psi = f_0(\mathbf{n} + i\mathbf{m})/\sqrt{2}$ , where  $f_0 = \sqrt{\alpha/\beta_1}$  and  $\mathbf{m}$ ,  $\mathbf{n}$  are arbitrary unit vectors satisfying  $\mathbf{n} \perp \mathbf{m}$ . The spin of the condensate is given by  $\mathbf{S} = f_0^2 \mathbf{l}$ ,  $\mathbf{l} \equiv \mathbf{n} \times \mathbf{m}$ . The phase is stable if  $\alpha > 0$ ,  $\beta_1 > 0$ , and  $\beta_2 > 0$ . The ground state free energy is  $-\alpha^2/2\beta_1$ . The ground state is highly degenerate. Each vacuum is specified by a choice of the orthonormal triad  $\mathbf{l}$ ,  $\mathbf{n}$ ,  $\mathbf{m}$ . The vacuum manifold is isomorphic to a group  $\text{SO}(3)$ . It consists of (1) arbitrary rotations of vector  $\mathbf{l}$  and (2) combined transformations: rotations of pair  $\mathbf{m}$ ,  $\mathbf{n}$ , around  $\mathbf{l}$  by angle  $\vartheta$  which are accompanied by gauge transformations  $e^{i\vartheta}$ .

To study magnetic effects we proceed to consider spa-

tially nonuniform configurations which are translationally invariant in the direction  $z$  of an external magnetic field. We use the following approach: we allow that the modulus of the order parameter  $|\boldsymbol{\psi}| \equiv f$  depends on spatial coordinates, while still keeping  $\boldsymbol{\psi} \sim \mathbf{n} + i\mathbf{m}$ . Then the free energy per unit length, measured from its vacuum value, reads

$$\mathcal{F} = \int d^2x \left[ \frac{\kappa^2}{2} (1 - f^2)^2 + (\partial_i f)^2 + \frac{1}{2} f^2 (\partial_i \mathbf{l})^2 + f^2 [\mathbf{n} \partial_i \mathbf{m} - A_i]^2 + B_i^2 - g f^2 l_i B_i \right]. \quad (3)$$

In Eq. (3) the dimensionless units were used such that  $|\boldsymbol{\psi}|$  is measured in units of  $f_0$ , length—in units of  $\lambda \equiv (c/|e^*|f_0)\sqrt{m^*/4\pi}$ , magnetic flux—in units of  $\Phi_0 \equiv hc/e^*$ , the line free energy—in units of  $\epsilon_0 \equiv (\Phi_0/4\pi\lambda)^2$ . GL parameter  $\kappa \equiv \lambda/\xi$  is defined via coherence length  $\xi \equiv \hbar/\sqrt{2\alpha m^*}$ . ZI strength is characterized by  $g \equiv (2m^*c/e^*\hbar)\mu$ . GL equations read:

$$n_i \nabla m_i - \mathbf{A} + \frac{g}{2} \nabla \times (f^2 \mathbf{l}) = \nabla \times \mathbf{B} \equiv \mathbf{j}, \quad (4)$$

$$\Delta \mathbf{l} - \mathbf{l}(\Delta \mathbf{l}) + 2j_k (\mathbf{l} \times \partial_k \mathbf{l}) + g[\mathbf{B} - \mathbf{l}(\mathbf{B}) - (\nabla \times \mathbf{l})_k (\mathbf{l} \times \partial_k \mathbf{l})] = 0, \quad (5)$$

where  $\mathbf{B} = \nabla \times \mathbf{A}$ . Equation (4) shows that the superconducting velocity is given by  $n_p \nabla m_p = -\nabla \vartheta$ , where the angle  $\vartheta$  specifies the direction of  $\mathbf{n}$  in the plane perpendicular to  $\mathbf{l}$ . Thus, it is  $\vartheta$  that takes the role of superconducting phase.

Let us determine finite energy solutions to GL equations on the whole  $x-y$  plane (solitons) which carry nonzero magnetic flux. It follows from Eq. (3) that they should satisfy:  $f \rightarrow 1, \partial_i f \rightarrow 0, \partial_i \mathbf{l} \rightarrow 0$  and  $-\nabla \vartheta \propto \mathbf{A}$  when  $|x| \rightarrow \infty$ . [In this paper we restrict consideration to the case  $B(\infty) = 0$ ]. Imposing the flux quantization condition restricts the vacuum manifold:  $SO(3) \rightarrow SO(2) \otimes S_2$  where  $S_2$  is the direction of  $\mathbf{l}$  and  $SO(2)$  is the phase  $\vartheta$ . For a given number of flux quanta  $N$ , the phase  $\vartheta$  makes  $N$  winds at infinity. The first homotopy group of this part is therefore fixed:  $\pi_1[SO(2)] = Z$ . If vector  $\mathbf{l}$  is fixed throughout the volume of the superconductor, there is no way to avoid singularity in the phase  $\vartheta$  [note the fourth term in integrand in Eq. (3)]. Accordingly, the modulus of the order parameter  $f$  has to vanish at some point and we arrive at the picture of a vortex. Energy of the vortex line is high for type II superconductors because it contains a large factor  $\log \kappa$ . However, the finite energy requirement for solutions tells us that  $\mathbf{l}$  should be fixed only at infinity. This effectively ‘‘compactifies’’  $x-y$  plane into  $S_2$ . The relevant homotopy group is  $\pi_2(S_2) = Z$ . The new topological number is  $Q = (1/8\pi) \int \epsilon_{ij} \mathbf{l} (\partial_i \mathbf{l} \times \partial_j \mathbf{l}) d^2r$ .<sup>3</sup>

Thus, all configurations fall into classes characterized by two integers  $N$  and  $Q$ . The presence of two topological numbers produces a very interesting effect: there exist nontrivial (spatially nonuniform) configurations in which  $f$  assumes its saturation value everywhere. We call these solutions magnetic skyrmions. For them magnetic flux number  $N$  and skyrmion number  $Q$  are related to each other:  $Q = N/2$ . This relation is obtained by integrating supercurrent equation [Eq. (4)] along a remote contour. Unlike vortices, magnetic skyr-

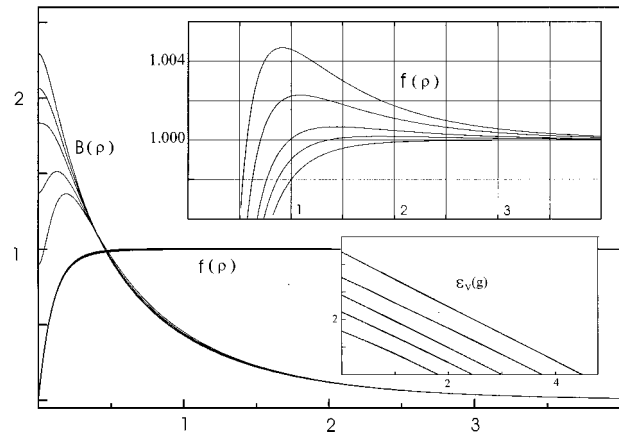


FIG. 1. Structure of a vortex. Modulus of the order parameter  $f$  and magnetic field  $B$  as functions of  $\rho$  for  $g$  from 0 to 1.5 and  $\kappa = 10$ . Distance  $\rho$  is measured in units of  $\lambda$ . Upper inset illustrates fine structure of  $f$ . Lower inset shows vortex line energy  $\epsilon_v$  as a function of  $g$  for  $\kappa$  from 2 to 50.

mions have no normal core and for small  $g$  their line energy should be smaller than vortex line energy by the factor  $\log \kappa$ .

We now turn to investigate the structure of topological solitons. We are concerned with well isolated line defects which are cylindrically symmetric. Accordingly, it is convenient to utilize polar coordinates  $\rho$  and  $\varphi$ . The lowest energy vortex has  $Q = 0$  and  $N = \pm 1$ . Using Ansatz  $\mathbf{l} = \mathbf{e}_z, \mathbf{n} = \mathbf{e}_x \cos \varphi + \mathbf{e}_y \sin \varphi$ , and  $\mathbf{m} = -\mathbf{e}_x \sin \varphi + \mathbf{e}_y \cos \varphi$  we can simplify Eqs. (4) and (5) considerably. Only two unknown functions are left:  $f(\rho)$  and  $A(\rho)$ . The corresponding system of differential equations is then solved numerically by the finite element method. The results are presented in Fig. 1. In a vortex, the magnetic field falls off exponentially at distances of the order  $\lambda$ , while  $f$  already changes considerably in the range of  $\xi$ . This is similar to vortices in conventional superconductors. The effect of ZI is considerable. It changes behavior of magnetic field  $B$  in the core, and also modifies the way  $f$  approaches its vacuum value and reduces the core size (see upper inset in Fig. 1). The most prominent consequence of ZI, however, is that the energy of a vortex acquires a bulk negative contribution. As a result, the vortex line energy  $\epsilon_v$  linearly decreases with  $g$  (see lower inset in Fig. 1). This finding is in agreement with earlier work by Toknyasu et al.<sup>4</sup> At large  $g$ , of the order of  $\log \kappa$ , the vortex line tension becomes zero indicating instability of a homogeneous superconducting state at  $H = 0$ . Thus, we arrive at the conclusion that a spontaneous vortex state, carrying a magnetic flux, could exist. Note that in our calculations we have set  $g > 0$ . However, all the above results on the dependence of vortex structure on  $g$  are not, in fact, affected by the sign of  $g$ . Let us fix the direction of  $B$  then, for any sign of  $g$  there always exists a vortex with the directions of  $\mathbf{l}$  such that the ZI contribution to the vortex energy is negative.

The magnetic skyrmion with the lowest energy satisfies  $N = 2Q = \pm 2$ . We employ Ansatz:  $f = f_0$  and  $\mathbf{l} = \mathbf{e}_z \cos \Theta + \mathbf{e}_\rho \sin \Theta, \mathbf{n} = \tau \sin \varphi + \mathbf{e}_\varphi \cos \varphi, \mathbf{m} = \tau \cos \varphi - \mathbf{e}_\varphi \sin \varphi$ , where  $\Theta = \mathbf{e}_z \cdot \mathbf{l}$  and  $\tau = \mathbf{e}_z \sin \Theta - \mathbf{e}_\rho \cos \Theta(\rho)$ . Again, only two unknown functions  $\Theta(\rho)$  and  $A(\rho)$  are left in Eqs. (5) and (4). The results of numerical integration are presented in Fig. 2.

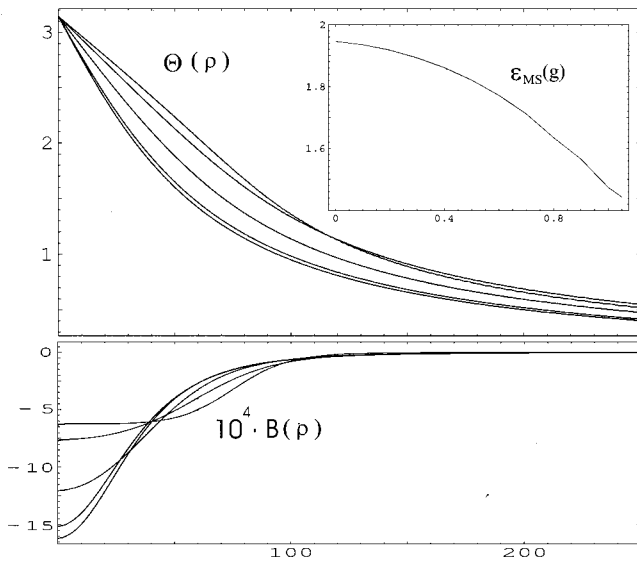


FIG. 2. Structure of a magnetic skyrmion. Upper and lower panels show azimuthal angle  $\Theta$  and magnetic field  $B$ , correspondingly, as functions of  $\rho$ . Distance  $\rho$  is measured in units of  $\lambda$ . Inset shows magnetic skyrmion line energy as a function of  $g$ .

We immediately observe that a magnetic skyrmion is a much larger object than a vortex. Magnetic field is localized within the range of  $50\lambda$ . Moreover, it decays with distance as a power. The line energy of the magnetic skyrmion,  $\epsilon_{MS}$ , is quite small and independent of  $\kappa$ . The dependence on  $g$  is very weak (see inset). We associate this with the fact that ZI contributes to  $\epsilon_{MS}$  with opposite signs on small and large distances. In fact, magnetic skyrmions are lighter than vortices of a wide range of  $g$  and, hence they will determine  $H_{c1}$  and dominate the physics in our model at low external magnetic fields. However, a magnetic skyrmion lattice is expected to be destroyed when distance between skyrmion becomes about several  $\lambda$ . Accordingly, at  $H$  of about several  $H_{c1}$  there should be a crossover to a vortex lattice.

The domain of vortex lattice existence is much larger because  $H_{c2}/H_{c1} \approx \kappa^2 \gg 1$  for type II superconductors. Therefore, it is useful to develop an analytical model of vortex lattice behavior. We proceed by assuming that vortex core size  $d$  depends on  $H$ . Such a point of view is actually supported by our numerical results on a single vortex (see upper inset in Fig. 1). For simplicity, the behavior of  $f$  is approximated by step function. The Gibbs free energy density in dimensionless units reads:

$$G = B \left[ B - \log \eta d \sqrt{B} + \frac{\kappa^2 d^2}{4} - 2H - g \left( 1 - \frac{d^2 B}{2} \right) \right]. \quad (6)$$

The first two terms represent the usual vortex interaction energy excluding ZI. It is obtained by standard methods of summing up all the interactions using transformation to the reciprocal space and replacement of the summation over the

reciprocal vortex lattice by appropriate integration (see Ref. 5). The third term is the energy lost in the core due to melting of the condensate. The fourth term is due to the external magnetic field. The terms with  $g$  summarize ZI contribution: energy gain in the region between vortices minus energy loss in vortex cores due to vanishing of effective spin  $S$  there. Coefficient  $\eta$  is an unknown quantity of order 1. The value of  $d$  is found from the condition  $dG/d\rho_c = 0$  that gives  $d_c(B) = \sqrt{2/(\kappa^2 + 2gB)}$ . We see that as the magnetic field increases the core shrinks. Shrinking of cores makes room for more vortices to squeeze in and allows internal magnetic field  $B$  to increase when  $H$  increases (of course, when the core becomes microscopic in size the whole approach ceases to be applicable). The magnetization curve is found from minimization of  $G$  with respect to  $B$ . This leads to the equation

$$H = B - \frac{1}{2} \log \frac{d_c(B) \sqrt{B}}{\eta} - \frac{g}{2} \left( 1 - \frac{d_c(B)^2 B}{2} \right), \quad (7)$$

that implicitly determines the magnetic induction  $B$  as a function of external magnetic field  $H$ . Note that for sufficiently large values of  $g$  Eq. (7) has solution  $B > 0$  even if  $H = 0$ . This agrees with our previous conclusion about the existence of a spontaneous vortex state. On the other hand, even at smaller values of  $g$ , as  $H$  increases magnetization  $M \equiv (B - H)/4\pi$  approaches a positive saturation value. For strongly type II superconductors (high  $\kappa$ ) parameter  $\eta$  can be estimated from the requirement that the magnetization curve given by Eq. (7) should smoothly join the magnetization curve which is derived by perturbative treatment of the free energy Eqs. (1) and (2) near upper critical field  $H_{c2}$ .<sup>5</sup> The saturation magnetization then reads:  $M_s = (1/16\pi)[(g-1) \times (1 + \beta_A)/\beta_A - \log g]$ , where  $\beta_A \approx 1.16$  is Abrikosov parameter.

In summary, a type II triplet superconductor can possess ferromagnetic properties due to a direct coupling of the spin of the condensate to a magnetic field. If this coupling is strong properties of the vortex matter are significantly modified. Response on high external magnetic field is positive. Above some critical value of  $g$ , a vortex state is created spontaneously. On the other hand the physics of a triplet superconductor at low magnetic field for sufficiently small  $g$  (weak ZI) is dominated by magnetic skyrmions, novel topological objects which are quite different from vortices.

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