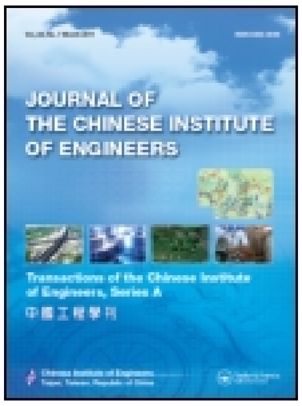

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Fractal extraction from a mixed fBm signal using discrete wavelet transforms

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FRACTAL EXTRACTION FROM A MIXED FBM SIGNAL USING DISCRETE WAVELET TRANSFORMS†

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Key Words: fractional Brownian motion, wavelet transform.

ABSTRACT

Since stationarity, ergodicity and self-similarity for the discrete wavelet transform of fractional Brownian motion (fBm) processes have been shown in previous works, these characteristics could be applied to extract the parameters of two fBm signals from a mixed fBm signal using time-average correlation functions. The goal of this paper is to distinguish the two identical power fBm signals from the mixed signal. In this case, we suppose that the mixed signal is available, but information on the parameters of the two fBm signals is not provided. A method is proposed to find the parameters of these two fBm signals. The smaller parameter can be detected from the fractal dimension of the mixed fBm signal. The parameter of the other fBm is estimated by processing the wavelet coefficients of the mixed fBm signal. Finally, the simulation results showed that this approach works well in increasing the difference between the parameters of the two fBms.

I. INTRODUCTION

Two fractional Brownian motion (fBm) signals with different fractal dimensions added have been shown to have multifractal behavior under the definition of multifractals with the q-th order information dimension. Arneodo *et al.* discussed an fBm signal with a single fractal dimension (1991). Two fBm signals with different fractal dimensions added were shown to have multifractal behavior under the definition of multifractals (Bunde and Havlin Eds, 1991) (Takayasu, 1992). The q-th order information dimension approaches the one with the smaller H. Various methods to estimate the single fractal dimension for an fBm signal were summarized by Gache, Frandrin and Garreau (1991). Kaplan and Kuo (1993) proposed a modified EM (estimate-maximize) algorithm to separate two fBm signals, i.e., one with H=0 and the other one with an unknown H. Wornell (1995) presented an optimal Bayesian detector to discriminate between Gaussian $\frac{1}{f}$ processes in a background of stationary white Gaussian noise, where the stationary white Gaussian noise was a special case corresponding to H=0. Fieguth and Willsky (1996) developed a multiscale Haar to estimate the Hurst parameter H of an fBm signal with noise. Chen, Erdol and Bao (1996) used a best matched wavelet tree to filter noise. However, the aforementaioned studies proposed estimator/detector/filter to distinguish an fBm's parameter from the environment with additional stationary white Gaussian noise, a special case of fBm corresponding to H=0. Here, we propose a method based on discrete wavelet transforms (DWT) to identify two identical power signals from a mixed fBm signal, where

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information on the parameters of the two fBm signals is not available. The combined fBm signal will be shown to have multifractal properties under the definition of the q-th order Renyi information dimension. The Renyi information dimension agrees with the capacity dimension when $q \to 0$ (Takayasu, 1992). The 0-th order Renyi information dimension of the combined fBm signal will be presented in Section II to approach the fBm signal with smaller H parameter as the increment of fBm approaches zero. Since the simulation results showed that the activity of the mixed fBm signal is similar to the fBm signal with a smaller H, the method proposed herein is estimating the 0-th order information dimension of the fBm with a smaller H by directly estimating the fractal dimension of the mixed signal. The other signal is predicted by using proportional power processing or the Wiener filter on discrete wavelet transforms. In this study, we propose two methods for the proportional power factors. One uses suboptimal filters to minimize the variance of the extraction error. The other uses proportional factors with dyadic decaying. In 1996, a de-noising function named "wden.m" was developed. This de-noising function, which is supported by *Matlab 5.0*, filters out noise for a given signal with noise. Here, we use the "de-noising" function to extract one of the fBm signals with a large H if the "signal" denotes the fBm signal we wanted, and the other fBm signal denotes the "noise". Simulation results are presented to compare the two kinds of filters and the "de-noising". Although the error of the extracted signal is not greater than *-20 db*, the fractal dimension of the extracted signal is close to the true dimension using the method of a Wiener filter on DWT.

The multifractal properties of a mixed fBm signal are derived in Section II. The extraction algorithms are proposed in Section III. Simulation results are shown in Section IV.

II. MULTIFRACTALS OF A COMBINED FBM SIGNAL

The multifractal characteristics of a mixed fBm signal are shown herein. The estimated fractal dimension for the mixed fBm signal will approach the dimension of the fBm signal with a smaller H. Let $B_1(t)$ and $B_2(t)$ be two fBm processes, then the increments, defined as

$$x_1(t,\tau) \equiv B_1(t+\tau) - B_2(\tau),$$

$$x_2(t,\tau) \equiv B_2(t+\tau) - B_2(\tau), \tag{1}$$

have a Gaussian distribution (Falconer, 1990; Peitgen and Saupe edt., 1988) with zero mean and variances of $\sigma_1^2 = \sigma^2|t|^{2H_1}$, $\sigma_2^2 = \sigma^2|t|^{2H_2}$, i.e., the probability density functions are

$$p_1(x_1) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{x_1^2}{2\sigma_1^2}\right)$$

$$= \frac{1}{\sqrt{2\pi}\sigma|t|^{H_1}} \exp\left(-\frac{x_1^2}{2\sigma^2|t|^{2H_1}}\right),$$

$$p_2(x_2) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{x_2^2}{2\sigma_2^2}\right)$$

$$= \frac{1}{\sqrt{2\pi}\sigma|t|^{H_2}} \exp\left(-\frac{x_2^2}{2\sigma^2|t|^{2H_2}}\right), \tag{2}$$

Suppose that $x_1(t,\tau)$ and $x_2(t,\tau)$ are independent and WLOG, $H_1 > H_2$. Then the composed signal

$$x(t,\tau) \equiv x_1(t,\tau) + x_2(t,\tau) \tag{3}$$

is also Gaussian with zero mean and a variance of $\sigma_x^2 = \sigma_1^2 + \sigma_2^2 = \sigma^2(|t|^{2H_1} + |t|^{2H_2})$, i.e., the probability density function of $x(t,\tau)$ is

$$p_x(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{x^2}{2\sigma_x^2}\right). \tag{4}$$

Based on the q-th order Renyi information dimension (Takayasu, 1992), let P_i denote the probability that a point belongs to the i-th cube with cube size $\Delta x = \delta^2$. Then

$$\sum_i P_i^q = \sum_i (p_x(x_i)\Delta x)^q = \sum_i p_x^q(x_i)\Delta x(\Delta x)^{q-1}$$

$$= \int p_x(x)dx(\Delta x)^{q-1}$$

$$= C_q|t|^{(1-q)H}(\Delta x)^{q-1} = C_q|t|^{(1-q)H}(\delta)^{2(q-1)}, \tag{5}$$

and the q-th order Renyi information dimension for the fBm process with parameter H_2 is written as

$$D_q(H_2) \equiv \lim_{\delta \to 0} \frac{\frac{1}{1-q}\log\{\sum_i P_i^q\}}{-\log\delta},$$

$$= \lim_{\delta \to 0} \frac{\frac{1}{1-q}\{\log(C_q|t|^{H_2(1-q)}) - 2(1-q)\log\delta\}}{-\log\delta},$$

(Since, t, $\delta \in \boldsymbol{R}$, there exists t such that $t = \delta$.)

$$= \lim_{\delta \to 0} \frac{\frac{1}{1-q}\{\log(C_q\delta^{H_2(1-q)}) - 2(1-q)\log\delta\}}{-\log\delta},$$

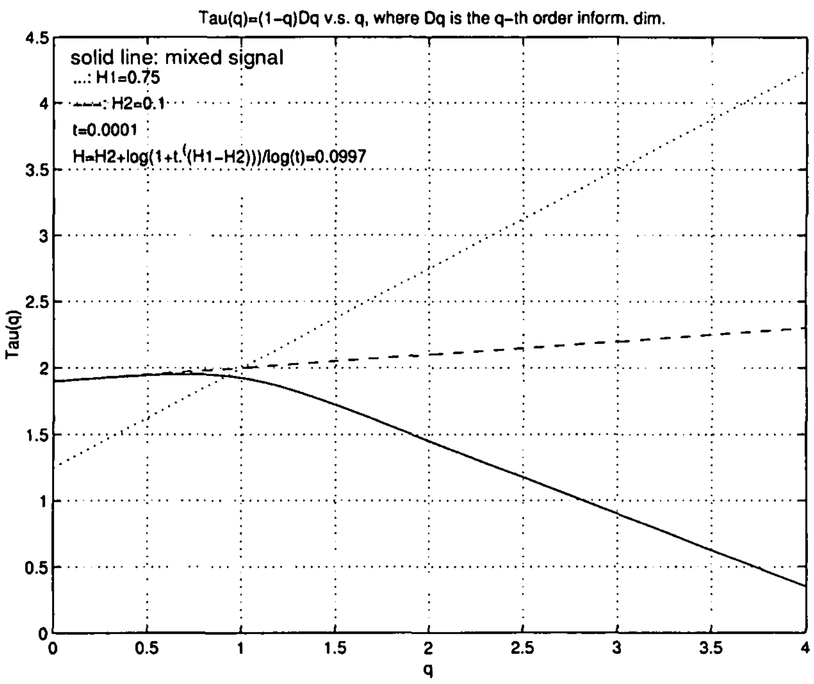

Fig. 1. Multifractal characteristics for the mixed signal, i.e., one with H=0.1, the other with H=0.75.

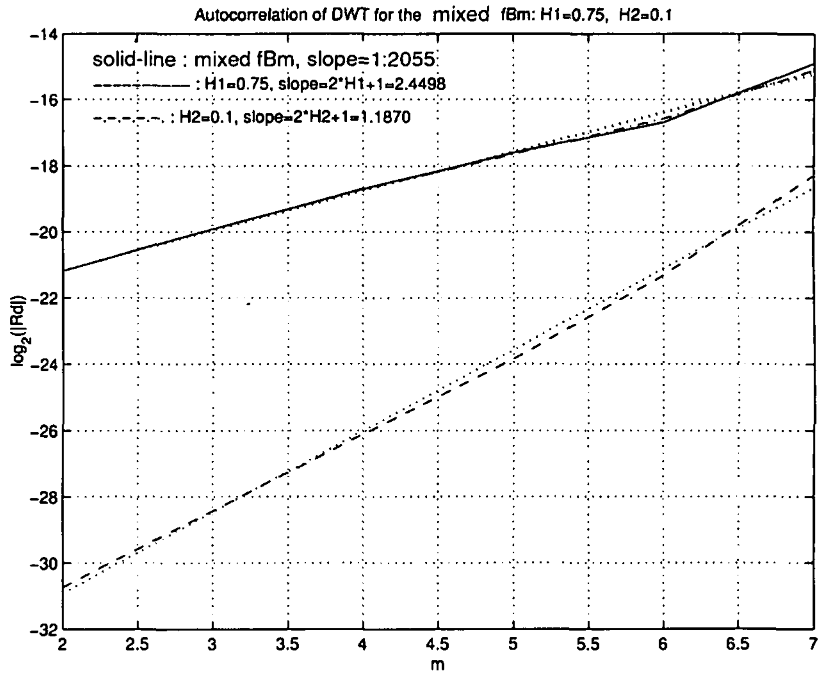

Fig. 2. Autocorrelation function of the wavelet coefficient for the mixed signal, i.e., one with H_1=0.75, the other with H_1=0.1.

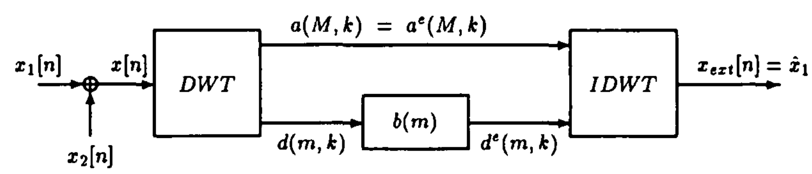

Fig. 3. Extraction using proportional power processing

$$=2-H_2, \tag{6}$$

where $C_q \equiv (\frac{1}{2\pi})^q\sqrt{\frac{2\pi}{q}}\sigma^{1-q}$. With the same definition, the q-th order information dimension for the two added fBm signals with H_1 and H_2 is manipulated by

$D_q(mixed)$

$$\equiv \lim_{\delta\to 0}\frac{\frac{1}{1-q}\{\log(C_q(|t|^{H_1(1-q)}+|t|^{H_2(1-q)}))-2(q-1)\log\delta\}}{-\log\delta},$$

$$=2-H_2-\lim_{\delta\to 0}\{\frac{\log(1+\delta^{H_1-H_2})}{-\log\delta}\}, \tag{7}$$

$$=2-H_2\sim D_q(H_2). \tag{8}$$

Eq. (7) shows that the mixed fBm signal has multifractal characteristics. Fig. 1 also shows this property. Eq. (8) expresses that the q-th order dimension of the mixed signal approximates the fBm with a smaller H as the increment t approaches zero.

III. FRACTAL EXTRACTIONS

In this case, we suppose that the mixed signal is available, but information regarding the parameters of the two fBm signals is not available. Since the simulation results showed that the activity of the mixed fBm signals is similar to the fBm signal with smaller H, (See Fig. 2), the method proposed herein can be utilized to estimate the q-th order information dimension of the fBm with a smaller H by directly computing the fractal dimension of the mixed signal. The other signal can be predicted by using proportional power processing or a Wiener filter on discrete wavelet transforms.

1. Proportional Power Process

The proportional power process diagram is shown in Fig. 3. The proportional factors $b(m)$, m=1, 2, ..., M, are chosen using two methods. One uses dyadic factors, the other is calculated under the minimum variance sense.

(i) Dyadic Factors

The dyadic factor process is summarized in the following. Let x_1 and x_2 denote two fBm signals. For x_1 with a large $H(H_1)$ and x_2 with a small $H(H_2)$, $x=x_1+x_2$, and $a(m,k)$ and $d(m,k)$ denote the scaling and wavelet coefficients, respectively. Let the extracted wavelet coefficients for the fBm with a large H, denoted by $d^e(m,k)_{H_1}$ be scaled from the wavelet coefficients of the mixed fBm signal $d(m,k)$ by factor $b(m)$ as

$$d^e(m,k)_{H_1}=b(m)d(m,k), \tag{9}$$

for m=1, 2, ..., M and $b(M)$=1. Also, let the extracted scaling coefficient at the resolution 2^{-M} be $a^e(M,k)_{H_1}=a(M,k)$. For experience, the proportional factor $b(m)$ can be given by ,

$$b(m)=\frac{1}{2^{M-m/2}}, \tag{10}$$

for m=1, 2, ..., M. Then, the signal $\hat{x}_1$ is extracted by iteratively reconstructing the discrete wavelet transform from the resolution 2^{-M} to 1.

(ii) Suboptimal Factors

The process is similar to the dyadic factor process, but the factor $b(m)$ is required to minimize the variance of the extracting errors, i.e.,

$$d^e(m,k)_{H_1}=b(m)d(m,k),$$

$$\min_{b(m)} \varepsilon\{(X_1^e-X_1)^T(X_1^e-X_1)\}, \tag{11}$$

where $X_1 \equiv [x_1[1]x_1[2]...x_1[N-1]]^T$ and $X_1^e \equiv [x_1^e[0]x_1^e[1]...x_1^e[N-1]]^T$, $x_1^e=x_{ext}=\hat{x}_1$. Using the linear operators $\boldsymbol{H}$ and $\boldsymbol{G}$ and the reconstruction formula (Su and Wu, 1998), X_1 and X_1^e can be expressed as

$$X_1=\boldsymbol{H}^T\boldsymbol{a}_1+\boldsymbol{G}^T\boldsymbol{d}_1$$

$$=\boldsymbol{H}^{T^M}\boldsymbol{a}_M^1+\boldsymbol{H}^{T^{M-1}}\boldsymbol{G}^T\boldsymbol{d}_M^1+\boldsymbol{H}^{T^{M-2}}\boldsymbol{G}^T\boldsymbol{d}_{M-1}^1+\cdots$$

$$+\boldsymbol{H}^T\boldsymbol{G}^T\boldsymbol{d}_2^1+\boldsymbol{G}^T\boldsymbol{d}_1^1 \tag{12}$$

$$X_1^e=\boldsymbol{H}^T\boldsymbol{a}_1^e+\boldsymbol{G}^T\boldsymbol{d}_1^e$$

$$=\boldsymbol{H}^{T^M}\boldsymbol{a}_M+\boldsymbol{H}^{T^{M-1}}\boldsymbol{G}^T\boldsymbol{d}_M b(M)+\boldsymbol{H}^{T^{M-2}}\boldsymbol{G}^T\boldsymbol{d}_{M-1}b(M-1)$$

$$+\cdots+\boldsymbol{H}^T\boldsymbol{G}^T\boldsymbol{d}_2 b(2)+\boldsymbol{G}^T\boldsymbol{d}_1 b(1). \tag{13}$$

Substitute Eq. (12) and Eq. (13) into Eq.(11) and take the partial derivative with respect to $b(m)$, then set it equal to zero, i.e.,

$$\frac{\partial}{\partial b(m)}\varepsilon\{(X_1^e-X_1)^T(X_1^e-X_1)\}$$

$$=2b(m)Trace\{\boldsymbol{H}^{T^{m-1}}\boldsymbol{G}^T\varepsilon\{d_m d_m^T\}\boldsymbol{G}\boldsymbol{H}^{m-1}\}+\cdots$$

$$+b(1)Trace\{\boldsymbol{H}^{T^{m-1}}\boldsymbol{G}^T\varepsilon\{d_m d_1^T\}\boldsymbol{G}$$

$$+b(1)Trace\{\boldsymbol{G}^T\varepsilon\{d_1 d_m^T\}\boldsymbol{G}\boldsymbol{H}^{m-1}\}$$

$$+Trace\{\boldsymbol{H}^{T^{m-1}}\boldsymbol{G}^T\varepsilon\{d_m d_M^T\}\boldsymbol{H}^M\}$$

$$+Trace\{\boldsymbol{H}^{T^M}\varepsilon\{a_M d_m^T\}\boldsymbol{G}\boldsymbol{H}^{m-1}\}-\cdots$$

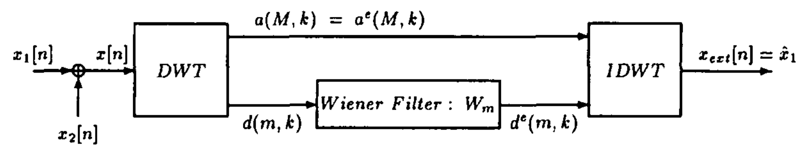

Fig. 4. Extraction using Wiener filter.

$$-Trace\{\boldsymbol{H}^{T^{m-1}}\boldsymbol{G}^T\varepsilon\{d_m^1 d_M^T\}\boldsymbol{G}\boldsymbol{H}^{m-1}\}$$

$$=0 \tag{14}$$

Here, we assume d_i, d_j and a_M are uncorrelated for $i \neq j$ to simplify the problem. Therefore, the optimal factor $b(m)$ is obtained as

$$b(m)=$$

$$\frac{Trace\{\boldsymbol{H}^T\boldsymbol{G}^T\varepsilon\{d_m d_{(1)M-1}^T\}\boldsymbol{G}\boldsymbol{H}^{M-2}+\cdots+\boldsymbol{H}^T\boldsymbol{G}^T\varepsilon\{d_m d_{(1)1}^T\}\boldsymbol{G}\}}{2Trace\{\boldsymbol{H}^{T^{m-1}}\boldsymbol{G}^T\varepsilon\{d_m d_m^T\}\boldsymbol{G}\boldsymbol{H}^{m-1}\}} \tag{15}$$

where $a_M \equiv [a(M,0)a(M,1)...a(M,\frac{N}{2^M}-1)]^T$, $d_m \equiv [d(m,0)d(m,1)...d(m,\frac{N}{2^m}-1)]^T$, and $d_{(1)m} \equiv [d^{(1)}(m,0)d^{(1)}(m,1)...d^{(1)}(m,\frac{N}{2^m}-1)]^T$ is $d^{(1)}(m,k)$ the wavelet coefficient of x_1. The linear operators $\boldsymbol{H}_{m-1}$ and $\boldsymbol{G}_{m-1}$ are two $\frac{N}{2^m}\times\frac{N}{2^{m-1}}$ matrices, $m \neq 1$, whose entries are defined as

$$[\boldsymbol{H}_{m-1}]_{k,i} \equiv h[i-2k] \text{ and } [\boldsymbol{G}_{m-1}]_{k,i} \equiv g[i-2k], \tag{16}$$

where h and g are the mirror filters for DWT. For simplicity, d_m and $d_{(1)i}$ are assumed to be uncorrelated for $i \neq m$ such that the suboptimal factor becomes

$$b(m)=\frac{Trace\{\boldsymbol{H}^T\boldsymbol{G}^T\varepsilon\{d_m d_{(1)m}^T\}\boldsymbol{G}\boldsymbol{H}^{M-1}\}}{2Trace\{\boldsymbol{H}^{T^{m-1}}\boldsymbol{G}^T\varepsilon\{d_m d_m^T\}\boldsymbol{G}\boldsymbol{H}^{M-1}\}}. \tag{17}$$

This method has one disadvantage which is that $d_{(1)m}$ must be given. It is useless for the implementation in actual practice since we assumed no prior information about x_1.

2. Wiener Filter

Since the wavelet coefficient of an fBm signal is stationary (Su and Wu, 1998; Wu and Su, 1998a) and ergodic (Wu and Su, 1998b), the Wiener filter can be applied to process the wavelet coefficient. The proportional power process with the Wiener filter diagram is shown as Fig. 4.

The Wiener filters (Widrow and Stearns, 1985) W_m are described as below:

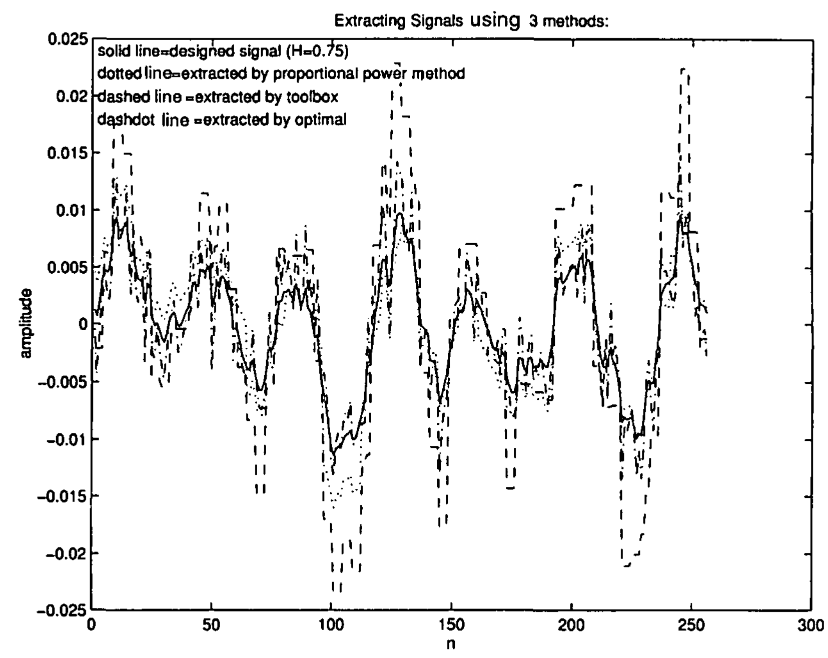

Fig. 5. Extracted signals using three methods: suboptimal factors, proportional factors and "de-noising" function in Matlab 5.0, where the solid-line denotes the designed signal, the dotted line denotes the proportional factors, the dashdot line denotes the suboptimal factors and the dashed line denotes the "De-noising" function.

$$W_m = R_m^{-1} P_m \equiv [W_m(0) W_m(1) \cdots W_m(\frac{N}{2^m} - 1)]^T ,$$

$$R_m \equiv \varepsilon\{d_m d_m^T\} ,$$

$$P_m \equiv \varepsilon\{d(m, \frac{N}{2^m} - 1) d_m\} ,$$

$$d_m \equiv [d(m,0) d(m,1) \cdots d(m, \frac{N}{2^m} - 1)]^T . \quad (18)$$

The procedures of the adaptation algorithm are summarized in the following:

Step 1. Take M-level DWT for the combined fBm signal, $x[n]=x_1[n]+x_2[n]$, $n=0, 1, ..., N-1$, to obtain the scaling coefficient $a(M,k)$ and the detail coefficients $d(m,k)$, $m=1, 2, ..., M$.

Step 2. Let $\hat{a}(M,k)=a(M,k)$.

Step 3. Process $\hat{d}(m,k)=W_m(k)d(m,k)$.

Step 4. Take the inverse DWT of $\hat{a}(M,k)$ and $\hat{d}(m,k)$, $m=1, 2, ..., M$ to obtain the extraction signal $x_{ext}[n]=\hat{a}(0,N-1)$.

Step 5. Update $x[n]=x[n+1]$, $n=0, 1, ..., N-2$, $n[N-1]=x[N]$, then repeat Step 2 to Step 5.

IV. SIMULATION RESULTS

Consider two sampled fBm processes (Kaplan and Kou, 1993), denoted respectively by $x_1[n]=B_{H_1}(n\Delta t)$ and $x_2[n]=B_{H_2}(n\Delta t)$, where $n\in Z$ and Δt is the sampling period. The autocorrelation of B is denoted by

$$R_B[n_1, n_2] = \frac{\sigma^2}{2}|\Delta t|^{2H}(|n_1|^{2H} + |n_2|^{2H} - |n_1 - n_2|^{2H}). \quad (19)$$

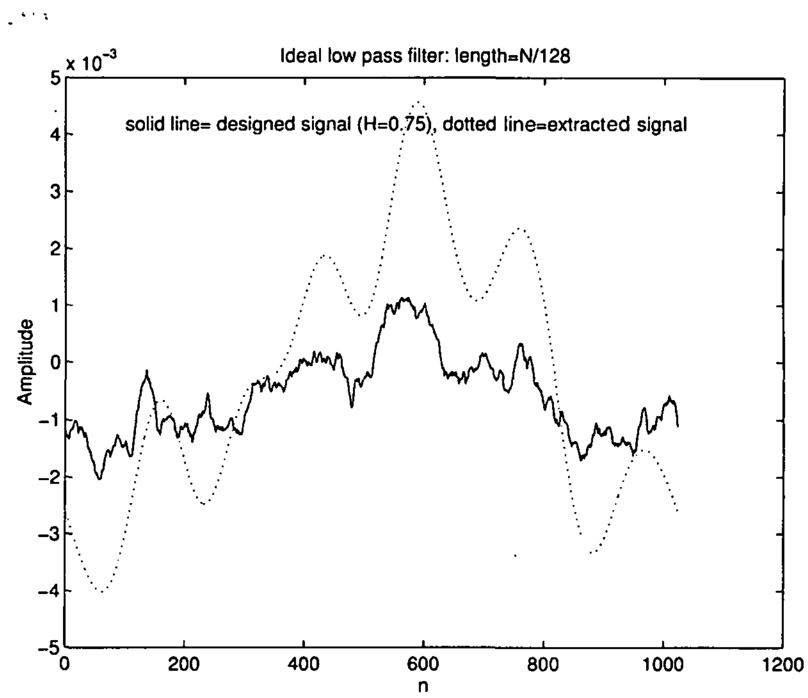

Fig. 6. Extraction signal by an ideal filter, where the solid-line denotes the designed signal and the dotted line denotes the extracted signal.

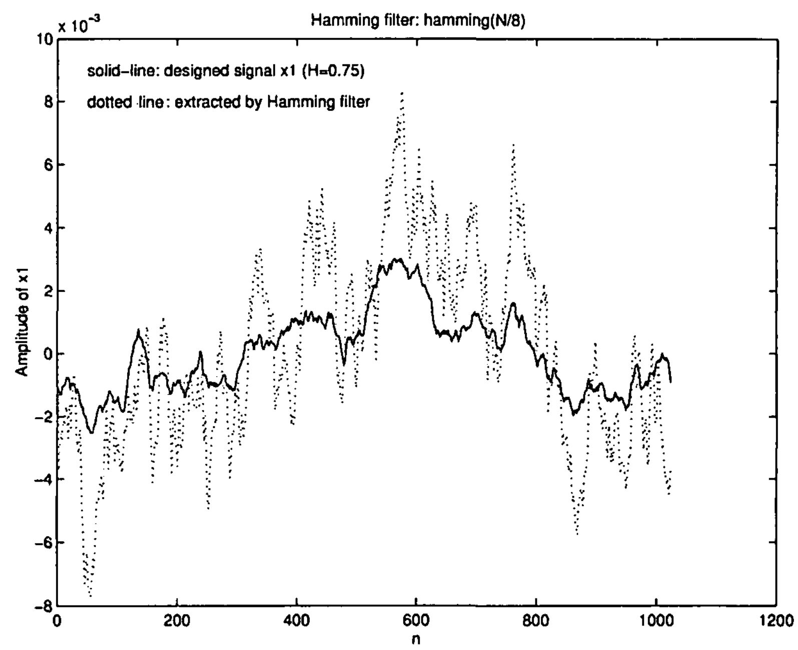

Fig. 7. Extraction signal using Hamming filter, where the solid-line denotes the designed signal and the dotted line denotes the extracted signal.

Let $\Delta t=1$ and choose Haar basis for simplicity, that is $h[0]=h[1]=g[0]=-g[1]=\frac{1}{\sqrt{2}}$. We used the spectral synthesis method (Peitgen and Saupe edt., 1988) to generate two independent fBm signals with different parameters and chose $H_1=0.75$ and $H_2=0.1$. The simulated data length was 2048. The extraction results shown in Fig. 5 represent the comparison of the dyadic factors, the suboptimal factors and the "de-noising" function supported by Matlab 5.0. The first two methods performed better than the "de-noising" function. Figs. 6 and 7 show the signals extracted by an ideal low-pass filter and a Hamming filter respectively. No matter what cut-off frequency was chosen for the ideal filter or what parameters were chosen for the Hamming filter, the extraction results were

Table 1. Fractal dimensions extracted from the mixed fBm signal with *dB* SNR uisng Wiener filters processed on wavelet coefficients

Mixed fBm signal							$\hat{H}_1$		$\hat{H}_2$		$\hat{r}$
$H_{1(D)}$	$H_{1(mean)}$	$H_{1(std)}$	$H_{2(D)}$	$H_{2(mean)}$	$H_{1(std)}$	r	*mean*	*Std.*	*mean*	*Std.*	
0.75	0.7387	0.0206	0.	0.1953	0.0147	0.0385	0.7234	0.0256	0.0050	0.0198	-0.2943
0.9	0.8449	0.0175	0.	0.1953	0.0147	0.0219	0.8394	0.0276	0.0030	0.0178	-0.2790
0.6	0.6200	0.0210	0.1	0.2545	0.0177	0.0153	0.6470	0.0646	0.1014	0.0187	-0.4027
0.75	0.7387	0.0206	0.1	0.2545	0.0177	-0.0114	0.7348	0.0138	0.1086	0.0388	-0.3930
0.9	0.8449	0.0175	0.1	0.2545	0.0177	0.0112	0.8586	0.0268	0.0858	0.0232	-0.6564
0.6	0.6200	0.021	0.2	0.3189	0.0135	0.0133	0.6508	0.0684	0.2040	0.0255	-0.6523
0.75	0.7387	0.0206	0.2	0.3189	0.0135	0.0108	0.7426	0.0326	0.1964	0.0231	-0.3548
0.9	0.8449	0.0175	0.2	0.3189	0.0135	0.0484	0.9022	0.0194	0.1762	0.0361	-0.6668
0.6	0.6200	0.0210	0.35	0.4260	0.0175	-0.0531	0.7003	0.0915	0.2490	0.0160	-0.4609
0.75	0.7387	0.0206	0.35	0.4260	0.0175	0.0054	0.7678	0.0213	0.3353	0.0276	-0.6782
0.9	0.8449	0.0175	0.35	0.4260	0.0175	0.0018	0.9139	0.0138	0.3432	0.0297	-0.6730
0.75	0.7387	0.0206	0.5	0.5413	0.0181	-0.0063	0.8156	0.0432	0.4786	0.0351	-0.5067
0.9	0.8449	0.0175	0.5	0.5413	0.0181	-0.0060	0.9317	0.0176	0.4923	0.0328	-0.5590

- std. = standard deviation. [11]MonteCarlo runs. (D) denotes the designed value.
- $H_{1(\text{or }2)(mean)}$ and $H_{1(\text{or }2)(std)}$ denote the calculated values for the generated fBm signals with parameter $H_{1(\text{or }2)(D)}$ using the box-counting method.
- r denotes the correlation coefficient.

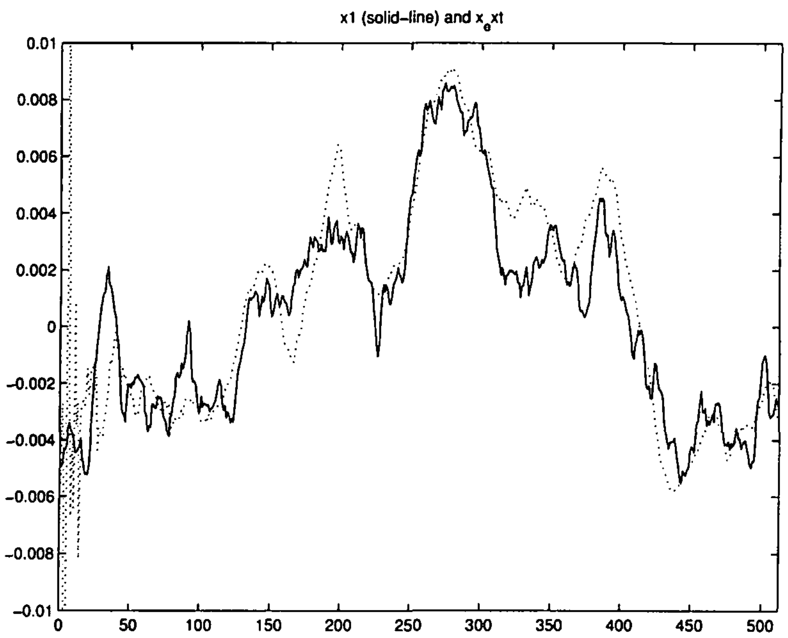

Fig. 8. Extracted signal using the Wiener filter, where the solid-line denotes the designed signal and the dotted line denotes the extracted signal.

worse. Fig. 8 shows the signal extracted by the Wiener filter using the detail coefficients method. The result of the Wiener filter method is very similar to the result of the dyadic factor seen in the simulation. Since the two fBm signals have the same power and the extracted signal x_{ext} still correlates with x_2 (see the correlation coefficient $\hat{r}$ in Table 1), it is difficult to extract the signal precisely without any prior information. However, it is possible to estimate the fBm parameters from the extracted signal x_{ext}. A simulation with 11 Monte-Carlo runs for the H extraction using the Wiener filter method for different fBm parameters is shown in Table 1. In the simulation, the parameters H_1 and H_2 were calculated by the box-counting method, and the parameters $\hat{H}_1$ and $\hat{H}_2$ were computed by the fractal extraction method useing the variance estimator. Table 1 shows that (1) H_2 can be estimated by the mixed fBm signal; (2) H_1 can be detected by the Wiener filter method on DWT; and (3) it is difficult to correctly extract the H_1 the parameter when H_2 approaches H_1.

White Gaussian noise with H_2 equal to zero is a special case. For this case, Wornell and Oppenheim (1992) and Kaplan and Kuo (1993) proposed a maximum likelihood method and a modified maximum likelihood method to estimate the parameter H from the fBm signal added white Gaussian noise with a 10 *dB* signal to noise ratio (SNR). Wornell and Oppenheim's (WO) algorithm, Kaplan and Kuo's (KK) method and the fractal extraction using the Wiener filter are summarized in Table 2 using simulated fBm data. Here, the parameter H estimation in the fractal extraction method uses the variance estimator. For a 10 *dB* SNR, the fractal extraction method does not perform as well as the WO and KK algorithms, but it still works to identify the fBm signal from the noisy background. Furthermore, it also works well in a 0 *dB* SNR enviroment. Since the method used in the WO and KK algorithms is better than the variance estimator to calculate the parameter H, the results will be better if the variance estimator in the fractal extraction

Table 2. Fractal dimensions extracted from white Gaussian noise background using Wiener filters processed on wavelet coefficients

Noise		True	WO		KK		Fractal Extract	
With/Without	SNR	H	mean	Std.	mean	Std.	mean	Std.
Without		0.9	0.846	0.021	0.899	0.017	0.826	0.021
		0.75	0.683	0.026	0.748	0.017	0.717	0.022
With	10 dB	0.9	0.894	0.032	0.917	0.019	0.874	0.012
		0.75	0.760	0.040	0.770	0.029	0.757	0.039
With	0 dB	0.9	--	--	--	--	0.839	0.028
		0.75	--	--	--	--	0.723	0.026

- std. = standard deviation. ''Monte Carlo runs for fractal extraction.
- The data for WO and KK simulation is from (Womell and Oppenheim, 1992) and the data length is 2048.
- The data length for the fractal extracted simulation is 2048.
- $H_{(mean)}$ and $H_{(std)}$ denote the calculated values for the generated fBm signals with parameter $H_{(D)}$ using the box-counting method.

is replaced by the method used in the WO or the KK algorithm.

V. CONCLUSION

The characteristics of stationarity, ergodicity and self-similarity for the DWT of fBm processes can be applied to extract fBm parameters from a mixed fBm signal. Information on the parameters of the two fBm signals is not provided. The signal to noise ratio will be improved to about 20 *dB* after the process, where the "signal" denotes the wanted fBm signal and the other fBm signal is referred to as "noise". In general, the error of this extraction method depends on the high frequency term of the "signal" plus the low frequency part of the "noise", under the uncorrelated assumption for the two fBm signals. This method is useful in comparisons with the traditional generalized cosine filter (ideal low pass/hamming/hanning/Kaiser). Although the extracted signals are not similar enough to the original signals, their parameters can be detected by the method herein. The potential application of our approach is to detection/noise cancellation in a communication field without any auxiliary noise sources.

NOMENCLATURE

$a(M,k)$	scaling coefficient with translation index k at M-th scale level
$a^e(M,k)$	extracted scaling coefficient
$\hat{a}(m,k)$	estimated scaling coefficient
$b(m)$	a scaling factor
$B_1(t)$, $B_2(t)$	fBm signals
C_q	constant value
$d(m,k)$	wavelet coefficient with translation index k at m-th scale level
$d^e(m,k)$	extracted wavelet coefficient
$\hat{d}(m,k)$	estimated wavelet coefficient
d_m,$d_{(1)m}$	vectors composed of $d(m,k)$, k=0, 1, 2, ..., $\frac{N}{2^m}-1$, for x and x_1, respectively
D_q	q-th order Renyi information dimension
$\varepsilon\{$	expected value
g	mirror filter
G_{m-1}	$\frac{N}{2^m}\times\frac{N}{2^{m-1}}$ matrix
h	mirror filter
H_{m-1}	$\frac{N}{2^m}\times\frac{N}{2^{m-1}}$ matrix
H	fBm parameter
$p(x)$	probability density function
P	probability
r	correlation coefficient for the designed signals
$\hat{r}$	correlation coefficient for the estimated signals
R_B	autocorrelation function of B
W_m	Wiener filter
Z	integer set
σ	standard deviation
δ	step size
$\Delta x=\delta^2$	cube size

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以離散小波轉換從混合訊號萃取碎形布朗運動

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摘 要

本文利用碎形布朗運動訊號經過離散小波轉換之後具有定態、恆均與自我相似的特性，將其運用於從混合訊號中分離出兩種不同的碎形布朗運動，且不需預先知曉此二碎形布朗運動的任何資訊，便可估測其參數。其應用方式爲計算混合訊號的碎形維度，即爲碎形布朗運動具有比較小參數之碎形維度估測值；另外，具有比較大參數碎形布朗運動則從混合訊號的小波係數處理估測得之。模擬的結果顯示，當被估測的碎形布朗運動參數差距越大，此方法越可行。

關鍵詞：碎形布朗運動，小波轉換。