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Negative parity states and octupole collectivity of even Ge isotopes

Der-San Chuu
Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan, Republic of China

S. T. Hsieh and H. C. Chiang
Department of Physics, National Tsing Hua University, Hsinchu Taiwan, Republic of China

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The negative parity energy levels of the even-even Ge isotopes with mass number between 64 and 76 are studied systematically by enlarging the model space of the interacting boson approximation model to include both collective and noncollective basis states. The basis states consist of N_B-1 *sd*-boson plus a *f*-boson configuration and N_B-1 *sd* boson plus a fermion pair configuration. The fermions are allowed to occupy the $f_{5/2}$ and $g_{9/2}$ single-particle orbitals, respectively. It was found that the negative parity energy levels of $^{64-76}$Ge nuclei can be described reasonably well. The intensities of the collective configuration in 3^- states increase when going from nucleus ^{64}Ge to nucleus ^{72}Ge and decrease from nucleus ^{72}Ge to nucleus ^{74}Ge. The $B(E3;3_1^-\rightarrow 0_1^+)$ values are calculated and compared with the available observed data.

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I. INTRODUCTION

In recent years, considerable progress has been made in extending the interacting boson approximation model (IBA) to study the negative parity states of even-mass nuclei and the high spin states anomaly in medium to heavy deformed nuclei [1–9]. Among these works very few are concentrated on the structure of energy levels of medium light nuclei such as the Ge isotopes. During the past few years, the observed nuclear properties of the negative parity energy states of even-mass Ge isotopes have been accumulated and considerable attention has been attracted [10–22]. It is known that the Ge nuclei are complex nuclear systems with unstable shapes. Both the coexistence of a shape transition from spherical to weakly deformed and a coexistence of different types of deformation are expected in these nuclei [23–25]. Qualitatively these features can be explained with the help of the Nilsson model calculation [26], shell model calculation [27], constrained Hartree-Fock calculation [28], two-quasiparticle plus- rotor calculation [13], and IBA model calculation [13].

Substantial experimental evidence suggests that extensive regions of statically octupole deformed nuclei occur in some mass regions [29–33]. The question whether certain nuclei can be octupole unstable has been a subject of much experimental and theoretical interest during the past ten years. Nazarewicz and collaborators [31,32] have proposed that the nuclei with the strongest octupole correlations (i.e., the best candidates for static octupole deformation) occur when *N* and *Z* are equal to 34, 56, 88, and 134. However, Cottle [30] analyzed systematically the behavior of 3_1^- states from the available observed data and identified the *N* and *Z* values equal to 40, 64, 88, and 134 for maximum octupole collectivity. It is interesting to study the problem of octupole collectivity from the point of view of the IBA model.

The purpose of this work is twofold. First, we want to present a systematic study of the negative parity energy levels of even-mass Ge isotopes. Second, we desire to investigate the octupole collectivity around the region of mass number $A\simeq 70$ by a hybrid of *sdf* IBA and IBAF models.

II. MODEL

The negative parity states of even mass Ge isotopes with $Z=32$ and $32\leq N\leq 44$ will be studied systematically. For this mass region, the ^{40}Ca nucleus or the ^{56}Ni nucleus can be treated as the core. In IBA calculation, it is known that the effect of using a different core can be absorbed in the interaction strengths. Therefore, we may take the ^{40}Ca nucleus or ^{56}Ni nuclei as the core in this work. By assuming ^{40}Ca nuclei as the core, the boson number for the isotopes ^{64}Ge and ^{66}Ge are $N_B=12$ and 13, respectively. For the other isotopes which pass the neutron midshell the neutron boson numbers are counted as one-half of the number of neutron holes. Thus, IBA model assumes valence boson numbers N_B as 13, 12, 11, 10, and 9 for the nuclei ^{68}Ge, ^{70}Ge, ^{72}Ge, ^{74}Ge, and ^{76}Ge, respectively. In this work, the model space is considered as the admixture of two subspaces: (1) the configuration of N_B-1 *sd* bosons plus one *f* boson; or (2) the configuration of N_B-1 *sd* bosons plus two fermions which are allowed to distribute in the $f_{5/2}$ and $g_{9/2}$ orbitals. The former subspace is of a more collective behavior while the later contains some single particle nature. To be more specific, the model space is spanned by the hybrid of two types of basis states:

$$|n_s n_d v a L, f; L_T M_T\rangle \text{ and } |n_s n_d v a L, j_1 j_2(J); L_T M_T\rangle$$

where $n_s+n_d=N_B-1$, $j_1,j_2=5/2$ or $9/2$, and $J=2,3,\ldots,7$.

The model Hamiltonian can be expressed as [8]

$$H=H_B+H_F+V_{BF}+V_N$$

where H_B is the IBA boson Hamiltonian

$$H_B=a_0\varepsilon_d+a_1p^\dagger\cdot p+a_2L\cdot L+a_3Q\cdot Q\ .$$

The octupole term $T_3\cdot T_3$ and the hexadecapole term $T_4\cdot T_4$ have been omitted in H_B since they are generally believed to be less important. The fermion Hamiltonian H_F is

$$H_F=\sum_{j,m}\varepsilon_j a_{jm}^\dagger a_{jm}+\frac{1}{2}\sum_{JM}\sum_{j_1,j_2}V^J(a_{j_1}^\dagger a_{j_2}^\dagger)^{JM}(\tilde{a}_{j_1}\tilde{a}_{j_2})^{JM}\ ,$$

where ε_j is the fermion single-particle energy, V^J's are the fermion-fermion interactions, $a_{jm}^\dagger$ (a_{jm}), and $\tilde{a}_{jm}=(-1)^{j-m}\,a_{jm}$ being the nucleon creation (annihilation) operator. The mixing Hamiltonian V_{BF} between the *sd* boson and the fermion is assumed:

$$V_{BF}=\alpha Q^B\cdot\sum_{j_1,j_2}(a_{j_1}^\dagger\tilde{a}_{j_2})^{(2)}$$

where

$$Q^B=(d^\dagger\tilde{s}+s^\dagger\tilde{d})^{(2)}-\sqrt{7}/2(d^\dagger\tilde{d})^{(2)}\ ,$$

and the Hamiltonian related to the f-boson part is

$$V_N=\varepsilon_f n_f+\gamma Q^B\cdot(f^\dagger\tilde{f})^{(2)}+\delta\sum_{j_1,j_2,J}Q^B\cdot[(a_{j_1}^\dagger a_{j_2}^\dagger)^{(J)}\tilde{f}+\text{H.c.}]^{(2)}$$

which includes the f-boson single-particle energy and mixing the Hamiltonian of the f-boson with the *sd* boson and with the fermions. The fermion potential is taken as the Yukawa type with the Rosenfeld mixture. The oscillation constant $\nu=0.96A^{-1/3}$ fm^{-2} with $A=70$ is assumed. The whole Hamiltonian is then diagonalized in the selected model space. Practically, we first performed a calculation for the positive parity energy levels of even mass $^{64-78}$Ge nuclei with the framework of extended IBA model [34,35]. For the calculation of the negative parity energy levels, the interaction strength parameters in Hamiltonian H_B are kept as the same values as those obtained from the calculation of the positive parity energy levels of $^{64-78}$Ge nuclei. The mixing parameters α, γ, δ, the single f-boson energy ε_f, and the single-fermion energies ($j=5/2$ and $9/2$) contained in the fermion Hamiltonian H_F, V_N, and V_{BF} were chosen to reproduce the negative parity energy spectra of isotopes $^{64-76}$Ge, respectively. The interaction strengths and single-particle energies for each isotope are allowed to be mass-number dependent.

III. RESULT AND DISCUSSION

Table I represents the final chosen values of the interaction strengths and single-particle energies. The values of γ which represents the interaction between the *sd* and the f-bosons are in general very small [8] and thus can be set to zero. It can be seen from Table I that the values of the mixing parameters are in general not too large. The smallness of the mixing parameters manifests the small mixings between the different configurations. The single f-boson energy as well as the single fermion energies has a minimum value around the mass number $A\simeq 68$. This means the effect of the f boson is most important around the ^{68}Ge nucleus. One may note that some of the effective interaction strength parameters and single particle energies (especially the ε_f) have a significant change around the nucleus ^{68}Ge. This is because in our work we assume ^{40}Ca nuclei as the core, the boson number for ^{64}Ge and ^{66}Ge are counted as one-half of the number of nucleons outside the core while other Ge isotopes ($^{68-76}$Ge) that pass the neutron midshell and the neutron boson numbers are counted as one half of the number of neutron holes. Therefore, we have particle-particle to particle-hole transitions from nucleus ^{66}Ge to nucleus ^{68}Ge. And this is the reason that we have an unusual change of the interaction parameters in the $A\simeq 68$ region. One can also note from Table I that the features of the variation of single fermion energies ($\varepsilon_{5/2}$ and $\varepsilon_{9/2}$) are similar to that of single f-boson energy. The values of $\varepsilon_{5/2}$ and $\varepsilon_{9/2}$ of the nuclei ^{74}Ge and ^{76}Ge are obtained from the extrapolation.

In the past few years, several observations on the negative parity states of Ge nuclei have been performed. de Lima *et al.* [13] studied the low and high spin states of ^{68}Ge through in-beam γ-ray spectroscopy via the ^{58}Ni(^{12}C,2p)^{68}Ge, ^{63}Cu(^{7}Li,2n)^{68}Ge, and ^{52}Cr(^{19}F,p2n)^{68}Ge reactions. They observed three positive parity bands and two negative parity bands up to two tentative 11^- states. Ardouin *et al.* [16] presented some negative parity states data from $^{70-76}$Ge(p,t)$^{68-74}$Ge reactions. The calculated negative parity energy spectra in this work are compared with the observed ones in Figs. 1 and 2. The levels marked with asterisks are not included in the least-

TABLE I. The interaction parameters (in MeV) adopted in this work.

	Parameter (MeV)								
Nucleus	a_0	a_1	a_2	a_3	α	δ	ε_f	$\varepsilon_{5/2}$	$\varepsilon_{9/2}$
^{64}Ge	0.298	-0.220	0.035	-0.016	-0.006	-0.4	2.080	0.996	2.235
^{66}Ge	0.254	-0.220	0.035	-0.008	-0.661	-0.4	1.295	3.245	4.547
^{68}Ge	0.156	-0.220	0.023	0.015	-0.005	-0.3	0.836	0.576	1.545
^{70}Ge	0.289	-0.155	0.023	-0.001	-0.527	-0.3	1.707	3.158	3.903
^{72}Ge	0.289	-0.102	0.023	-0.001	-0.527	-0.3	2.116	3.974	4.461
^{74}Ge	0.396	-0.035	0.023	-0.001	-0.527	-0.3	2.760	4.791	5.020
^{76}Ge	0.400	-0.025	0.023	-0.001	-0.527	-0.3	2.850	5.607	5.578

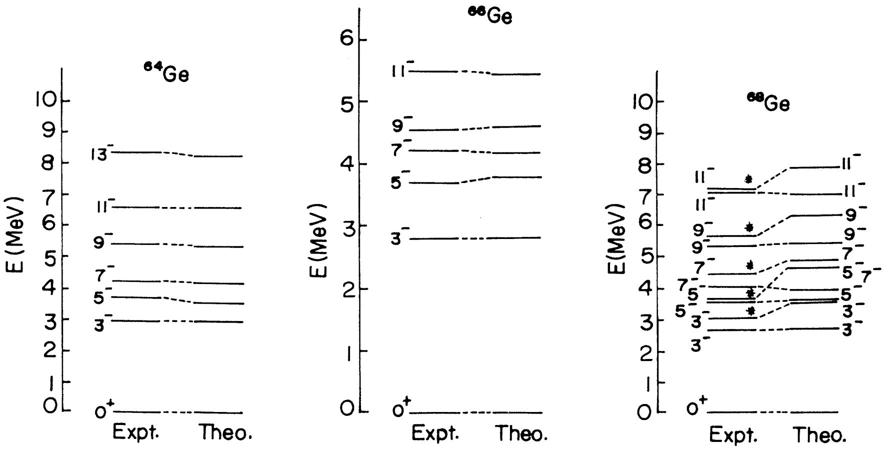

FIG. 1. The calculated and observed negative parity energy spectra for the nuclei ^{64}Ge, ^{66}Ge, and ^{68}Ge. The experimental data are taken from Refs. [10–15].

squares fitting. Figure 1 shows the calculated and the observed negative parity states of the three lighter mass nuclei $^{64-68}$Ge. Figure 2 presents those of the three heavier mass nuclei $^{70-74}$Ge. Several levels not yet observed are presented in the figures for future reference.

Nazarewicz *et al.* [32] performed a Strutinsky calculation that included octupole as well as qaudruple and hexadecapole shape degrees of freedom and indicated that nuceli in the light Ge-Se region would be stable but rather soft with respect to octupole deformation. They predicted that the largest octupole softness would occur for ^{64}Ge. The calculation of Görres *et al.* [10] found a modest decrease in the 3_1^- energy in ^{66}Ge followed by a sharper drop in ^{64}Ge. From a microscopic point of view, octupole collectivity originates in the interaction between the unique parity orbit in a major shell and the common parity orbit having both orbital and total angular momentum $3\hbar$ less than that of the unique parity orbit [24]. The nuclei in which the strongest octupole effects occur are those in which Fermi surfaces of both neutrons

TABLE II. The relative intensities of the (N_B-1)-boson plus one f-boson configuration and the (N_B-1)-boson plus a fermion pair configuration for negative parity states of Ge nuclei. The total intensity of these two configurations for each state is normalized to 1.0. The numbers shown are the intensities for boson plus f-boson intensities.

	Nucleus						
States	^{64}Ge	^{66}Ge	^{68}Ge	^{70}Ge	^{72}Ge	^{74}Ge	^{76}Ge
3_1	0.799	0.919	0.932	0.946	0.953	0.819	0.947
3_2			0.804		0.502	0.764	0.931
3_3					0.939	0.882	
3_4					0.781	0.879	
1_1					0.591	0.611	
1_2					0.702	0.848	
2_1					0.780	0.673	0.892
2_2					0.572	0.775	
2_3					0.850	0.810	
2_4					0.786		
4_1						0.785	0.925
4_2						0.800	0.912
5_1	0.679	0.736	0.781	0.839			0.894
5_2			0.795				
6_1				0.732			0.899
7_1	0.000	0.135	0.000	0.154			
8_1				0.011			
9_1	0.000	0.029	0.000	0.022			
10_1				0.011			
11_1	0.000	0.009	0.000	0.009			
13_1	0.000		1.000				

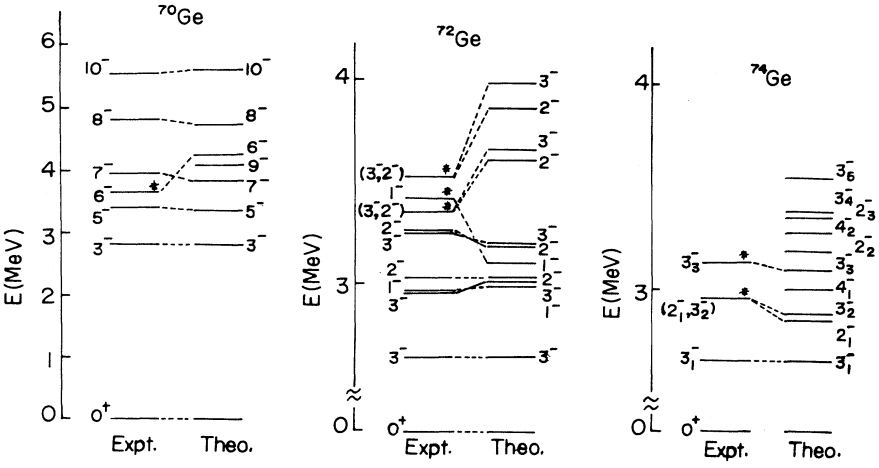

FIG. 2. The calculated and observed negative parity state energy spectra for the nucleus ^{70}Ge, ^{72}Ge, and ^{74}Ge. The experimental data are taken from Refs. [16–21].

and protons lie between the two interacting orbits. Based on the above argument, Nazarewicz *et al.* proposed that the nuclei with the strongest octupole correlations occur when N and Z are equal to 34, 56, 88, and 134. Cottle [30] performed a systematic observation regarding the energies of 3_1^- states which are octupole vibrational states in most nuclei and found that instead of those nuclei proposed by Nazarewicz *et al.* [32], the strongest octupole correlations seem to occur at nuclei with N and Z being equal to 40, 64, 88, and 134. Our plot of 3_1^- states of Ge nuclei versus the neutron number N is shown in Fig. 3. The octupole deformation behavior can be investigated by the behavior of 3_1^- states because nuclei which have the greatest degree of octupole collectivity have the lowest 3_1^- state energies. It can be noted from the figure that the 3_1^- state of the ^{72}Ge nucleus lies at the minimum position. We also analyzed our calculated wave functions. In Fig. 3, we list the relative wave function intensities of the (N_B-1)-*sd*-boson plus a f-boson configuration for the lowest 3_1^- states of Ge isotopes in the parentheses. The total intensity of the (N_B-1)-boson plus one f-boson configuration and the (N_B-1)-boson plus two fermions configuration for each state has been normalized to 1000. One can note from the figure that the intensities of the f-boson configuration increase when going from nucleus ^{64}Ge to nucleus ^{72}Ge and then decrease when going from ^{72}Ge to ^{74}Ge. Our calculation is equivalent to enlarge the model space to consider both collective and noncollective negative parity states. The relative wave-function intensities of the (N_B-1)-*sd*-boson plus a f-boson configuration for the negative parity energy levels of Ge nuclei are shown in Table II. One can note from Table II that, in general, the states with smaller angular momenta $(J_\pi<5^-)$ are dominated by the (N_B-1)-*sd*-boson plus a f-boson configuration while the states with higher angular momenta are dominated by the (N_B-1)-*sd*-boson plus

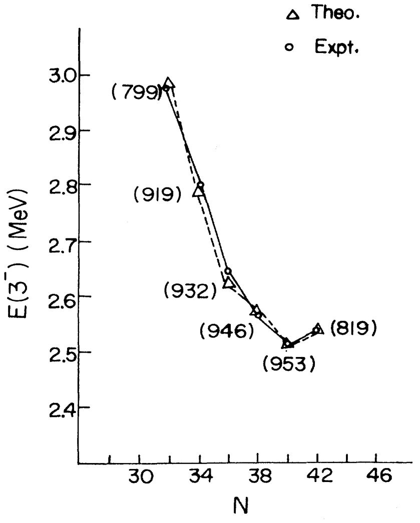

FIG. 3. The energies of 3_1^- states for the Ge nuclei are plotted as a function of the neutron number of Ge isotopes.

TABLE III. The calculated and observed $B(E3;0_1^+\rightarrow 3_1^-)$ values. The experimental data are taken from Ref. [33].

	$B(E3;0_1^+\rightarrow 3_1^-)$	
Nucleus	Expt.	Theo.
^{64}Ge		31.577
^{66}Ge		29.762
^{68}Ge		30.440
^{70}Ge	35.850	23.616
^{72}Ge	23.751	32.028
^{74}Ge	8.830	9.140
^{76}Ge	8.730	5.720

two fermions configuration. The mixings between the two kinds of configurations are in general not too large.

In order to check our wave functions, we calculated the $B(E3;0_1^+\rightarrow3_1^-)$ values. There are only a few experimental $B(E3;0_1^+\rightarrow3_1^-)$ values available [33]. For future observation, we present also some theoretical values for which the experimental counter parts are not available now. In our calculation the fermion effective charge e^F is assumed to be 0.5e. The average boson effective charge e^B which is determined by normalizing the calculated $B(E2)$ value [36] to the corresponding observed data for the transition $2_1^+\rightarrow0_1^+$ is about 0.24e. It can be noted from Table III that our calculated $B(E3;0_1^+\rightarrow3_1^-)$ values agree satisfactorily with the experimental data, since the $B(E3)$ values are very sensitive to the wave functions. The reasonable agreement of $B(E3;0_1^+\rightarrow3_1^-)$ values shows that our wave function is also reasonably good.

IV. SUMMARY

In summary, we have investigated the structure of the negative parity energy spectra of the Ge isotope with mass numbers between 64 and 76. The IBA model space is enlarged to consider the collective and noncollective negative parity states. It was found that the mixings of the configuration of (N_B-1)-sd-boson plus a f-boson and the configuration of (N_B-1)-sd-boson plus two fermions in the negative parity states of Ge isotopes are in general small. The octupole softening is also analyzed by studying the energy values, the wave functions, and the $B(E3;0_1^+\rightarrow3_1^-)$ transition rates of the 3_1^- states.

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