Billing Strategies and Performance Analysis for PCS Networks

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***Abstract*—It is predicted that by the year 2000, U.S. cellular carriers will invest billions of dollars for cellular billing and customer care. Two of the most desirable attributes of the cellular billing systems are the flexibility of upgrade and ability to inform the billing experts quickly about the status of the system to minimize any possible fraud and improve customer service. Although the second attribute can be realized by reporting the billing customer record in real time, this tends to congest the signaling channel, which is not desirable. In this paper, we propose several schemes for the provision of a quick billing status report which maintains low-signaling traffic. The performance of these schemes is derived analytically. We expect that these results may provide a guideline for the future design of on-line billing systems in personal communication services (PCS's) networks.**

***Index Terms*—Billing, cell residence times, PCS networks, real-time updating.**

I. Introduction

BILLING WORLD [11] predicts that by the year 2000, U.S. cellular carriers will invest U.S. $1.6 billion for cellular billing and customer care. According to the managers from the top 20 cellular carriers, two of the most desirable attributes of the cellular billing systems are:

- flexibility of upgrade;
- capability to inform the in-house billing experts quickly about the status of the system.

Telecommunication services are "culture sensitive," and the ways of service charging will significantly affect customers' behavior. For example, in many U.S. cellular services, cellular subscribers are charged for the cellular usage, whether they are the calling or the called parties. Thus, cellular customers will tend to share their cellular phone numbers with only a small group of people to avoid "junk calls." On the other hand, the calling party is always charged for the cellular usage in some countries, for instance, in Taiwan. Therefore, cellular subscribers (especially people using cellular phones for business) tend to distribute their cellular phone numbers as widely as possible to enhance their business opportunities. To maximize profits, a cellular carrier will therefore need to offer a variety of billing plans for same services and may need change the plans from time to time to adapt to changing "customer culture." Thus, flexibility with upgrade is considered a highly important attribute of the billing system.

Another important attribute is the provision of quick billing status report, which is essential for the monitoring and diagnosing the billing system. One way to achieve this attribute is to report the customer billing records in real time. Unfortunately, this feature is not supported in most existing cellular billing systems. By comparison, in the public switched telephone network (PSTN) (wireline network), real-time billing information is possible. Typically, billing information will be delivered in the signaling system no. 7 (SS7) messages [9] during the call setup/release processes. The billing information is produced from the automatic message accounting (AMA) records. During the call setup/release processes, the monitor system tracks SS7 messages of the call and generates a call detail record (CDR) in AMA format when the call is completed. The CDR record will be stored in Bellcore accounting format (BAF), and the data can be transfered to the rating and billing systems.

The difficulty of providing real-time cellular customer billing records is due to the fact that the cellular users may roam from their "home systems" (HS's) to the "visited systems" (VS's). When a cellular user is in a VS, the billing records for all call activities are kept in the VS. In the existing cellular roaming management/call control protocols [7], there is no interaction between the VS and the HS at the end of a call. Typically the billing information is kept in the VS as a "roam-type" cellular intercarrier billing exchange record (CIBER). The roam CIBER's will be batched and periodically sent to a clearinghouse electronically or via mail in a tape format and later forwarded by the clearinghouse to the customer's HS. The whole process may take from five days to more than two weeks. To speed up the billing information transmission, a cellular billing transmission standard called EIA/TIA IS-124 has been developed by working group four of TIA's TR 45.2 committee [1]. IS-124 will allow real-time billing information exchange, which will help control fraud by reducing the lag time created by the use of overnighted tape messages. Version A of IS-124 will also accommodate both US AMPS and international GSM carriers, which is desirable for heterogeneous personal communication services (PCS's) integration [8].

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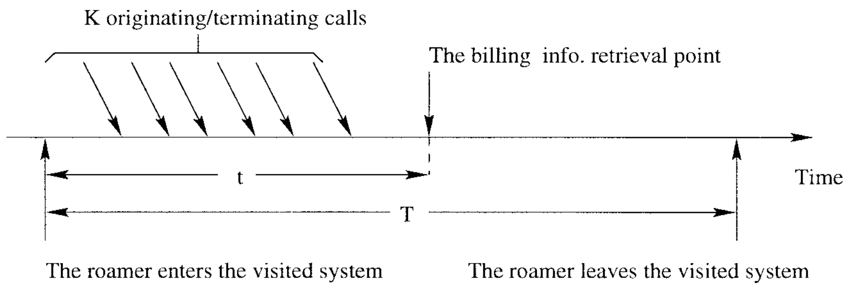

Fig. 1. The timing diagram for the checkpointing model based on the number of calls.

An important performance issue of cellular billing information transmission is the frequency of the billing information exchange. In the ideal case, records would be transmitted for every phone call to achieve the real-time operation. However, real-time transmission would significantly increase the cellular signaling traffic and seriously overload the signaling network of the PSTN. In order to achieve quick billing status report, a tradeoff is therefore needed between the frequency of the billing information transmission and the signaling traffic.

In this paper, we propose three strategies for controlling the billing traffic for future PCS networks, which can be easily implemented in the billing systems of current cellular systems. The first strategy is to update the billing information when a fixed number of calls have been handled. The second strategy is to report the customer billing record in fixed real-time intervals. The third strategy is a combination of these two strategies. We provide performance analysis of these strategies. Our study provides the guidelines to select the appropriate frequency of transmissions for different network engineering requirements.

II. The Billing Checkpointing Models

The purpose of real-time billing information transmission is to ensure that when one (either a billing expert of the HS or a querying customer) checks the customer billing records, the information is up to date. The time of checking is referred to as the *billing information retrieval point* or *retrieval point*. Very often the billing record observer may tolerate certain degree of information outstandingness. We define an observation at a retrieval point as *k-call outstanding* if the most recent k call records have not been received by the HS at the retrieval point. We can also define an observation at a retrieval point as *x-time outstanding* if the time elapsed at the retrieval point from the last record update is no more than x time units. It is very important to know the distributions of the number of the outstanding calls and the outstanding time. Since the billing system makes decisions based on the billing information received from the VS's, the degree of "outstandingness" of the billing information affects the accuracy of the billing decisions. Thus, it is desirable to know the average number of outstanding calls and the average total calling time of outstanding calls for the performance evaluation of billing systems.

We propose three strategies for the provision of quick billing status report while keeping the signaling traffic due to the on-line billing update in a certain tolerable level.

A. Checkpointing Model Based on the Number of Calls

Assume that the billing information of a roamer is sent back to the HS for every n calls. We refer to n as the *checkpointing interval*, where $0 \leq k < n$. The real-time requirement in the cellular billing system specification may be "the probability of more than k-call outstanding observation should be less than $\theta\%$." Thus, based on the mobility and call activities of a user, an appropriate n value can be selected to satisfy the above requirement.

Note that when the user leaves the VS, the HS will send a *deregistration* or *cancellation* message to the VS informing it that the user left the VS, and the VS will acknowledge the deregistration ([12]). We assume that the not-yet checkpointed billing records will be sent back to the HS through the acknowledgment, and no extra billing transmission message will be created.

Suppose that the roamer enters a VS at time zero. The roamer resides at the VS for a period T as shown in Fig. 1. We call T as the VS *residence time*. Suppose that the roamer's billing information is retrieved at the HS at time t, i.e., an expert or a customer of the HS queries the billing information at time t. If there are K phone calls to the roamer during $[0, t]$, then the billing retrieval is k-call outstanding if $K = in + k$ for some $i \geq 0$. Let T have a general distribution with the Laplace transform $f^*(s)$ and the mean $1/\eta$, the calls to the roamer be a Poisson process with arrival rate λ, and the billing retrieval be a random observer. We note that a billing record for a call is created when the call is completed. Thus, "call arrival time" in this context means that the time when the call record is created. Let $\Pr[k]$ be the probability that a billing retrieval is k-call outstanding. We use $u^{(i)}(s)$ in the subsequent development, to denote the ith order derivative of function $u(s)$ at point s. Then $\Pr[k]$ is derived in Appendix I [see (18)] as

$$\Pr[k] = \frac{\eta}{\lambda} \sum_{i=0}^{\infty} \left[1 - \sum_{j=0}^{in+k} \frac{(-\lambda)^j}{j!} f^{*(j)}(\lambda) \right].$$

Let N_o denote the number of outstanding calls, i.e., the number of arriving calls during the interval between the last billing update and the retrieval point. These calls are not available at the retrieval point at HS. Let $t_{\rm tot}$ be the total calling time of these outstanding calls, and let the arriving calls have the expected call holding time $1/\mu$. Then the expected number of outstanding calls is given by

$$E[N_o] = \sum_{k=0}^{n-1} k \, \Pr[k].$$

Let t_i denote the call holding time of an arriving call, then the total calling time of outstanding calls is given by

$$t_{\text{tot}} = \sum_{i=1}^{N_o} t_i.$$

From Wald's equation ([14]), we obtain

$$E[t_{\text{tot}}] = E[t_i]E[N_o] = E[N_o]/\mu.$$

Remark: If $1/\mu$ is interpreted as the average call holding times for possible fraudulent calls, then the maximum fraudulent calling time is $E[t_{\text{tot}}]$.

Furthermore, if the VS residence time T has an Erlang distribution with the shape parameter m and the scale parameter $\alpha = m\eta$ (and thus the variance of the distribution is $V = 1/m\mu^2$), then the probability $\Pr_{(m,\alpha)}[k]$ of k-call outstanding billing retrieval for Erlang [with parameters (m, α)] VS residence time is derived in Appendix I as [see (21)] given in the equation at the bottom of the page. Specifically from the derivations in Appendix I [see (22), (24), and (26)], we have

$$\Pr_{(1,\alpha)}[k] = \left(\frac{\eta}{\lambda+\alpha}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^k \left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^n\right]^{-1} \quad (\alpha = \eta)$$

$$\Pr_{(2,\alpha)}[k] = \left\{\left(\frac{n\alpha}{\lambda+\alpha}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^n \left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^n\right]^{-1} + \left[\frac{(k+1)\alpha}{\lambda+\alpha}\right] + 1\right\}\Pr_{(1,\alpha)}[k] \qquad (\alpha = 2\eta)$$

$$\begin{aligned}\Pr_{(3,\alpha)}[k] = {} & \frac{\eta\lambda^k\alpha^2}{(\lambda+\alpha)^{k+3}} \\ & \times \left\{ n^2\left(\frac{\lambda}{\lambda+\alpha}\right)^n \left[1 + \left(\frac{\lambda}{\lambda+\alpha}\right)^n\right] \cdot \left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^n\right]^{-3} \right. \\ & + n(2k+3)\left(\frac{\lambda}{\lambda+\alpha}\right)^n \left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^n\right]^{-2} \\ & \left. + (k+2)(k+1)\left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^n\right]^{-1} \right\} \\ & + \Pr_{(2,\alpha)}[k], (\alpha = 3\eta). \end{aligned} \tag{1}$$

It is observed that when η is sufficiently small, i.e., the roamer stays in that visited area for sufficiently long time, the number of outstanding calls are equally probable, i.e., uniformly distributed. Indeed, we can easily show that

$$\lim_{\eta \to 0} \Pr_{(1,\alpha)}[k] = \frac{1}{n} \qquad \lim_{\eta \to 0} \Pr_{(2,\alpha)}[k] = \frac{1}{n}.$$

We conjecture that this observation is valid for any nonlattice distribution $f(T)$.

When the VS residence time is exponentially distributed, from Appendix I we obtain

$$\begin{aligned} E[N_o] &= \frac{\lambda}{\eta}\left[1 - \left(\frac{\lambda}{\lambda+\eta}\right)^n\right]^{-1} \cdot \left[1 - n\left(\frac{\lambda}{\lambda+\eta}\right)^{n-1} + (n-1)\left(\frac{\lambda}{\lambda+\eta}\right)^n\right] \\ E[t_{\text{tot}}] &= \frac{\lambda}{\eta\mu}\left[1 - \left(\frac{\lambda}{\lambda+\eta}\right)^n\right]^{-1} \cdot \left[1 - n\left(\frac{\lambda}{\lambda+\eta}\right)^{n-1} + (n-1)\left(\frac{\lambda}{\lambda+\eta}\right)^n\right]. \end{aligned}$$

B. Checkpointing Model Based on Real-Time Interval

In the previous section, we applied a threshold to the number of calls in the update of billing status. This approach may have a disadvantage when the call holding times are long. For example, when a fraudulent user, using another customer's identification, makes a call to a 900 number in the United States, the call holding time tends to be quite long. During a typical billing period, the number of such calls may not be high, therefore, the previous strategy may not be appropriate. One solution to overcome this is to use real-time interval (threshold in time) for updating of billing record. In this section, we analyze this strategy.

Assume now that the billing information is sent back to the HS from the VS for every I time units. As in the previous section, we assume that the VS residence time T has a nonlattice density function $f(T)$ with Laplace transform $f^*(s)$ and with expected residence time $1/\eta$. Let $F(T)$ be its distribution function. Let t be the retrieval point. Let t_o denote the outstanding time, i.e., the time between the retrieval point and the last update instant. Since the customer billing record is forwarded to the HS every I time units, t_o is distributed in the interval $[0, I]$. Let t_o have density function $f_o(x)$ $(0 \le x \le I)$. Then, from Appendix III, we have

$$f_o(x) = \eta \sum_{i=0}^{\infty} [1 - F(iI + x)].$$

$$\Pr_{(m,\alpha)}[k] = \begin{cases} \sum_{i=0}^{\infty} \eta \binom{in+k+m-1}{m-1} \frac{\lambda^{in+k}\alpha^{m-1}}{(\lambda+\alpha)^{in+k+m}} + \Pr_{(m-1,\alpha)}[k], & \text{for } m \ge 1 \\ 0, & \text{for } m = 0 \end{cases}$$

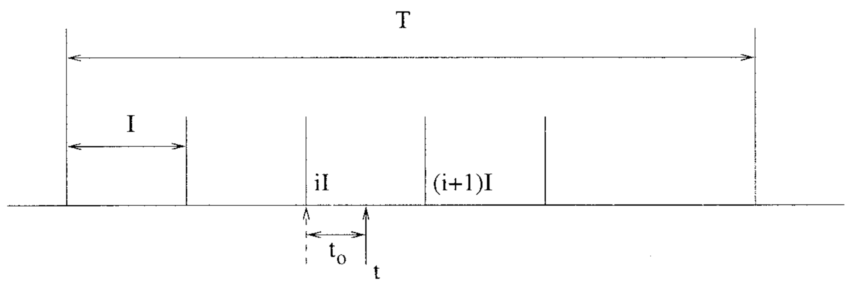

Fig. 2. Time diagram for checkpointing model based on real-time interval.

Suppose that $f^*(s)$ has only isolated singular points. Let σ_p denote the set of poles of $f^*(s)$. Let $\mathrm{Res}_{s=p}$ denote the residue operator at the pole $s = p$ ([6]). From Appendix III, $f_o(x)$ can be expressed as follows:

$$f_o(x) = -\eta \sum_{p\in\sigma_p} \mathrm{Res}_{s=p} \frac{f^*(s)e^{sx}}{s(1-e^{sI})}. \tag{2}$$

Let N_o be the number of outstanding calls as defined in the last section and t_{tot} the total calling time of outstanding calls, let $1/\mu$ denote the average call holding time of the outstanding calls. We can obtain (Appendix III)

$$E[N_o] = -\lambda\eta \sum_{p\in\sigma_p} \mathrm{Res}_{s=p} \frac{[1+(sI-1)e^{sI}]f^*(s)}{s^3(1-e^{sI})} \tag{3}$$

$$E[t_{\mathrm{tot}}] = -\frac{\lambda\eta}{\mu} \sum_{p\in\sigma_p} \mathrm{Res}_{s=p} \frac{[1+(sI-1)e^{sI}]f^*(s)}{s^3(1-e^{sI})}. \tag{4}$$

When the VS residence time is Erlang distributed, simple analytic results can be obtained. For simplicity, let

$$g(s;x) = \frac{e^{sx}}{s(1-e^{sI})} \qquad h(s) = \frac{1+(sI-1)e^{sI}}{s^3(1-e^{sI})}.$$

Let $\alpha = m\eta$, and then we obtain (Appendix III)

$$f_o(x) = -\frac{\eta\alpha^m}{(m-1)!} \frac{\partial^{m-1}}{\partial s^{m-1}} g(s;x)\bigg|_{s=-\alpha} \tag{5}$$

$$E(N_o) = -\frac{\lambda\eta\alpha^m}{(m-1)!} h^{(m-1)}(-\alpha) \tag{6}$$

$$E(t_{\mathrm{tot}}) = -\frac{\lambda\eta\alpha^m}{(m-1)!\mu} h^{(m-1)}(-\alpha). \tag{7}$$

In particular, when $m = 1$, i.e., the VS residence time is exponentially distributed with parameter η, we have

$$f_o(x) = \frac{\eta e^{-\eta x}}{1-e^{-\eta I}}$$
$$E(N_o) = \frac{\lambda}{\eta} \cdot \frac{1-(1+\eta I)e^{-\eta I}}{1-e^{-\eta I}}$$
$$E(t_{\mathrm{tot}}) = \frac{\lambda}{\eta\mu} \cdot \frac{1-(1+\eta I)e^{-\eta I}}{1-e^{-\eta I}}. \tag{8}$$

When $m = 2$, we have (Appendix III) (9), given at the bottom of the page. It is not difficult to show that when η is sufficiently small, the density function $f_o(x)$ for either $m = 1$ or $m = 2$ approaches $1/I$, which implies that the outstanding time is uniformly distributed when the roamer stays in the visited area for sufficiently long. We conjecture that this is true for any nonlattice distributed VS residence time.

C. Checkpointing Model Based on Number of Calls and Real-Time Interval

Another strategy is to use both the number of calls and real-time interval for updating billing record. The thresholds used in this case (n and I) may be less restricted (larger) than those in previous strategies. In this strategy, the billing record update works as follows. Choose threshold n for the number of calls and threshold I for the real-time interval. The billing record at the VS keeps the records of both time, say, t_3 and the number of calls, say, n_3. When a customer enters a visited area, the billing record for it starts, and setting $t_3 = 0$ and $n_3 = 0$, the clock for t_3 starts. If there are calls from or to the customer, n_3 is incremented by one. Whenever either t_3 or n_3 reaches the threshold, the billing record is forwarded to the HS at the same time t_3 and n_3 are reset to zeros and starts over again if the customer is still in the VS area. The methods used in previous two sections can be modified to analyze the performance of this model.

$$f_o(x) = \frac{\eta[(1+2\eta x)(1-e^{-2\eta I}) + 2\eta I e^{-2\eta I}]}{(1-e^{-2\eta I})^2} e^{-2\eta x}$$
$$E(N_o) = \frac{\lambda\{[1-(1+2\eta I)e^{-2\eta I}][3(1-e^{-2\eta I}) + 2\eta I e^{-2\eta I}] - (2\eta I)^2(1-e^{-2\eta I})e^{-2\eta I}\}}{4\eta(1-e^{-2\eta I})^2}$$
$$E(t_{\mathrm{tot}}) = \frac{\lambda\{[1-(1+2\eta I)e^{-2\eta I}][3(1-e^{-2\eta I}) + 2\eta I e^{-2\eta I}] - (2\eta I)^2(1-e^{-2\eta I})e^{-2\eta I}\}}{4\eta\mu(1-e^{-2\eta I})^2} \tag{9}$$

D. How to Choose the Thresholds

In the above strategies, we need to determine how often the customer's billing record should be updated, i.e., how to choose the thresholds, since if the billing record is forwarded from VS to the HS one by one in real time, the signaling channel may be congested.

To this end we assume that the call arrivals of a customer have Poisson distribution with parameter λ and let $\tau_1, \tau_2, \cdots$ denote the interarrival times whose expected value is $1/\lambda$.

We consider the first strategy. In this strategy, the number of outstanding calls is related to the call arrival process. Suppose that the billing system may tolerate the outstandingness of time for a period of $T_h > 0$, which is determined by the PCS network design objective and the availability of the bandwidth for signaling channel. Then the total time that n call arrives is given by

$$T_{\text{tot}} = \tau_1 + \tau_2 + \cdots + \tau_n$$

and we need $E[T_{\text{tot}}] \le T_h$, which is equivalent to

$$n \le \lambda T_h.$$

So the threshold can be chosen as the integral part of the positive number λT_h.

In the second strategy, the billing system will be tolerant of a fixed number of outstanding calls. This is interpreted as the *probability of more than, say, K calls in the interval of length I is less than a preassigned number, say, θ* $(0 < \theta < 1)$. Suppose that there are N calls in the updating interval with length I, then the above criterion can be translated into the following:

$$\Pr(N > K) = \sum_{i=K+1}^{\infty} \Pr(N = i) = \sum_{i=K+1}^{\infty} \frac{(\lambda I)^i}{i!} e^{-\lambda I} < \theta$$

i.e.,

$$p(I) \triangleq \sum_{i=0}^{K} \frac{(\lambda I)^i}{i!} e^{-\lambda I} \ge 1 - \theta. \tag{10}$$

Notice that since $(dp(I)/dI) = -((\lambda I)^K / K!)\lambda e^{-\lambda I} < 0$ for any $I > 0$, so $p(I)$ is a strictly decreasing function of I. Also, $p(0) = 1$ and $p(+\infty) = 0$, and then for any $0 \le \theta < 1$, we can always find solution for (10). For very small T, (10) is always true, but the small value for I may not be desirable due to the channel traffic intensity requirement. The best value would be the largest I that satisfies (10). It is obvious that there is a unique solution for this value, which is denoted by $T_{\max}$. We can choose $I = T_{\max}$ in the second strategy. The thresholds in the third strategy can use either (T_h, K), a conservative choice, or (I, n) from the previous two strategies based on the value (T_h, K).

III. Illustrative Examples

We first consider the model based on the number of outstanding calls. Based on (1), Fig. 3 illustrates the effect of the Erlang shape parameter m on the k-call outstanding probability $\Pr_{(m,\alpha)}[k]$. In the figures, the mean VS residence time is $E[T] = (1/\eta)$. $E[T]$ can be several hours, days, or months. For simplicity, the call arrival rate λ is normalized by η in our numerical examples. Each figure plots the $\Pr[k]$ curves for $m = 1,\ 2$, and 3 (i.e., $\alpha = \eta, 2\eta$, and 3η) and $n = 5,\ 10$, and $\lambda = 5\eta,\ 50\eta$. That is, we consider the Erlang VS residence times T with the same mean $1/\eta$, but different variances $1/\eta^2$, $1/(2\eta^2)$, and $1/(3\eta^2)$, respectively. The figures indicate that the shape parameter (or the variance) of the Erlang VS residence times do not have significant effect on the k-call outstanding probability. This result is true for different n values and for large λ values. Figs. 4 and 5 plot $\Pr[k]$ for different λ values, where $m = 1$ and $n = 5$ and 10, respectively. These figures indicate that

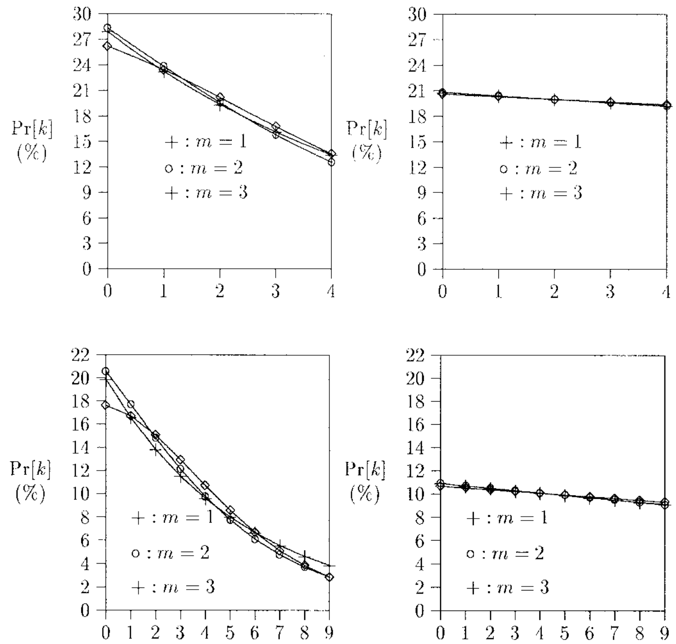

Fig. 3. Effects of m on $\Pr[k]$.

$$\Pr[k] > \Pr[k+1], \qquad \text{for } 0 \le k < n-1. \tag{11}$$

Let $\rho = \lambda/\eta$ be the call to mobility ratio. When $\rho < 3n$, $\Pr[k]$ may significantly larger than $\Pr[k+1]$. On the other hand, if $\rho \gg n$, then $\Pr[k] \simeq \Pr[k+1]$. This phenomenon is explained as follows. In the period T, the last checkpointing is performed when the roamer moves out of the VS, and no more than n call records are sent back to the HS. Thus, if a billing retrieval falls in this last checkpointing period, it is likely to have a small k-call outstanding observation. For the other "normal" n-checkpointing intervals, $\Pr[k]$ tends to be uniformly distributed (because the billing retrieval is a random observer). When $\rho \gg n$, the end effect of the last checkpointing interval becomes insignificant and $\Pr[k] \simeq \Pr[k+1]$. On the other hand, when ρ is not much larger than n, the end effect results in large $\Pr[k]$ for a small k. Note that the situations of small ρ are often observed in the existing system [1], and the end effect cannot be ignored. Suppose that checkpointing is performed for every n phone calls (i.e., the checkpointing interval is n). Define $\theta_{k,n}$ as the probability that the retrieval is less than k-call outstanding and N_n as the number of checkpointing operations performed during T. In

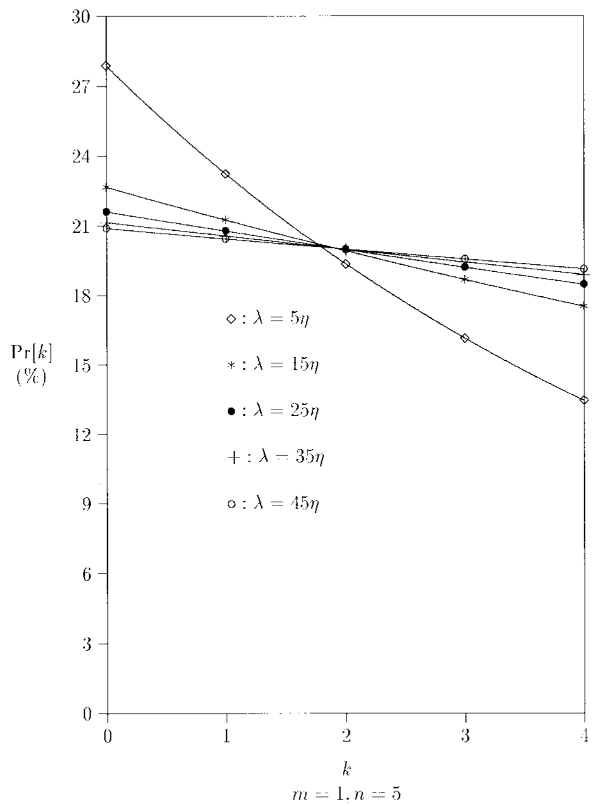

Fig. 4. Effects of λ on $\Pr[k]$ $(n = 5)$.

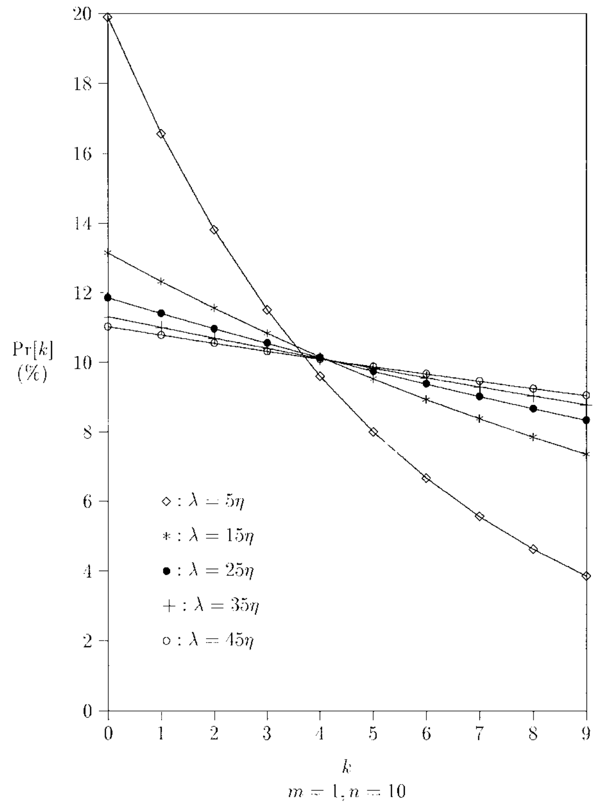

Fig. 5. Effects of λ on $\Pr[k]$ $(n = 10)$.

cellular network engineering, maximizing $\theta_{k,n}$ and minimizing N_n are two conflict goals.

It is easy to derive that

$$\theta_{k,n} = \left[1 - \left(\frac{\lambda}{\lambda+\eta}\right)^{k+1}\right]\left[1 - \left(\frac{\lambda}{\lambda+\eta}\right)^{n}\right]^{-1}$$

for $m = 1$. From (27) in Appendix II

$$E[N_n] = \left(\frac{\lambda}{\lambda+\eta}\right)^{n}\left[1 - \left(\frac{\lambda}{\lambda+\eta}\right)^{n}\right]^{-1}$$

form $m = 1$. Figs. 6 and 7 plot $\theta_{2,n}$ and $E[N_n]$ for $\rho = 5$ and 45, respectively. From these two figures, several engineering questions can be answered. For example, when $\rho = 5$, if n is changed from 12 to 8, then $\theta_{2,n}$ is increased by 16% and $E[N_n]$ is increased by 130%. With appropriate weighting factors specific to the network under study, one can use the above data to determine whether it is beneficial to change n from 12 to 8. Note that $\theta_{2,n}$ is more sensitive to the change of n when ρ is large than when ρ is small. On the other hand, $E[N_n]$ is more sensitive to the change of n when ρ is small than when ρ is large. When $\rho = 45$, if n is changed from 12 to 8, then $\theta_{2,n}$ is increased by 44%, and $E[N_n]$ is increased by 57%. Thus, it is more cost effective to decrease n when ρ is large than when ρ is small.

Next, we consider the billing model based on the real-time interval. Fig. 8 illustrates the density function for different the average VS residence time $1/\eta$ for the case when the VS residence time is exponentially distributed. As in the previous case, the density function is a decreasing function of time elapsed from the last updating. When the roamer stays sufficiently long at that visited area, the outstanding time is uniformly distributed. When the roamer stays sufficiently short at the visited area, the outstanding time is approximately exponentially distributed.

Fig. 9 shows the expected number of outstanding calls for different call arrival rate λ. As expected, the expected number of outstanding calls is decreasing as η is increasing, which implies that the shorter the roamer stays, the fewer the outstanding calls. The expected number of outstanding calls is also decreasing as the call arrival rate λ is decreasing, i.e., the fewer the arrival calls, the fewer the outstanding calls. These observations are consistent with our intuition.

Fig. 10 shows the density function of outstanding time is not much affected by the variance of the VS residence time (which is uniquely determined by m).

IV. Conclusions

This paper has proposed a number of strategies for expediting billing record updates in the PCS networks. These strategies have been designed as a compromise between the requirements of traffic for network signaling and the timeliness of the roamer's billing record (i.e., the accuracy of the billing information) at the HS. Performance analysis was performed for all proposed strategies and was shown to be useful in

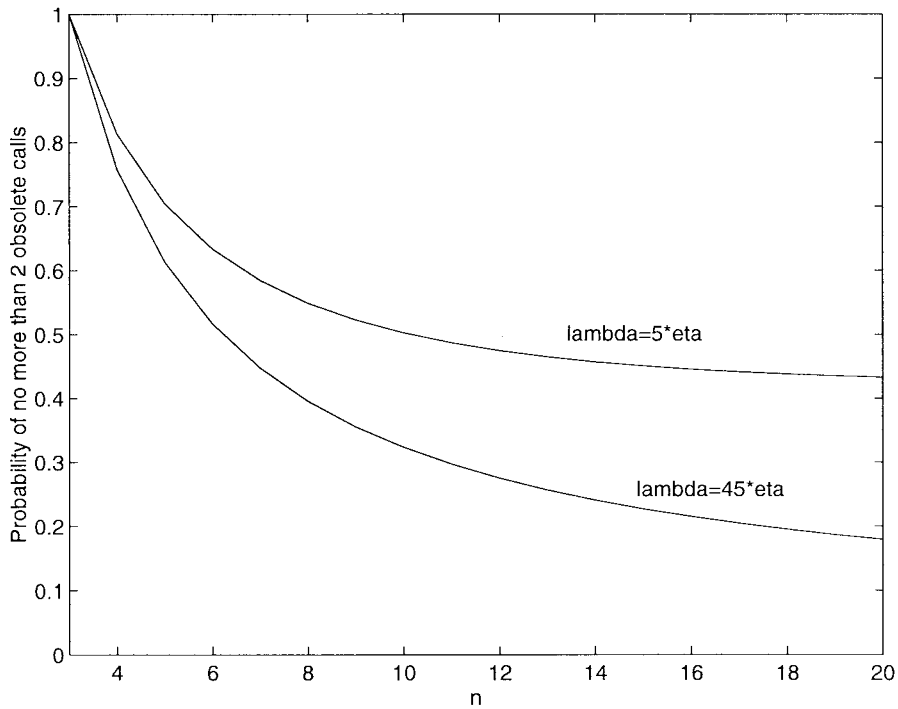

Fig. 6. The effect of n on $\theta_{2,n}$ $(m = 1)$.

optimizing the strategies performance by proper setting of timing thresholds to balance control traffic overhead and the usefulness of the billing process. We therefore believe that the results presented here can be used in the future design of billing systems for PCS networks.

APPENDIX I

Consider Fig. 1. Assume that T has a general density function $f(T)$ with the Laplace transform $f^*(s) = \int_{T=0}^{\infty} f(T)e^{-sT}\, dT$ and the mean $E[T] = 1/\eta$. Let the calls to the roamer be a Poisson process with arrival rate λ and the billing retrieval point be a random observer. Let $r(t)$ and $r^*(s)$ be the density function and the Laplace transform of the interval t in Fig. 1. If $f(T)$ is nonlattice, then from the residual life theorem [10] we have

$$r(t) = \eta \int_{\tau=t}^{\infty} f(\tau)\, d\tau \qquad \text{and} \qquad r^*(s) = \frac{\eta}{s}[1 - f^*(s)]. \tag{12}$$

The billing retrieval is k-call outstanding if there are $K = in + k$ $(0 \le k < n, i \ge 0)$ call arrivals during the period t. Since the call arrivals during the period t are a Poisson process, the k-call outstanding probability $\Pr[k]$ is expressed as

$$\begin{aligned}\Pr[k] &= \int_{t=0}^{\infty} \sum_{i=0}^{\infty} \frac{(\lambda t)^{in+k}}{(in+k)!} e^{-\lambda t} r(t)\, dt \\ &= \sum_{i=0}^{\infty} \frac{\lambda^{in+k}}{(in+k)!} \int_{t=0}^{\infty} t^{in+k} r(t) e^{-\lambda t}\, dt \\ &= \sum_{i=0}^{\infty} \frac{(-\lambda)^{in+k}}{(in+k)!} \left[\frac{d^{in+k} r^*(s)}{ds^{in+k}}\right]\Bigg|_{s=\lambda}. \end{aligned} \tag{13}$$

From (12), (13) is rewritten as

$$\begin{aligned}\Pr[k] &= \sum_{i=0}^{\infty} \frac{\eta(-\lambda)^{in+k}}{(in+k)!} \\ &\quad \cdot \left\{ \frac{d^{in+k}}{ds^{in+k}}\left(\frac{1}{s}\right) - \frac{d^{in+k}}{ds^{in+k}}\left[\frac{f^*(s)}{s}\right]\right\}\Bigg|_{s=\lambda} \\ &= \sum_{i=0}^{\infty} \frac{\eta(-\lambda)^{in+k}}{(in+k)!}[A - B]|_{s=\lambda} \end{aligned} \tag{14}$$

where

$$A = \frac{d^{in+k}}{ds^{in+k}}\left(\frac{1}{s}\right) = -\frac{(in+k)!}{(-s)^{in+k+1}} \tag{15}$$

and

$$B = \frac{d^{in+k}}{ds^{in+k}}\left[\frac{f^*(s)}{s}\right]. \tag{16}$$

For two functions $u(s)$ and $v(s)$, we have

$$\frac{d^n[u(s)v(s)]}{ds^n} = \sum_{j=0}^{n} \binom{n}{j} \left[\frac{d^j u(s)}{ds^j}\right]\left[\frac{d^{n-j} v(s)}{ds^{n-j}}\right]$$

and letting $u(s) = (1/s)$ and $v(s) = f^*(s)$, (16) is rewritten as

$$B = -\sum_{j=0}^{in+k} \binom{in+k}{j} \left[\frac{j!}{(-s)^{j+1}}\right]\left[\frac{d^{in+k-j} f^*(s)}{ds^{in+k-j}}\right]. \tag{17}$$

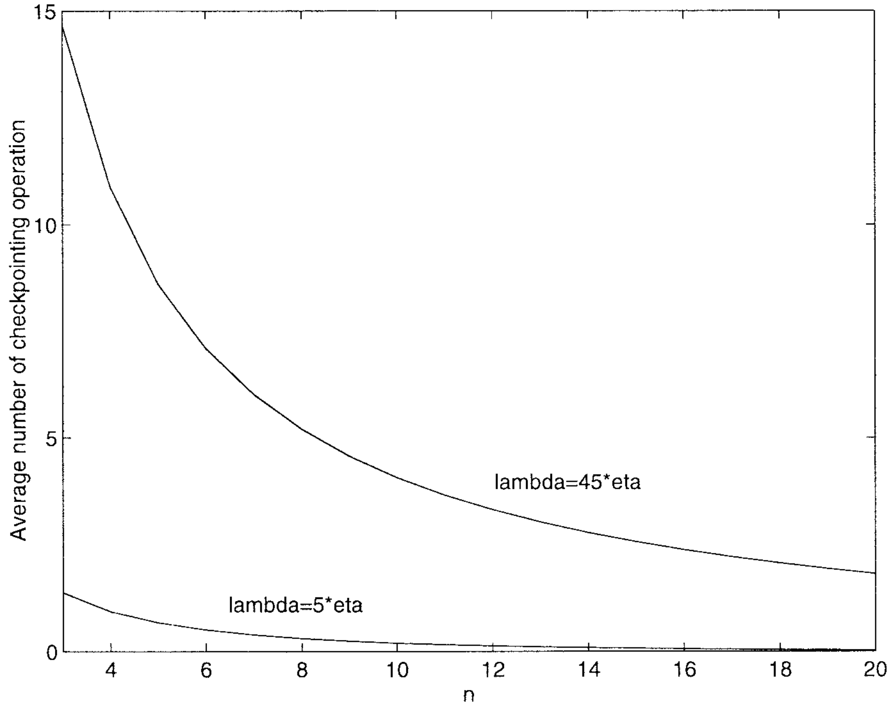

Fig. 7. The effect of n on $E[N_n]$ $(m = 1)$.

From (14), (15), and (17), we have

$$\begin{aligned}\Pr[k] &= \sum_{i=0}^{\infty} \frac{\eta(-1)^{in+k+1}\lambda^{in+k}}{(in+k)!} \\ &\quad \times \left\{ \frac{(in+k)!}{(-s)^{in+k+1}} - \sum_{j=0}^{in+k} \binom{in+k}{j} \right. \\ &\quad \left. \cdot \left[\frac{j!}{(-s)^{j+1}}\right]\left[\frac{d^{in+k-j} f^*(s)}{ds^{in+k-j}}\right] \right\}\Bigg|_{s=\lambda} \\ &= \sum_{i=0}^{\infty} \eta \left\{ \left(\frac{\lambda^{in+k}}{s^{in+k+1}}\right) - \sum_{j=0}^{in+k} (-1)^{in+k-j} \right. \\ &\quad \left. \cdot \left[\frac{\lambda^{in+k}}{(in+k-j)!s^{j+1}}\right]\left[\frac{d^{in+k-j} f^*(s)}{ds^{in+k-j}}\right] \right\}\Bigg|_{s=\lambda} \\ &= \sum_{i=0}^{\infty} \eta \left\{ \left(\frac{1}{\lambda}\right) - \frac{1}{\lambda} \sum_{j=0}^{in+k} (-1)^{in+k-j} \right. \\ &\quad \left. \cdot \left[\frac{\lambda^{in+k-j}}{(in+k-j)!}\right]\left[\frac{d^{in+k-j} f^*(s)}{ds^{in+k-j}}\right]\Bigg|_{s=\lambda} \right\} \\ &= \sum_{i=0}^{\infty} \eta \left\{ \left(\frac{1}{\lambda}\right) - \frac{1}{\lambda} \sum_{j=0}^{in+k} (-1)^{j} \left[\frac{\lambda^j}{j!}\right] f^{*(j)}(\lambda) \right\} \\ &= \frac{\eta}{\lambda} \sum_{i=0}^{\infty} \left[1 - \sum_{j=0}^{in+k} \frac{(-\lambda)^j}{j!} f^{*(j)}(\lambda) \right].\end{aligned}$$

From this, we can develop an algorithm to compute the probability $\Pr[k]$. Since $f^*(s)$ is an analytic function at $s = \lambda$, hence

$$f^*(s) = \sum_{j=0}^{\infty} \frac{f^{*(j)}(\lambda)}{j!}(s-\lambda)^j$$

$$1 = f^*(0) = \sum_{j=0}^{\infty} \frac{f^{*(j)}(\lambda)}{j!}(-\lambda)^j$$

hence from (18), we obtain

$$\begin{aligned}\Pr[k] &= \frac{\eta}{\lambda} \sum_{i=0}^{\infty} \sum_{j=in+k+1}^{\infty} \frac{(-\lambda)^j}{j!} f^{*(j)}(\lambda) \\ &= \frac{\eta}{\lambda} \sum_{i=0}^{\infty} (i+1) \left(\sum_{j=in+k+1}^{(i+1)n+k} \frac{(-\lambda)^j}{j!} f^{*(j)}(\lambda) \right). \qquad (18)\end{aligned}$$

Thus, we first expand the $f^*(s)$ at $s = \lambda$ as a Taylor series, using this to express $f^*(0)$ as $f^*(0) = \Sigma_{i=0}^{\infty} \alpha_j$ and then using the sequence $\{\alpha_j\}$ and (18) to compute $w_i = \Sigma_{j=in+k+1}^{(i+1)n+k} \alpha_j$, hence, $\Pr[k]$ can be computed by $\Pr[k] = \Sigma_{i=0}^{\infty}(i+1)w_i$. This algorithm is numerically easy to implement.

For special VS residence time distributions, an analytical expression may be obtained. For example, if the VS residence times have an Erlang distribution with the shape parameter m and the scale parameter $\alpha = m\eta$, then

$$f(t) = \frac{(\alpha t)^{m-1}}{(m-1)!}\alpha e^{-\alpha t} \qquad \text{and} \qquad f^*(s) = \left(\frac{\alpha}{s+\alpha}\right)^m. \qquad (19)$$

Denote $\Pr_{(m,\alpha)}[k]$ as the probability $\Pr[k]$ when the Erlang VS residence distribution has the shape parameter m and the scale parameter α. Substitute (19) in (18) and after careful rearrangement using the combinatorial identity techniques

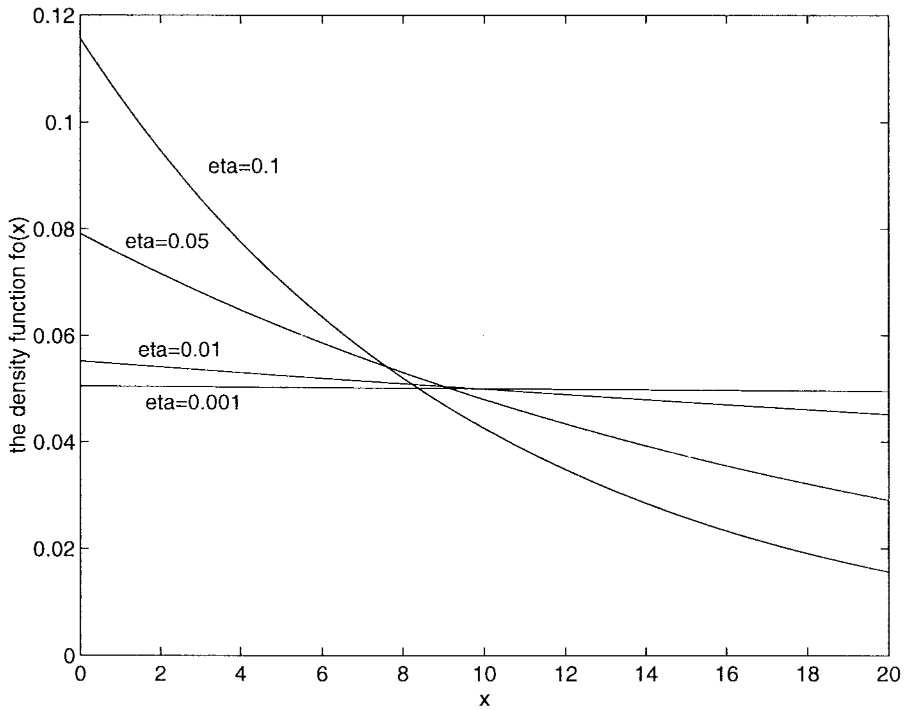

Fig. 8. The density function of outstanding time for different η $(m = 1)$.

[13], we have

$$\Pr_{(m,\alpha)}[k] = \sum_{i=0}^{\infty}\sum_{l=0}^{m-1} \eta \binom{in+k+l}{l} \frac{\lambda^{in+k}\alpha^{l}}{(\lambda+\alpha)^{in+k+l+1}}. \tag{20}$$

Equation (20) can be presented in a recursive format as (21), given at the bottom of the page. For $m = 1$, we have

$$\begin{aligned}\Pr_{(1,\alpha)}[k] &= \sum_{i=0}^{\infty} \frac{\eta\lambda^{in+k}}{(\lambda+\alpha)^{in+k+1}}\\ &= \left(\frac{\eta}{\lambda+\alpha}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^{k}\left\{\sum_{i=0}^{\infty}\left[\left(\frac{\lambda}{\lambda+\alpha}\right)^{n}\right]^{i}\right\}\\ &= \left(\frac{\eta}{\lambda+\alpha}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^{k}\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{n}\right]^{-1}\\ &\qquad (\alpha = \eta).\end{aligned} \tag{22}$$

For $m = 2$, we have

$$\begin{aligned}\Pr_{(2,\alpha)}[k] &= \sum_{i=0}^{\infty} \frac{\eta\alpha(in+k+1)\lambda^{in+k}}{(\lambda+\alpha)^{in+k+2}} + \Pr_{(1,\alpha)}[k]\\ &= \sum_{i=0}^{\infty} \frac{in\eta\alpha\lambda^{in+k}}{(\lambda+\alpha)^{in+k+2}}\\ &\quad + \sum_{i=0}^{\infty} \frac{(k+1)\eta\alpha\lambda^{in+k}}{(\lambda+\alpha)^{in+k+2}} + \Pr_{(1,\alpha)}[k].\end{aligned} \tag{23}$$

Since

$$\begin{aligned}&\sum_{i=0}^{\infty} \frac{in\eta\alpha\lambda^{in+k}}{(\lambda+\alpha)^{in+k+2}}\\ &\quad = \frac{n\eta\alpha\lambda^{k}}{(\lambda+\alpha)^{k+2}}\left\{\sum_{i=0}^{\infty} i\left[\left(\frac{\lambda}{\lambda+\alpha}\right)\right]^{i}\right\}\\ &\quad = \left[\frac{n\eta\alpha\lambda^{k}}{(\lambda+\alpha)^{k+2}}\right]\left(\frac{\lambda}{\lambda+\alpha}\right)^{n}\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{n}\right]^{-2}\\ &\quad = \left(\frac{n\alpha}{\lambda+\alpha}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^{n}\\ &\qquad \cdot \left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{n}\right]^{-1} \Pr_{(1,\alpha)}[k]\end{aligned}$$

and

$$\begin{aligned}&\sum_{i=0}^{\infty} \frac{(k+1)\eta\alpha\lambda^{in+k}}{(\lambda+\alpha)^{in+k+2}}\\ &\quad = \frac{(k+1)\eta\alpha\lambda^{k}}{(\lambda+\alpha)^{k+2}}\left\{\sum_{i=0}^{\infty}\left[\left(\frac{\lambda}{\lambda+\alpha}\right)^{n}\right]^{i}\right\}\\ &\quad = \left[\frac{(k+1)\eta\alpha\lambda^{k}}{(\lambda+\alpha)^{k+2}}\right]\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{n}\right]^{-1}\\ &\quad = \left[\frac{(k+1)\alpha}{\lambda+\alpha}\right]\Pr_{(1,\alpha)}[k].\end{aligned}$$

$$\Pr_{(m,\alpha)}[k] = \begin{cases} \displaystyle\sum_{i=0}^{\infty}\eta\binom{in+k+m-1}{m-1}\frac{\lambda^{in+k}\alpha^{m-1}}{(\lambda+\alpha)^{in+k+m}} + \Pr_{(m-1,\alpha)}[k], & \text{for } m \ge 1\\ 0, & \text{for } m = 0\end{cases} \tag{21}$$

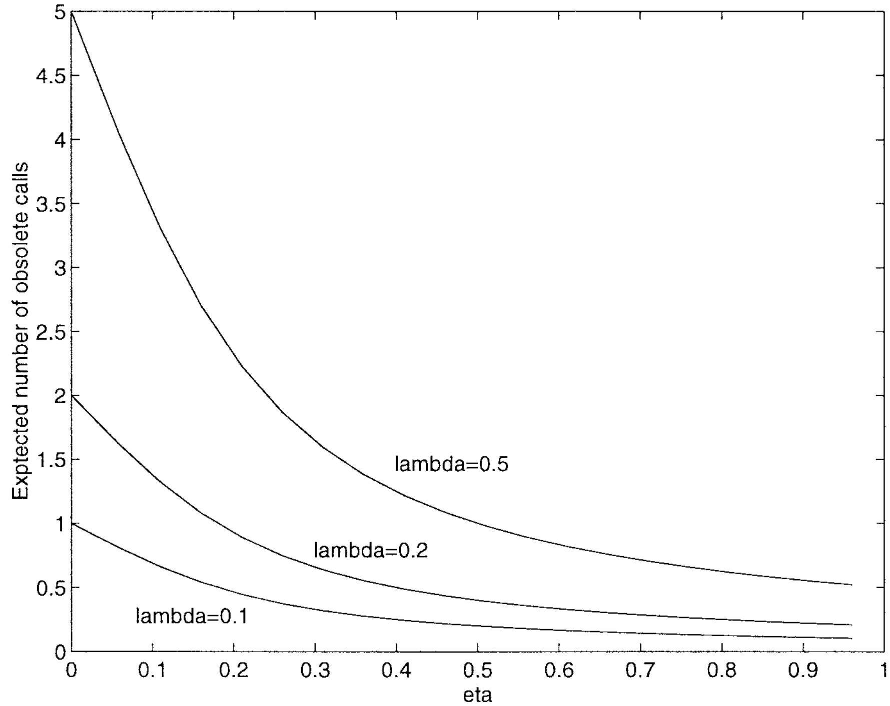

Fig. 9. The expected number of outstanding calls for different arrival rate $(m = 1)$.

Equation (23) is rewritten as

$$\Pr_{(2,\alpha)}[k] = \left\{ \left(\frac{n\alpha}{\lambda+\alpha}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^n \left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^n\right]^{-1} + \left[\frac{(k+1)\alpha}{\lambda+\alpha}\right] + 1 \right\} \Pr_{(1,\alpha)}[k] \qquad (\alpha = 2\eta). \tag{24}$$

For $m = 3$

$$\begin{aligned} \Pr_{(3,\alpha)}[k] &= \sum_{i=0}^{\infty} \eta(in+k+2)(in+k+1) \cdot \left[\frac{\lambda^{in+k}\alpha^2}{(\lambda+\alpha)^{in+k+3}}\right] + \Pr_{(2,\alpha)}[k] \\ &= \left[\frac{\eta\lambda^k\alpha^2}{(\lambda+\alpha)^{k+3}}\right] \sum_{i=0}^{\infty} [i^2 n^2 + (2k+3)in + (k+2)(k+1)] \left[\left(\frac{\lambda}{\lambda+\alpha}\right)^n\right]^i \\ &\quad + \Pr_{(2,\alpha)}[k]. \end{aligned} \tag{25}$$

Since

$$\sum_{i=0}^{\infty} i^2 x^i = \frac{x(1+x)}{(1-x)^3}$$

(25) is rewritten as

$$\begin{aligned} \Pr_{(3,\alpha)}[k] &= \frac{\eta\lambda^k\alpha^2}{(\lambda+\alpha)^{k+3}} \times \left\{ n^2 \left(\frac{\lambda}{\lambda+\alpha}\right)^n \left[1+\left(\frac{\lambda}{\lambda+\alpha}\right)^n\right] \cdot \left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^n\right]^{-3} \right. \\ &\quad + n(2k+3)\left(\frac{\lambda}{\lambda+\alpha}\right)^n \left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^n\right]^{-2} \\ &\quad \left. + (k+2)(k+1)\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^n\right]^{-1} \right\} \\ &\quad + \Pr_{(2,\alpha)}[k] \qquad (\alpha = 3\alpha). \end{aligned} \tag{26}$$

When the VS residence time is exponentially distributed, i.e., $m = 1$, we have

$$\begin{aligned} E[N_o] &= \sum_{k=0}^{n-1} k \Pr[k] \\ &= \sum_{k=0}^{n-1} \left(\frac{\eta}{\lambda+\eta}\right)\left(\frac{\lambda}{\lambda+\eta}\right)^k \left[1-\left(\frac{\lambda}{\lambda+\eta}\right)^n\right]^{-1} \\ &= \left(\frac{\eta}{\lambda+\eta}\right)\left[1-\left(\frac{\lambda}{\lambda+\eta}\right)^n\right]^{-1}\left(\frac{\lambda}{\lambda+\eta}\right) \cdot \sum_{k=1}^{k-1} k\left(\frac{\lambda}{\lambda+\eta}\right)^{k-1} \\ &= \frac{\lambda}{\eta}\left[1-\left(\frac{\lambda}{\lambda+\eta}\right)^n\right]^{-1} \cdot \left[1-n\left(\frac{\lambda}{\lambda+\eta}\right)^{n-1} + (n-1)\left(\frac{\lambda}{\lambda+\eta}\right)^n\right] \end{aligned}$$

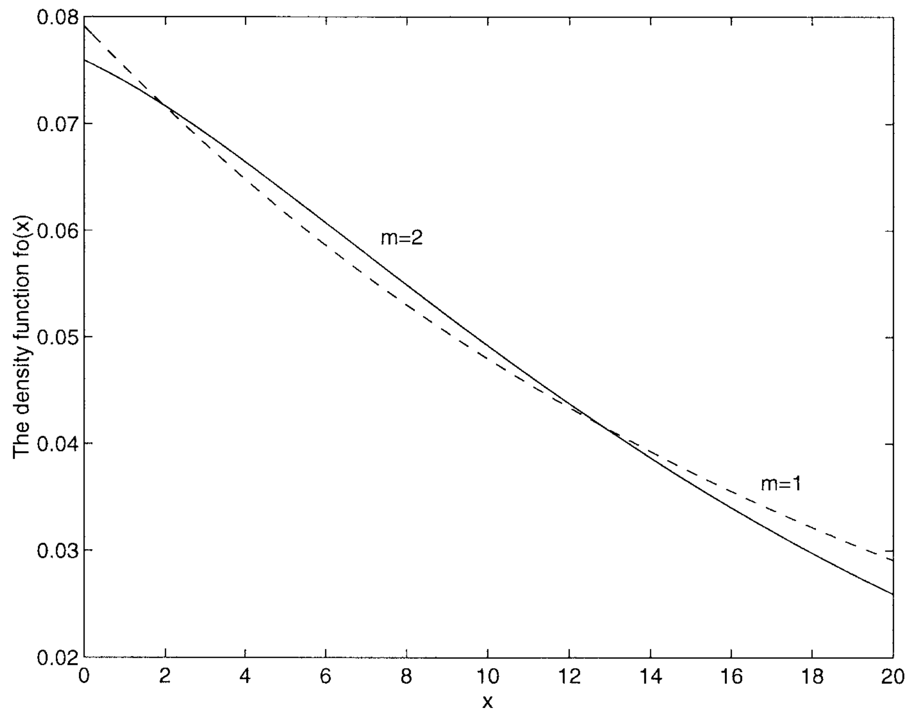

Fig. 10. The density function for different m.

and

$$E[t_{\rm tot}] = \frac{\lambda}{\eta\mu}\left[1-\left(\frac{\lambda}{\lambda+\eta}\right)^n\right]^{-1} \cdot \left[1-n\left(\frac{\lambda}{\lambda+\eta}\right)^{n-1}+(n-1)\left(\frac{\lambda}{\lambda+\eta}\right)^n\right].$$

APPENDIX II

Let $\beta[i]$ be the probability that there are i checkpointing operations during T. Then

$$\begin{aligned}\beta[i] &= \sum_{k=0}^{n-1}\int_{t=0}^{\infty}\frac{(\lambda t)^{in+k}}{(in+k)!}e^{-\lambda t}f(t)\,dt \\ &= \sum_{k=0}^{n-1}\frac{\lambda^{in+k}}{(in+k)!}\int_{t=0}^{\infty}t^{in+k}f(t)e^{-\lambda t}\,dt \\ &= \sum_{k=0}^{n-1}\frac{\lambda^{in+k}}{(in+k)!}(-1)^{in+k}\left[\frac{d^{in+k}f^*(s)}{ds^{in+k}}\right]\Bigg|_{s=\lambda}.\end{aligned}$$

If $f(t)$ is exponentially distributed, then we have

$$\begin{aligned}\beta[i] &= \sum_{k=0}^{n-1}\frac{\eta\lambda^{in+k}}{(in+k)!}\,\frac{(in+k)!}{(\lambda+\eta)^{in+k+1}} \\ &= \frac{\eta\lambda^{in}}{(\lambda+\eta)^{in+1}}\left[\sum_{k=0}^{n-1}\left(\frac{\lambda}{\lambda+\eta}\right)^k\right] \\ &= \left[1-\left(\frac{\lambda}{\lambda+\eta}\right)^n\right]\left[\left(\frac{\lambda}{\lambda+\eta}\right)^n\right]^i\end{aligned}$$

and

$$\begin{aligned}E[N_n] &= \sum_{i=0}^{\infty} i\beta[i] \\ &= \left[1-\left(\frac{\lambda}{\lambda+\eta}\right)^n\right]\sum_{i=0}^{\infty} i\left[\left(\frac{\lambda}{\lambda+\eta}\right)^n\right]^i \\ &= \left(\frac{\lambda}{\lambda+\eta}\right)^n\left[1-\left(\frac{\lambda}{\lambda+\eta}\right)^n\right]^{-1}. \qquad (27)\end{aligned}$$

APPENDIX III

As before, let T be the VS residence time and t be the billing information retrieval time instant. Let $f(t)$ and $F(t)$ be the density function and distribution function of T, respectively, and $f^*(s)$ denotes its Laplace transform. Let $r(t)$ and $r^*(s)$ be the density function and Laplace transform of the interval t. Let I denote the updating interval, i.e., the billing information will be forwarded from the VS to HS every I time units whenever the roamer is still in that visited area. Using the residual life theorem ([10]), we have

$$r(t) = \eta(1-F(t)) \quad \text{and} \quad r^*(s) = \frac{\eta}{s}(1-f^*(s)). \qquad (28)$$

Let t_o denote the time during which the outstanding calls may happen as shown in Fig. 2, i.e., the time between the last updating instant, say, iI and the checking point t, so

$$t_o = t - iI = t(\text{mod } I)$$

where $(\text{mod } I)$ here means the residue modulo I. Let $f_o(t)$ and $F_o(t)$ denote the density function and distribution function of t_o, respectively. Then for any $0 \le x \le I$, we have

$$\begin{aligned} F_o(t) &= \Pr(t_o \le x) = \Pr(t(\text{mod } I) \le x) \\ &= \sum_{i=0}^{\infty} \Pr(t(\text{mod } I) \le x, iI \le t \le (i+1)I) \\ &= \sum_{i=0}^{\infty} \Pr(t - iI \le x, iI \le t \le (i+1)I) \\ &= \sum_{i=0}^{\infty} \Pr(iI \le t \le iI + x) \\ &= \sum_{i=0}^{\infty} \int_{iI}^{iI+x} r(t)\, dt = \sum_{i=0}^{\infty} \eta \int_{iI}^{iI+x} (1 - F(t))\, dt \end{aligned}$$

so the density function of t_o is given by

$$f_o(x) = \sum_{i=0}^{\infty} r(iI + x) = \eta \sum_{i=0}^{\infty} [1 - F(iI + x)]. \tag{29}$$

Suppose that the Laplace transform of the VS residence time $f^*(s)$ has only isolated singular points in the open left-half complex plane, let σ_p be the set of poles of $f^*(s)$. Let $\sigma < 0$ be such real number such that all poles have real parts smaller than σ. It is obvious that $r^*(s)$ and $f^*(s)$ have the same poles. From the inverse Laplace transform formula and the Residue Theorem ([6]), we have (the contour used in the Residue Theorem is chosen to be the domain in the left-half plane to the left of vertical line $\text{Re}(s) = \sigma$)

$$\begin{aligned} f_o(x) &= \sum_{i=0}^{\infty} \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} r^*(s) e^{s(iI+x)}\, ds \\ &= \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} r^*(s) \left(\sum_{0}^{\infty} e^{iIs} \right) e^{sx}\, ds \\ &= \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{r^*(s) e^{sx}}{1 - e^{sI}}\, ds \\ &= \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{\eta(1 - f^*(s)) e^{sx}}{s(1 - e^{sI})}\, ds \\ &= \sum_{p \in \sigma_p} \text{Res}_{s=p} \frac{\eta(1 - f^*(s)) e^{sx}}{s(1 - e^{sI})} \\ &= -\eta \sum_{p \in \sigma_p} \text{Res}_{s=p} \frac{f^*(s) e^{sx}}{s(1 - e^{sI})} \end{aligned} \tag{30}$$

where $\text{Res}_{s=p}$ denotes the residue at the pole $s = p$.

Assume that the call arrivals for the roamer is Poisson with arrival rate λ. Let N_o denote the number of outstanding calls (the number of calls arriving after the last billing update instant and before the retrieval point, the billing records of these outstanding calls are not available at the checking point), and then the probability that there are k outstanding calls is given by

$$\begin{aligned} \Pr(N_o = k) &= \int_0^I \Pr(N_o = k | t_o = x) f_o(x)\, dx \\ &= \int_0^I \frac{(\lambda x)^k}{k!} e^{-\lambda x} f_o(x)\, dx. \end{aligned} \tag{31}$$

Following a similar procedure as in the derivation of (30), the expected number of outstanding calls is given by

$$\begin{aligned} E(N_o) &= \sum_{k=0}^{\infty} k \Pr(N_o = k) \\ &= \sum_{k=0}^{\infty} \int_0^I \left(k \frac{(\lambda x)^k}{k!} \right) e^{-\lambda x} f_o(x)\, ds \\ &= \int_0^I \lambda x f_o(x)\, dx = \lambda E(t_o) \\ &= \lambda \sum_{i=0}^{\infty} \int_0^I x r(iI + x)\, dx \\ &= \frac{\lambda}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} r^*(s) \int_0^I x e^{sx} \left(\sum_{i=0}^{\infty} e^{isI} \right) dx\, ds \\ &= \frac{\lambda}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} r^*(s) \int_0^I \frac{x e^{sx}}{1 - e^{sI}}\, dx\, ds \\ &= \frac{\lambda}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{r^*(s)[(-1 + sI)e^{sI} + 1]}{s^2(1 - e^{sI})}\, ds \\ &= \frac{\lambda}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{\eta(1 - f^*(s))[(-1 + sI)e^{sI} + 1]}{s^3(1 - e^{sI})}\, ds \\ &= \lambda\eta \sum_{p \in \sigma_p} \text{Res}_{s=p} \frac{(1 - f^*(s))[(-1 + sI)e^{sI} + 1]}{s^3(1 - e^{sI})} \\ &= -\lambda\eta \sum_{p \in \sigma_p} \text{Res}_{s=p} \frac{[1 + (sI - 1)e^{sI}] f^*(s)}{s^3(1 - e^{sI})}. \end{aligned} \tag{32}$$

Let $t_1, t_2, \cdots, t_k, \cdots$ be the call holding times of the roamer and assume that this random sequence is independently identically distributed with expected value $1/\mu$. Then the total outstanding calling time (the summation of total calling times for all outstanding calls) is given by

$$t_{\text{tot}} = \sum_{i=1}^{N_0} t_i. \tag{33}$$

$$\begin{aligned} E(N_o) &= -\lambda\eta(2\eta)^2 h^{(1)}(-2\eta) \\ &= \frac{\lambda\{[1 - (1 + 2\eta I)e^{-2\eta I}][3(1 - e^{-2\eta I}) + 2\eta I e^{-2\eta I}] - (2\eta I)^2 (1 - e^{-2\eta I}) e^{-2\eta I}\}}{4\eta(1 - e^{-2\eta I})^2}. \end{aligned}$$

$$E(t_{\text{tot}}) = \frac{\lambda\{[1 - (1 + 2\eta I)e^{-2\eta I}][3(1 - e^{-2\eta I}) + 2\eta I e^{-2\eta I}] - (2\eta I)^2 (1 - e^{-2\eta I}) e^{-2\eta I}\}}{4\eta\mu(1 - e^{-2\eta I})^2}.$$

From Wald's equation [14], we obtain the expected total outstanding time is given by

$$
\begin{aligned}
E(t_{\text{tot}}) &= E(N_o)E(t_i) \\
&= -\frac{\lambda\eta}{\mu}\sum_{p\in\sigma_p}\operatorname{Res}_{s=p}\frac{[1+(sI-1)e^{sI}]f^*(s)}{s^3(1-e^{sI})}. \quad (34)
\end{aligned}
$$

For simplicity, let

$$
g(s;x)=\frac{e^{sx}}{s(1-e^{sI})} \qquad h(s)=\frac{1+(sI-1)e^{sI}}{s^3(1-e^{sI})}. \quad (35)
$$

If the VS residence times have an Erlang distribution with the shape parameter m and the scale parameter $\alpha = m\eta$, then

$$
f^*(s)=\left(\frac{\alpha}{s+\alpha}\right)^m.
$$

This function has only one pole $s=-\alpha$, i.e., $\sigma_p=\{-\alpha\}$. From (30), (32), and (34), we obtain

$$
f_o(x) = -\frac{\eta\alpha^m}{(m-1)!}\frac{\partial^{m-1}}{\partial s^{m-1}}g(s;x)\bigg|_{s=-\alpha} \quad (36)
$$

$$
E(N_o) = -\frac{\lambda\eta\alpha^m}{(m-1)!}h^{(m-1)}(-\alpha) \quad (37)
$$

$$
E(t_{\text{tot}}) = -\frac{\lambda\eta\alpha^m}{(m-1)!\mu}h^{(m-1)}(-\alpha) \quad (38)
$$

where $h^{(i)}(z)$ denotes the ith order derivative of function $h(z)$ at point z.

In particular, when $m=1$, i.e., the VS residence times are exponentially distributed, and we have

$$
\begin{aligned}
f_o(x) &= \frac{\eta e^{-\eta x}}{1-e^{-\eta I}} \\
E(N_o) &= \frac{\lambda}{\eta}\cdot\frac{1-(1+\eta I)e^{-\eta I}}{1-e^{-\eta I}} \\
E(t_{\text{tot}}) &= \frac{\lambda}{\eta\mu}\cdot\frac{1-(1+\eta I)e^{-\eta I}}{1-e^{-\eta I}}.
\end{aligned}
$$

When $m=2$, the outstanding time distribution is given by

$$
\begin{aligned}
f_o(x) &= -\eta(2\eta)^2\frac{\partial}{\partial s}g(s;x)\bigg|_{s=-2\eta} \\
&= \frac{\eta[(1+2\eta x)(1-e^{-2\eta I})+2\eta I e^{-2\eta I}]}{(1-e^{-2\eta I})^2}e^{-2\eta x}.
\end{aligned}
$$

The expected number of outstanding calls and the total outstanding calling time are given by the equations at the bottom of the previous page, respectively.

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