Exact solution of the Ginzburg-Landau equation for the upper critical field of a $d_{x^2-y^2}$ superconductor

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(Received 16 October 1998)

A $d_{x^2-y^2}$ superconductor is modeled as the superconducting layers in the a-b plane, whose coupling in the c direction is approximated by the effective mass, within the Ginzburg-Landau theory. In this work, this model is applied to the system where the coherence length along the c direction is greater than the layer spacing. Based on our model, we calculate the upper critical field in a magnetic field lying in a-c plane and tilted by an angle from the c axis. According to our results, the curvature of $H_{c2}(T)$ is upward, and the slope $-dH_{c2}(T)/dT$ depends on the angle between the c axis and the external field. The most interesting feature is that the ratio of the c-direction parameter related to the effective mass to the a-b plane parameter connected with the effective mass can influence H_{c2}. As the ratio is decreased, H_{c2} becomes increased. We also find that there is no admixture of s-wave component in the critical regime and believe that the upward curvature of the $H_{c2}(T)$ is the characteristic property of a d-wave superconductor. [S0163-1829(99)01914-1]

I. INTRODUCTION

A fundamental issue that links microscopic theory and experimental phenomenology of high-T_c superconductors is the symmetry of the pairing state.[1–3] The symmetry of the order parameter provides further insight into the mechanism of superconductivity. In mean-field theory, exact calculations,[4] and variational calculations,[5] it appears that a d-wave symmetry is slightly lower in free energy. Many experiments directly or indirectly prove the pairing state in high-T_c temperature superconductors HTSC. For example, Mathai *et al.* have found consistent unambiguous proof of a time-reversal invariant order parameter with a π shift in $YBa_2Cu_3O_{7-\delta}$ (YBCO) by using a scanning SQUID (superconducting quantum interference device) microscope.[6] Wollman *et al.* measured the phase shift of the order parameter between the orthogonal **a** and **b** directions in YBCO crystals by studying the phase coherence of YBCO-Pb dc SQUID's.[3] Tsuei *et al.* used the concept of flux quantization in superconducting $YBa_2Cu_3O_{7-\delta}$ rings with 0, 2, and 3 grain-boundary Josephson junctions to test the pairing symmetry in high-T_c superconductors.[7] All these experimental results support the d-wave pairing in high-T_c cuprate superconductors, especially, $d_{x^2-y^2}$ symmetry.

Experimental measurements, such as photoemission studies,[8] Josephson interference,[9] and c-axis Josephson tunneling experiment,[10] give evidence as a predominant d-wave order parameter and minor s-wave symmetry. Kouznetsov *et al.* report c-axis Josephson tunneling between YBCO and experimental results to give a direct evidence for mixed d- and s-wave pairing in YBCO.[11] On the theoretical side, Volovik[12] and Soininen, Kallin, and Berlinsky[13] have derived the vortex structure of a d-wave superconductor. They predicted that the s- and d-wave components coexist. Betouras and Joynt show the existence of the s-wave and d-wave components for the order parameters.[14] Franz *et al.* derive phenomenologically the Ginzburg-Landau theory for the d-wave superconductor and study the properties of a single vortex and of the Abrikosov vortex lattice.[15] Müller proposed that there exist in the copper oxide superconductors two condensates with different symmetry but the same transition temperature, namely, there are two kinds of superconducting gap.[16] Ren, Xu, and Ting assumed a d-wave pairing interaction together with a repulsive on-site Coulomb interaction.[17] Using the finite temperature Green's-function method,[18] they derived microscopically the Ginzburg-Landau equations for a superconductor with $d_{x^2-y^2}$ symmetry in the presence of a magnetic field. The structure of a single vortex is analyzed, but no upper critical field is derived.

The Ginzburg-Landau theory of d-wave superconductors in a magnetic field is quite rich. The measures of the upper critical fields H_{c2} of high-T_c superconductors are extremely important because they provide the direct information about microscopic parameters. For example, the coherence length ξ is one of the characteristic parameters and fails to be observed directly. It is often indirectly derived from the expression $H_{c2}(0)=\phi_0/2\pi\xi^2$, where $H_{c2}(0)$ is the upper critical magnetic field measured near $T=0$, and ϕ_0 is the magnetic flux quantum. Many experimental observations have been reported from resistive transition curves[19] and dc magnetization measurements.[20] These experimental results showed a positive curvature of $H_{c2}(T)$ versus T plots, including layer compounds,[21] the cubic bismuthates,[22] the "electron-doped" cuprates,[23] and the Fe-doped $YBa_2Cu_3O_7$.[24] Suzuki and Hikata indicated an upward curving $H_{c2}(T)$ for the field oriented parallel to the c axis.[25] But Samuely *et al.*[26] determined the upper critical field of the fully three-dimensional system of $Ba_{1-x}K_xBiO_3$ by using tunneling measurement and found that $H_{c2}(T)$ followed the WHH (Werthamer-

Helfand-Hohenberg) model revealing a saturation at low temperature. Hence the positive curvature of $H_{c2}(T)$ for all high-T_c cuprates remains to be tested. On the theoretical side, Ovchinnikov and Kresin[27,28] based their approach on the method of integrated Green's functions including the effect of magnetic impurities for cuprates in HTSC to determine the exotic temperature dependence of $H_{c2}(T)$. Their results gave an evidence that the positive curvature of $H_{c2}(T)$ and also linear temperature dependence near $T\rightarrow 0$ were due to magnetic impurities and their correlations at low temperature. They also showed that $H_{c2}(T)$, in the absence of the magnetic impurities, has negative curvature.[29] Kim, Zhu, and Ting[30] calculated $H_{c2}(T)$ of a mixed *d*- and *s*-wave superconductor based on the linearized gap equation. The results demonstrate that the value of $H_{c2}(T)$ of the mixed *d*- and *s*-wave superconductor was larger than that of a pure *d*-wave superconductor, but $H_{c2}(T)$ still showed negative curvature. Joynt explained this characteristic property with help of the phenomenological Ginzburg-Landau theory.[31] Maki and Beal-Monod derived the Ginzburg-Landau equation microscopically within a $(d+s)$-wave superconductivity model including the effect of the an orthorhombic distortion,[32] and got the upper critical field H_{c2} for various applied magnetic-field orientations. Affleck, Franz, and Sharifzadeh Amin presented a simple model by expressing *s* component in terms of higher-order derivative terms in the *d* component.[33] In a similar approximation, Chang *et al.* calculated the upper critical field $H_{c2}(T)$ by perturbed method.[34] Takanaka and Kuboya calculated the angular dependence of the upper critical field in the $LaSrCuO_4$ in the *a-b* plane, taking account of the effect of the gap anisotropy and nonlocal correction.[35] All $H_{c2}(T)$ results based on the Ginzburg-Landau (GL) theory for two-dimensional *d*-wave superconductors show that some approximation is made in the derivation and show positive curvature. Although the upper critical field of a *d*-wave superconductor have been derived within the phenomenological and microscopical GL model, the weak-coupling model, and linearized gap model, none of them includes the effects of the external field lying *a-c* plane and the parameters related to the effective mass. In order to investigate the effect of the anisotropic effective-mass approximation of a *d*-wave superconductor rather than the single-layered two-dimensional *d*-wave superconductors, we here present H_{c2} with the external field in an anisotropic effective-mass approximation of $d_{x^2-y^2}$ superconductor based on the Ginzburg-Landau theory.

In Sec. II, we use anisotropic Ginzburg-Landau equations to derive the upper critical field for some directions of the external field relative to *a-c* plane. By following the similar approach in the simple harmonic-oscillator equation, the creation and annihilation operators are defined, and the two recursion relations are presented. According to Sundaram and Joynt's description,[36] we can solve these relations by standard methods[37] and obtain an exact solution of the problem of the upper critical field. Our relations do not close and, therefore, we reduce them to a matrix problem. By neglecting the higher-order terms, the asymptotic solutions for some cases are derived. We set the lowest eigenvalue to vanish and obtain H_{c2}. It is noteworthy that the upper critical field depends not only on the angle between the external field and the crystal axes, but also on the ratios of the effective masses. In Sec. III, we discuss our work and give some interesting conclusions. Choosing the ratio of the slopes $dH_{c2}(T)/dT$ parallel to perpendicular directions relative to the *x* axis to exceed 5, we show that there is an extreme large anisotropy in H_{c2}.

II. CALCULATIONS OF THE UPPER CRITICAL FIELD

The Ginzburg-Landau (GL) theory for a superconductor with $d_{x^2-y^2}$ symmetry has been microscopically described by Ren, Xu, and Ting.[17] They considered a repulsive ''on-site'' interaction and a nearest-neighbor attractive interaction. It has been shown that the interactions give rise to a pure *d*-wave superconductor for a uniform system if the ''on-site'' repulsion is large. Finally, the generic Ginzburg-Landau equations, as expressed in terms of the *s*-wave and *d*-wave components of the order parameter, are

$$2(1+2V_s/V_d)\Delta_s^* + \alpha\lambda_d\left[\frac{1}{2}v_F^2\Pi^2\Delta_s^* + \frac{1}{4}v_F^2(\Pi_x^2-\Pi_y^2)\Delta_d^* + 2|\Delta_s|^2\Delta_s^* + 2|\Delta_d|^2\Delta_s^* + \Delta_d^{*2}\Delta_s\right]=0, \tag{1}$$

$$-\lambda_d\Delta_d^*\ln(T_c/T) + \alpha\lambda_d\left[\frac{1}{4}v_F^2\Pi^2\Delta_d^* + \frac{1}{4}v_F^2(\Pi_x^2-\Pi_y^2)\Delta_s^* + 2|\Delta_s|^2\Delta_d^* + \Delta_s^{*2}\Delta_d + \frac{3}{4}|\Delta_d|^2\Delta_d^*\right]=0, \tag{2}$$

with $\alpha = 7\zeta(3)/8(\pi T_c)^2$, $\lambda_d = \frac{1}{2}N(0)V_d$, and the operator $\Pi = -i\nabla_R - e^*A_R$. Here V_s and V_d correspond, respectively, to the *s*-wave and *d*-wave interactions and are positive, $N(0)$ is the density of states at the Fermi surface, and v_F is the Fermi velocity. Equations (1) and (2) are valid for two-dimensional *d*-wave superconductivity. In order to meet the reality, we use the effective mass to include *z*-direction effect such as $\gamma_2\Pi_z^2\Delta_d^*$; in Eq. (1) and $\gamma_2\Pi_z^2\Delta_d^*$; in Eq. (2). Such an approximation is valid for the coherent length larger than the layer spacing. Near the upper critical field H_{c2}, the amplitudes of the order parameters are small. We may ignore the nonlinear terms and obtain the linearized Ginzburg-Landau equations

$$\alpha_s\Delta_s^* + 2\gamma_1(\Pi_x^2+\Pi_y^2)\Delta_s^* + \gamma_2\Pi_z^2\Delta_s^* + \gamma_1(\Pi_x^2-\Pi_y^2)\Delta_d^* = 0, \tag{3}$$

$$\alpha_d\Delta_d^* + \gamma_1(\Pi_x^2+\Pi_y^2)\Delta_d^* + \gamma_2\Pi_z^2\Delta_d^* + \gamma_1(\Pi_x^2-\Pi_y^2)\Delta_s^* = 0, \tag{4}$$

where $\alpha_s = 2(1+2V_s/V_d)$ and $\alpha_d = -\lambda_d\ln(T_c/T)$. It was shown that α_d is negative when the pure *d*-wave state is thermodynamically stable.[15] In Eqs. (3) and (4), it is convenient to use the symbols $\gamma_i' s$ to replace the corresponding coefficients in Eqs. (1) and (2).

Now, we want to determine the upper critical field. By assuming the applied field $\mathbf{H} = H(\sin\theta\hat{x} + \cos\theta\hat{z})$, in which θ is the angle from the $\hat{z}$ axis (**c** axis), the commutative relations are

$$\ell^2[\Pi_x,\Pi_y] = i\cos\theta, \quad \ell^2[\Pi_y,\Pi_z] = i\sin\theta,$$

$$\ell^2[\Pi_x,\Pi_z]=0, \tag{5}$$

where ℓ^2 is the magnetic length and equals to $1/e^*H$ and e^* is the charge associated with the "super electrons." Next, we rotate the operator Π and get

$$p_1=\Pi_x \sin\theta+\Pi_z\cos\theta,\quad p_2=-\Pi_x\cos\theta+\Pi_z\sin\theta,$$
$$p_3=\Pi_y. \tag{6}$$

The new commutative relations become

$$[p_2,p_3]=\frac{-i}{\ell^2},\quad [p_1,p_2]=[p_1,p_3]=0. \tag{7}$$

The linear Ginzburg-Landau equations are associated with the Schrödinger equation for a simple harmonic oscillator system. To solve the problem, we define the creation and annihilation operators

$$a_s=\ell(u_s p_2-iv_s p_3),$$
$$a_s^+=\ell(u_s p_2+iv_s p_3), \tag{8}$$

and similarly for a_d and a_d^+. In order to handle the problem easily, we set the commutative relations $[a_i,a_i^+]=1$ for $i=s,d$ and the corresponding specific parameter values are allowed as

$$2u_s v_s=1,\quad v_s^4=\frac{1}{2(2\cos^2\theta+\gamma_{12}\sin^2\theta)}, \tag{9}$$

$$2u_d v_d=1,\quad v_d^4=\frac{1}{4(\cos^2\theta+\gamma_{12}\sin^2\theta)}, \tag{10}$$

with $\gamma_{12}=\gamma_2/\gamma_1$. Thus the Ginzburg-Landau equations become

$$\left[\alpha_s+\frac{\gamma_1}{\ell^2 v_s^2}(2a_s^+a_s+1)\right]\Delta_s^*+\left[\left(\frac{\gamma_1 v_d^2\cos^2\theta}{\ell^2}+\frac{\gamma_1}{4\ell^2 v_d^2}\right)\times(a_d^{+2}+a_d^2)+\left(\frac{\gamma_1 v_d^2}{\ell^2}\cos^2\theta-\frac{\gamma_1}{4\ell^2 v_d^2}\right)\times(2a_d^+a_d+1)\right]\Delta_d^*=0, \tag{11}$$

$$\left[\alpha_d+\frac{\gamma_1}{\ell^2 v_d^2}(2a_d^+a_d+1)\right]\Delta_d^*+\left[\left(\frac{\gamma_1 v_s^2\cos^2\theta}{\ell^2}+\frac{\gamma_1}{4\ell^2 v_s^2}\right)\times(a_s^{+2}+a_s^2)+\left(\frac{\gamma_1 v_s^2}{\ell^2}\cos^2\theta-\frac{\gamma_1}{4\ell^2 v_s^2}\right)\times(2a_s^+a_s+1)\right]\Delta_s^*=0. \tag{12}$$

These two equations are the Hermitian operators acting on the states Δ_s^* and Δ_d^*. The lowest eigenvalue for the Hermitian operators determines the upper critical field. In contrast to the Schrödinger equations the GL equations are hard to solve because the two states, Δ_s^* and Δ_d^*, are coupled each other. By taking Δ_s^* and Δ_d^* in the occupation number representations,

$$\Delta_s^*=\sum_{n=0}^{\infty}a_n|n\rangle_s,\quad \Delta_d^*=\sum_{n=0}^{\infty}b_n|n\rangle_d, \tag{13}$$

we have the properties

$$a_s|0\rangle_s=0,\quad a_s^+a_s|n\rangle_s=n|n\rangle_s,$$
$$a_s^{+2}|n\rangle_s=\sqrt{(n+1)(n+2)}|n+2\rangle_s,$$
$$a_s^2|n\rangle_s=\sqrt{n(n-1)}|n-2\rangle_s,\quad n\geqslant 2 \tag{14}$$

and similarly for a_d and a_d^+. By substituting for Δ_s^* and Δ_d^*, we obtain two recursion relations

$$\left[\alpha_s+\frac{\gamma_1}{\ell^2 v_s^2}(2n+1)\right]a_n+\left(\frac{\gamma_1 v_d^2}{\ell^2}\cos^2\theta+\frac{\gamma_1}{4\ell^2 v_d^2}\right)\times[\sqrt{n(n-1)}c_{n-2,n-2}b_{n-2}+\sqrt{(n+1)(n+2)}c_{n+2,n+2}b_{n+2}]+\left(\frac{\gamma_1 v_d^2}{\ell^2}\cos^2\theta-\frac{\gamma_1}{4\ell^2 v_d^2}\right)(2n+1)c_{nn}b_n=0, \tag{15}$$

$$\left[\alpha_d+\frac{\gamma_1}{2\ell^2 v_d^2}(2n+1)\right]b_n+\left(\frac{\gamma_1 v_s^2}{\ell^2}\cos^2\theta+\frac{\gamma_1}{4\ell^2 v_s^2}\right)\times[\sqrt{n(n-1)}c_{n-2,n-2}a_{n-2}+\sqrt{(n+1)(n+2)}c_{n+2,n+2}a_{n+2}]+\left(\frac{\gamma_1 v_s^2}{\ell^2}\cos^2\theta-\frac{\gamma_1}{4\ell^2 v_s^2}\right)(2n+1)c_{nn}a_n=0, \tag{16}$$

where $a_n=b_n=c_{nn}=0$ for $n<0$ and c_{nn} represents the inner product of two state functions, Δ_s^* and Δ_d^*, depending on the angle θ. For example,

$$c_{00}=\frac{\sqrt{2}(u_s/v_s)^{1/4}(u_d/v_d)^{1/4}}{(u_s/v_s+u_d/v_d)^{1/2}} \tag{17}$$

for $0\leqslant\theta\leqslant\pi/2$. Different from $c_{00}(\theta=0)=1$, we can prove $c_{00}<1$ for $0<\theta\leqslant\pi/2$. According to these two recursion relations [Eqs. (15) and (16)], the convergence of the large-n asymptotic of a_n and b_n can be shown. These relations do not close and Joynt treats the problem in perturbation theory.[31] According to standard method,[37] the convergence can be shown to be exponential, and is extremely rapid as $n\rightarrow\infty$. The Airy equation has the analogous properties. Therefore we can truncate the relations at $n=5$, and obtain the eigenvalue correction to one part in 10^{-3}. Equations (15) and (16) can be reduced to a matrix problem. For $\theta=0$, the lowest eigenvalue is produced when $|0\rangle_d$ couples to $|2\rangle_s$ and $|4\rangle_d$. For $0<\theta\leqslant\pi/2$, the lowest eigenvalue is produced when $|0\rangle_d$ couples to $|0\rangle_s$ and $|2\rangle_s$.

By neglecting the higher-order terms, which are small, we can simplify the calculation to solve the two recursion relations and then derive the asymptotic solutions. The H_{c2} equation corresponding to the d-wave component is

$$\left(\alpha_s+\frac{\sqrt{2}\gamma_1(2\cos^2\theta+\gamma_{12}\sin^2\theta)^{1/2}}{\ell^2}\right)\left(\alpha_d+\frac{\gamma_1(\cos^2\theta+\gamma_{12}\sin^2\theta)^{1/2}}{\ell^2}\right)$$
$$=\frac{c_{00}^2\gamma_2^2\sin^4\theta}{4\sqrt{2}\ell^4(\cos^2\theta+\gamma_{12}\sin^2\theta)^{1/2}(2\cos^2\theta+\gamma_{12}\sin^2\theta)^{1/2}}. \tag{18}$$

Finally, we express the upper critical field H_{c2} for all directions of the external field in the a-c plane as

$$H_{c2}(\theta,T)=\frac{2\alpha_s\alpha_d}{e^*\{-\gamma_1[\sqrt{2}(2\cos^2\theta+\gamma_{12}\sin^2\theta)^{1/2}\alpha_d+(\cos^2\theta+\gamma_{12}\sin^2\theta)^{1/2}\alpha_s]-\sqrt{D}\}}, \tag{19}$$

where

$$D=\gamma_1^2[\sqrt{2}(2\cos^2\theta+\gamma_{12}\sin^2\theta)^{1/2}\alpha_d+(\cos^2\theta+\gamma_{12}\sin^2\theta)^{1/2}\alpha_s]^2+\frac{\alpha_s\alpha_d c_{00}^2\gamma_2^2\sin^4\theta}{\sqrt{2}(\cos^2\theta+\gamma_{12}\sin^2\theta)^{1/2}(2\cos^2\theta+\gamma_{12}\sin^2\theta)^{1/2}}. \tag{20}$$

It is easy to see that the upper critical field is upward curvature from the slope of H_{c2}. As $T\to T_c$,

$$H_{c2}(\theta,T)=\frac{\lambda_d\left(1-\frac{T}{T_c}\right)}{e^*\gamma_1^{1/2}(\gamma_1\cos^2\theta+\gamma_2\sin^2\theta)^{1/2}}. \tag{21}$$

There is no admixture of s wave in this limit. The curves show linear temperature dependence.

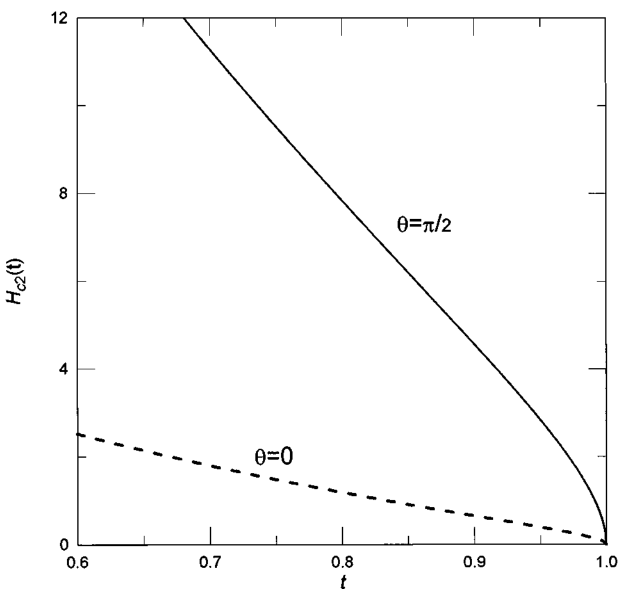

FIG. 1. Upper critical fields versus reduced temperatures $t=T/T_c$ for $\theta=0$ and $\theta=\pi/2$, where θ is the angle between the applied field and the c axes. The solid line represents $\theta=\pi/2$ and the dashed line labels $\theta=0$. We choose $\gamma_1=0.526$, $\gamma_2=0.017$, $\lambda_d=1$, and $\alpha_s=2$. We note that the upper critical field for $\theta=\pi/2$ is larger than that for $\theta=0$. There exists extreme anisotropy in H_{c2}.

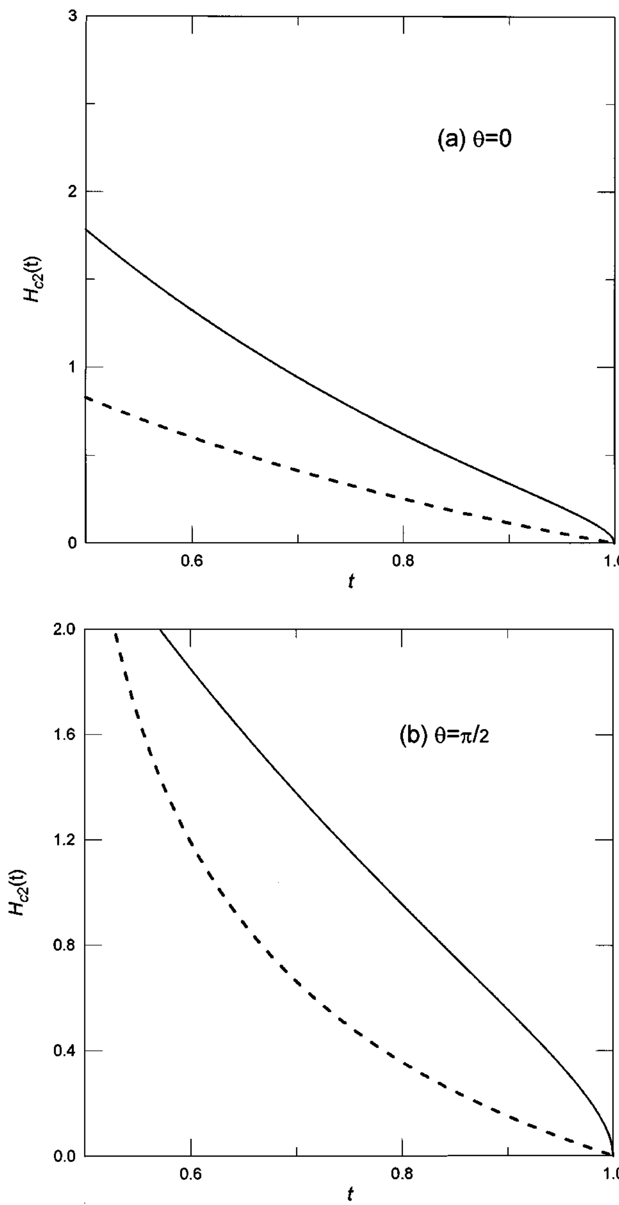

FIG. 2. Upper critical fields as a function of reduced temperature t: (a) $\theta=0$ and (b) $\theta=\pi/2$, where θ is the angle between the applied field and the c axes. The solid line represents our results and the dashed line corresponds to the asymptotic solution. We chose $\gamma_1=1$, $\gamma_2=0.588$, $\lambda_d=1$, and $\alpha_s=2$.

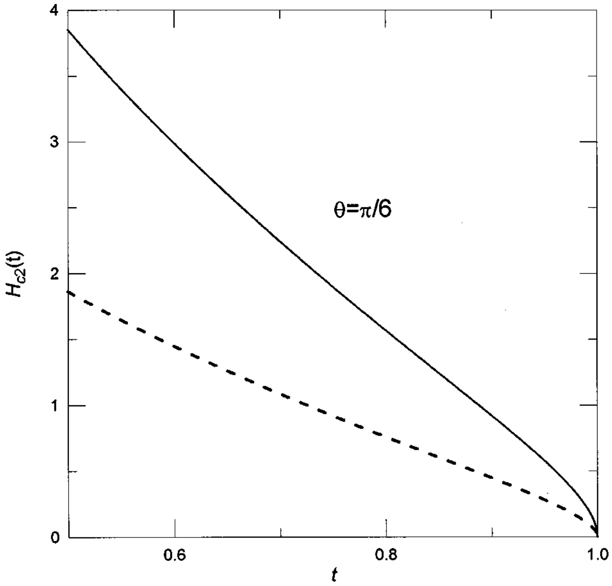

FIG. 3. Upper critical fields versus reduced temperature for $\theta=\pi/6$: The solid line represents $\gamma_1=0.526$, $\gamma_2=0.017$, and the dashed line corresponds to $\gamma_1=1$, $\gamma_2=0.588$. We also chose $\lambda_d=1$ and $\alpha_s=2$.

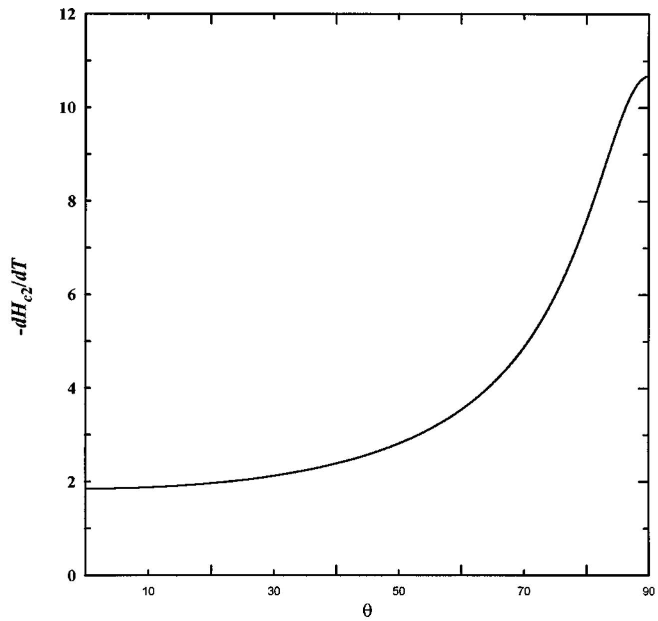

FIG. 4. Dependence of $-dH_{c2}/dT$ (evaluated at $T_c=90$ K) on angle from the $\hat{z}$ axis corresponding to the ratio of $\gamma_{12}=0.03$.

For $\theta=0$, only the space spanned by $\{|0\rangle_d,|2\rangle_s\}$ is in the ground state. The H_{c2} equation corresponding to the d-wave component is

$$\left(\alpha_d+\frac{\gamma_1}{\ell^2}\right)\left(\alpha_s+\frac{5\gamma_1}{2\ell^2}\right)=2\frac{\gamma_1^2}{\ell^4}. \tag{22}$$

For example, $\theta=\pi/2$, no γ_2 appears in this expression. But for other θ values, H_{c2} depends on γ_2. The H_{c2} equation corresponding to the d-wave component is

$$\left(\alpha_d+\frac{(\gamma_1\gamma_2)^{1/2}}{\ell^2}\right)\left(\alpha_s+\frac{(2\gamma_1\gamma_2)^{1/2}}{\ell^2}\right)=\frac{\gamma_1\gamma_2}{4\sqrt{2}\ell^4}c_{00}^2. \tag{23}$$

Here $c_{00}<1$ can be easily proved. These results are in complete accord with experimental measurements.[20]

Of course, we can use the variational calculation to estimate the lowest eigenvalue of Eqs. (3) and (4). For $0<\theta\leqslant\pi/2$, the variational solutions of ground state have the forms

$$\Delta_d^*=d_0e^{i\sigma_1(-x\cos\theta+z\sin\theta)}e^{-1/2e^*H\sigma_1(\cos^2\theta+\gamma_2/\gamma_1\sin^2\theta)^{1/2}y^2}$$

and

$$\Delta_s^*=s_0e^{i\sigma_2(-x\cos\theta+z\sin\theta)}e^{-1/2e^*H\sigma_2(\cos^2\theta+\gamma_2/2\gamma_1\sin^2\theta)^{1/2}y^2},$$

where σ_1 and σ_2 are the variational parameters. Substituting these functions into Eqs. (3) and (4), we can obtain the same consequences as we have done in asymptotic case. The result for $H_{c2}(\theta=0)$ is the same as that of Franz *et al.*[15]

III. CONCLUSIONS

Measurements of $H_{c2}(T)$ are beyond accessible laboratory magnetic fields as a ten-hundredth decrease in temperature, and so they are limited to temperatures near T_c. Thus the Ginzburg-Landau theory is an appropriate one to investigate the behavior of $H_{c2}(T)$. Based on the Ginzburg-Landau theory, we express the upper critical field with the external field lying in the a-c plane. Different from the work,[30] the nonzero s-wave order parameter in our model is only induced by spatial variations of the d-wave component. From the results measured by U. Welp *et al.*,[20] the critical-field slopes from the linear fits are -1.9 and -10.5 T/K for fields along the $\hat{z}$ and $\hat{x}$ axis, respectively. Corresponding to our model, γ_1 and γ_2 are equal to 0.526 and 0.017, respectively. By following the result of Monthoux and Pines,[38] who found λ_d close to 1 as $T_c=90$ K in a spin-fluctuation model, we set $\lambda_d=1$. We plot in Fig. 1 as a function of reduced temperature $t=T/T_c$ for $\theta=0$ and $\theta=\pi/2$. The upper critical field for $\theta=\pi/2$ is larger than that for $\theta=0$. This result points out that we can call our model, adding the term Π_z^2, a pseudo-two-dimensional superconductor. The measurements of electrical properties of single crystal $(La_{1-x}Sr_x)CuO_4$ evidenced that the superconducting oxide systems favored extremely large anisotropy in upper critical magnetic fields.[39]

We also derived the expressions of the asymptotic solutions. Comparing with the experimental result,[40] the ratio γ_2/γ_1 corresponds to the ratio of the effective masses perpendicular and parallel to the **c** axis. We set $\gamma_1=1$ and $\gamma_2=0.588$, $\lambda_d=1$ and $\alpha_s=2$. The H_{c2} versus t for our result and asymptotic result is plotted in Fig. 2 at various angles including $\theta=0$ and $\pi/2$. The upward curvature in Figs. 1 and 2 are expected for a superconductor with a tetragonal structure for an order parameter of combined s-wave and d-wave symmetry. On the other hand, orthorhombic distortion in a pure d-wave superconductor can also have such curvatures. In summary, the nonzero s-wave component, derived by a mixed gradient coupling to the d-wave component, not only forms fourfold symmetry but creates the upward curvature of H_{c2}. In Fig. 3, H_{c2} as functions of t for $\gamma_{12}=0.032$ and 0.588 are shown. Note that as the ratio of γ_{12} is decreased, H_{c2} becomes increased. The most striking aspect is the variation of slope from low to high fields. The change in slope of H_{c2} represents a crossover from a d wave at low fields to $(s+d)$-wave at high fields. Finally, the slope $-dH_{c2}/dT$ near T_c at various angles is plotted in Fig. 4.

Thermodynamic fluctuations in the order parameter bring about meaningful correction for the $d_{x^2-y^2}$ superconductors. The effect of fluctuations may be important in high magnetic field as expected for BCS mean-field model.[41] In the near future, we will study the effect of thermodynamic fluctuations to magnetic properties for the d-wave superconductors (upper critical field, specific heat, etc.).

ACKNOWLEDGMENTS

M.C. and T.J. thank the National Science Council of the Republic of China which gave financial support for this work through Grant No. NSC 88-2112-M-009-003. T.J. would like to thank faculty of the Texas Center for Superconductivity at the University of Houston for their hospitality and partial financial aid when he visited there last summer. C.S. would like to thank the Texas Center for Superconductivity at the University of Houston, and the Robert A. Welch Foundation for a financial support for this work.

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