Dielectric breakdown patterns and active walker model

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Simulations based on the active walker model are used successfully to reconstruct the dielectric breakdown patterns observed in a cell with parallel-plate electrodes. Different types of patterns can be obtained with suitable parameters. These parameters correspond to the electrical and environmental conditions during the breakdown. [S1063-651X(99)09602-6]

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I. INTRODUCTION

Pattern formation is one of the central problems of nonlinear sciences. In particular, patterns observed in dielectric breakdown (DB) are of immense interest. The DB process is a highly nonlinear phenomenon. During this process many variables such as the material contents, environmental temperature and pressures, as well as the electric field distribution are changing interactively. All these variables will determine where the next breakdown is going to happen. The system is one of the so-called complex systems [1]. Kuchinski [2] and Suehle and Chaparala [3] elucidated the microscopic mechanisms for DB in many situations. Experimental observations and theoretical modeling of the DB patterns have also been studied by a number of authors [4–9]. The configuration of electrodes employed in most of these works is of the needle-plate type. The electric field distribution is not uniform and changes with the development of DB pattern in this configuration. On the other hand, simulations have been carried out for both local critical field effect and nonlinear polarizability effect. Nonetheless, in all of these simulations, the electric field needs to be recalculated at each step of the simulations [4–7].

We have previously studied experimentally the DB patterns with electrodes configured as in a parallel-plates capacitor [10,11]. The dielectric liquid is inserted in between the electrodes. In this configuration, the electric field between the electrodes is essentially perpendicular to the pattern plane. The field distribution is always uniform. We have shown that the track patterns left on the electrodes after DB can be categorized qualitatively as either ''dense and winding,'' ''radial,'' or ''radial center with dense and winding tips.'' For convenience, we call them type I, type II, and type III patterns, respectively. Typical patterns for these three types are shown in Fig. 1. Significantly, these patterns are similar to those created by other groups in the works mentioned above. This suggests that the field distribution may not be the direct factor for the pattern formations in DB. There could be a more general and fundamental mechanism to explain the patterns obtained.

In this paper we show that it is possible to reconstruct our experimental results [10,11] by simulation using the active walker model (AWM). The AWM is a commonly used tool to model a complex system such as DB. The micromechanisms for the observed DB patterns are assumed to be the ''thermal effect and active material depletion effect.'' The ''thermal effect'' is used as a general factor favoring the DB process, which includes the effects due to temperature, field, charge, etc. On the other hand, the ''active material depletion'' is the general term for all of the mechanisms against DB, such as depletion of the active material and increasing pressure. Applying these mechanisms in the AWM for pattern generation allows us to reconstruct the DB pattern. All types of patterns can be generated with distinctive sets of parameters. These parameters have physical meaning corresponding to the DB conditions. The results are in good agreements with experimental measurements.

II. EXPERIMENTAL DETAILS

The experimental methods have been described previously [10,11]. Briefly, a pair of glass plates with transparent conductive indium tin oxide (ITO) coating, $3.5\,\mathrm{cm}\times4.0\,\mathrm{cm}$ in area, was employed as electrodes. Either olive oil or mineral oil was inserted between the glass plates with the ITO coating on the two inner surfaces. To control the cell thickness, we put two pieces of 6-μm-thick Mylar spacers with shapes ''['' and '']'' respectively, between the glass plates along two opposite edges of the cell. Thus a rectangular enclosure with two openings at opposite ends was formed. The openings allowed the oil and/or vapor to escape during the DB process, preventing any possible explosion of the cell. The cell was held together with clamps.

A single loop circuit consisting of a voltage supply, a small resistor (0.2 Ω), a switch, and the cell as a parallel-plate capacitor was used. We applied voltage across the cell by using thin conductive wires attached to the coated inner surfaces. DB was achieved by one of two methods: (I) The circuit was closed first; a dc voltage was turned up gradually from zero to the threshold at which DB occurred and then was maintained a constant; (II) a dc voltage above the DB threshold was turned on before the circuit was closed. A digital storage oscilloscope was used to record the time variations of voltage across the cell and the 0.2 Ω resistor. The latter gives the current information through the cell.

III. AWM SIMULATIONS

Active walker models have been used to describe many pattern formation problems [12–14]. In these models, not only the moves of walkers are subject to the influences of the environment; they also change the environment locally.

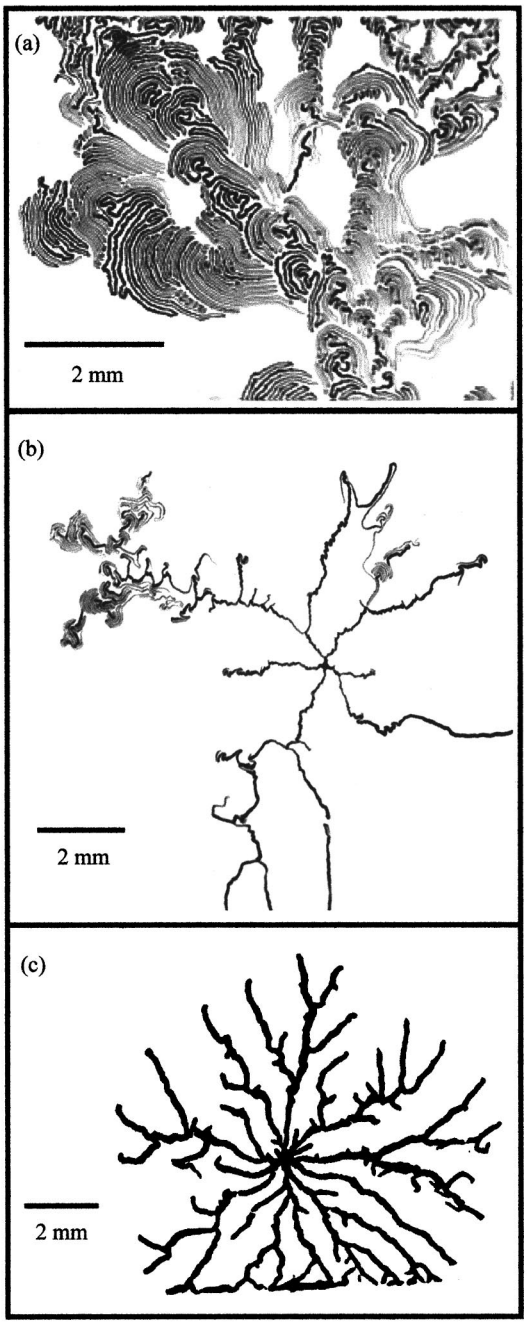

FIG. 1. Type I (a), type II (b), and type (III) (c) dielectric breakdown patterns.

Many model functions are used to simulate the real systems. In our work, the discharging positions are moving; the patterns left on the electrode plates are the tracks of the discharging points. We consider the moving discharging positions as the active walkers.

For the convenience of the readers, we will first summarize the theoretical framework of the active walker model in Ref. [12] as pertinent to the present problem. In this model, the motion of a general walker at position R is described by the Langevin equation

$$m d^2\mathbf{R}/dt^2 = -\eta d\mathbf{R}/dt - \int d\mathbf{r}\,\delta(\mathbf{r}-\mathbf{R})\nabla V(\mathbf{r},t) + \mathbf{F}(t), \tag{1}$$

where $\mathbf{F}(t)$ is a rapid changing force with its time average $\langle \mathbf{F}(t)\rangle = 0$. The potential function (or landscape function) $V(\mathbf{r},t)$ is either a fixed function or a function changing with the walker. The walker moves from a position of high potential to that of low potential. In AWM, the walker changes the environment by modifying the potential function with a landscaping function W:

$$\partial V(\mathbf{r},t)/\partial t = D\nabla^2 V(\mathbf{r},t) + W(\mathbf{r}-\mathbf{R}(t),t), \tag{2}$$

where D is the diffusion constant of the potential (e.g., heat diffusivity for a potential function related to temperature). The function W can be anisotropic and time dependent. Sometimes, it is more convenient to use the Fokker-Plank equations to treat the same problem. They are

$$\partial p(\mathbf{r},t)/\partial t = \boldsymbol{\nabla}\cdot[p(\mathbf{r},t)\boldsymbol{\nabla} V(\mathbf{r},t)] + D_w\nabla^2 p(\mathbf{r},t), \tag{3}$$

and

$$\partial V(\mathbf{r},t)/\partial t = D\nabla^2 V(\mathbf{r},t) + \int dr'\, W(\mathbf{r}-\mathbf{r}',t)p(\mathbf{r}',t), \tag{4}$$

where $p(\mathbf{r},t)$ is the probability density for the walkers found at position $\mathbf{r}$ and time t, while D_w is the diffusion constant of the walker.

None of the equations above is easy to solve. Even when they are solved, only the statistical properties of the walker's tracks can be obtained. In pattern formation problems, however, it is the morphology of the tracks that is of interest. Thus a discrete version of the Langevin equation for computer simulation is needed.

In such a computer model the single-valued landscape $V(i,n)$ is defined at every site i at time n, where $n = 0,1,2,\ldots$. Furthermore,

$$V(i,n) = V_0(i,n) + V_1(i,n), \tag{5}$$

where V_0 is the external background; V_1 evolves in time due to the action of the walker on the environment. At $n=0$, a walker (or multiple walkers) is placed on the initial landscape $V_0(i,0)$. V_1 is then updated according to the landscaping rule

$$V_1(i,n+1) = V_1(i,n) + W(\mathbf{r}_i - \mathbf{R}(n),n), \tag{6}$$

with $V_1(i,0)=0$; $\mathbf{r}_i$ is the position vector of site i, $\mathbf{R}(n)$ the position of the walker at time n. The time difference dt in Eqs. (1)–(4) is taken to be unity here. Each time after the landscape is updated, the walker takes a step according to the stepping rule specified by P_{ij}, which is the probability for the walker to step from its present site i to site j. The latter belongs to the set of available surrounding sites A_i.

Many forms of P_{ij} may be used. Three methods had been suggested in Ref. [12]. Those are (i) deterministic active walk: the walker goes to the lowest potential site in A_i, (ii) Boltzmann active walk: P_{ij} is proportional to $\exp\{[V(i,n) - V(j,n)]/T\}$, where T is a characteristic ''temperature,'' and (iii) the probabilistic active walk: P_{ij} is proportional to $[V(i,n)-V(j,n)]^\eta$ if $V(i,n)>V(j,n)$ and $P_{ij}=0$ otherwise, where η is a constant.

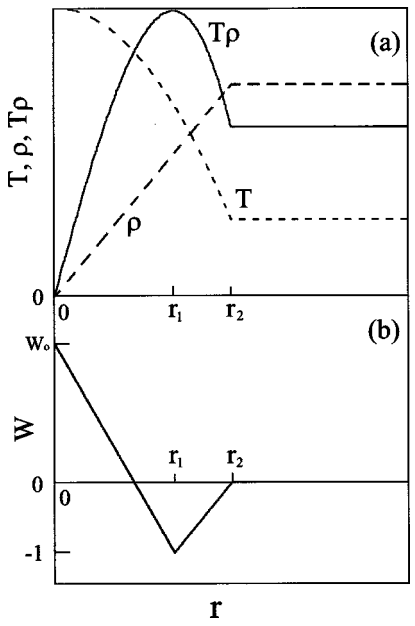

FIG. 2. (a) Temperature T and active chemical material density ρ around a breakdown point. (b) The landscaping function, r is the distance between the site position r_i and the walker position $R(t)$.

In Eqs. (5) and (6), the diffusion constant D is set to zero. Nevertheless, it would be easy to incorporate it in V in the computer program when it is needed. The inertial effect in Eq. (1) implies that $P_{ij}(n)$ should also depend on $\mathbf{R}(n)-\mathbf{R}(n-1)$. This can be done by either modifying P_{ij} to give an enhanced stepping probability along $\mathbf{R}(n)-\mathbf{R}(n-1)$ or by letting the landscaping function be anisotropic.

In this work we consider a 200×200 lattice with a ''landscape potential'' $V(i,n)$ defined at each site i and a discrete instant of time n. Initially, $V(i,0)=0$. Several walkers are set at some sites in the lattice. When a walker is moved to a new position $\mathbf{R}(n)$ at time n, the potential is changed from $V(i,n)$ to $V(i,n+1)$ with an isotropic time-independent landscaping function W, i.e.,

$$V(i,n+1)=V(i,n)+W(r),$$

where $r=|\mathbf{r}_i-\mathbf{R}(n)|$ and W is shown in Fig. 2. Three parameters W_0, r_1, and r_2 are used to define this function. We choose this form of landscaping function because it is simple and agrees with our model for the DB process. We assume

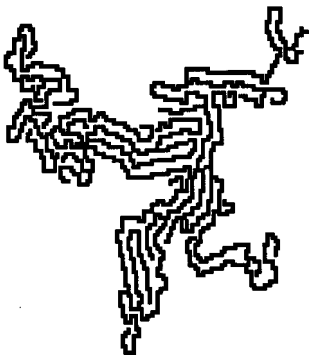

FIG. 3. Simulated type I DB pattern (dense and winding).

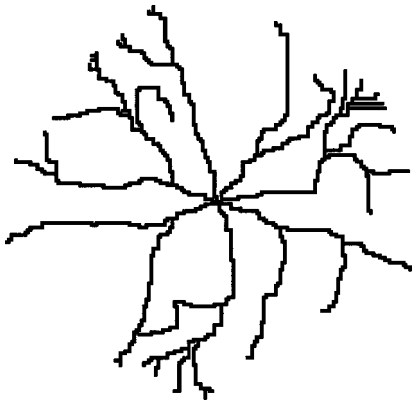

FIG. 4. Simulated type III DB pattern (radial).

that the temperature T of the material can enhance the dielectric breakdown process [2,3] and the track is left due to a chemical reaction caused by DB [10]. Therefore the point at a higher temperature and with more active chemical material has a better chance for being the next DB point. In Fig. 2 we show a simple model for the temperature and the active material density ρ, around a breakdown point. The product of these two factors is also shown in the figure. Near $r=0$, most active materials are consumed. As a result, the chance for reaction is low. The potential must be high there. The point having maximum value of $T\rho$ at r_1 corresponds to a point with the best chance being the next DB point. Therefore this point should correspond to a lower potential. For $r\geqslant r_2$, $T\rho$ is unchanged, so the potential should also be unchanged. We let the minimum value of $W(r)$ be -1, then the value W_0 corresponds to the ratio of maximum value of $W(r)$ at $r=0$ and the absolute value of the minimum value of $W(r)$ at r_1. The minimum value of $W(r)$ is fixed because only the ratio of potentials for any two points is significant. The stepping rule is according to a probability linear with the potential difference (see also the discussions in the next paragraph).

The probability P_{ij} in this work is the probabilistic active walk with $\eta=1$. Let $D_n(j)$ be the potential drop from walker position $\mathbf{R}(n)$ to site j. The next step for the walker is to walk into one of the not-yet-visited neighboring sites j with positive $D_n(j)$, i.e., sites with lower potential V. If there are more than one site having positive $D_n(j)$, their chances to be the next walker position are proportional to $D_n(j)$.

To accommodate the branching property of the DB patterns, we allow the walkers to branch to more walkers, when the potentials of the neighboring sites are close enough. A branching factor γ is used to determine the likelihood that branching in the DB pattern will occur. If j is the walker position chosen for the next step by the above procedure and $D_n(k)\geqslant\gamma D_n(j)$, then another new walker will be created at k. That is, branching in the dielectric breakdown pattern occurs if the branching probabilities of two walker positions for the next step are close enough. On the other hand, an existing walker can also be terminated when all of the neighboring sites either have been visited already or have negative $D_n(j)$. The walker will also be terminated when it reaches the edge of a circle specified in the simulation program. The simulation is stopped automatically when all the walkers are terminated.

With the model described above, we can obtain either

TABLE I. AWM parameters for obtaining the simulated patterns.

Pattern	Number of walkers	Branching factor	W_0	r_2	r_1/r_2
Type I (Fig. 3)	6	0.96	3	5	0.8
Type III (Fig. 4)	6	0.9	100	20	0.8
Type II (Fig. 5)	6	0.96	50	as shown in Fig. 5(b)	0.8

type I (dense and winding) or type III (radial) patterns by using different values for r_2 while keeping the ratio of r_2/r_1 the same. Figures 3 and 4 are examples of simulated Type I and type III patterns, respectively. The parameters used for these two simulations are listed in Table I. ''Number of walkers'' means the number of walkers put in the lattice initially.

We have also varied r_1 during the simulation as the patterns are developing. Simulated type II pattern has been obtained in this way. Figure 5(a) shows the simulated pattern; while the change of r_1 is shown in Fig. 5(b).

IV. DISCUSSION

Examining Table I, we can see that the smaller W_0 and r_1 values cause the ''dense and winding'' type I patterns, while larger W_0 and r_1 values produce the ''radial'' type III patterns. The W_0 and r_1 values for the ''radial center with dense and wind tips'' type II patterns are large in the beginning, but r_1 values then decrease as the patterns evolve. These results can be understood by examining the properties and the parameters of the landscaping function $W(r)$. Large W_0 in $W(r)$ means a large raise in potential near the previous walker positions. New walkers experience a repulsive force near these points. The parameter r_1 in $W(r)$ affects the size of the repulsive region. Therefore the type III patterns with radial features should have larger W_0 and r_1 values than those of type I patterns should.

The physical meanings of these two parameters can be related to the dependence of DB formation with the temperature elevation and the available active material density. When the discharge is fast and the associated reaction is drastic, more energy is released and more active material is consumed. Then, the temperature elevation is higher and the affected area is larger. A larger affected area corresponds to larger r_1 and r_2. More active material consumed means that the nearby points are less likely to be the next discharge points. This requires larger W_0 values for the development of DB patterns. On the other hand, a mild discharge would be associated with smaller values of W_0, r_1, and r_2.

In our previous work [10] we have found that the type I patterns are formed when the voltages across the plates are lower. We can assume that the energy released is smaller in these cases. Therefore the smaller values for W_0 and r_1 in our simulation of type I patterns agree with our experimental observations. On the other hand, the threshold voltage is usually larger and the electric current was initially a constant and then decreased gradually to smaller values during the DB process when a type II pattern was obtained [10]. Figure 6 shows an example of the measured current vs time curve recorded during one of the DB process. The gradually decreasing currents suggested a decreasing heat releasing rate

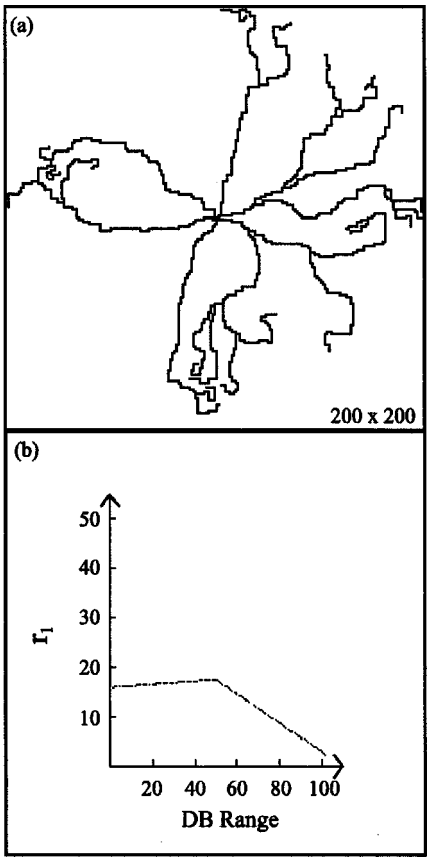

FIG. 5. Simulated type II DB pattern (radial center with dense and winding tips). The parameter r_1 is varied during this simulation process, as shown in (b).

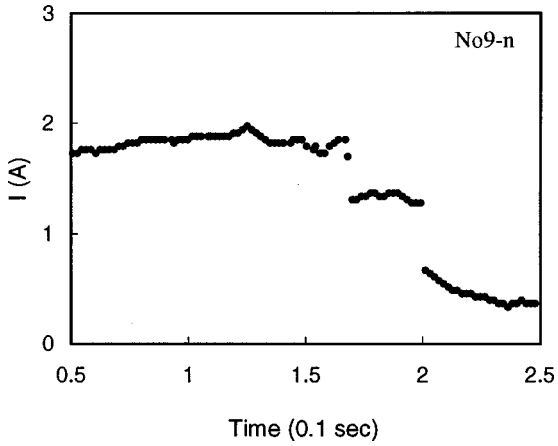

FIG. 6. Time dependence of electric current during a DB process. The data in the first 0.05 sec were not recorded due to the triggering and delay of the oscilloscope.

and the decreasing of active material depletion. This would correspond to the decreasing of r_1. The variation of r_1 in Fig. 5 is consistent with the trend of current variation.

The ''thermal effect and active material depletion effect'' mechanism is proposed to explain the various patterns obtained in this work. Using the model, we show that dielectric breakdown patterns can be successfully simulated. In the dielectric breakdown problem, however, the detailed mechanisms may be much more complex. We do not imply that $T\rho$ is the only possible variable. In our work all the factors favoring the process are combined into the ''thermal effect'' and all the other factors that disfavor the process into the ''active material depletion effect.'' The simulation is based on the Langevin equation, the type of the landscaping function W, and the stepping rule. We suggest that, if any problem can be structured in the same manner, the patterns generated will be the same.

V. CONCLUSIONS

We are able to reproduce all three types of two-dimensional DB patterns observed for the parallel-plate electrodes by numerical simulation with the active Walker model. The same model is used for all patterns, only with different parameters. These parameters are correlated with experimental measured currents and voltages. The parameters responsible for the dense and winding structure correspond to lower voltage or lower electrical current, while the parameters generating radial patterns correspond to higher voltage or higher electrical current. The most common DB patterns we observed are of the type with radial center and dense and winding tips. These can be simulated with parameters changing with time, e.g., slowly decreasing current. Further work is in progress to simulate the growth speed of DB patterns.

ACKNOWLEDGMENTS

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