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A design approach to the multi-objective facility layout problem

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A new multi-objective heuristic algorithm for resolving the facility layout problem is presented in this paper. It incorporates qualitative and quantitative objectives and resolves the problem of inconsistent scales and different measurement units. We consider suboptimal solutions since optimal methods are computationally unfeasible to large layout problems. In this paper, we develop the dominant index (DI) and incorporate it in our heuristic algorithm to guarantee the quality of proposed solutions. Moreover, a new measure of solution quality, dominant probability (D_p), is offered to determine the probability that one layout is better than the others. Computational results show that our proposed heuristic algorithm is an efficient method for obtaining good-quality solutions.

1. Introduction

The facility layout problem deals with finding the most effective physical arrangement of facilities, personnel, and any resources required to facilitate the production of goods or services. It has attracted the attention of many researchers because of its practical utility and interdisciplinary importance. Historically, two basic approaches have most commonly been used to generate desirable layouts: a qualitative one and a quantitative one. These approaches are usually used one at a time when solving a facility layout problem.

With qualitative approaches, layout designers provide subjective evaluations of desired closeness between departments. Then, overall subjective closeness ratings between various departments are maximized. These subjective closeness ratings can be used: A (absolutely necessary), E (essentially important), I (important), O (ordinary), U (unimportant) and X (undesirable), to indicate the respective degrees of necessity that two given departments be located close together. Layout designers may then assign numerical values to the ratings such that they have the ranking $A > E > I > O > U > X$. Seehof and Evans (1967), Lee and More (1967), Muther and McPherson (1970) and Muther (1973) have developed algorithms based on qualitative criteria to obtain final layouts. These different qualitative approaches are distinguished primarily by the scoring methods used for the closeness ratings. For example, the numerical values used by Sule (1994) and Harmonosky and Tothero (1992) for these ratings are $A = 4$, $E = 3$, $I = 2$, $O = 1$, $U = 0$ and $X = -1$. Another example, the ALDEP procedure presented by Seehof and Evans (1967) used the numerical values: $A = 64$, $E = 16$, $I = 4$, $O = 1$, $U = 0$ and $X = -1024$.

Quantitative approaches involve primarily the minimization of material handling costs between various departments. The quadratic assignment problem (QAP) for-

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mulation for assigning n facilities to n mutually exclusive locations is the most typical model used. Gilmore (1962), Lawler (1963) and Gavett and Plyter (1966) have offered exact solution procedures using branch-and-bound techniques. However, the QAP formulation belongs to the class of NP-complete problems (Garey and Johnson 1979), and no known method can arrive at an optimal solution in a reasonable time when 15 or more facilities are considered. Consequently, many heuristic algorithms have been developed for achieving a trade-off between computation time and the efficiency of the final solution (Kusiak and Heragu 1987) and our proposed approach is also a heuristic one.

Many researchers have questioned the appropriateness of selecting a single-criterion objective to solve the facility layout problem because qualitative and quantitative approaches each have advantages and disadvantages. The major limitations on quantitative approaches are that they consider only relationships that can be quantified and to not consider any qualitative factors. The shortcoming of qualitative approaches is their strong assumption that all qualitative factors can be aggregated into one criterion. In real life, the facility layout problem must consider quantitative and qualitative criteria and this falls into the category of the multi-objective facility layout (MOFL) problem.

The primary purpose in solving the MOFL problem is to generate efficient alternatives that can then be presented to the decision maker for his or her selection. Malakooti (1989) classified three types of methods for solving the MOFL problem: (*a*) generate the set of efficient layout alternatives and then present it to the decision maker; (*b*) assess the decision maker's preferences first, and then generate the best layout alternative, and (*c*) use an interactive method to find the best layout alternative. Our proposed approach in this paper falls into the category of the type (*a*) methods in terms of generating good-quality solutions using an effective heuristic algorithm. With respect to type (*a*) methods, Rosenblatt (1979), Dutta and Sahu (1982), Fortenberry and Cox (1985), Waghodekar and Sahu (1986), Urban (1987, 1989) and Houshyar (1991), Harmonosky and Tothero (1992) all developed QAP formulations by specifying different objective weights to generate the best layout. However, there are two inadequacies in these approaches:

(1) all factors may not be represented on the same scale;
(2) measurement units used for objectives may be incomparable.

In this paper we present an effective approach that overcomes the above-mentioned inadequacies by reasonably normalizing all objectives of the MOFL problem, and handling qualitative and quantitative information in similar fashion. Because existing optimization methods are computationally inefficient when large numbers of facilities are involved, heuristic methods are more appropriate for generating effective layouts. Kusiak and Heragu (1987) have suggested a measure for assessing the quality of solutions for the single-objective facility layout problem. However, evaluating various solutions to the MOFL problem is difficult because of the lack of a suitable measure for effectiveness with respect to multiple objectives. In this paper, a new measure for the MOFL problem, dominant probability (D_{p}), is presented that determines the probability that one layout is better than the others. Moreover, we develop the dominant index (DI) and combine it with our heuristic algorithm to guarantee the quality of solutions.

In section 2 we give an overview of MOFL models; our heuristic approach is presented in section 3. The heuristic algorithm for the proposed approach is presented in section 4, and a numerical example is given in section 5. In section 6, the effectiveness of the proposed approach is demonstrated by solving some problems cited in the literature. Section 7 concludes the paper.

2. Review of past approaches

The QAP formulation of the MOFL problem is shown in equations (1) to (4)

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} a_{ijkl} X_{ij} X_{kl} \tag{1}$$

subject to

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad j = 1,\ldots,n \tag{2}$$

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad i = 1,\ldots,n \tag{3}$$

$$X_{ij} \in \{0,1\} \quad i,j = 1,\ldots,n, \tag{4}$$

where

$$X_{ij} = \begin{cases} 1 & \text{if facility } i \text{ is assigned to location } j,\\ 0 & \text{otherwise,}\end{cases}$$

A_{ijkl} = the cost of locating facility i at location j and facility k at location l.

A_{ijkl} in equation (1) is a cost variable representing the combination of quantitative and qualitative measures in MOFL models. Equation (2) ensures that each location contains only one facility. Equation (3) ensures that each facility is assigned to only one location. We divided these models presented in previous studies into four categories:

(1) Rosenblatt (1979) and Dutta and Sahu (1982) defined the cost term as:

$$A_{ijkl} = WcC_{ijkl} - W_R R_{ijkl}, \tag{5}$$

where C_{ijkl} is the total material handling cost, R_{ijkl} is the total closeness rating score, and W_c and W_R are weights assigned to the total material handling cost and to the total rating score.

(2) Foretenberry and Cox (1985) defined the cost term as:

$$A_{ijkl} = f_{ik} \times d_{jl} \times r_{ik}, \tag{6}$$

where f_{ik} is the work flow between two facilities, d_{jl} is the distance between two locations and r_{ik} is the closeness rating desirability of the two facilities.

(3) Urban (1987, 1989) defined the cost term as:

$$A_{ijkl} = d_{jl} \times (f_{ik} + C \times r_{ik}), \tag{7}$$

where C is a constant weight that determines the importance of the closeness rating to the work flow.

(4) Khare *et al.* (1988b) defined the cost term as:

$$A_{ijkl} = W_1 \times r_{ik} \times d_{jl} + W_2 \times f_{ik} \times d_{jl}, \tag{8}$$

where W_1 and W_2 are weights assigned to the work flow and to the closeness rating.

The listed models are similar in nature, and vary only in stating the relationship between the cost term A_{ijkl} and the quantitative and qualitative measures. Although these models have been applied to the MOFL problem, they all have two inadequacies.

(1) All factors may not be represented on the same scale: for example, values for work flow may range from zero to a tremendous amount, while closeness rating values may range from −1 to 4. As a result, the closeness ratings would be dominated by work flow and have little impact on the final layout (Harmonosky and Tothero 1992).
(2) Measurement units used for objectives may be incomparable: the closeness rating represents an order preference indicating the necessity that given facilities be located close together. The total closeness rating score is only an ordinal value; on the other hand, the material flow handling is measured according to cost. Combining these two values with different measurement units in an algebraic operation is unsuitable.

For the reasons cited above, Harmonosky and Tothero (1992) suggested an approach that normalizes all factors, before combining them. To normalize a factor, each relationship value is divided by the sum of all relationship values for that factor, as shown in equation (9):

$$T_{ikf} = \frac{S_{ikf}}{\sum_i^n \sum_k^n S_{ikf}}, \tag{9}$$

where

S_{ikf} is the relationship value between departments i and k for factor f, and
T_{ikf} is the normalized relationship value between departments i and k for factor f.

Next, all values are multiplied by weights representing the relative importance of each factor f. Then, the sum of all values for each pair of departments is calculated. The resulting objective function is shown in equation (10):

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{f=1}^{t} \alpha_f T_{ikf} d_{jl} X_{ij} X_{kl}, \tag{10}$$

where α_f is the weight for factor f.

Harmonosky and Tothero (1992) proposed a methodology for normalizing all factors into comparable units on the same scale. However, the scaling problem remains unresolved. Note first that values for work flow may range from zero to a very large positive value, while closeness rating values may range from a negative value to a positive value. After using equation (9), most normalized relationship values of the larger scaling factor are lower than those of the smaller scaling factor. As a result, the larger scaling factor would have very little effect on the final layout. Second, different scoring values for the closeness ratings may cause some inadequacies. For example, we take the same sample problem from Harmonosky and Tothero (1992) and show it in figure 1. Then we take the values used by ALDEP (Seehof and Evans 1967) to quantify the qualitative factor, and the

	Quantitative							Qualitative					
	1	2	3	4	5	6		1	2	3	4	5	6
1		4	6	2	4	4	1		E	O	U	A	I
2	4		4	2	2	8	2	E		E	U	A	U
3	6	4		2	2	6	3	O	E		X	U	X
4	2	2	2		6	2	4	U	U	X		U	U
5	4	2	2	6		10	5	A	A	U	U		A
6	4	8	6	2	10		6	I	U	X	U	A	

Figure 1. Sample problem.

	Quantitative							Qualitative					
	1	2	3	4	5	6		1	2	3	4	5	6
1		4	6	2	4	4	1		16	1	0	64	4
2	4		4	2	2	8	2	16		16	0	64	0
3	6	4		2	2	6	3	1	16		-1024	0	-1024
4	2	2	2		6	2	4	0	0	-1024		0	0
5	4	2	2	6		10	5	64	64	0	0		64
6	4	8	6	2	10		6	4	0	-1024	0	64	

Figure 2. Quantitative and qualitative factors expressed numerically.

resulting qualitative matrix is shown in figure 2, along with the quantitative matrix. The result of normalization using equation (9) for this problem is shown in figure 3. In this figure, value A in the qualitative matrix of figure 1 is converted into the largest negative value and X is converted into the largest positive value. In this case, it is inadequate to use the resulting qualitative matrix to generate solutions to this problem.

In order to resolve the scaling and measurement problems simultaneously, we developed an effective alternative approach to normalize reasonably all objectives before combining them.

3. Development of a new multi-objective approach

Wallace *et al.* (1976) presented a method for computing the variance of the cost distribution associated with the facility layout problem using only the basic data on flow, distance and problem size. Subsequently, refinements were given in Sahu and Sahu (1979), and Dutta and Sahu (1981). However, they considered only the forward movement of materials. Khare *et al.* (1988a) extended these methods to consider both forward and backward movement of materials, and showed that the layout cost distribution closely approximates a normal distribution. The general expression for variance proposed by Khare *et al.* (1988a) is shown in equation (11). The mean of all feasible layout costs can also be computed (Nugent *et al.* 1968):

	Quantitative					
	1	2	3	4	5	6
1		0.0313	0.0469	0.0156	0.0313	0.0313
2	0.0313		0.0313	0.0156	0.0156	0.0625
3	0.0469	0.0313		0.0156	0.0156	0.0469
4	0.0156	0.0156	0.0156		0.0469	0.0156
5	0.0313	0.0156	0.0156	0.0469		0.0781
6	0.0313	0.0625	0.0469	0.0156	0.0781	

	Qualitative					
	1	2	3	4	5	6
1		-0.0044	-0.0003	0.0000	-0.0176	-0.0011
2	-0.0044		-0.0044	0.0000	-0.0176	0.0000
3	-0.0003	-0.0044		**0.2815**	0.0000	**0.2815**
4	0.0000	0.0000	**0.2815**		0.0000	0.0000
5	-0.0176	-0.0176	0.0000	0.0000		-0.0176
6	-0.0011	0.0000	**0.2815**	0.0000	-0.0176	

Figure 3. Normalized relationships.

$$\begin{aligned} V_{\mathrm{c}} = & \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} f^2(i,j) \times \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} d^2(i,j) - n^2(n-1)^2 M_{\mathrm{f}}^2 M_{\mathrm{d}}^2 \\ & + \frac{4}{n(n-1)(n-2)} P_n f P_n d + \frac{16 Q_n f Q_n d}{n(n-1)(n-2)(n-3)} + \frac{4P'_n f P'_n d}{n(n-1)(n-2)} \\ & + \frac{4}{n(n-1)} \left[\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f(i,j) \times f(j,i) \right] \left[\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} d^2(i,j) \right] \\ & + \frac{16 Q'_n f Q'_n d}{n(n-1)(n-2)(n-3)}, \end{aligned} \tag{11}$$

where

V_{c}	is the cost distribution variance,
$f(i,j)$	is a flow matrix element,
$d(i,j)$	is a distance matrix element,
M_{f}	is the flow matrix mean,
M_{d}	is the distance matrix mean,
$P_n f, P'_n f, Q_n f, Q'_n f$	are the products of flow matrix elements,
$P_n d, P'_n d, Q_n d, Q'_n d$	are the products of distance matrix elements.

In order to achieve normalization, we subtract the mean of the layout cost distribution from each objective value and divide the result by the standard deviation of all feasible layout costs, as shown in equation (12):

$$H_m = \frac{\sum_i^n \sum_j^n \sum_k^n \sum_l^n S_{ijklm - M_m}}{V_m^{1/2}}, \tag{12}$$

where

S_{ijklm} is the objective value of locating facility i at location j and facility k at location l for objective m (for $m = 1, \ldots, t$),
M_m is the mean value of the layout cost distribution for objective m,
V_m is the variance of the layout cost distribution for objective m, and
H_m is the normalized value for objective m.

In this paper, we consider only those objectives with distance-weighted attributes such as flow and closeness rating. Therefore, we propose this approach for solving the MOFL problem which is based on minimization of distance-weighted objectives, namely minimization of total flow cost (TFC) and minimization of total numerical rating (TNR). The TNR presented by Khare *et al.* (1988b) is given as:

$$\text{TNR} = \sum_i \sum_j r_{ij} \times d_{ij}. \tag{13}$$

Hence, all distance-weighted objective functions in this paper possess the cost variance shown in equation (11) and can be characterized as a normal distribution. Using equation (12), we reasonably normalize all objectives, and resolve both the different scale and measurement unit problems. The values obtained are then multiplied by weights (W_m) representing the relative importance of each objective. In our proposed model, the A_{ijkl} in equation (1) is represented by equation (14), and the resulting objective function is shown in equation (15). To simplify the following discussion, we call the objective function, Z, the facility layout score (FLS):

$$A_{ijkl} = \sum_{m=1}^{t} W_m H_m, \tag{14}$$

$$\text{FLS} = Z = \sum_i^n \sum_j^n \sum_k^n \sum_l^n \sum_m^t W_m H_m X_{ij} X_{kl}. \tag{15}$$

Since the layout cost distribution closely approximates a normal distribution (Khare *et al.* 1988a), the variable H_m is approximately a (standard) normal distribution with mean zero and variance 1. Therefore, the variable, FLS, is approximately normally distributed with mean zero and variance $\sum W_m^2$.

In order to guarantee the quality of solutions, we develop a criterion called the dominant index (DI), as shown in equation (16):

$$\text{DI} = Z_\alpha \left(\sum_m W_m^2 \right)^{1/2}, \tag{16}$$

where

α the tolerance probability that the final layout can be dominated (for $0 < \alpha < 1$), and
Z_α the standard normal value leaving an area of α to the left.

Using the DI, we develop a heuristic algorithm for obtaining good-quality solutions. These solutions are guaranteed to be better than other solutions with a probability of at least $1-\alpha$. We call the fraction $1-\alpha$ the dominance confidence coefficient. Moreover, we present a new measure, dominant probability (D_{p}), for determining the probability that one solution is better than the others. It is given by:

$$D_{\mathrm{p}} = 1 - P\left(Z \leq \frac{\mathrm{FLS}}{(\sum W_m^2)^{1/2}}\right). \tag{17}$$

4. Proposed heuristic algorithm

Once these objectives have been reasonably normalized and composed, the MOFL problem can be solved as a single-objective problem. A heuristic algorithm based on the DI is used to generate an effective layout. Our algorithm is a multi-pass pairwise exchange similar to those presented by Dutta and Sahu (1982) and Fortenberry and Cox (1985). This proposed heuristic algorithm is detailed below.

Step 0. Read the input data (flow matrix, size of problem n, relationship matrix, decision weights, a random initial layout, the dominance confidence coefficient $1-\alpha$).
Step 1. Compute the mean and variance for each objective, the dominant index (DI), and the facility layout score (FLS).
Step 2. Set $I = 1$ and $J = 2$.
Step 3. Exchange facility I and J.
Step 4. Compute a new FLS.
Step 5. If the new FLS is less than the previous FLS, then go to step 6; otherwise, go to step 7.
Step 6. Set the previous FLS to the new FLS, record the new layout, and go to step 8.
Step 7. Exchange facilities I and J.
Step 8. If $J = n$, go to step 9; otherwise, go to step 10.
Step 9. If $I = n-1$, go to step 12; otherwise, go to step 11.
Step 10. Set $J = J + 1$. Go to step 3.
Step 11. Set $I = I + 1$ and $J = I + 1$; go to step 3.
Step 12. If FLS has been reduced, go to step 2; otherwise, go to step 13.
Step 13. If FLS is less than DI, go to step 15; otherwise, go to step 14.
Step 14. Read new input data and compute FLS, go to step 2.
Step 15. Output the best layout, FLS, and the dominant probability D_{p}.
Step 16. Stop.

5. Numerical example

Consider the following plant with 12 departments. These departments will be configured in the following (3×4) rectangle. Distance between department locations

1	2	3	4
5	6	7	8
9	10	11	12

is rectilinear and the width of each location is one unit. The values of work flow and closeness rating between departments are given in figures 4 and 5.

The dominance confidence coefficient $(1 - \alpha)$ is set equal to 0.999, and the weight for total material handling cost (W_1) is set to 0.5. According to equation (16), we get DI $= -2.1850$. After the proposed procedure, we get a solution with a facility layout score of FLS $= -3.2782$, which is better than DI.

5	3	6	7
2	8	11	1
4	9	10	12

Solution

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	15	0	0	0	5	5	10	15	10	0	15
2	40	0	25	40	100	90	80	70	40	160	85	60
3	0	20	0	0	0	0	0	0	0	0	0	0
4	10	60	0	0	0	0	20	30	210	280	0	10
5	50	120	600	0	0	40	0	60	0	0	340	50
6	5	90	350	0	40	0	0	20	60	160	270	60
7	10	80	400	40	20	0	0	30	0	140	320	45
8	10	70	350	30	10	10	60	0	10	150	310	40
9	15	110	215	60	10	15	10	20	0	210	180	110
10	30	160	0	450	160	350	380	300	400	0	0	680
11	0	300	700	40	550	510	400	410	250	0	0	20
12	20	90	0	10	50	60	50	30	410	680	30	0

Figure 4. Work flow matrix.

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	1	0	0	0	0	3	4
2	0	0	2	4	0	-1	1	2	0	0	0	0
3	0	2	0	0	3	3	1	2	2	3	0	0
4	0	4	0	0	1	0	0	1	0	0	0	0
5	0	0	3	1	0	3	0	0	0	-1	0	0
6	1	-1	3	0	3	0	2	0	0	3	0	0
7	0	1	1	0	0	2	0	0	0	0	0	0
8	0	2	2	1	0	0	0	0	1	1	1	0
9	0	0	2	0	0	0	0	1	0	1	2	0
10	0	0	3	0	-1	3	0	1	1	0	3	0
11	3	0	0	0	0	0	0	1	2	3	0	4
12	4	0	0	0	0	0	0	0	0	0	4	0

Figure 5. Closeness rating matrix.

The dominant probability of this solution is given below:

$$D_{\mathrm{p}} = 1 - P\left(Z \leq \frac{\mathrm{FLS}}{(\sum W_m^2)^{1/2}}\right)$$

$$= 1 - P\left(Z \leq \frac{-3.2782}{(0.5^2 + 0.5^2)^{1/2}}\right)$$

$$= 1 - P(Z \leq -4.6361) = 0.999\,998.$$

Since the dominant probability is extremely large in this example, the layout planner can consider accepting the solution.

6. Performance evaluation

The proposed method is evaluated using two standard performance criteria. One is computation time, and the other is the quality of solution. We make a comparison with Harmonosky and Tothero's procedure (1992) using the two test problems in their paper. Further comparisons are made with eight test problems (Nugent *et al.* 1968) solved using other heuristic methods. These comparisons show that our proposed method provides acceptable suboptimal solutions in reasonable amounts of computing time. Our proposed algorithm was programmed in the FORTRAN language and run on a DEC VAX-8650 computer.

6.1. *Comparison with Harmonosky and Tothero's procedure*

Harmonosky and Tothero (1992) have shown the superiority of their procedure over previous algorithms presented by Rosenblatt (1979), Dutta and Sahu (1982), Fortenberry and Cox (1985) and Urban (1987). Therefore, our comparison was made with the results obtained by Harmonosky and Tothero (1992). All layouts for each weight combination generated by our approach were listed and scores were compared. These results are shown in tables 1–4. Tables 1 and 2 summarize the results for the eight-department problem. Table 1 presents results based on our scoring system, and table 2 is based on Harmonosky and Tothero's scoring system. Tables 3 and 4 summarize the results for the 12-department problem. All tables show that our proposed procedure is superior to Harmonosky and Tothero's procedure.

6.2. *Test problems*

The non-symmetric flow matrices for the problems of sizes 8, 12, 15 and 20 were taken directly from Khare *et al.* (1988a). The corresponding closeness rating matrices for these problems, which were generated by a random number generator are shown in the Appendix. We ran our proposed method using five random initial layouts for each test problem with given weight combinations. We show the best solution for each case in tables 5–8. According to the dominant probability shown in these tables, our method is capable of obtaining good-quality solutions.

6.3. *Computational effort*

To test computational effort, we generated eight random instances of 10-, 12-, 15-, 20-, 25-, 30-, 36- and 40-facility problems. For each problem size, 100 problems were randomly generated in which work flow values were taken from a discrete uniform distribution with range [0, 500] and closeness rating values generated at

Weights W_1	Weights W_2	Scores for the H+ T† layout	Scores for the proposed layout	Layout by the proposed method	Improvement (%)
1.0	0.0	−4.4235	−4.4235	2 7 6 4 1 5 8 3	0.00
0.9	0.1	−3.1377	−4.1631	2 7 6 4 1 5 8 3	24.63
0.8	0.2	−2.9408	−3.9028	2 7 6 4 1 5 8 3	24.65
0.7	0.3	−2.7483	−3.6425	2 7 6 4 1 5 8 3	24.67
0.6	0.4	−2.6124	−3.3822	2 7 6 4 1 5 8 3	22.76
0.5	0.5	−3.1941	−3.1941	6 7 2 4 8 5 1 3	0.00
0.4	0.6	−3.4654	−3.4654	6 7 2 4 8 5 1 3	0.00
0.3	0.7	−3.7368	−3.7368	8 5 1 3 6 7 2 4	0.00
0.2	0.8	−4.0082	−4.0082	8 5 1 3 6 7 2 4	0.00
0.1	0.9	−4.2795	−4.2795	8 5 1 3 6 7 2 4	0.00
0.0	1.0	−4.5508	−4.5508	8 5 1 3 6 7 2 4	0.00
Average improvement					9.67

† H+ T is a symbol representing Harmonosky and Tothero.

Table 1. Comparison procedure for the eight-department problem using our scoring system.

Weights W_1	Weights W_2	Scores for the H+ T layout	Scores for the proposed layout	Improvement (%)
1.0	0.0	1.467	1.467	0.00
0.9	0.1	1.616	1.490	8.46
0.8	0.2	1.633	1.513	7.93
0.7	0.3	1.651	1.537	7.42
0.6	0.4	1.632	1.560	4.62
0.5	0.5	1.512	1.512	0.00
0.4	0.6	1.459	1.459	0.00
0.3	0.7	1.405	1.405	0.00
0.2	0.8	1.352	1.352	0.00
0.1	0.9	1.299	1.299	0.00
0.0	1.0	1.245	1.245	0.00
Average improvement				2.58

Table 2. Comparison procedure for the eight-department problem using H+ T's scoring system.

Weights		Scores for the H+ T layout	Scores for the proposed layout	Layout by the proposed method	Improvement (%)
W_1	W_2				
1.0	0.0	−3.9721	−4.0990	3 5 12 2 8 7 10 6 1 11 9 4	3.10
0.8	0.2	−3.0705	−3.6943	2 8 5 10 3 6 7 12 1 4 11 9	16.89
0.6	0.4	−2.4046	−3.4892	1 4 11 9 3 6 7 12 2 8 5 10	33.00
0.5	0.5	−3.0514	−3.3777	2 8 5 10 1 6 7 12 3 4 11 9	9.66
0.4	0.6	−3.4190	−3.4190	9 7 6 4 12 11 5 3 10 8 2 1	0.00
0.3	0.7	−3.6684	−3.6684	9 7 6 4 12 11 5 3 10 8 2 1	0.00
0.2	0.8	−3.9178	−3.9178	9 7 6 4 12 11 5 3 10 8 2 1	0.00
0.1	0.9	−4.1672	−4.2073	10 8 11 9 1 2 5 6 3 12 4 7	0.95
0.0435	0.9565	−4.3081	−4.4794	7 4 12 3 6 5 2 1 9 11 8 10	3.82
0.0	1.0	−4.4166	−4.6889	9 12 8 10 6 5 2 1 7 4 11 3	5.81
Average improvement					7.32

Table 3. Comparison procedure for the 12-department problem using our scoring system.

random. Each problem was run individually with an initially random layout. The results shown in table 9 are the average results obtained for the 100 problems of each problem size.

Further proving the effectiveness of our proposed algorithm, another important comparison was made with other published heuristic approaches to the eight commonly used test problems proposed in Nugent *et al.* (1968). For the eight single-objective problems, our solutions were obtained by setting the value of the qualitative weight equal to 0. Comparisons were made in terms of the quality of the solutions obtained and the computation time required. With respect to the solution quality, Kusiak and Heragu (1987) took it as (OV × 100)/LB, where OV is the objective value and LB is the lower bound as given by Nugent *et al.* (1968). Thus, the lower the value of the solution quality measure, the better the solution.

Weights				
W_1	W_2	Scores for the H+ T layout	Scores for the proposed layout	Improvement (%)
1.0	0.0	1.991	1.979	0.61
0.8	0.2	2.019	1.967	2.64
0.6	0.4	2.036	1.936	5.17
0.5	0.5	1.916	1.907	0.47
0.4	0.6	1.818	1.818	0.00
0.3	0.7	1.759	1.759	0.00
0.2	0.8	1.701	1.701	0.00
0.1	0.9	1.643	1.619	1.48
0.0435	0.9565	1.610	1.573	2.35
0.0	1.0	1.585	1.538	3.06
Average improvement				1.58

Table 4. Comparison procedure for the 12-department problem using H+ T's scoring system.

Weights							
W_1	W_2	Proposed layout				Proposed FLS	Dominant probability (%)
1.0	0.0	6 2	3 4	5 1	8 7	−3.8730	99.9946
0.9	0.1	7 8	1 5	4 3	2 6	−3.4312	99.9924
0.8	0.2	7 8	5 1	4 3	2 6	−3.0038	99.9865
0.7	0.3	7 8	5 1	4 3	2 6	−2.6113	99.9697
0.6	0.4	7 8	5 1	4 3	2 6	−2.2188	99.8954
0.5	0.5	2 6	4 3	5 1	8 7	−2.0705	99.8295
0.4	0.6	2 8	4 5	3 7	6 1	−2.0199	99.7454
0.3	0.7	2 8	4 5	3 7	6 1	−2.2302	99.8296
0.2	0.8	8 5	2 4	3 7	6 1	−2.5735	99.9098
0.1	0.9	8 5	2 4	3 7	6 1	−2.9899	99.9286
0.0	1.0	5	4	7	1	−3.4063	99.9671
		8	2	3	6	Average	99.9135

Table 5. Problem size $n = 8$ (area limited to two rows and four columns).

Weights							
W_1	W_2	Proposed layout				Proposed FLS	Dominant probability (%)
		7	10	3	5		
1.0	0.0	6	11	1	8	−4.2233	99.9988
		2	4	12	9		
		5	1	10	6		
0.9	0.1	8	11	3	7	−3.7797	99.9985
		9	12	4	2		
		9	12	4	2		
0.8	0.2	8	11	3	7	−3.4235	99.9983
		5	1	10	6		
		8	1	11	10		
0.7	0.3	9	12	3	4	−3.1550	99.9983
		5	2	7	6		
		8	11	4	3		
0.6	0.4	9	1	6	10	−3.0986	99.9991
		5	12	2	7		
		4	3	11	8		
0.5	0.5	2	10	1	12	−3.1647	99.9996
		7	6	9	5		
		5	9	6	7		
0.4	0.6	12	1	10	2	−3.3025	99.9998
		8	11	3	4		
		4	3	11	8		
0.3	0.7	2	10	1	12	−3.4403	99.9997
		7	6	9	5		
		12	1	10	2		
0.2	0.8	5	9	6	7	−3.7008	99.9996
		8	11	3	4		
		4	3	11	10		
0.1	0.9	2	6	8	1	−4.0271	99.9996
		7	5	9	12		
		8	11	3	4		
0.0	1.0	12	9	6	5	−4.4277	99.9995
		10	1	2	7		
						Average	99.9992

Table 6. Problem size $n = 12$ (area limited to three rows and four columns).

We ran our proposed method with 100 random initial layouts for each test problem. The comparison results are shown in tables 10 and 11. Table 10 gives a comparison of the quality of the best solutions obtained with these heuristic methods, and table 11 gives a comparison of the average solution quality obtained with these heuristic methods. Tables 10 and 11 show that our proposed solutions are better than those provided by other heuristic methods, or are at least as good. As mentioned by Kusiak and Heragu (1987), the computation time provided in table 11 cannot be directly used for comparison because the computation time for each of the

Weights							
W_1	W_2	Proposed layout					Proposed FLS
		10	5	3	2	4	
1.0	0.0	13	9	1	8	11	−5.7751
		12	7	14	15	6	
		10	5	3	2	4	
0.9	0.1	13	9	1	8	11	−5.0971
		12	7	14	15	6	
		10	5	3	2	4	
0.8	0.2	13	9	1	8	11	−4.4392
		12	7	14	15	6	
		15	4	2	3	10	
0.7	0.3	7	9	8	13	5	−4.1012
		12	6	14	1	11	
		12	6	14	1	11	
0.6	0.4	7	9	8	13	5	−4.0121
		15	4	2	10	3	
		3	10	2	4	15	
0.5	0.5	5	13	8	9	7	−3.9556
		11	1	14	6	12	
		15	9	4	13	10	
0.4	0.6	7	6	2	8	5	−3.9674
		12	14	1	11	3	
		12	7	6	14	5	
0.3	0.7	15	10	2	8	3	−4.3027
		9	4	13	1	11	
		12	7	6	14	5	
0.2	0.8	15	10	2	8	3	−4.7276
		9	4	1	13	11	
		15	7	6	12	14	
0.1	0.9	9	10	2	8	5	−5.232
		4	1	13	11	3	
		15	7	6	12	14	
0.0	1.0	9	10	2	8	5	−5.8092
		4	1	13	11	3	

Note. All the dominant probabilities are greater than 0.999 999.

Table 7. Problem size $n = 15$ (area limited to three rows and five columns).

algorithms depends on factors such as the programmer's efficiency, the computer system used, etc. However, we can see our proposed method does yield solutions of very competitive quality in reasonable computation time.

6.3.1. *Empirical results*

While, in general, the improvement algorithm is exponential in the worst case, the algorithm in practice behaves very well. As an improvement algorithm, the computing time is mainly spent on: (1) the number of major iterations; (2) the computing efforts required for each iteration. In order to examine the efficiency of our algor-

Weights							
W_1	W_2	Proposed layout					Proposed FLS
1.0	0.0	16	7	1	10	14	−5.6513
		13	2	4	3	12	
		17	5	8	15	19	
		20	18	11	6	9	
0.9	0.1	13	5	15	19	12	−4.9734
		17	18	2	4	3	
		16	8	7	1	10	
		20	11	9	6	14	
0.8	0.2	19	5	4	18	20	−4.5214
		12	10	1	14	16	
		15	3	2	11	17	
		6	9	7	8	13	
0.7	0.3	20	6	11	15	8	−4.3692
		12	4	14	3	9	
		18	1	10	2	7	
		5	19	16	17	13	
0.6	0.4	20	10	12	18	5	−4.3766
		4	1	14	2	19	
		6	11	9	3	15	
		8	7	16	17	13	
0.5	0.5	5	18	1	4	20	−4.5022
		19	14	10	12	11	
		15	2	9	3	6	
		17	13	7	16	8	
0.4	0.6	5	1	18	4	20	−4.8599
		19	10	14	12	11	
		15	2	9	16	6	
		17	7	3	13	8	
0.3	0.7	15	7	3	13	8	−5.2405
		17	2	9	16	6	
		19	1	14	11	4	
		5	10	18	12	20	
0.2	0.8	5	1	18	11	4	−5.8506
		19	10	14	12	20	
		17	2	9	16	6	
		15	7	3	13	8	
0.1	0.9	15	7	3	13	8	−6.4512
		17	9	16	12	6	
		19	2	14	11	20	
		5	10	1	18	4	
0.0	1.0	6	4	12	15	5	−7.0729
		11	20	14	18	1	
		13	16	9	2	10	
		8	3	7	17	19	

Note. All the dominant probabilities are greater than 0.999999.

Table 8. Problem size $n = 20$ (area limited to four rows and five columns).

Problem size (n)	FLS	CPU time (s)	Dominant probability (%)
10	−2.5403	0.1063	99.9836
12	−3.0398	0.1995	99.9991
15	−3.5842	0.3557	> 99.9999
20	−4.2929	0.9033	> 99.9999
25	−4.9634	1.9067	> 99.9999
30	−5.5365	3.8185	> 99.9999
36	−6.4063	7.6159	> 99.9999
40	−7.1134	10.8260	> 99.9999

Note. The weight W_1 was set = 0.5.

Table 9. Computational effort.

n	H63	H63-66	CRAFT	Biased sampling	STEP	Proposed method	Best known
5	100.0	116.0	100.0	100.0	104.0	100.0	100.0
6	104.9	104.9	104.9	104.9	112.2	104.9	104.9
7	114.9	110.4	110.4	110.4	110.4	110.4	110.4
8	119.8	117.6	117.6	117.6	122.0	117.6	117.6
12	123.9	125.1	118.9	118.9	118.9	118.9	118.9
15	128.8	120.7	121.7	120.0	123.6	120.0	120.0
20	136.5	130.1	130.6	128.6	129.9	128.1	126.7
30	145.0	141.2	140.7	138.2	139.2	137.4	136.8
Computer system	GE 265	GE 265	GE 265	GE 265	VAX 6000	VAX 8650	

Note 1. Results for H63, H63-66, CRAFT, Biased sampling were obtained from Nugent *et al.* (1968).
Note 2. Results for STEP were obtained from Li and Smith (1995).
Note 3. For $n \leq 15$, the best known results ($n = 5$–8) are global optimal solutions obtained from Nugent *et al.* (1968), the best known results ($n = 12$, 15) are obtained from Burkard and Stratmann (1978) and the best known results ($n = 20$, 30) were obtained from Burkard and Bönninger (1983).

Table 10. Comparison of the best solution qualities for the eight test problems in Nugent *et al.* (1968).

ithm, we will re-examine Nugent *et al.*'s test problems and record the number of major iterations (step 2–step 13) of our proposed algorithm after the initial assignment. The results are shown in table 12. As this table shows, the maximum number of major iterations was less than $2n/3$, where the problem size was n. Table 12 shows that the number of major iterations required by our algorithm does not increase significantly when the problem size is increased. This result is indicative of the potential of our algorithm for applications to problems of substantially large dimensions.

7. Conclusions

In this paper, a new multi-objective heuristic algorithm for resolving the facility layout problem is presented. It incorporates qualitative and quantitative objectives and resolves the problem of inconsistent scales and different measurement units. We

	H63		H63–66		CRAFT		Biased sampling		FATE		Proposed method			Best known
												CPU time (s)		
n	Solution quality	CPU time (s)	Solution quality	CPU time (s)	Solution quality	CPU time (s)	Solution quality	CPU time (s)	Solution quality	CPU time (s)	Solution quality	Mean	std.	Solution quality
5	110.4	6.00	117.6	10.00	112.8	1.00	107.2	11.00	104.0	2.80	101.3	0.020	0.009	100.0
6	107.8	7.00	107.8	9.00	107.8	2.00	106.3	21.00	123.4	2.40	104.9	0.028	0.012	104.9
7	117.6	15.00	117.0	12.00	118.8	5.00	111.6	57.00	116.4	3.10	113.8	0.039	0.013	110.4
8	125.7	14.00	121.1	14.00	124.6	10.00	117.6	109.00	139.2	3.30	120.2	0.057	0.017	117.6
12	130.6	55.00	127.7	19.00	121.9	70.00	120.6	658.00	134.3	4.80	123.3	0.167	0.042	118.9
15	132.1	78.00	125.3	40.00	126.5	160.00	121.1	2192.00	138.0	6.10	124.5	0.295	0.059	120.0
20	138.1	168.00	132.6	75.00	132.1	528.00	129.4	6915.00	141.6	11.90	132.0	0.738	0.170	126.7
30	146.1	398.00	143.3	285.00	142.5	3150.00	139.6	4224.00	151.5	32.40	142.2	2.795	0.660	136.8
Computer system	GE 265		GE 265		GE 265		GE 265		ICL 1903T		VAX 8650			

Note 1. Results for H63, H63–66, CRAFT, Biased sampling were obtained from Nugent *et al.* (1968).
Note 2. Results for FATE were obtained from Lewis and Block (1980).

Table 11. Comparison of the average solution qualities for the eight test problems in Nugent *et al.* (1968).

Problem size (n)	5	6	7	8	12	15	20	30
Number of major iterations	3	4	3	4	6	7	7	8

Table 12. The number of major iterations required by our proposed method.

develop the dominant index (DI) to guarantee the quality of proposed solutions. Moreover, a new measure of solution quality, dominant probability (D_p), is offered to determine the probability that one layout is better than the others. The proposed approach seems simple, applicable and computationally efficient. We are optimistic that our approach will be helpful in assisting layout planners select good-quality solutions to practical facility layout problems. In this paper we considered only departments of equal area. In future research, we plan to take unequal-area departments into account.

Appendix

Data sets created at random for closeness rating values in 8-, 12-, 15- and 20-facility problems. The corresponding work flow matrices are from Khare *et al.* (1988a).

8-department problem

0	−1	2	2	0	4	2	0
−1	0	1	0	0	0	1	3
2	1	0	2	0	3	2	0
2	0	2	0	4	0	2	3
0	0	0	4	0	0	−1	3
4	0	3	0	0	0	0	0
2	1	2	2	−1	0	0	0
0	3	0	3	3	0	0	0

12-department problem

0	4	−1	0	2	0	0	3	2	3	2	3
4	0	1	0	1	3	3	0	0	1	1	1
−1	1	0	2	0	4	0	1	−1	1	4	0
0	0	2	0	0	1	1	0	1	0	0	0
2	1	0	0	0	3	0	1	2	0	0	1
0	3	4	1	3	0	0	0	4	0	1	0
0	3	0	1	0	0	0	0	3	−1	0	0
3	0	1	0	1	0	0	0	3	0	3	2
2	0	−1	1	2	4	3	3	0	1	3	2
3	1	1	0	0	0	−1	0	1	0	0	2
2	1	4	0	0	1	0	3	3	0	0	−1
3	1	0	0	1	0	0	2	2	2	−1	0

15-department problem

0	2	0	3	0	−1	2	0	3	1	4	−1	2	2	0
2	0	3	1	2	3	2	4	0	3	0	1	1	0	1
0	3	0	0	3	1	0	2	−1	0	4	0	2	1	0
3	1	0	0	−1	0	1	0	4	4	0	1	2	0	2
0	2	3	−1	0	0	1	3	0	2	1	0	1	0	0
−1	3	1	0	0	0	2	0	0	3	0	1	3	3	0
2	2	0	1	1	2	0	0	0	0	−1	2	1	0	1
0	4	2	0	3	0	0	0	1	4	3	2	3	3	2
3	0	−1	4	0	0	0	1	0	3	0	2	1	0	3
1	3	0	4	2	3	0	4	3	0	1	0	1	0	1
4	0	4	0	1	0	−1	3	0	1	0	3	2	1	1
−1	1	0	1	0	1	2	2	2	0	3	0	−1	1	1
2	1	2	2	1	3	1	3	1	1	2	−1	0	0	1
2	0	1	0	0	3	0	3	0	0	1	1	0	0	0
0	1	0	2	0	0	1	2	3	1	1	1	1	0	0

20-department problem

0	2	1	2	1	0	1	1	1	1	2	0	−1	4	0	0	−1	2	4	0
2	0	0	0	0	−1	1	0	4	3	3	−1	3	3	3	0	1	3	3	1
1	0	0	2	−1	0	3	1	2	1	1	0	4	2	1	3	4	0	0	0
2	0	2	0	1	2	0	0	1	0	1	2	1	2	0	0	−1	1	0	3
1	0	−1	1	0	0	−1	0	1	2	0	1	−1	2	2	0	2	2	1	0
0	−1	0	2	0	0	0	0	3	0	3	4	1	0	2	2	1	0	0	2
1	1	3	0	−1	0	0	2	3	3	1	2	0	1	1	2	3	0	0	0
1	0	1	0	0	0	2	0	2	0	0	0	0	0	1	1	0	0	0	3
1	4	2	1	1	3	3	2	0	3	0	2	1	2	2	4	3	2	2	2
1	3	1	0	2	0	3	0	3	0	2	3	1	−1	0	1	0	3	4	0
2	3	1	1	0	3	1	0	0	2	0	1	3	2	0	2	0	3	0	3
0	−1	0	2	1	4	2	0	2	3	1	0	0	2	1	3	2	3	2	3
−1	3	4	1	−1	1	0	0	1	1	3	0	0	4	0	2	0	2	0	−1
4	3	2	2	2	0	1	0	2	−1	2	2	4	0	1	3	3	3	2	3
0	3	1	0	2	2	1	1	2	0	0	1	0	1	0	0	0	1	2	0
0	0	3	0	0	2	2	1	4	1	2	3	2	3	0	0	2	0	2	3
−1	1	4	−1	2	1	3	0	3	0	0	2	0	3	0	2	0	2	4	1
2	3	0	1	2	0	0	0	2	3	3	3	2	3	1	0	2	0	0	3
4	3	0	0	1	0	0	0	2	4	0	2	0	2	2	2	4	0	0	0
0	1	0	3	0	2	0	3	2	0	3	3	−1	3	0	3	1	3	0	0

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