LETTERS

Alternating Matrices

I would like to bring to your attention, concerning the article "On the product of two alternating matrices" (The MONTHLY, 98 (1991), 935–936) by D. Ž. Đoković, the fact that a complete characterization of the products of two alternating (skew-symmetric) matrices has already appeared in the paper "Pairs of alternating forms and products of two skew-symmetric matrices" (Linear Algebra Appl., 63 (1984), 119–132) by R. Gow and T. J. Laffey: The n by n matrix A is such a product if and only if

(1) the elementary divisors of A corresponding to nonzero eigenvalues have even multiplicity, and

(2) the elementary divisors of A corresponding to zero eigenvalue are of the form

$$x^{k_1}, x^{k'_1}, \dots, x^{k_s}, x^{k'_s} \quad \text{or} \quad x^{k_1}, x^{k'_1}, \dots, x^{k_s}, x^{k'_s}, x,$$

where $k'_i = k_i$ or k_{i+1}, $1 \leqslant i \leqslant s$.

The main results in Đoković's article are of course easy corollaries of this.

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Bessel Functions

Readers of the article *Bessel Functions and Kepler's Equation* by Peter Colwell in the January, 1992, *Monthly* may be interested to know that there is a discussion of Kepler's equation in Section 89 of the book *Théorie des Résidues* by H. Laurent (Gauthier-Villars, Paris, 1865). Laurent considers *une équation que l'on rencontre en Astronomie*: $z = x + t \sin z$. In Section 87, he investigates solutions of Lagrange's equation $w = a + t\phi(w)$, described in Colwell's article. There is no discussion, however, of the work of Bessel and Carlini. Laurent's book appears to be one of the first giving an exposition of Cauchy's theory of residues.

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Fundamental Theorem

Professor Joseph Bennish's admirably concise and elementary proof of the Fundamental Theorem of Algebra ["Another Proof of the Fundamental Theorem of Algebra", *American Mathematical Monthly*, 99, (May 1992), p. 426] may become still clearer with one ending comment. Some proper subsets of the plane have both a non-empty interior and a boundary consisting of only finitely many points, for example, the plane punctured at the origin. Because the Riemann sphere is compact, however, so must be its image under a continuous map, for instance, a polynomial. As a compact subset of the Riemann sphere, the image is also closed, which excludes the possibility of any isolated point on the boundary of its interior, which such points would puncture. Consequently, from Bennish's proof that the boundary consists of at most finitely many points follows that the boundary is empty. Therefore, the interior of the image is open, non-empty, and has an empty boundary, which means that it covers the entire connected Riemann sphere.

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Corrigendum

I write to call attention to a serious typographical error in the paper, "Reliability, Recursion, and Risk," by L. B. Page and J. E. Perry (the *Monthly*, v. 98 #10, Dec. 1991, pp. 937–946).

Because the paper presented a novel but accessible technique for computing probabilities of moderately-complex events, I chose to discuss it in an undergraduate course for computer-science students here at Brock University. Our discussion was greatly complicated by the above-mentioned error; the paper's Figure 5 presents its major example, with node #21 (which should be a **union**) shown as an **intersection**. (The symbols used in the diagrams, although distinct, are unreasonably similar.) With that correction, we reproduced all the paper's results.

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