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The anomalous Hall effect for a mixed *s*- and *d*-wave symmetry superconductor

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Abstract

By taking account of the complex relaxation times for *s*- and *d*-wave order parameters in the time-dependent Ginzburg–Landau equations, our results indicate that the imaginary parts of the relaxation times can change the sign of the Hall angle for mixed *s*- and *d*-wave superconductors. The effect of the concentration of the nonmagnetic impurities on the Hall angle is investigated, and it is found that the concentration of the nonmagnetic impurities can affect the parameters in the Ginzburg–Landau free energy and the relaxation times. The results of the anomalous Hall effect arising from the nonmagnetic impurities are discussed.

Keywords: A. High-T_c superconductors; D. Electronic transport; D. Galvanomagnetic effects

The pairing symmetry of the order parameter in high-T_c superconductors (HTS) attracts attention and a number of experimental observations strongly support the *d*-wave pairing symmetry [1–3]. Most of the theories proposed that the superconducting state might be relevant to the heavy fermion and the high-T_c superconductors [4–6]. Based on group theory arguments, the phenomenological Ginzburg–Landau (GL) theory with many unknown parameters has been studied [4,5,7]. Ren et al. [8,9] microscopically derived the GL equations for the pure *d*-wave as well as mixed *s*- and *d*-wave superconductors in the framework of the weak coupling theory.

Recently attention has been focussed on the anomalous behavior of the Hall effect, which appears to undergo a sign change in the superconducting mixed state [10–15]. The phenomenological theories about the vortex motion have been proposed to investigate the anomalous Hall effect of superconductors [16–20]. Dorsey [20] introduced a complex relaxation time in the time-dependent Ginzburg–Landau (TDGL) equation and then derived the equation of motion for a single vortex

$$V_{s1} \times \hat{z} = \alpha_1 V_L + \alpha_2 V_L \times \hat{z}, \tag{1}$$

where (α_1 and α_2 are functions of the parameters that appear in the TDGL equation. The results showed that, if $\alpha_2 < 0$, the Hall effect in the vortex state will change its sign, which is opposite to the sign of the normal-state Hall effect. Experimental measurements in $Tl_2Ba_2CaCuO_8$ systems also showed a consistent description of the behavior of the imaginary part of the order parameter relaxation time [21]. However, the problems of the Hall effect for *d*-wave superconductors have not been fully understood. Following the generalized London theory, derived by Affleck et al. [22], Alvarez, Domínguz and Bleeder

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studied the dynamics of vortices in d-wave superconductors. They found that an intrinsic Hall effect depended on $\sin(4\xi)$ with an angle ξ with respect to the b crystal axes [23]. Dai and Yang [19] have elucidated that both the imaginary parts of the relaxation times for the order parameters and the parameters in the GL free energy functional can affect the anomalous Hall effect for a pure d-wave superconductor. Here we will investigate the Hall effect for mixed s- and d-wave superconductors from the TDGL equations.

The phenomenological TDGL equations in the dimensionless form for $d_{x^2-y^2}$ superconductors can be expressed as

$$\eta_d(\partial_t + \mathrm{i}\tilde{\phi})d = \alpha_d d + \frac{8}{3}\left(\frac{\beta_2}{\beta_1}\right)|d|^2 d + \frac{4}{3}\left(\frac{\beta_3}{\beta_1}\right)|s|^2 d + \frac{8}{3}\left(\frac{\beta_4}{\beta_1}\right)d^* s^2 + 2\left(\frac{\gamma_d}{\gamma_s}\right)\Pi'^2 d + 2\left(\frac{\gamma_\nu}{\gamma_s}\right)(\Pi_x'^2 - \Pi_y'^2)s, \quad (2)$$

$$\eta_s(\partial_t + \mathrm{i}\tilde{\phi})s = \alpha_s s + \frac{8}{3}|s|^2 s + \frac{4}{3}\left(\frac{\beta_3}{\beta_1}\right)|d|^2 s + \frac{8}{3}\left(\frac{\beta_4}{\beta_1}\right)s^* d^2 + 2\Pi'^2 s + 2\left(\frac{\gamma_\nu}{\gamma_s}\right) \times (\Pi_x'^2 - \Pi_y'^2)d, \quad (3)$$

where $\Pi' = -\mathrm{i}\nabla/\kappa - A$, $\tilde{\phi} = 2e\phi$, s and d are the order parameters, α_s and α_d depend on the temperature, and β_1, β_2, β_3, β_4, γ_s, γ_d and γ_ν are assumed to be positive. One should treat the order parameters not as wave functions but as a thermodynamic variable like the magnetic moment in an Ising magnet or like the volume in an expanding gas. The parameters γ are related to the effective masses with $\gamma_i = \hbar^2/2m_i^*$ for $i = s, d, \nu$. These parameters in the TDGL equations can determine the thermodynamic stability. The dimensionless order-parameter relaxation time η_d and η_s can be defined by

$$\eta_d \equiv \eta_{d1} + \mathrm{i}\eta_{d2}, \qquad \eta_s \equiv \eta_{s1} + \mathrm{i}\eta_{s2}. \quad (4)$$

The imaginary part of the complex relaxation time breaks the particle-hole symmetry in superconductors, resulting in nonvanishing Hall current [10,20]. As these nonlinear TDGL equations are complicated, we shall derive an equation of motion for a single vortex in the limit $h \ll H_{c2}$ Namely, the magnetic fields are slightly above the lower critical field H_{c1}. According to this limit, the vortices are well separated and may be studied individually. The normal current J_n and the supercurrent J_s can be rewritten in the form

$$J_n = \sigma^{(n)} \cdot \left[-\frac{1}{\kappa}\nabla\tilde{\phi} - \partial_t A \right], \quad (5)$$

$$J_s = 2s^*(\Pi' s) + 2\left(\frac{\gamma_d}{\gamma_s}\right)d^*(\Pi' d) + 2\left(\frac{\gamma_\nu}{\gamma_s}\right)\{\hat{x}[s^*(\Pi_x' d) + d(\Pi_x' s)^*] - \hat{y}[s^*(\Pi_y' d) + d(\Pi_y' s)^*]\} + h.c. \quad (6)$$

The Hall conductivity $\sigma_{xy}^{(n)}$ of the normal-state conductivity tensor produces a Hall effect due to the transverse response of the normal fluid to the electric fields, generated in the vortex core for type-II superconductors [20].

First, we express the complex order parameters d and s in terms of an amplitude and a phase, $d(r,t) = f(r,t)\mathrm{e}^{\mathrm{i}\theta_d(r,t)}$ and $s(r,t) = g(r,t)\mathrm{e}^{\mathrm{i}\theta_s(r,t)}$, to discuss an isolated vortex. In a superconductor the gradient of the phases can determine the observable quantity J_s. Affleck et al. [22] have derived the supercurrent of a d-wave superconductor by way of the generalized London theory. If $\gamma_\nu = 0$, these TDGL equations have the same solutions as those presented in conventional superconductors. This case is not interesting to us. We attempt to solve these equations for $\gamma_\nu \neq 0$. By substituting the complex order parameters and Eq. (4) for Eqs. (2), (3), (5) and (6), we can separate the real and imaginary parts from the TDGL equations. In order to solve these nonlinear TDGL equations, three essential steps are applied. First, we assume that the vortices translate uniformly. Therefore all the quantities, which characterize the vortex system, are functions of the quantity $r - V_L t$ with V_L the vortex line velocity and can be expanded in first order of V_L

$$\psi(r,t) = \psi^{(0)}(r - V_L t) + \psi^{(1)}(r - V_L t). \quad (7)$$

Herein the correction terms $\psi^{(1)}$ are small relative to the velocity of motion V_L. Next, the equations can be expanded in powers of V_L. The terms of order of unit and order of V_L correspond to the equilibrium GL

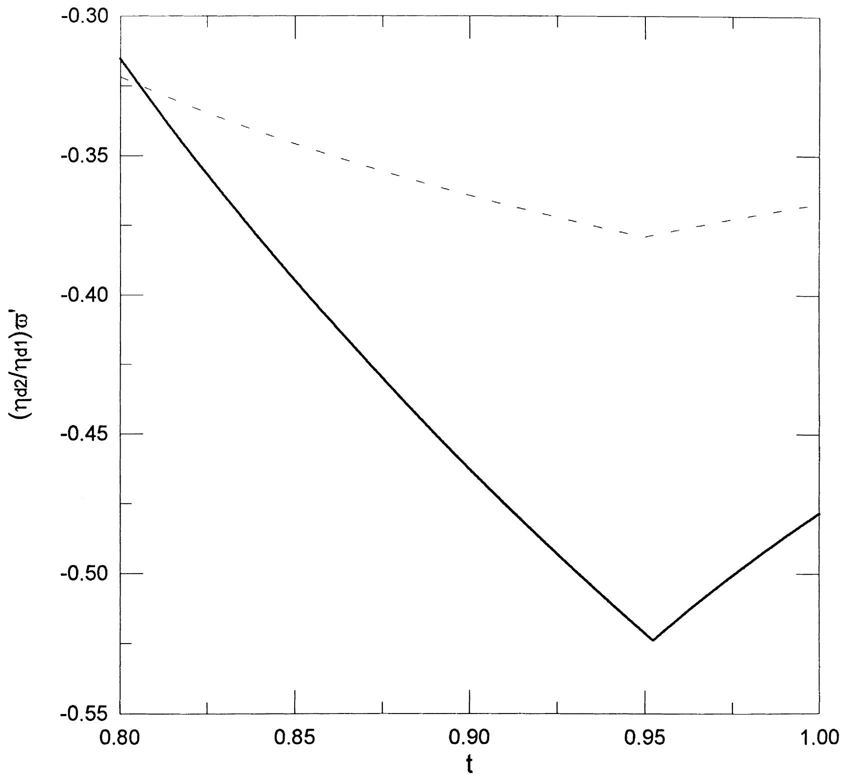

Fig. 1. The function $(\eta_{d2}/\eta_{d1})\overline{\omega'}$ versus the reduced temperature $t = T/T_{d0}$. The solid line represents the condition in the absence of impurities and the dashed line labels one in the presence of the low impurity concentration doped. We choose $T_{s0} = 0.5T_{d0}$ and $(\eta_s/\eta_d) = 2$.

equations and a set of inhomogeneous differential equations, respectively. The equilibrium GL equations have been done well [8,9,24]. We need to solve the $O(V_L)$ equations. Following [25] the time-independent GL equations possess translational invariance. As the magnetic field can be neglected in the large-κ limit, the distributions of the field and current can be investigated far from and close to the vortex cores, respectively (see Ref. [19] for a detailed derivation). Finally, we obtain an equation of motion for the vortices by deriving a solvability condition

$$\frac{1}{2\kappa}\int \mathrm{d}S\cdot[J_s^{(1)}\theta_d^{(\ell)} + J_s^{(1)}\theta_s^{(\ell)} - J_s^{(\ell)}\theta_d^{(1)} - J_s^{(\ell)}\theta_s^{(1)}]$$
$$= \int \mathrm{d}^2r\{\eta_{d1}f_v f_\ell + \eta_{d2}f_0 f_\ell P_d + \eta_{s1}g_v g_\ell$$
$$+ \eta_{s2}g_0 g_\ell P_s + \Sigma_2(V_L) - [\eta_{d1}f_0^2 P_d - \eta_{d2}f_0 f_v$$
$$+ \eta_{s1}g_0^2 P_s - \eta_{s2}g_0 g_v - \Sigma_1(V_L)]\times(\theta_d^{(\ell)} + \theta_s^{(\ell)})\}, \quad (8)$$

where ℓ is an infinitesimal translation, $f_v \equiv V_L\cdot\nabla f_0$, $g_v \equiv V_L\cdot\nabla g_0$, $f_\ell \equiv \ell\cdot\nabla f_0$, $g_\ell \equiv \ell\cdot\nabla g_0$, $\theta_d^\ell \equiv \ell\cdot\nabla\theta_d^{(0)}$, and $\theta_s^\ell \equiv \ell\cdot\nabla\theta_s^{(0)}$. $\Sigma_1(V_L)$ and $\Sigma_2(V_L)$ come from the imaginary and real parts of the TDGL equations, respectively, and include multitudes of functions, which consist of the characteristic functions. This expression, satisfied by the first order in V_L, is equal in effect to derive an equation of motion for the vortices. The inhomogeneous equations are solvable while this solubility condition for steady vortex motion in Eq. (8) holds.

We choose a system of coordinates with the z direction along the applied magnetic field, the applied transport current J_t to be in the x direction; the direction of the vortex in motion at an angle θ_H with respect to the $-y$ direction and the origin of r–φ plane (in the polar coordinates) at the center of a vortex. The displacement vector ℓ makes an angle χ respect to the x-axis. Numerical and asymptotic solutions for the single vortex have been carried out [8,9,24]. At a distance of several coherence lengths ξ_d from the core the *s*-wave order parameter is much less than *d*-wave order parameter so that the *s*–*d* coupling terms are weak enough to be neglected. For the purpose of investigating how the *s*–*d* coupling terms affect the sign of the Hall effect, we choose the region close to the vortex core. The interaction between two vortices can be neglected as $r \gg \lambda_d$ with λ_d the penetration depth of the *d*-wave order parameter. The integration regions in Eq. (8), therefore, are cutoff at λ_d and we obtain

$$\tan\theta_{\mathrm{H}} = \overline{\omega_0} + \overline{\omega'}, \tag{9}$$

where

$$\overline{\omega_0} = \frac{-\dfrac{8\sigma_{xx}^{(n)}}{\kappa^2\lambda_d^2}p_{d2}^{(1)} + \dfrac{8\sigma_{xy}^{(n)}}{\kappa\lambda_d^2}h_0}{\dfrac{8\sigma_{xx}^{(n)}}{\kappa^2\lambda_d^2}p_{d1}^{(1)} + \eta_{d1}c_1^2 + \eta_{s1}c_2^2 - \dfrac{4}{\kappa^2}\left[b_2c_2 + \left(\dfrac{\gamma_d}{\gamma_s}\right)a_2c_1 - \left(\dfrac{\gamma_\nu}{\gamma_s}\right)(b_2c_1 + a_2c_2)\right]}, \tag{10a}$$

$$\overline{\omega'} = \frac{-\eta_{d2}c_1^2 + \eta_{s2}c_2^2 - \dfrac{4}{\kappa^2}\left[b_1c_2 + \left(\dfrac{\gamma_d}{\gamma_s}\right)a_1c_1 - \left(\dfrac{\gamma_\nu}{\gamma_s}\right)(b_1c_1 + a_1c_2)\right]}{\dfrac{8\sigma_{xx}^{(n)}}{\kappa^2\lambda_d^2}p_{d1}^{(1)} + \eta_{d1}c_1^2 + \eta_{s1}c_2^2 - \dfrac{4}{\kappa^2}\left[b_2c_2 + \left(\dfrac{\gamma_d}{\gamma_s}\right)a_2c_1 - \left(\dfrac{\gamma_\nu}{\gamma_s}\right)(b_2c_1 + a_2c_2)\right]}, \tag{10b}$$

$$a_1 = \kappa^2\left[6\eta_{d2}c_1 - 8\left(\frac{\gamma_\nu}{\gamma_s}\right)\eta_{s2}c_2\right]/\Delta_0,$$

$$a_2 = \kappa^2\left[-6\eta_{d1}c_1 - 8\left(\frac{\gamma_\nu}{\gamma_s}\right)\eta_{s1}c_2\right]/\Delta_0,$$

$$b_1 = \kappa^2\left[6\left(\frac{\gamma_d}{\gamma_s}\right)\eta_{s2}c_2 - 8\left(\frac{\gamma_\nu}{\gamma_s}\right)\eta_{d2}c_1\right]/\Delta_0,$$

$$b_2 = \kappa^2\left[6\left(\frac{\gamma_d}{\gamma_s}\right)\eta_{s1}c_2 + 8\left(\frac{\gamma_\nu}{\gamma_s}\right)\eta_{d1}c_1\right]/\Delta_0,$$

$$\Delta_0 = 36\left(\frac{\gamma_d}{\gamma_s}\right) - 64\left(\frac{\gamma_\nu}{\gamma_s}\right)^2, \qquad c_2 = \frac{1}{2}\left(\frac{|\alpha_d|}{\alpha_s}\right)\left(\frac{\gamma_\nu}{\gamma_d}\right)c_1. \tag{10c}$$

Here the scalar potentials P_{d1}, P_{d2}, P_{s1} and P_{s2} satisfy a set of homogeneous equations related to $\mathrm{O}(V_L)$ as $r \to 0$. $\overline{\omega_0}$ is independent of the imaginary parts of the relaxation times and positive due to the large-κ limit. If $\overline{\omega'}$ is negative and satisfies

$$|\overline{\omega'}| > \overline{\omega_0},$$

we can get the sign change of the Hall effect.

Xu et al. [26,27] derived the Ginzburg–Landau equations for a mixed $s + d$ symmetry superconductor with nonmagnetic impurities and showed that the transition temperature for *s*-wave order parameter can only be affected by the magnetic impurity scattering while the transition temperature for *d*-wave order parameter is dominantly affected by the nonmagnetic scattering. Hence, the parameters, α_s and α_d, are given by

$$\alpha_s = \ln\left[\frac{T}{T_{s0}}\right],$$
$$\alpha_d = -\frac{1}{2}\left[\ln\frac{T_{d0}}{T} + \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \frac{c\alpha}{\pi T}\right)\right], \tag{11}$$

where T_{s0} andT_{d0} are the critical temperatures of a

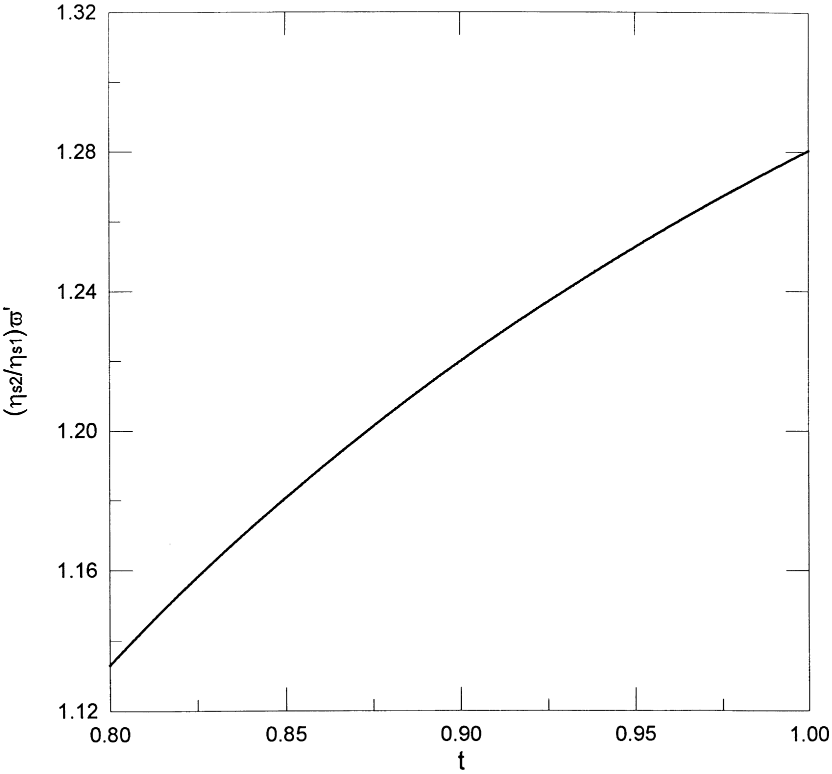

Fig. 2. The function $(\eta_{s2}/\eta_{s1})\overline{\omega'}$ versus reduced the temperature $t = T/T_{d0}$ for high nonmagnetic impurity density doped. We choose $T_c = 0.55T_{d0}$ and $T_{s0} = 0.5T_{d0}$.

clean superconductor, c is the concentration of the impurities, $\Psi(z)$ is the digamma function, and $\alpha = \pi U^2/N_0(0)$ and $1/\pi N_0(0)$ for the Born limit and the unitary limit, respectively. (U is the nonmagnetic potential due to the static impurities and $N_0(0)$ is the normal state electron density of states at the Fermi level.) When the concentration of the impurities increases, the parameter α_d will change from linear T behavior to T^2 behavior. The high scattering strength can also affect the ratio of the parameters γ, related to the effective masses. In this limit, one can find that

$$\gamma_s \gg \gamma_d > \gamma_\nu.$$

In order to illustrate the effect of nonmagnetic impurities on $\overline{\omega'}$, we take $\gamma_s/\gamma_d = 2$, $T_c = 0.95T_{d0}$ and $T_{s0} = 0.5T_{d0}$. In Fig. 1 we plot the function $(\eta_{d2}/\eta_{d1})\overline{\omega'}$ versus $t(= T/T_{d0})$ in the absence of impurities and in the presence of the low impurity density doped. In Fig. 2 we plot the function $(\eta_{s2}/\eta_{s1})\overline{\omega'}$ versus t for the high scattering strength. Fig. 1 shows that $\overline{\omega'}$ is negative near the critical temperature. $\overline{\omega'} < 0$ can lead to the sign change of the Hall angle. Also, the impurity concentrations increasing from low to high, the lowest Hall angle shifts to lower temperature and becomes nonnegative. This implies that the anomalous Hall effect also depends on the concentration of the impurities. This feature is consistent qualitatively with the experimental results.

In conclusion, we show that the imaginary parts of the complex relaxation times can give rise to the anomalous Hall effect for mixed s- and d-wave symmetry superconductors. We decompose the Hall angle into two parts: $\overline{\omega_0}$ and $\overline{\omega'}$. The former part is independent of the imaginary parts of the relaxation times and positive in the large-κ limit. The sign of the latter part can be affected by the relaxation times. If the real and imaginary parts of the relaxation times have the same sign, $\overline{\omega'}$ will be negative near the critical temperature. We also show that the doped nonmagnetic impurities can influence the temperature range of the anomalous Hall effect. In addition, it is noteworthy that the concentration of the impurities can affect the Hall effect. When the impurity concentrations are increasing from low to high, the dependence of the Hall angle changes from the *d*-component to the *s*-component.

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References

[1] D.A. Wollman, D.J. Van Harlingen, W.C. Lee, D.M. Ginsberg, A.J. Leggett, Phys. Rev. Lett. 71 (1993) 2134.
[2] A. Mathai, Y. Gim, R.C. Black, A. Amar, F.C. Wellstoo, Phys. Rev. Lett. 74 (1995) 4523.
[3] C.C. Tsuei, J.R. Kirtley, C.C. Chi, L.S. Yu-Jahnes, A. Gupta, T. Shaw, J.Z. Sun, M.B. Ketchen, Phys. Rev. Lett. 73 (1994) 593.
[4] P. Kumar, P. Wolfle, Phys. Rev. Lett. 59 (1987) 1954.
[5] R. Joynt, Phys. Rev. B 41 (1990) 4271.
[6] Q.P. Li, B.E.C. Koltenbah, R. Joynt, Phys. Rev. B 48 (1993) 437.
[7] M.M. Doria, H.R. Brand, H. Dleiner, Physica C 159 (1989) 46.
[8] Young Ren, J.-H. Xu, C.S. Ting, Phys. Rev. Lett. 74 (1995) 3680.
[9] Young Ren, J.-H. Xu, C.S. Ting, Phys. Rev. B 52 (1995) 7663.
[10] A.T. Dorsey, M.P.A. Fisher, Phys. Rev. Lett. 68 (1992) 694.
[11] J. Luo, T.P. Orlando, J.M. Graybeal, X.D. Wu, R. Muenchausen, Phys. Rev. Lett. 68 (1992) 690.
[12] S.J. Hagen, C.J. Lobb, R.L. Greene, M. Eddy, Phys. Rev. B 43 (1991) 6246.
[13] T.R. Chien, T.W. Jing, N.P. Ong, Z.Z. Wang, Phys. Rev. Lett. 66 (1991) 3075.
[14] S.J. Hagen, C.J. Lobb, R.L. Greene, M.G. Forrester, J.H. Kang, Phys. Rev. B 41 (1990) 11630.
[15] Y. Iye, S. Nakamura, T. Tamegai, Physica C 159 (1989) 616.
[16] E.H. Brandt, Rep. Prog. Phys. 58 (1995) 1465.
[17] Z.D. Wang, Q. Wang, P.C.W. Fung, J. Supercond. Sci. Technol. 9 (1996) 333.
[18] A. van Otterlo, M. Feigel'man, V. Geshkenbein, G. Blatter, Phys. Rev. Lett. 75 (1995) 3736.
[19] M.C. Dai, T.J. Yang, unpublished.
[20] A.T. Dorsey, Phys. Rev. B 46 (1992) 8376.
[21] A.V. Samoilov, Z.G. Ivanov, L.G. Johansson, Phys. Rev. B 49 (1994) 3667.
[22] I. Affleck, M. Franz, M.H. Sharifzadeh Amin, Phys. Rev. B 55 (1997) R704.
[23] J.J. Vicente Alvarez, D. Dominquez, C.A. Balseriro, Phys. Rev. Lett. 79 (1997) 1373.
[24] M. Franz, C. Kallin, P.I. Soininen, A.J. Berlinsky, A.L. Fetter, Phys. Rev. B 53 (1996) 5795.
[25] L.P. Gor'kov, N.B. Kopnin, Sov. Phys Usp. 18 (1975) 496.
[26] W. Xu, W. Kim. Y. Ren, C.S. Ting, Phys. Rev. B 54 (1996) R12693.
[27] J.X. Zhu, W. Kim, C.S. Ting, C.-S. Hu, Phys. Rev. B 58 (1998) 15020.