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Reception of coherent signals with steering vector restoral beamformer

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Abstract

A beamforming scheme is proposed for reception of multiple coherent signals. The scheme consists of three stages. First, estimates of coherent source directions are used to restore the composite steering vector (CSV). Second, a transformation removes the coherent signals with the interference and noise retained. Finally, optimum beamforming is performed based on the CSV and transformed data to produce the maximum output signal-to-interference-plus-noise ratio (SINR). Numerical examples confirm the efficacy of the proposed beamformer in combining the coherent signals. © 1999 Elsevier Science B.V. All rights reserved.

Zusammenfassung

Dieser Beitrag stellt ein Beamforming Programm zum Empfang mehrerer kohärenter Signale vor. Das Programm umfaßt drei Stufen. Zuerst werden Schätzungen von Richtungen kohärenter Sender genutzt um den zusammengesetzten Richtungsvektor (CVS) wiederherzustellen. Im zweiten Schritt entfernt eine Transformation die kohärenten Signale, wobei die Störung und das Rauschen erhalten bleiben. Schließlich wird optimales Beamforming, basierend auf dem CVS und den transformierten Daten, eingesetzt, um das maximale Ausgangs Signal-zu-Störung-plus-Rausch-Verhältnis (SINR) zu erzeugen. Numerische Beispiele bestätigen die Wirksamkeit des vorgestellten Beamformers zur Verbindung kohärenter Signale. © 1999 Elsevier Science B.V. All rights reserved.

Résumé

Un schéma de formation de rayons est proposé pour la réception de signaux cohérents multiples. Le schéma consiste en trois étapes. Tout d'abord, des estimations des directions des sources cohérentes sont utilisées pour restaurer le vecteur directeur composite (CSF en anglais). Ensuite, une transformation retire les signaux cohérents, en retenant les interférences et le bruit. Finalement, une formation de rayon optimale est effectuée sur la base du CSV et des données transformées pour produire un rapport signal sur bruit plus interférence maximal en sortie. Des exemples numériques confirment l'efficacité de la formation de rayons proposée en combinant des signaux cohérents © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Beamforming; Array processing; Coherent signals

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Notations

θ_i direction of desired signal
$\hat{\theta}_i$ estimate of direction of desired signal
σ_d^2 desired signal power
σ_n^2 noise power
ρ_i complex amplitude of coherent signal
λ pseudo-noise power

1. Introduction

The minimum-variance distortionless response (MVDR) beamformer can achieve the maximum output signal-to-interference-plus-noise ratio (SINR) in the absence of coherent interference [4]. With coherent interference, the MVDR beamformer breaks down as a result of desired signal cancellation. To remedy this, one can generate 'hard nulls' in the directions-of-arrival (DOAs) of the coherent interferers [6]. However, in order to fully utilize the information of the coherent signals, a beamformer should 'constructively' combine these signals instead of cancelling all but one of them. For narrowband case, constructive combining can be done by working with the composite steering vector (CSV) associated with the coherent sources. The use of CSV has been discussed in the problem of blind beamforming [1].

This paper proposes a new beamformer working with the CSV. Specifically, a scheme is developed first to obtain an estimate of the CSV using the estimated coherent source DOAs. A transformation is then employed to remove the coherent signals, with the uncorrelated interference and noise retained as much as possible. The transformed array data are then sent to an MVDR beamformer with a gain constraint matched to the estimated CSV to compute the weight vector yielding the maximum output SINR. In order to lessen the sensitivity of the proposed method to model errors, high-order constraints and pseudo noise injection are incorporated to create a robust mode of operation.

2. Array model and beamforming issues

The scenario involves an M-element array and a desired group of J narrowband coherent sources from directions θ_i, $i = 1,\dots,J$. At the nth sampling instant, the array receives the data vector:

$$\boldsymbol{x}(n) = \boldsymbol{b}_d s_d(n) + \boldsymbol{i}(n) + \boldsymbol{n}(n), \tag{1}$$

where

$$\boldsymbol{b}_d = \sum_{i=1}^{J} \rho_i \boldsymbol{a}(\theta_i). \tag{2}$$

The scalar $s_d(n)$ represents the desired signal with $\sigma_d^2 = E\{|s_d(n)|^2\}$ being the signal power, and ρ_i's denote the complex amplitudes of the coherent signals. It is assumed that source 1 is the main source with $\rho_1 = 1$, and $|\rho_i| \leqslant 1$, $i = 2,\dots,J$. The vector $\boldsymbol{a}(\theta)$ is the steering vector of the array, $\boldsymbol{i}(n)$ is the interference vector, and $\boldsymbol{n}(n)$ is the noise vector whose entries are spatially white with power σ_n^2. Note that $\boldsymbol{b}_d$ is the CSV due to the J coherent sources.

A quantity frequently used in adaptive beamforming is the array data correlation matrix defined by

$$\boldsymbol{R} = E\{\boldsymbol{x}(n)\boldsymbol{x}^H(n)\} = \sigma_d^2 \boldsymbol{b}_d \boldsymbol{b}_d^H + \boldsymbol{R}_{in}, \tag{3}$$

where $(\cdot)^H$ denotes conjugate transposition, and $\boldsymbol{R}_{in} = E\{\boldsymbol{i}(n)\boldsymbol{i}^H(n)\} + \sigma_n^2 \boldsymbol{I}$ is the interference-plus-noise correlation matrix, with $\boldsymbol{I}$ being the identity matrix. For the coherent scenario, the output SINR of a beamformer using the weight vector $\boldsymbol{w}$ is defined by

$$\mathrm{SINR} = \frac{|\boldsymbol{w}^H \boldsymbol{b}_d|^2 \sigma_d^2}{\boldsymbol{w}^H \boldsymbol{R}_{in} \boldsymbol{w}}. \tag{4}$$

This expression indicates that the maximum output SINR can be achieved by the MVDR beamformer, which minimizes the output power subject to the constraint $\boldsymbol{w}^H \boldsymbol{b}_d = 1$ [1]. This leads to the well-known expression for the optimum weight vector:

$$\boldsymbol{w} = \boldsymbol{R}^{-1} \boldsymbol{b}_d. \tag{5}$$

In practice, the CSV $\boldsymbol{b}_d$ is not available, and needs to be estimated before beamforming.

3. Development of new beamformer

In the proposed beamforming scheme, the CSV is first estimated based on the use of a transformation $\boldsymbol{T}_1$ to eliminate $\boldsymbol{a}(\theta_i)$, $i = 2, \ldots, J$, i.e.,

$$\boldsymbol{T}_1 \boldsymbol{a}(\theta_i) = \boldsymbol{0}, \quad i = 2, \ldots, J, \tag{6}$$

where $\boldsymbol{0}$ is the zero vector. It follows from Eqs. (1) and (2) that

$$\boldsymbol{T}_1 \boldsymbol{x}(n) = \boldsymbol{T}_1 \boldsymbol{a}(\theta_1) s_{\mathrm{d}}(n) + \boldsymbol{T}_1 \boldsymbol{i}(n) + \boldsymbol{T}_1 \boldsymbol{n}(n). \tag{7}$$

Comparing this with Eq. (1) indicates that the CSV $\boldsymbol{b}_{\mathrm{d}}$ has been replaced by $\boldsymbol{T}_1 \boldsymbol{a}(\theta_1)$. As a result, if the error between $\boldsymbol{T}_1 \boldsymbol{x}(n)$ and $\boldsymbol{x}(n)$ is minimized, then we should have $\boldsymbol{T}_1 \boldsymbol{a}(\theta_1) \approx \boldsymbol{b}_{\mathrm{d}}$. Given the DOA estimates $\hat{\theta}_i$, $i = 1, \ldots, J$, a simple way of forcing $\boldsymbol{T}_1 \boldsymbol{x}(n) \approx \boldsymbol{x}(n)$, under the constraints of Eq. (6), is to solve the mean-square error problem

$$\min_{\boldsymbol{T}_1} E\{\|\boldsymbol{T}_1 \boldsymbol{x}(n) - \boldsymbol{x}(n)\|^2\} \equiv \mathrm{tr}\{(\boldsymbol{T}_1 - \boldsymbol{I})\boldsymbol{R}(\boldsymbol{T}_1 - \boldsymbol{I})^{\mathrm{H}}\}$$
$$\text{subject to} \quad \boldsymbol{T}_1 \hat{\boldsymbol{A}}_{\mathrm{c}} = \boldsymbol{O}, \tag{8}$$

where $\|\cdot\|$ denotes the Euclidean norm, tr denotes the trace, $\boldsymbol{O}$ is the zero matrix, and $\hat{\boldsymbol{A}}_{\mathrm{c}} = [\boldsymbol{a}(\hat{\theta}_2), \boldsymbol{a}(\hat{\theta}_3), \ldots, \boldsymbol{a}(\hat{\theta}_J)]$. Some matrix algebra gives the solution [2]

$$\boldsymbol{T}_1 = \boldsymbol{I} - \hat{\boldsymbol{A}}_{\mathrm{c}}(\hat{\boldsymbol{A}}_{\mathrm{c}}^{\mathrm{H}} \boldsymbol{R}^{-1} \hat{\boldsymbol{A}}_{\mathrm{c}})^{-1} \hat{\boldsymbol{A}}_{\mathrm{c}}^{\mathrm{H}} \boldsymbol{R}^{-1}. \tag{9}$$

Finally, the CSV estimate is given by $\hat{\boldsymbol{b}}_{\mathrm{d}} = \boldsymbol{T}_1 \boldsymbol{a}(\hat{\theta}_1)$.

The quality of the CSV estimate can be examined by substituting Eqs. (1) and (7) in Eq. (8):

$$E\{\|\boldsymbol{T}_1 \boldsymbol{x}(n) - \boldsymbol{x}(n)\|^2\}$$
$$= \sigma_{\mathrm{d}}^2 \|\boldsymbol{T}_1 \boldsymbol{a}(\theta_1) - \boldsymbol{b}_{\mathrm{d}}\|^2 + \mathrm{tr}\{(\boldsymbol{T}_1 - \boldsymbol{I})\boldsymbol{R}_{\mathrm{in}}(\boldsymbol{T}_1 - \boldsymbol{I})^{\mathrm{H}}\}, \tag{10}$$

which indicates that $\boldsymbol{T}_1$ seeks a joint minimization of the two error terms. It is thus conceivable that a good CSV estimate can be obtained if σ_{d}^2 is comparable in size to the interference-plus-noise power. On the other hand, the constraints in Eq. (8) is effective only when the DOA estimates are accurate enough. Unfortunately, it is generally difficult to locate coherent sources accurately. A remedy to this would be to broaden the 'region of operation' of $\boldsymbol{T}_1$ by incorporating high-order derivative constraints [3]. In particular, with the Lth-order constraints incorporated, Eq. (6) should be extended to

$$\boldsymbol{T}_1 \boldsymbol{a}^{(k)}(\theta_i) = \boldsymbol{0}, \quad i = 2, \ldots, J, \quad k = 0, \ldots, L, \tag{11}$$

where $\boldsymbol{a}^{(k)}(\theta) = \partial^k \boldsymbol{a}(\theta)/\partial \theta^k$.

With $\hat{\boldsymbol{b}}_{\mathrm{d}}$ available, it is ready to implement the MVDR beamformer in accordance with Eq. (5). However, to avoid signal cancellation due to the mismatch between $\hat{\boldsymbol{b}}_{\mathrm{d}}$ and the true CSV $\boldsymbol{b}_{\mathrm{d}}$, the desired signals should be removed before beamforming [5]. This can be done by using a transformation $\boldsymbol{T}_2$ which satisfies

$$\boldsymbol{T}_2 \boldsymbol{a}(\hat{\theta}_i) = \boldsymbol{0}, \quad i = 1, \ldots, J, \tag{12}$$

such that $\boldsymbol{T}_2 \boldsymbol{x}(n) \approx \boldsymbol{T}_2 \boldsymbol{i}(n) + \boldsymbol{T}_2 \boldsymbol{n}(n)$. For the beamformer to work properly with the transformed data, $\boldsymbol{T}_2$ should be chosen such that $\boldsymbol{T}_2 \boldsymbol{x}(n) \approx \boldsymbol{x}(n)$ as much as possible. Similar to Eq. (8), we set up the following problem:

$$\min_{\boldsymbol{T}_2} E\{\|\boldsymbol{T}_2 \boldsymbol{x}(n) - \boldsymbol{x}(n)\|^2\} \equiv \mathrm{tr}\{(\boldsymbol{T}_2 - \boldsymbol{I})\boldsymbol{R}(\boldsymbol{T}_2 - \boldsymbol{I})^{\mathrm{H}}\}$$
$$\text{subject to} \quad \boldsymbol{T}_2 \hat{\boldsymbol{A}}_{\mathrm{d}} = \boldsymbol{O}, \tag{13}$$

where $\hat{\boldsymbol{A}}_{\mathrm{d}} = [\boldsymbol{a}(\hat{\theta}_1), \boldsymbol{a}(\hat{\theta}_2), \ldots, \boldsymbol{a}(\hat{\theta}_J)]$. The solution to Eq. (13) is identical in form to Eq. (9), except that $\hat{\boldsymbol{A}}_{\mathrm{c}}$ is replaced by $\hat{\boldsymbol{A}}_{\mathrm{d}}$. We refer to $\boldsymbol{T}_2$ as the desired signal removal (DSR) transformation.

The DSR transformation can be made robust to DOA estimation errors by incorporating high order constraints as in the case of CSV estimation. It can be also made robust to other model errors by adding a pseudo-noise term to $\boldsymbol{R}$ to 'mask' the desired signal [1]. In so doing, we replace $\boldsymbol{R}$ by $\boldsymbol{R}_\lambda = \boldsymbol{R} + \lambda \boldsymbol{I}$, where λ is the pseudo noise power. It should be chosen large enough to deemphasize the desired signal, but not too large to distort the interference-plus-noise scenario. A suitable choice which has been confirmed by simulation is $\lambda = \sigma_{\mathrm{d}}^2 \sim M\sigma_{\mathrm{d}}^2$.

With DSR transformation, the beamforming weight vector is determined as in Eq. (5), with $\boldsymbol{R}$ replaced by $\boldsymbol{T}_2 \boldsymbol{R} \boldsymbol{T}_2^{\mathrm{H}}$ and $\boldsymbol{b}_{\mathrm{d}}$ by $\hat{\boldsymbol{b}}_{\mathrm{d}}$. Since $\boldsymbol{T}_2$ is singular, and the noise correlation in the transformed data (which is $\sigma_{\mathrm{n}}^2 \boldsymbol{T}_2 \boldsymbol{T}_2^{\mathrm{H}}$) is no longer the same as in the original data (which is $\sigma_{\mathrm{n}}^2 \boldsymbol{I}$), a whitening process is performed to construct the 'transformed and whitened' correlation matrix $\boldsymbol{R}_{\mathrm{w}} = \boldsymbol{T}_2 \boldsymbol{R} \boldsymbol{T}_2^{\mathrm{H}} - \sigma_{\mathrm{n}}^2 \boldsymbol{T}_2 \boldsymbol{T}_2^{\mathrm{H}} + \sigma_{\mathrm{n}}^2 \boldsymbol{I}$. Finally, with $\boldsymbol{R}_{\mathrm{w}}$ substituted, we obtain the weight vector

$$\hat{\boldsymbol{w}} = \boldsymbol{R}_{\mathrm{w}}^{-1} \hat{\boldsymbol{b}}_{\mathrm{d}}. \tag{14}$$

In summary, the estimation of CSV requires a matrix inversion of order M, a matrix inversion of order LJ and some matrix multiplications of order M. Approximately, the same amount of computation is required by the DSR transformation and computation of weight vector. Given the CSV estimate, the only increase in complexity of the proposed beamformer as compared to the regular MVDR beamformer is that involved in the computation of $\boldsymbol{T}_2$ and $\boldsymbol{R}_w$.

4. Simulation results

Computer simulations were conducted using a linear array of 16 elements uniformly spaced by a half-wavelength, with θ measured with respect to the broadside. The scenario involved a desired group of two sources at $\theta_1 = 0°$ and $\theta_2 = 25°$, with $\sigma_d^2 = 1$ and $\rho_2 = j$, and two uncorrelated interferers at $-30°$ and $-45°$. The input SNR and SIR (signal-to-interference ratio) were fixed at 10 dB and -10 dB, respectively. The true noise power $\sigma_n^2 = 0.1$ was used for computing $\boldsymbol{R}_w$.

The first set of simulations demonstrates the effects of DOA estimation errors. In this case, the true correlation matrix $\boldsymbol{R}$ was used. In Fig. 1(a)–(b), the output SINR versus (ED_1,ED_2) are given for $L = 0{,}3$ and $\lambda = 0$, where $\mathrm{ED}_i = \hat{\theta}_i - \theta_i$. These plots show that the use of third order constraints significantly enhances the beamformer. The simulation was then repeated with $\lambda = 5$ to demonstrate the efficacy of pseudo noise injection. The results given in Fig. 1(c)–(d) show that much improvement is achieved for both $L = 0$ and 3.

The second set of simulations examines the convergence behavior of the proposed beamformer. In this case, the sample estimate $\hat{\boldsymbol{R}} = (1/N_s)\sum_{n=1}^{N_s}\boldsymbol{x}(n)\boldsymbol{x}^H(n)$ replaced the true correlation matrix, (ED_1,ED_2) = (0°,0°), $L = 3$ and $\lambda = 5$. All signals were assumed to be zero-mean Gaussian, and each result was obtained by averaging over 30 independent trails. For comparison, we also included the results obtained with (1) MVDR beamformer working with $\hat{\boldsymbol{w}} = \hat{\boldsymbol{R}}^{-1}\hat{\boldsymbol{b}}_d$ (2) MSNR (maximum SNR) beamformer working with $\hat{\boldsymbol{w}} = \hat{\boldsymbol{R}}_{in}^{-1}\hat{\boldsymbol{b}}_d$, where $\hat{\boldsymbol{R}}_{in}$ is the sample estimate of $\boldsymbol{R}_{in}$. The MSNR beamformer is an ideal beamformer in that it does not exhibit signal cancellation in the presence of model errors. This is

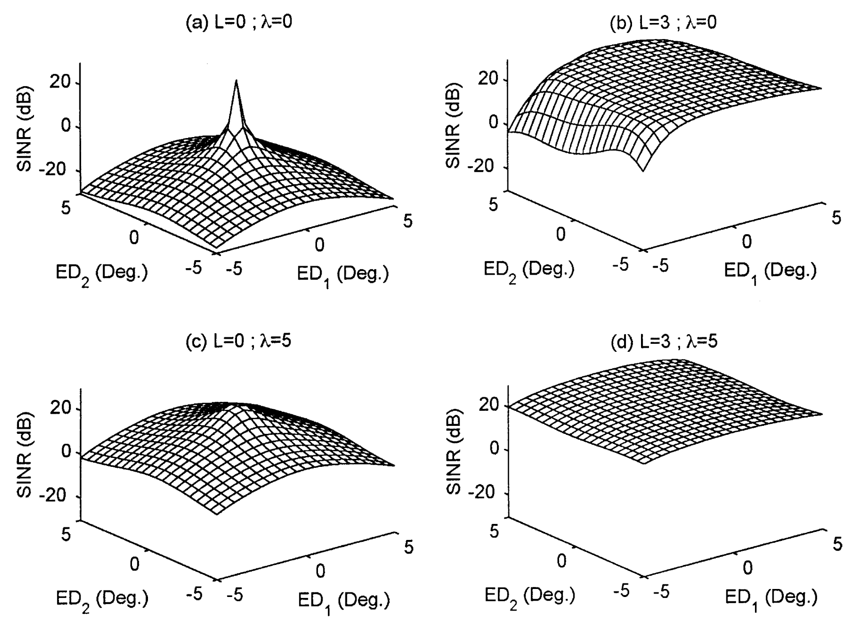

Fig. 1. Effects of DOA estimation error on proposed beamformer. (a) $L = 0$, $\lambda = 0$, (b) $L = 3$, $\lambda = 0$, (c) $L = 0$, $\lambda = 5$, (d) $L = 3$, $\lambda = 5$.

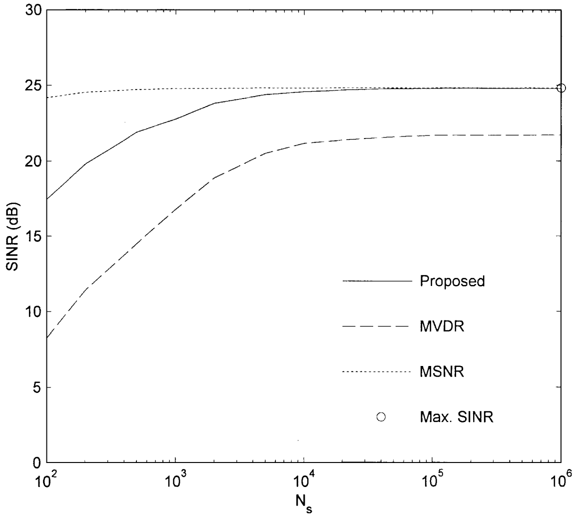

Fig. 2. Comparison of convergence behaviors of proposed, MVDR and MSNR beamformers. Solid line: Proposed beamformer. Dashed line: MVDR beamformer. Dotted line: MSNR beamformer. Small circle: Maximum SINR.

demonstrated in Fig. 2, which shows that the MSNR beamformer converges much faster than the other two, achieving the maximum SINR with a small sample size N_s. The poor behavior of the MVDR beamformer is again due to signal cancellation, and can be enhanced remarkably with the incorporation of the DSR transformation in the proposed method.

5. Conclusion

A new adaptive beamformer for combining multiple coherent signals was proposed. The beamformer was developed based on a three-stage procedure: (1) estimation of the composite steering vector (2) removal of desired signal (3) MVDR beamforming. To enhance the robustness of the beamformer against model errors, high order constraints and pseudo noise injection were incorporated. Computer simulations confirm that the proposed beamformer is quite reliable so long as the model errors are reasonably small.

References

[1] M.C. Doğan, J.M. Mendel, Cumulant-based blind optimum beamforming, IEEE Trans. Aerospace Electron. Systems 30 (3) (July 1994) 722–741.
[2] D.G. Luenberger, Optimization by Vector Space Methods, Wiley, New York, 1969, Chapter 4, pp. 84–86.
[3] A.K. Steele, Comparison of directional and derivative constraints for beamformers subject to multiple linear constraints, IEE Proc.-F 130 (1) (February 1983) 41–45.
[4] M. Wax, Y. Anu, Performance analysis of the minimum variance beamformer, IEEE Trans. Signal Process. 44 (4) (April 1996) 928–937.
[5] B. Widrow, K.M. Duvall, R.P. Gooch, W.C. Newman, Signal cancellation phenomena in adaptive antennas: causes and cures, IEEE Trans. Antennas and Propagation 30 (5) (May 1982) 469–478.
[6] C.C. Yeh, W.D. Wang, Coherent interference suppression by an antenna array of arbitrary geometry, IEEE Trans. Antennas and Propagation 37 (10) (October 1989) 1317–1322.