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Development and Implementation of a Semi-Adaptive H_∞ Active Control Algorithm for Duct Noise

A hybrid active noise control (ANC) scheme for suppressing duct noise based on the H_∞ control synthesis is proposed. The controllers are designed in terms of performance, stability, and robustness using a general framework of the H_∞ robust control theory. In addition to the fixed controller, the system is further enhanced by introducing an adaptive compensator based on the least-mean-square (LMS) algorithm. Youla parameterization is employed in designing the adaptive compensator so that the resulting system is stable. Experimental investigations demonstrate that the proposed methods are effective in suppressing broadband random noise in a finite-length duct.

Introduction

Active noise control (ANC) techniques have attracted research attention because it provides advantages over traditional passive methods in attenuating low-frequency noises (Elliot and Nelson, 1993). However, many problems with practical applications remain to be solved prior to full commercialization of the ANC techniques. Besides the difficulty of obtaining an upstream correlated reference, acoustic feedback has been a plaguing problem that usually arises in feedforward structure. An excellent review concerning acoustic feedback can be found in the book by Kuo and Morgan (1995).

This paper adopts a new approach to tackle acoustic feedback. Semi-adaptive H_∞ controllers based on a hybrid (feedback/feedforward) scheme is proposed. The H_∞ synthesis incorporates the acoustic feedback path into the control design. In addition to the fixed controllers, the system is further enhanced by introducing an adaptive compensator based on the least-mean-square (LMS) algorithm.

The Fixed H_∞ Controllers

The H_∞ robust control theory is employed for the design of fixed controllers. The details can be found in the paper by Bai and Lin (1997). Let $C(z)$ be the controller and $P(z)$ the primary plant. The controlled variable $v(k)$ corresponds to various control objectives $z_1(k)$, $z_2(k)$, and $z_3(k)$, and the extraneous input $w(k)$ consists of the reference $r(k)$, the disturbance $d(k)$, and the noise $n(k)$. The signal $u(k)$ and $e(k)$ are the control input to the plant and the measured output from the plant, respectively.

Because acoustic feedback generally causes detrimental effects to the ANC systems, positive feedback loop must be taken into account in the design of controllers. The beauty of the H_∞ general framework lies in the fact that it needs no special treatment insofar as acoustic feedback is concerned.

Although the feedforward controller is effective in broadband noise attenuation, its performance could vary drastically with system perturbations. It is desirable to develop an ANC controller capable of accommodating these effects. To this end, we introduce another feedback controller to the feedforward system in order to improve the system robustness. The block diagram of the hybrid structure with acoustic feedback is illustrated in Fig. 2. $C_1(z)$ and $C_2(z)$ are the feedback and the feedforward parts of the hybrid controller. To find a H_∞ controller, we weight the error signal $e(k)$ by $W_1(z)$, the control input $u(k)$ by $W_2(z)$, and the downstream sensor output $y(k)$ by $W_3(z)$. The input-output relation of the augmented plant corresponding to the hybrid structure can be expressed as

$$
\begin{bmatrix} Z_1(z) \\ Z_2(z) \\ Z_3(z) \\ X(z) \\ E(z) \end{bmatrix}
=
\begin{bmatrix}
W_1(z) & -W_1(z)P(z) & -W_1(z)S(z) \\
0 & 0 & W_2(z) \\
0 & W_3(z)P(z) & W_3(z)S(z) \\
1 & -P(z) & -S(z) \\
0 & 1 & F(z)
\end{bmatrix}
\begin{bmatrix} R(z) \\ D(z) \\ U(z) \end{bmatrix},
\tag{1}
$$

here $P(z)$ is the primary path, $S(z)$ is the secondary path, $F(z)$ is the acoustic feedback path. The suboptimal condition of the H_∞ hybrid controller reads

where

$$
\left\| \begin{bmatrix}
W_1(z)FS(z) & -W_1(z)\{P(z)[1 - C_2(z)F(z)] + C_2(z)S(z)\}FS(z) \\
W_2(z)FR(z) & W_2(z)[C_2(z) - C_1(z)P(z)]FS(z) \\
W_3(z)FT(z) & W_3(z)\{P(z)[1 - C_2(z)F(z)] + C_2(z)F(z)\}FS(z)
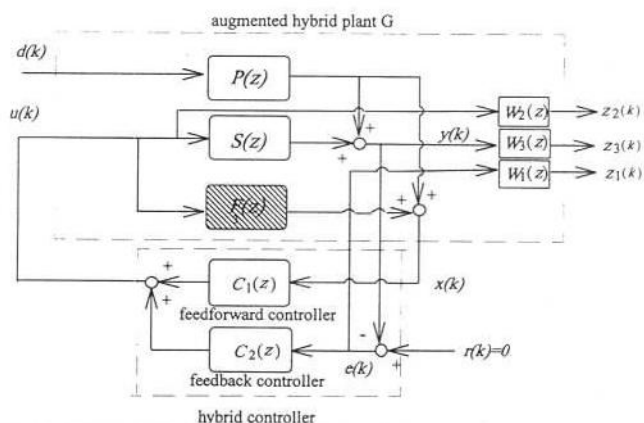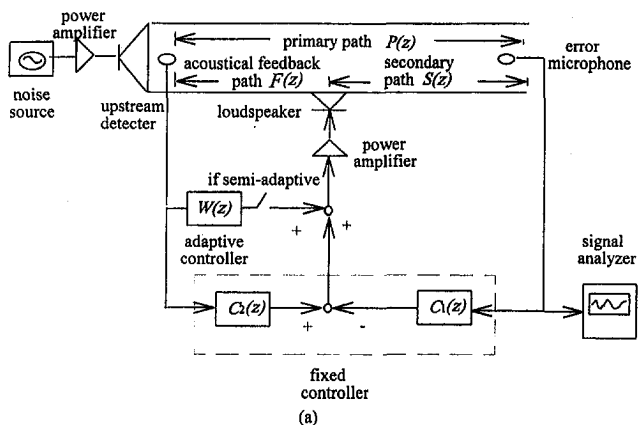\end{bmatrix} \right\|_\infty < 1.
\tag{2}
$$

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$$
FS(z) = \frac{1 - C_2(z)F(z)}{1 - C_2(z)F(z) + C_1(z)S(z)},
\tag{3}
$$

$$
FR(z) = \frac{C_2(z) - P(z)C_1(z)}{1 - C_2(z)F(z) + C_1(z)S(z)},
\tag{4}
$$

and

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Fig. 1 Hybrid ANC structure including an acoustical feedback path (a) Experimental setup, (b) control block diagram

$$FT(z) = \frac{P(z)[1 - C_1(z)F(z)] + C_1(z)S(z)}{1 - C_2(z)F(z) + C_1(z)S(z)}. \quad (5)$$

In H_∞ control synthesis, the optimal tradeoff between performance and stability is achieved by tuning the weighting functions $W_1(z)$, $W_2(z)$, and $W_3(z)$ that generally chosen to be lowpass, constant, and high-pass, respectively.

The Semi-Adaptive H_∞ Controllers

In practical applications, ANC schemes with more flexibility should be sought to further enhance the performance of the system. This can be done by nesting an adaptive compensator based on LMS algorithm to the original fixed controller. Because of the coupling between the fixed controller and the adaptive compensator, we refer the scheme as the *semi-adaptive* controller.

The semi-adaptive hybrid structure is shown in Fig. 2. The primary plant, $P(z)$, is the transfer function formed by the primary acoustic path, the power amplifier, the upstream sensor, and the error microphone. The secondary plant $S(z)$ is the transfer function of the cancellation path formed by the canceling loudspeaker, the power amplifier, the secondary acoustic path, and the error microphone. The transfer function $F(z)$ is the acoustical feedback path formed by the canceling loudspeaker, the power amplifier, the acoustical feedback path, and the upstream sensor. The transfer functions $C_1(z)$ and $C_2(z)$ are the fixed feedforward controller and the feedback controller obtained by H_∞ synthesis. Note that $T(z)$ is a transfer function

describing the source dynamics formed by the primary source, a power amplifier, an acoustical path, and the upstream microphone. Finally, $W(z)$, represents the adaptive compensator that is aimed to *fine-tune* the fixed controllers. Hence, the model-matching problem can be expressed as

$$\min_W \left\| T\left[P + S \frac{(C_1 + W) - PC_2}{1 - F(C_1 + W) + SC_2} \right] \right\|. \quad (6)$$

Here, $T(z)$ may not be measurable so that a heuristic guess is needed.

Instability may arise after the introduction $W(z)$. The compensator $W(z)$ must be designed with care under the constraint that the resulting system is internally stable. The key of undoing this difficulty is the Youla's parametrization (Youla et al., 1976). For a stable plant $K(z)$, the Youla's theorem indicates that the controllers $W(z)$ that guarantee the internal stability of the closed-loop system can be parametrized as

$$W(z) = \frac{Q(z)}{1 - K(z)Q(z)} \quad (7)$$

where the parameter $Q(z)$ is a stable, proper, and real-rational function (denoted as $Q(z) \in RH^\infty$) and

$$K(z) = \frac{-F(z)}{1 - C_2(z)F(z) + C_1(z)S(z)}. \quad (8)$$

Substituting Eq. (7) into the minimization problem in Eq. (6) leads to

$$\min_{Q \in RH^\infty} \left\| T\left[P + S \frac{(1 - KQ)(C_1 - PC_2) + Q}{(1 - KQ)(1 - FC_1 + SC_2) + FQ} \right] \right\|. \quad (9)$$

The optimal solution of Eq. (16), $Q_{opt}(z)$, can be obtained by using the LMS algorithm. The update formula for the LMS algorithm can be shown to be

$$Q(k + 1) = Q(k) - \mu x'(k)e(k), \quad (10)$$

where $x'(k) = m(k)*x(k)$ is the filtered input signal, $x(k)$ is the upstream microphone signal, and $m(k)$ is the impulse response of
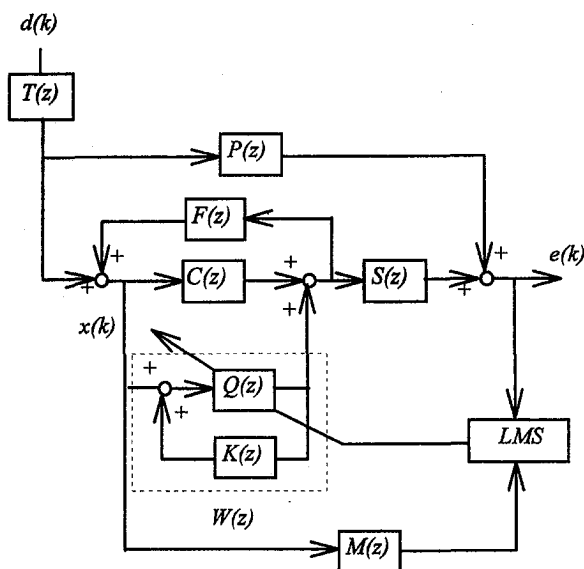

Fig. 2 Block diagram of the semi-adaptive H_∞ hybrid controller

$$M(z) = \frac{S(z)}{1 - C_2(z)F(z) + C_1(z)S(z)} .$$

Experimental Investigations

A rectangular duct of cross-section 0.25×0.25 m and length 1 m is used for the experiments, which renders the cutoff frequency 690 Hz. The sampling rate is selected to be 2 kHz. The upstream reference microphone and error microphone are located at 0.25 m and 0.75 m, respectively, from the source end. On the other hand, the error microphone is placed in the close vicinity of the control loudspeaker to form the so-called collocated control.

The forgoing H_∞ and LMS designs are applied to obtain the fixed and adaptive controllers. The controllers are then coded into digital filters using TMS320C31. Gaussian white noise is chosen in the experiment. The pressure spectra of the residual noise at the error microphone position with and without active control are shown in Fig. 3. The semi-adaptive H_∞ hybrid controller shows effectiveness in suppressing the white noise. Approximately 3–10 dB and 5–23 dB noise attenuation have been achieved in two bands 50–100 Hz and 170–400 Hz, respectively. The total attenuation within the band 0–400 Hz is 4.85 dB.

Conclusions

The semi-adaptive hybrid active noise controller has been developed for suppressing duct noise. The controller design is based on a general framework of the H_∞ theory that takes into account performance, stability, and robustness of the control system. The performance has been improved by introducing the
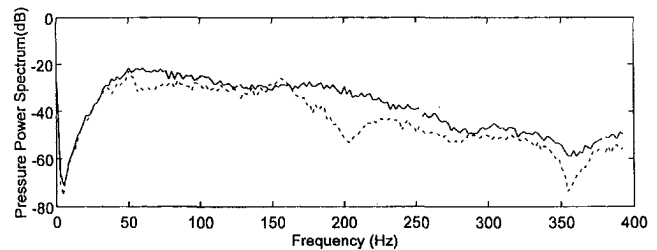

Fig. 3 The residual sound pressure spectra obtained by ANC. The Gaussian white noise is used as the primary source. The ANC method is the semi-adaptive H_∞ hybrid controller. Active control off _; control on.

feedback compensator and the LMS compensator. The proposed method shows potential in suppressing broadband noise.

Acknowledgments

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