Pergamon

Ocean Engineering 26 (1999) 147–160

OCEAN ENGINEERING

An explicit approximation to the wavelength of nonlinear waves

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Received 1 February 1997; received in revised form 21 May 1997; accepted 1 June 1997

Abstract

An explicit and concise approximation to the wavelength in which the effect of nonlinearity is involved and presented in terms of wave height, wave period, water depth and gravitational acceleration. The present approximation is in a rational form of which Fenton and Mckee's (1990, Coastal Engng 14, 499–513) approximation is reserved in the numerator and the wave steepness is involved in the denominator. The rational form of this approximation can be converted to an alternative form of a power-series polynomial which indicates that the wavelength increases with wave height and decreases with water depth. If the determined coefficients in the present approximation are fixed, the approximating formula can provide a good agreement with the wavelengths numerically obtained by Rienecker and Fenton's (1981, J. Fluid Mech. 104, 119–137) Fourier series method, but has large deviations when waves of small amplitude are in deep water or all waves are in shallow water. The present approximation with variable coefficients can provide excellent predictions of the wavelengths for both long and short waves even, for high waves. © 1998 Elsevier Science Ltd. All rights reserved.

Keywords: Explicit approximation; Wavelength; Nonlinear waves

Nomenclature

A	Coefficient undetermined in the present approximation
d	Water depth
$E(m)$	Elliptic integral of the second kind

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PII: S0029-8018(97)10031-2

g	Gravitational acceleration
H	Wave height
$K(m)$	Elliptic integral of the first kind
k_l	Wave number of linear wave theory
L	Wavelength
L_0	Wavelength in deep water
L_s	Wavelength obtained by Stokes wave theory
L_c	Wavelength obtained by Cnoidal wave theory
L_n	Wavelength obtained by Rienecker and Fenton's (1981) Fourier series method
m	Parameter of elliptic functions and integrals
n	Ratio of group velocity to wave celerity
T	Wave period
U_s	Ursell number
λ	$(1 - m^2)/m^2$
μ	$E(m)/m^2/K(m)$
ν	Coefficient undetermined in the present approximation

1. Introduction

Water wave motion in general occurs where disturbances propagate in various directions interacting nonlinearly over a varying permeable bed. These grouping waves may be decomposed into a number of components with different frequency harmonics. Different wave numbers will lead to different wave speeds. Therefore, in a synthesis of many components by superposition, components with different wave numbers disperse as time goes on. The dispersion relation can be obtained to specify wave speeds as a function of wave numbers in the dispersive waves.

A common assumption that there is only one component wave train will give the simplest case of gravity waves propagating steadily with a constant period and wave height over an impermeable and flat bed. In this case, the two-dimensional flow is irrotational because it is assumed to be homogeneous and incompressible. The resulting dispersion relation for a linear wave will become a rougher approximation to the physical problem. In such a case the wave is assumed to be so small that the effect of wave height on wave speed is ignored. The dispersion relation for linear waves is still a nonlinear transcendental equation for wavelength when both wave period and water depth are specified. The computational disadvantage prevents engineers from solving many practical wave problems. Nevertheless, to avoid time-consuming numerical computations, many attempts have been made to obtain explicit expressions for the wavelength so as to provide a fast and more flexible method to predict the wavelength.

Various methods of approximate solution for the dispersion relation of linear waves including Eckart's dispersion relation, iterative techniques, power series, and Padé asymptotic form were developed. Those approximations are almost all excellent, but have their limitations for waves which are longer or shorter than a certain length. For long waves, the approximations to wavelength given by Olson (1973), Venezian (1980), Nielsen (1982) and Wu and Thornton (1986) are all highly accurate for waves longer than $L/d \approx 3$ where L is wavelength and d is the water depth. Those formulae are based on the same versions of the power series approximation. Turning to the short wave formulae, Nielsen (1984) and Wu and Thornton (1986) proposed excellent approximations only for $L/d < 4$. Hunt (1979) proposed a polynomial solution of Padé form and this polynomial solution is valid for waves in all water depths. Both Eckart (1952) and Fenton and Mckee (1990) provided accurate approximations in a similar form over all wavelengths. The accuracy and validity of each approximation were reviewed by Fenton and Mckee (1990) in detail.

All formulae mentioned above do not allow for any effect of nonlinearities. In problems where the waves are not very high or where great accuracy is not required, it is more reasonable to use an approximate explicit solution, such as cnoidal wave theories for shallow water or Stokes wave theories for deep water. However, the Stokes wave theories are an acceptable method to use to predict the wavelength of short waves, but are a poor method to use to predict the wavelength of long waves. In contrast, cnoidal wave theories are an acceptable method to use to predict the wavelength of long waves, but are a poor method to use to predict the wavelength of short waves. Because of the nonlinear nature of the governing equations, highly accurate solutions for steady high water waves have been found when these solutions have been only based on numerical methods, such as Schwartz (1974), Williams (1981) and Rienecker and Fenton (1981) etc. The Fourier approximation method introduced by Rienecker and Fenton (1981) is usually applied to solve the wave problem of a gravity wave propagating over a constant depth. This approximation method is obtained as a function of position and it can provide a comprehensive solution for the values of stream function and surface elevations. This numerical model is known for its mathematical validity because it can satisfy the governing equation and boundary conditions to the required degree of accuracy. Therefore, the Fourier approximation method, hereafter called the RF method, was applied with high accuracy to calculate the wavelengths of gravity waves in a constant water depth, taking into consideration the effects of nonlinearities.

The numerical method is not convenient for practical applications because it needs computer programming and gives an implicit solution. A simple yet accurate formula of approximation to wavelength is then required for engineering purposes. An approximate formula for the wavelength of a nonlinear gravity wave will be introduced in this paper. This approximation is easy to explicitly calculate when the wave height and wave period are given. Meanwhile, this approximation is superior to the existing theories because it does not require time-consuming computation and because it is valid for all water depths.

2. The effect of wave height on wavelength

The motion of progressive waves of permanent form which can be uniquely defined by water depth d, wave height H, and wave period T (or wavelength L) are steady in a frame of reference moving at the phase speed C. Stokes (1847) used the perturbation method to determine the solutions of finite amplitude waves both in water of finite constant depth and in deep water. The first-order solution of Stokes wave theory coincides with the small amplitude wave theory proposed first by Airy (1845). The local wavelength, $L = L_l$, for linear progressive waves over constant depth can be determined by the dispersion relation, which for a wave with period, T, in water of depth, d, may be written as

$$L_l = \frac{g}{2\pi} T^2 \tanh kd \tag{1}$$

where k is the wave number and is defined to be $k = 2\pi/L$. The strongly nonlinear interactions occur in waves when the wave height increases so that the linear wave theory is not longer valid for higher waves. Therefore, higher order solutions are required so that better predictions of large wave-amplitude motion can be made.

Two of the fifth-order solutions obtained by Isobe et al. (1978) and Fenton (1985) are convenient for numerical calculation, since wave steepness ($kH/2$) is used as the perturbation parameter instead of the unknown Fourier coefficient ka, where a is a length scale which is equal to the amplitude of the wave in the linear wave theory. The dispersion relation with no current of the fifth-order wave theory presented by Isobe et al. (1978) based on the second definition of wave celerity is written as

$$L_s = \frac{g}{2\pi} T^2 \tanh kd \left[1 + \left(\frac{kH}{2}\right)^2 \left(\frac{9ct^4 - 10ct^2 + 9}{16} - \frac{ct}{2kd}\right) + \left(\frac{kH}{2}\right)^4 \left(\frac{-405ct^{10} - 117ct^8 + 2454ct^6 - 2194ct^4 + 351ct^2 + 39}{1024} - \frac{ct(-9ct^6 - 3ct^4 - 7ct^2 + 3)}{64}\right)\right]^2 \tag{2}$$

where

$$ct = \coth kd. \tag{3}$$

The wavelength is quantified to the fifth order, as indicated by Eq. (2) in which the neglected terms of order $(kH/2)^6$ and higher are not shown. By omitting the higher order terms in Eq. (2), the wavelength simply approaches Eq. (1).

Provided T, H and d are known, the wavelength can be found from Eq. (1) or Eq. (2) by a numerical method for solving transcendental equations. From a comparison with the terms of Eqs. (1) and (2), the wavelength is shown to quadratically increase with wave height. Eq. (2) is valid for waves which are not so high as to be close to breaking and for waves which are not long. Ursell parameter should be smaller than 25 for the fifth-order Stokes wave theory to be valid.

If the waves are long, but not higher than about 40% of the water depth, the cnoidal wave theory can be used to give a better dispersion relation than Stokes wave theory. The first-order cnoidal wave theory was derived by Korteweg and de

Vries (1895) for a shallow water wave by taking into account the effects of finite depth and finite amplitude. Various versions of higher-order cnoidal wave theories were presented, such as Laitone (1960), Chappelear (1962), Isobe et al. (1978) and Fenton, 1979), etc. The dispersion relation of the third-order cnoidal wave theory proposed by Isobe et al. (1978) is given as

$$L_c = T\sqrt{gd}[1 + \left(\frac{H}{d}\right)\left(\frac{1 + 2\lambda - 3\mu}{2}\right) + \left(\frac{H}{d}\right)^2\left(\frac{-6 - 16\lambda + 5\mu - 16\lambda^2 + 10\lambda\mu + 15\mu^2}{40}\right) + \left(\frac{H}{d}\right)^3(150 + 1079\lambda - 203\mu + 2337\lambda^2 - 2653\lambda\mu + 350\mu^2 + 1558\lambda^3 - 2653\lambda^2\mu + 700\lambda\mu^2 + 175\mu^3)/2800] \quad (4)$$

where $\lambda = (1 - m^2)/m^2$, $\mu = E(m)/m^2/K(m)$ and $K(m)$ and $E(m)$ are the complete elliptic integrals of the first and second kind, respectively. The m is the modulus of the elliptic integrals. Once d, T, and H are given, the modulus m can be implicitly determined by the following formula

$$U_s\left[1 + \left(\frac{H}{d}\right)\left(\frac{-1 - 2\lambda}{4}\right) + \left(\frac{H}{d}\right)^2\left(\frac{8 + 33\lambda - 10\mu + 33\lambda^2 - 20\lambda\mu}{40}\right)\right] - \frac{16m^2K^2(m)}{3} = 0 \quad (5)$$

where U_s is the Ursell parameter in the long wave approximation and is defined to be $U_s = gHT^2/d^2$. L_c denotes the wavelength calculated from Eq. (4) in cnoidal wave theory to distinguish it from the wavelength, L_s, as a result of Stokes wave theory. It is a tedious computation to find L_c, because the extensive Jacobian elliptic functions and its integrals are difficult to calculate. In the case of the limit of $m \rightarrow 1$, the L_c is particularly difficult to calculate because of very poor numerical convergence. The wavelength, L_c, of cnoidal wave theory is shown to linearly increase with wave height, but not like Stokes wave theory to perform a quadratic increase with wave height. The terms of higher order quantities in Eq. (4) are dropped out when the nonlinearities are ignored. Then, Eq. (4) becomes

$$L_c = T\sqrt{gd} \quad (6)$$

Eq. (6) is a well-known formula for linear long waves. In the case of long waves, when $kd \rightarrow 0$ and $\tanh kd \rightarrow kd$, Eq. (1) is easily shown to have the same form of Eq. (6).

Since the expansion of cnoidal wave theory is affected by $(d/L)^2$, the cnoidal wave theory is not suitable in the conditions of deep water in a manner complementary to that in which Stokes theory is not suitable in the conditions of shallow water. Both Eqs. (2) and (4) indicate that the wavelength depends on not only wave period and water depth, but also wave height. However, the drawbacks of both Stokes and cnoidal wave theories have been a major reason that they have not been widely believed to be accurate for all waves. To overcome the limitation of those wave theories and to obtain highly accurate results, it would be necessary to obtain very high-order expansions for physical quantities in waves.

The Fourier series is a very common method and is capable of accurately approximating any periodic quantity. However, expanding to even higher orders by the perturbation method becomes extremely formidable. For this reason, it is desirable to have wave theories that could be developed on the computer to any order. The first such theory was developed by Chappelear (1961) who used the velocity potential and introduced a Fourier series for the surface elevations. Dean (1965) used the stream function to develop the stream function wave theory, which was computationally simpler than Chappelear's technique.

Rienecker and Fenton (1981) presented a method and a computer program that gave somewhat simpler equations, which were identically satisfied with both dynamic and kinematic boundary conditions at a number of points on the surface, rather than Dean's stream function wave theory which minimizes errors there. Moreover, it is valid for all depths and for finite-amplitude waves. The Fourier series of only 10–20 terms, even for waves close to the highest, was demonstrated by Rienecker and Fenton (1981) to be an accurate solution to water wave problems. Their comparison of experimental data of particle velocity shows that the theoretical predictions satisfactorily correspond to the experimental results. This method provides a stable convergence and therefore, when accuracy is important in steady water wave problems, it is the best method to use.

In this paper, the RF method with a series of 32 terms was used to obtain more accurate wavelengths with considerations of strong nonlinearities. The wavelength obtained by the RF method is denoted by L_n. There were 492 cases of waves calculated with a period $T = 8$ s of different amplitudes in various water depths. Twenty-five relative depths were considered, d/L, approximately varying from a range of 1/35 ~ 1/2 and included the conditions from shallow water to deep water. About 20 waves ranging from small amplitude to the highest were chosen for each water depth.

3. A new approximation

A convenient approximation to the wavelength from the linearized dispersion relation was introduced by Fenton and Mckee (1990) as follows:

$$L_1 = L_0 \tanh^{\frac{1}{\nu}} \left(\frac{2\pi d}{L_0} \right)^{\frac{\nu}{2}} \tag{7}$$

in which $L_0 = gT^2/2\pi$ is the deep-water wavelength. The value of ν was found to be 3/2 by minimizing the value of the maximum error over all wavelengths of linear waves. Eq. (7) satisfies both long and short wave limits and has a maximum relative error of only 1.7% between those limits when the effect of nonlinearities is ignored.

Eqs. (2) and (4) are demonstrated to relate the wavelength to wave height with a linear or quadratic increase and as a function of coth *kd*. We can use the 492 wavelengths obtained by the RF method to give a rational approximation which can provide a concise expression. Following the Fenton and Mckee's (1990) approximation to wavelength for linear waves, we suggest a Padé approximation in which the Fenton and Mckee's (1990) approximation is preserved in the numerator and the wave height is involved in the denominator as in the form of

$$L = \frac{L_0 \tanh^{\frac{1}{\nu}}\left(\frac{2\pi d}{L_0}\right)^{\frac{\nu}{2}}}{1 - A\left(\frac{H}{L_0}\right)\coth^{\frac{2}{\nu}}\left(\frac{2\pi d}{L_0}\right)^{\frac{\nu}{2}}} \tag{8}$$

where A and ν are the coefficients to be determined. The expansion of Eq. (8) in terms of a power-series polynomial when the value of $A\left(\frac{H}{L_0}\right)\coth^{\frac{2}{\nu}}\left(\frac{2\pi d}{L_0}\right)^{\frac{\nu}{2}}$ is small gives an alternative form of Eq. (8) as

$$L = L_0 \tanh^{\frac{1}{\nu}}\left(\frac{2\pi d}{L_0}\right)^{\frac{\nu}{2}}\left\{1 + \left[A\left(\frac{H}{L_0}\right)\coth^{\frac{2}{\nu}}\left(\frac{2\pi d}{L_0}\right)^{\frac{\nu}{2}}\right] + \left[A\left(\frac{H}{L_0}\right)\coth^{\frac{2}{\nu}}\left(\frac{2\pi d}{L_0}\right)^{\frac{\nu}{2}}\right]^2 + \left[A\left(\frac{H}{L_0}\right)\coth^{\frac{2}{\nu}}\left(\frac{2\pi d}{L_0}\right)^{\frac{\nu}{2}}\right]^3 + \ldots\right\}. \tag{9}$$

That the wavelength increases with wave height is clearly shown in Eq. (9) and implies that in Eq. (8) the wavelength increases also with wave height. After trying many values of A and ν, we found that the minimum value of maximum error over 492 wavelengths in Eq. (8) was $A = 1.13$ and $\nu = 1.19$. This approximation has a maximum error of 6.5% and a mean error of only 1.9% for 492 wavelengths. The approximating wavelengths are depicted by open circles in Fig. 1, in which the abscissa denotes the relative wavelengths obtained by the RF method and the ordinate indicates the predicted wavelengths by Eq. (8). Small deviations from the L_n are shown from Fig. 1 by a declined slope for small-wavelength waves in each shallow water. However, the predicted wavelength is underestimated for waves of small amplitude but large wavelength. For an excellent agreement between the results obtained by the RF method and by using Eq. (8), the open circles in Fig. 1 will lie on a line of which the slope has a value of 1. By best linearly fitting to the open circles, we find that the equation of the line can be given by $L/L_0 = 1.023L_n/L_0 - 0.014$. This linear equation has a slope of 1.023 which is near the value of 1. For all wavelengths, Eq. (8) provides the same satisfactory results as obtained by the RF method.

To improve the accuracy of Eq. (8), we find the coefficients $A's$ and $\nu's$ from the wavelengths computed by Rienecker and Fenton's method for each water depth. The values of A and ν vary with the water depths and their trends related to relative depth, d/L_0, were plotted in Fig. 2 by symbols of solid circles and open circles,

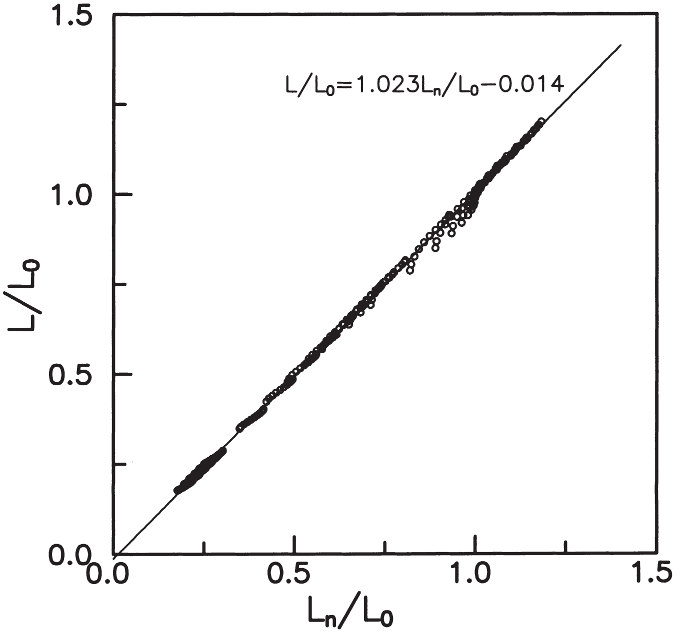

Fig. 1. Comparison of the computed wavelength by the present approximation with constant coefficients of A = 1.13 and ν = 1.19 with those obtained by Rienecker and Fenton's (1981) method for all waves in both shallow water and deep water.

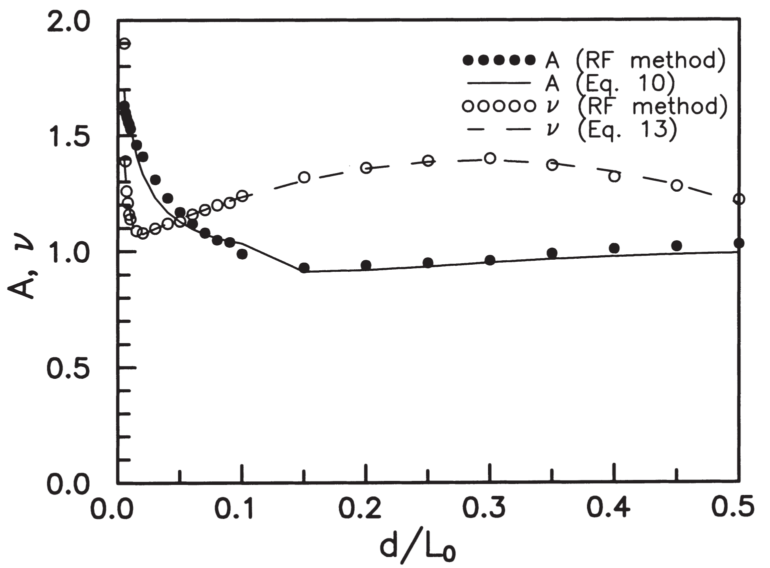

Fig. 2. Predicted A and ν and their best fitting values in different water depths.

respectively. The performance of coefficient A is similar to the shoaling coefficient of the linear wave theory. The coefficient ν has a steep decrease when $d/L_0 < 0.015$ and then increases to reach a maximum value. The characteristics of coefficient A and ν as function of d/L_0 can be represented by formulae to estimate their values for practical use. The suggested approximating formula for coefficient A is given as

$$A = \sqrt{\frac{1}{n}\frac{1}{\tanh k_l d}} + 0.1,\ 0.008 \le d/L_0 \le 0.1$$

$$= \sqrt{\frac{1}{n}\frac{1}{\tanh k_l d}},\ d/L_0 > 0.1 \tag{10}$$

$$n = \frac{1}{2}\left(1 + \frac{2k_l d}{\sinh 2k_l d}\right) \tag{11}$$

$$k_l = \frac{2\pi}{L_0 \tanh^{2/3}\left(\frac{2\pi d}{L_0}\right)^{3/4}} \tag{12}$$

where k_l is the wave number in which the approximate wavelength was obtained by Eq. (7) when the linear dispersion relation was used and n is the ratio of group velocity to wave celerity for linear waves. Eq. (10) indicates that the value of A deviates the shoaling coefficient from 0.1 when d/L_0 lies between 0.008 and 0.1. The coefficient νs are divided into two regions. One region is for the value of d/L_0 being smaller than 0.02 and the other is for the value of d/L_0 larger than 0.02. The values of ν for the shallow water region can be expressed by a inversed power law. Using the least-square method, a quadratic polynomial can be used to represent the values for deep water. The ν can be conclusively given as

$$\nu = \frac{0.342}{(d/L_0)^{0.266}},\ d/L_0 < 0.02$$

$$= -4.30(d/L_0)^2 + 2.51(d/L_0) + 1.03,\ d/L_0 \ge 0.02 \tag{13}$$

Fig. 2 shows that there is a good agreement between the RF method results, and Eqs. (10) and (13) for the values of coefficients of A and ν, respectively. The purpose of Eqs. (10) and (13) is to obtain the coefficients of A and ν. After A and ν are known for any wave of a given wave height in any water depth, it is easy to obtain the wavelength by applying Eq. (8), using the variable values of coefficients A and ν. The wavelength predicted by using Eqs. (8), (10) and (13) were compared with those obtained by the RF method and were plotted in Fig. 3.

The relationship between the wavelengths predicted by using Eq. (8) with variable values of A and ν for different waves and those predicted by using the RF method can make a line of which its slope is 0.996, as shown in Fig. 3. The acceptable deviation is less than 4.5% and its mean error is 0.8%. The slight disparity in Fig. 1 for both small and large relative wavelengths is adapted as shown in Fig. 3. Fig.

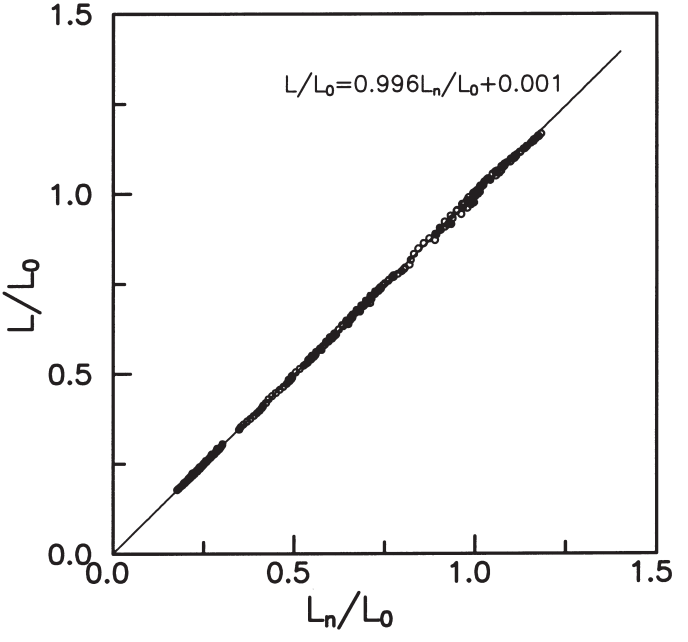

Fig. 3. Comparison of the wavelength computed by the present approximation with variable coefficients with those obtained by the Rienecker and Fenton's (1981) method for all waves in both shallow water and deep water.

3 shows a minimal disparity and this verifies the high merit of using Eq. (8) with variable values of coefficients A and ν to approximate wave lengths for nonlinear waves.

We will compare the validity of the different wave theories and approximations for wavelength of nonlinear waves by choosing four kinds of water depth ranging from shallow water to deep water. The wave theories chosen for wavelength are the Stokes wave theory for short waves, the cnoidal wave theory for long waves and the RF method for all waves. The wavelength formulae in Stokes wave theory and cnoidal wave theory are equated by Eq. (2) and Eq. (4), respectively. The first case is for the relative depth, d/L, approaching 1/2. The computed wavelengths of the small-amplitude and of the highest waves by using different wave theories and the present approximations are depicted in Fig. 4. In Fig. 4, open circles denote the wavelength obtained by the RF method; the solid line with solid circles is by the Stokes wave theory; and the solid line and the dashed line indicate the present approximation, Eq. (8), with constant or variable coefficients, respectively. The change of wavelength due to wave heights is seen in Fig. 4 by a 17% increase from the shortest wavelength to the largest one. The cnoidal wave theory fails to give any prediction for wavelength in this case. The present approximation with variable coefficients A and ν provides the closest prediction to the RF's wavelengths. The present approximation with constant A and ν performs slightly worse than the present

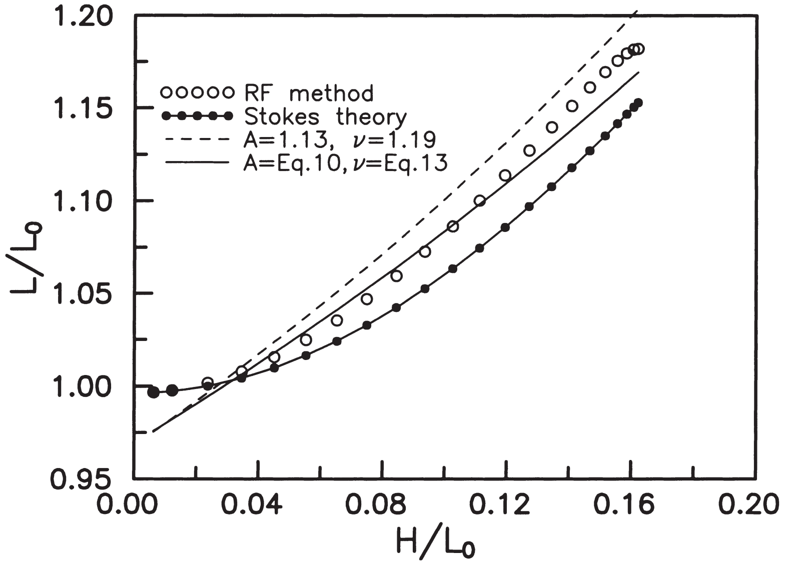

Fig. 4. The wavelength predicted by the Stokes wave theory, the RF method and the present approximations with constant and with variable coefficients in a water depth $d/L \approx 1/2$ ($d = 50$ m, $T = 8$ s).

approximation with variable coefficients. The wavelengths computed by the Stokes wave theory have remarkable deviations from those computed by the RF method. The present approximation to wavelengths no matter whether the coefficients of A and ν are constant or variable, appears to be not accurate for waves of very small amplitude in deep water.

Fig. 5 shows the wavelengths of different wave heights in a water depth of $d/L \approx 1/4.4$. The present approximation with variable coefficients is also the best one to predict the wavelengths. However, the present approximation with constant coefficients has slight disparity for the small-amplitude waves.

The wavelengths in a water of relative depth being $d/L \approx 1/10$ are depicted in Fig. 6. The cnoidal wave theory can provide the worst prediction to wavelengths among the considered models. A fair agreement between wavelengths obtained by the Stokes wave theory and those computed by the RF method is found for waves of small-amplitude. Furthermore, Stokes wave theory fails to predict the wavelengths of large waves in this water depth. For the present approximation, very small deviations are found in Fig. 6, whether the coefficients of A and ν are constant or not.

When the water depth becomes shallow, the waves belong to long waves. Fig. 7 shows the wavelengths of long waves propagating in a water of $d/L \approx 1/25$. The Stokes wave theory fails even for small-amplitude waves. The cnoidal wave theory is valid only for small-amplitude waves, but fails for large waves in shallow water. The present approximation with variable coefficients is the best prediction to wavelengths for waves from small-amplitude to highest. When the approximation with constant coefficients is used to predict wavelength in shallow water, a increasing deviation with wave height is found in Fig. 7.

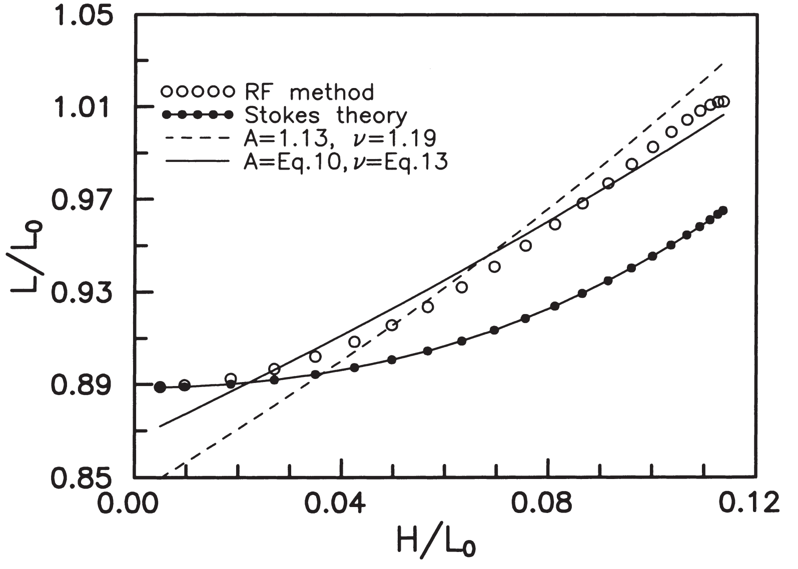

Fig. 5. The wavelength predicted by the Stokes wave theory, the RF method and the present approximations with constant and with variable coefficients in a water depth $d/L \approx 1/4.4$ ($d = 20$ m, $T = 8$ s).

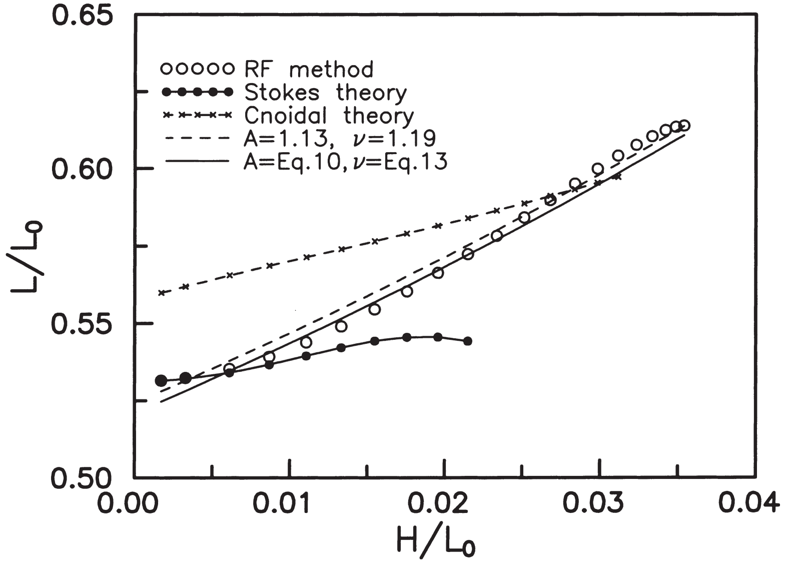

Fig. 6. The wavelength predicted by the Stokes wave theory, the RF method and the present approximations with constant and with variable coefficients in a water depth $d/L \approx 1/10$ ($d = 5$ m, $T = 8$ s).

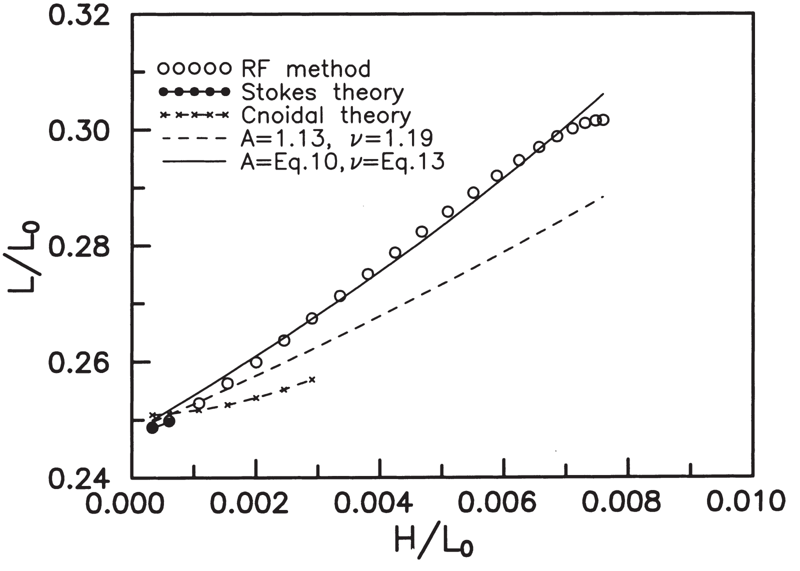

Fig. 7. The wavelength predicted by the Stokes wave theory, the RF method and the present approximations with constant and with variable coefficients in a water depth $d/L \approx 1/25$ (d = 1 m, T = 8 s).

4. Conclusion

In coastal hydrodynamics, many physical quantities, such as group velocity, radiation stress etc., will be obtained until wavelengths of waves are known. An explicit approximation to the wavelength is needed for practical engineering use to avoid numerical programming and algorithm in order to solve a nonlinear equation of dispersion relation. The effects of wave height on wavelength are rather important from comparing wavelengths of small amplitude waves with those of higher waves. There is a difference between these wavelengths of about 15–20%. A simple and efficient approximation to the wavelength in which the effect of nonlinearity is considered is introduced in a rational form. The present approximation with variable coefficients, which were determined by the best fitting, can give an excellent prediction to wavelengths for both long and short waves, even for high waves. The limitations of Stokes wave theory and cnoidal wave theory to predict wavelengths considering the effects of nonlinearities are also constrained to deep water and shallow water, respectively. Meanwhile, the Stokes wave theory and the cnoidal wave theory are not valid to predict wavelengths for large waves in intermediate water. The present approximation has no limiting disadvantages and is valid for all waves in all water depths.

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