Wireless Networks 5 (1999) 479–488

Performance modeling of wireless PBX systems

Wei-Ru Lai and Yi-Bing Lin

Department of Computer Science and Information Engineering, National Chiao Tung University, Hsinchu, Taiwan 30050, ROC

This paper studies the effects of cell residence times on wireless private branch exchange (WPBX) systems. In a WPBX, several base stations are connected to a PBX, and the PBX is connected to the Public Switched Telephone Network. It is important to determine the external line capacity and the radio channel capacities of a WPBX to optimize its system performance. Our previous study utilized an analytic model to study WPBX without user mobility. This paper proposes both the analytic and the simulation models to investigate the performance for WPBX with user mobility. Specifically, we study the effects of handoff and the variance of the cell residence times on resource planning. Based on the workload to the WPBX, our study provides several guidelines to determine the capacities for the PBX and the base stations.

1. Introduction

A *Private Branch Exchange* (PBX) is a switch that connects the telephones from a company to the *Public Switched Telephone Network* (PSTN). Since only a small number of the so-connected telephones are expected to be used at the same time, the number of links from a PBX to the PSTN is smaller than the number of telephones connected to the PBX. Such a configuration results in a cost effective approach to reduce the number of lines that a business has to lease from the local telephone company.

In a modern enterprise, employees are often away from their assigned wired phones but are still in their offices or other locations of the company. To remove the restriction that every telephone has a fixed location, a PBX system may integrate with cordless technologies (for example, DECT [3,4], PHS [7,10], and CT-2 [11]) to support user mobility at the workplace [1]. Such a system is referred to as the *Wireless PBX* (WPBX). We have introduced WPBX in [2,8]. For the reader's benefit, the descriptions are reiterated here.

The WPBX architecture is illustrated in figure 1, where the PBX is connected to k *base stations* (BSs) instead of wireline telephones (in figure 1, $k = 4$). A BS is a radio site that communicates with *handsets* or *mobile phones* through *radio channels*. BS i is equipped with c_i radio channels for handset communication and c_i circuits for connection to the PBX (that is, at most c_i handsets can be connected to the PBX through BS i). The PBX is connected to the PSTN with C external lines.

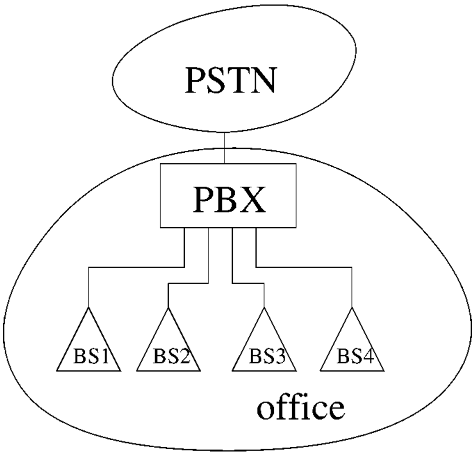


Figure 1. A WPBX system.

To establish an external call (between a WPBX user and a remote party in the PSTN), the WPBX allocates one external line and one radio channel to this call (see path (a) in figure 2). On the other hand, an internal call (between two WPBX users) consumes two radio channels (see path (b) in figure 2). During a conversation, a handoff request occurs when the user moves from a cell (the coverage area of one BS) to another cell. To support handoff, a radio channel of the new BS is assigned to the call and the radio channel of the old BS is released. If no free

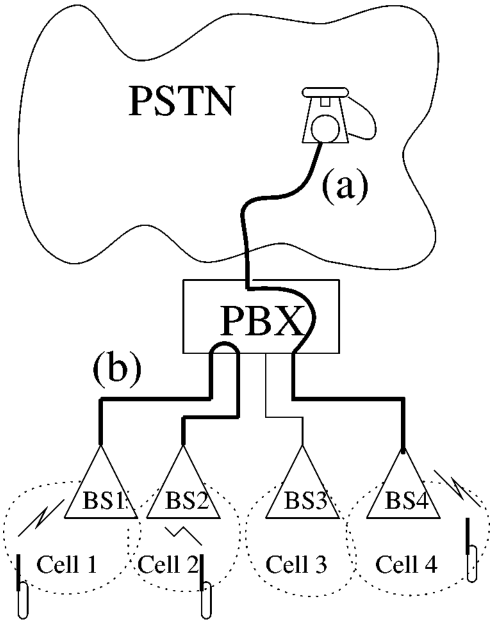


Figure 2. External and internal calls.

radio channel is available in the new BS, the call is force-terminated.

The probability that a call setup fails (due to the lack of an external line or radio channels) is referred to as the *new call blocking* probability. The probability that a handoff request is rejected is referred to as the *forced termination* probability. The *call incompletion* probability is the probability that a call is a blocked new call or a force-terminated handoff call. In designing a WPBX system, it is important to select the "optimal" C and c_i values to minimize the call incompletion probability with a reasonable amount of resources.

In [2], we proposed an analytic model to study WPBX systems without mobility. In this paper, we propose an analytic model with mobility to investigate the performance of WPBX systems that support handoff. Specifically, we study the effects of user mobility and the variance of the cell residence times on resource planning.

2. The analytic model

This section describes the WPBX resource planning problem and then provides our analytic solution.

2.1. The model without mobility

We assume that no handoff is allowed in the WPBX system, i.e., a call is always complete if it is successfully connected. The model described in this subsection is the same as the one described in [2] except that intra-BS call traffic is not considered here. We will re-iterate the model for the reader's benefit. In the next subsection, we extend this model to accommodate WPBX systems that support handoff. The following assumptions are made.

Assumption 1(a). The external call arrivals between BS i and the PSTN (initiated either by WPBX users or by the remote PSTN parties) form a Poisson process with an arrival rate λ_i.

Assumption 1(b). The external call holding times for BS i have a general distribution with mean $1/\mu_i$.

Assumption 2(a). The internal call arrivals between BS i and BS j form a Poisson process with an arrival rate $\lambda_{i,j}$, $1 \leqslant i < j \leqslant k$.

Assumption 2(b). The internal call holding times for BS i and BS j have a general distribution with mean $1/\mu_{i,j}$.

The effect of intra-BS traffic (for an intra-BS call, both the calling party and the called party are at the same cell) is ignored because the cell sizes in many office environments are small, and it is unlikely that a person makes a call to another person within, e.g., 10-meter distance. The intra-BS traffic is specifically investigated in [2].

The described WPBX system can be represented by a circuit-switched network that consists of several nodes and links, where the nodes represent the PSTN, the PBX and the BSs in figure 2, and the links represent the external lines and the internal lines. Note that in this model the radio channels in a BS are equivalent to the internal lines between the BS and the PBX. In figure 2, there are 4 external routes $r_i = \{\mathrm{BS}i \leftrightarrow \mathrm{PBX} \leftrightarrow \mathrm{PSTN}\}$ for $1 \leqslant i \leqslant 4$, and 6 internal routes $r_{j,l} = \{\mathrm{BS}j \leftrightarrow \mathrm{WPBX} \leftrightarrow \mathrm{BS}l\}$ for $1 \leqslant j < l \leqslant 4$. By convention, $r_{j,l} = r_{l,j}$, and we only consider the notation $r_{j,l}$ for $j < l$. Let $\mathfrak{R}$ be the set of routes. Then $\mathfrak{R} = \{r_1, r_2, \ldots, r_4, r_{1,2}, r_{1,3}, \ldots, r_{3,4}\}$ for the WPBX system shown in figure 2.

From assumptions 1–2, the calls on route r form a Poisson process with the arrival rate λ_r, and the call holding times on route r have a general distribution with mean μ_r^{-1}. A call is accepted if and only if the route has enough resources to accommodate this call. In our example, one circuit from each link on the route is required to connect a call.

For a WPBX system with k BSs, the vector **n** with size $k(k+1)/2$ is used to represent the number of outstanding calls on each route. A vector **n** can be expressed as

$$\mathbf{n} = \big((m_1, m_2, \ldots, m_i, \ldots, m_k),\\ (m_{1,2}, m_{1,3}, \ldots, m_{j,l}, \ldots, m_{k-1,k})\big),$$

where m_i represents the number of calls for route r_i, where $1 \leqslant i \leqslant k$, and $m_{j,l}$ represents the number of calls for route $r_{j,l}$, where $1 \leqslant j < l \leqslant k$. It is clear that the total amount of circuits used by the outstanding calls should be no more than the link capacities of the routes. Consider a stochastic process that represents the outstanding calls in the WPBX. A state in the process can be represented by the vector **n**, where a legal state in the state space must satisfy the following inequalities:

$$m_1 + m_2 + \cdots + m_k \leqslant C, \tag{1}$$

$$m_i + \sum_{1\leqslant j<i} m_{j,i} + \sum_{i<l\leqslant k} m_{i,l} \leqslant c_i, \quad \text{for } 1 \leqslant i \leqslant k. \tag{2}$$

Inequality (1) indicates that the number of busy external lines should be no more than the capacity C. In inequality (2), the number of busy radio channels of BS i (m_i for the external traffic and $\sum_{1\leqslant j<i} m_{j,i} + \sum_{i<l\leqslant k} m_{i,l}$ for the internal traffic) should be no more than the capacity c_i for $1 \leqslant i \leqslant k$. The state space S of the stochastic process is

$$S = \big\{\mathbf{n} \mid m_i \text{ and } m_{j,l} \text{ satisfy inequalities (1) and (2)}\big\}.$$

Let ρ_i be the offered load for the route r_i (between the PSTN and BS i) and $\rho_{j,l}$ be the offered load for the route $r_{j,l}$ (between BS j and BS l), then

$$\rho_i = \frac{\lambda_i}{\mu_i} \quad \text{for } 1 \leqslant i \leqslant k,$$

and

$$\rho_{j,l} = \frac{\lambda_{j,l}}{\mu_{j,l}} \quad \text{for } 1 \leqslant j < l \leqslant k.$$

According to Zachary [12] and Kelly [5], the stationary probability of the state $\mathbf{n} = ((m_1, \ldots, m_k), (m_{1,2}, m_{1,3}, \ldots, m_{k-1,k}))$ can be computed as

$$p(\mathbf{n}) = G^{-1}\left(\prod_{1 \leqslant i \leqslant k} \frac{\rho_i^{m_i}}{m_i!}\right)\left(\prod_{1 \leqslant j < l \leqslant k} \frac{\rho_{j,l}^{m_{j,l}}}{m_{j,l}!}\right), \tag{3}$$

where G is

$$G = \sum_{\mathbf{n} \in S}\left[\left(\prod_{1 \leqslant i \leqslant k} \frac{\rho_i^{m_i}}{m_i!}\right)\left(\prod_{1 \leqslant j < l \leqslant k} \frac{\rho_{j,l}^{m_{j,l}}}{m_{j,l}!}\right)\right]. \tag{4}$$

The second and the third terms of the right hand side of (3) are the weights contributed by external traffic and internal traffic, respectively, and G in (4) is a normalized factor to ensure that $\sum_{\mathbf{n} \in S} p(\mathbf{n}) = 1$. In this analytic model, the stationary probabilities are affected by the call holding time distribution only through its mean [6]. In other words, the call holding time distribution can be arbitrary.

The following output measures are computed by using the stationary probability $p(\mathbf{n})$:

- $P_{\mathrm{oEx}}(i)$ is the new call blocking probability of an external call between the PSTN and BS i, where $1 \leqslant i \leqslant k$. Let

  $$\overline{c}_i(\mathbf{n}) = m_i + \sum_{1 \leqslant j < i} m_{j,i} + \sum_{i < l \leqslant k} m_{i,l}$$

  be the number of occupied radio channels in BS i (where $1 \leqslant i \leqslant k$), and

  $$\overline{C}(\mathbf{n}) = \sum_{1 \leqslant i \leqslant k} m_i$$

  be the number of occupied external lines for the state $\mathbf{n} = ((m_1, m_2, \ldots, m_k), (m_{1,2}, m_{1,3}, \ldots, m_{k-1,k}))$. We have

  $$P_{\mathrm{oEx}}(i) = \sum_{\forall \mathbf{n} \in S,\ \overline{C}(\mathbf{n}) = C \text{ or } \overline{c}_i(\mathbf{n}) = c_i} p(\mathbf{n}), \tag{5}$$

  where $\overline{C}(\mathbf{n}) = C$ implies that all external lines are busy at state $\mathbf{n}$, and $\overline{c}_i(\mathbf{n}) = c_i$ implies that all radio channels in BS i are busy.
- $P_{\mathrm{oIn}}(j, l)$ is the new call blocking probability of an internal call between BS j and BS l, where $1 \leqslant j < l \leqslant k$. We have

  $$P_{\mathrm{oIn}}(j, l) = \sum_{\forall \mathbf{n} \in S,\ \overline{c}_j(\mathbf{n}) = c_j \text{ or } \overline{c}_l(\mathbf{n}) = c_l} p(\mathbf{n}). \tag{6}$$

Without loss of generality, we consider the homogeneous case that $c_i = c$, $\lambda_i = \lambda$, $\lambda_{j,l} = \lambda^*$ and $\mu_i = \mu_{j,l} = \mu$, for $1 \leqslant i \leqslant k$ and $1 \leqslant j < l \leqslant k$. In this case, P_{oEx} for all BSs are the same (also true for P_{oIn}), and we use the notations $P_{\mathrm{oEx}} = P_{\mathrm{oEx}}(i)$ for $1 \leqslant i \leqslant k$ and $P_{\mathrm{oIn}} = P_{\mathrm{oIn}}(j, l)$ for $1 \leqslant j < l \leqslant k$.

2.2. The analytic model with mobility

Based on the no-mobility model in section 2.1, we describe the model with mobility as follows. Besides assumptions 1(a) and 2(a), the following assumptions are made.

Assumption 3 (Moving pattern). The probability that a WPBX user moves from one cell to any one of its neighboring cells is the same.

Assumption 4 (Call holding time). The call holding times have an Exponential density function $f_{\mathrm{c}}(t_{\mathrm{c}})$ with mean $1/\mu$, i.e., $f_{\mathrm{c}}(t_{\mathrm{c}}) = \mu \mathrm{e}^{-\mu t_{\mathrm{c}}}$.

Assumption 5 (Handset residence time). The residence time of a handset in a cell has a general density function $f(t)$ with mean $1/\eta$.

Consider the timing diagram in figure 3. Suppose that an external call occurs when the handset P is in Cell 0. Suppose that this call successfully hands off n times and is complete in Cell n. Let t_{c} be the call holding time of the call. In this figure, t_i ($0 \leqslant i \leqslant n$) denotes the residence time of the handset in Cell i. For an external call, only one handset may move during the conversation. From assumption 5, $t_0, t_1, \ldots, t_n$ are independent and identically distributed random variables with density function $f_1(t_i) = f(t)$, the mobility rate η and the Laplace Transform $f_1^*(s) = \int_{t_i=0}^{\infty} f_1(t_i)\mathrm{e}^{-st}\,\mathrm{d}t_i$.

For an internal call, both handsets may move during the conversation. Therefore, the internal call model is approximated by the external call model where the mobility rate is 2η. That is, we assume that $t_0, t_1, \ldots, t_n$ for internal calls have density function $f_2(t_i)$ which is $f(t)$ with the mobility rate 2η and the Laplace Transform $f_2^*(s)$.

Let τ be the period between when the call arrives and when the handset moves out of Cell 0. Let $t_{c,i}$ be the period from the time when the handset moves into Cell i to the time when the call is complete. An idle radio channel is assigned to the handset when a new call arrives or when the handset makes a handoff request. The radio channel is released when the call is complete or when the handset moves out of the cell. Let t_{cn} be the channel occupation time for a new call in Cell 0. Then

$$t_{\mathrm{cn}} = \min(t_{\mathrm{c}}, \tau).$$

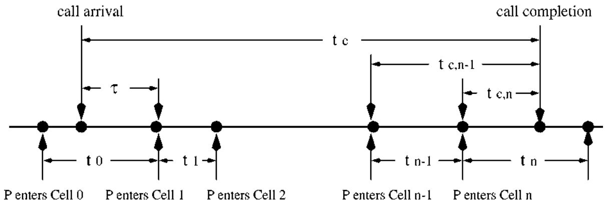


Figure 3. The timing diagram.

By using [9, equation (7)], the expected value $E[t_{\text{cn}}]$ of an external call is

$$E[t_{\text{cn}}] = \frac{1}{\mu} - \frac{\eta}{\mu^2}\bigl[1 - f_1^*(\mu)\bigr]. \tag{7}$$

Due to the memoryless property of the exponential distribution, the excess life $t_{\text{c},i}$ of a call has the same distribution as the original call holding time t_{c} for $1 \leqslant i \leqslant n$. Let t_{ch} be the channel occupation time of an external handoff call. Then

$$t_{\text{ch}} = \min(t_{\text{c},i}, t_i) = \min(t_{\text{c}}, t_i).$$

From [9, equation (9)], the expected value of t_{ch} of an external call is

$$E[t_{\text{ch}}] = \frac{1}{\mu}\bigl[1 - f_1^*(\mu)\bigr]. \tag{8}$$

A handoff request at Cell i is rejected if no radio channel is available at that cell. From section 2.1, the forced termination probability P_{f} is

$$P_{\text{f}} = \sum_{\mathbf{n}\in S,\ \overline{c}_i(\mathbf{n})=c_i} p(\mathbf{n}). \tag{9}$$

From [9, equation (13)], the handoff call arrival rate γ for external calls to a cell is

$$\gamma = \frac{\eta(1 - P_{\text{oEx}})[1 - f_1^*(\mu)]\lambda}{\mu[1 - f_1^*(\mu) + P_{\text{f}} f_1^*(\mu)]} \tag{10}$$

and the handoff call arrival rate γ^* for internal calls is

$$\gamma^* = \frac{2\eta(1 - P_{\text{oIn}})[1 - f_2^*(\mu)]\lambda^*}{\mu[1 - f_2^*(\mu) + P_{\text{f}} f_2^*(\mu)]}. \tag{11}$$

The external offered load ρ and the internal offered load ρ^* to the system can be derived from (7), (8), (10) and (11):

$$\begin{aligned}\rho &= \lambda E[t_{\text{cn}}] + \gamma E[t_{\text{ch}}]\\ &= \frac{\lambda}{\mu} - \frac{\lambda}{\mu}\\ &\quad\times \frac{\eta[1 - f_1^*(\mu)]\{P_{\text{oEx}}[1 - f_1^*(\mu)] + P_{\text{f}} f_1^*(\mu)\}}{\mu[1 - f_1^*(\mu) + P_{\text{f}} f_1^*(\mu)]}\end{aligned} \tag{12}$$

and

$$\begin{aligned}\rho^* &= \frac{\lambda^*}{\mu} - \frac{\lambda^*}{\mu}\\ &\quad\times \frac{2\eta[1 - f_2^*(\mu)]\{P_{\text{oIn}}[1 - f_2^*(\mu)] + P_{\text{f}} f_2^*(\mu)\}}{\mu[1 - f_2^*(\mu) + P_{\text{f}} f_2^*(\mu)]}.\end{aligned} \tag{13}$$

From [9, equation (15)], the external call incompletion probability P_{ncEx} can be expressed as

$$\begin{aligned}P_{\text{ncEx}} &= P_{\text{oEx}} + \frac{\gamma}{\lambda}P_{\text{f}}\\ &= P_{\text{oEx}} + \frac{\eta(1 - P_{\text{oEx}})[1 - f_1^*(\mu)]P_{\text{f}}}{\mu[1 - f_1^*(\mu) + P_{\text{f}} f_1^*(\mu)]}.\end{aligned} \tag{14}$$

Similarly, the internal call incompletion probability P_{ncIn} is expressed as

$$P_{\text{ncIn}} = P_{\text{oIn}} + \frac{2\eta(1 - P_{\text{oIn}})[1 - f_2^*(\mu)]P_{\text{f}}}{\mu[1 - f_2^*(\mu) + P_{\text{f}} f_2^*(\mu)]}. \tag{15}$$

We can iteratively compute the external and the internal new call blocking probabilities and forced termination probability with the following six steps.

Input parameters: C, c, λ, λ^*, μ, η, $f_1(t)$ and $f_2(t)$.

Output parameters: P_{oEx}, P_{oIn}, P_{f}, P_{ncEx} and P_{ncIn}.

Step 1. Set the initial values $P_{\text{oEx}} = 0$, $P_{\text{oIn}} = 0$ and $P_{\text{f}} = 0$.

Step 2. Compute ρ and ρ^* by using the equations (12) and (13), respectively.

Step 3. Let $P_{\text{oEx,old}} \leftarrow P_{\text{oEx}}$, $P_{\text{oIn,old}} \leftarrow P_{\text{oIn}}$, and $P_{\text{f,old}} \leftarrow P_{\text{f}}$.

Step 4. Compute $P_{\text{oEx}}, P_{\text{oIn}}$ and P_{f} by using the equations (5), (6) and (9) based on the analytic model described in section 2.1.

Step 5. If

$$\max\left(\left|\frac{P_{\text{oEx}} - P_{\text{oEx,old}}}{P_{\text{oEx}}}\right|, \left|\frac{P_{\text{oIn}} - P_{\text{oIn,old}}}{P_{\text{oIn}}}\right|, \left|\frac{P_{\text{f}} - P_{\text{f,old}}}{P_{\text{f}}}\right|\right) > \delta,$$

go to step 2. Otherwise, go to step 6 (the values for $P_{\text{oEx}}, P_{\text{oIn}}$, and P_{f} converge). Note that δ is a pre-defined value.

Step 6. Compute P_{ncEx} and P_{ncIn} by using equations (14) and (15).

The analytic model is validated a simulation described in appendix A. The figures in the next section indicate that the analytic and simulation models are consistent.

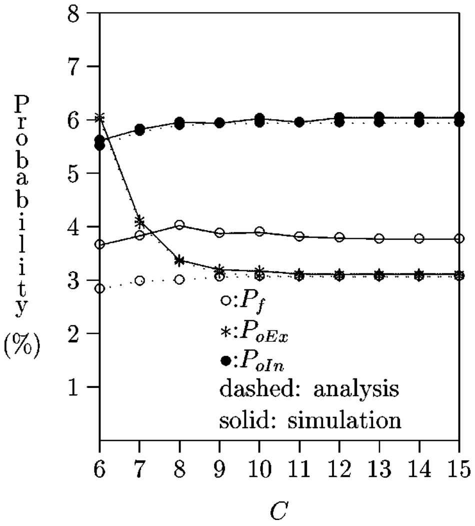
3. Numerical results

Based on the models described in section 2 and appendix A, we study the effects of the input parameters on the new call blocking, the forced termination and the incompletion probabilities.

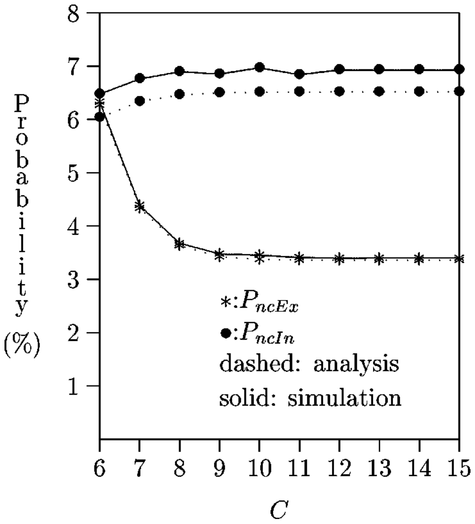
3.1. The effect of C

Figures 4(a) and (b) plot $P_{\text{oEx}}, P_{\text{oIn}}$, P_{f}, P_{ncEx}, and P_{ncIn} as functions of C. In these figures, $6 \leqslant C \leqslant 15$, $c = 4$, $k = 4$, $\lambda = 0.7\mu$, $\lambda^* = 0.2\mu$ and $\eta = 0.1\mu$. The dashed curves are for analytic results, and the solid curves are for simulation results. These figures indicate that the analytic results are almost identical to the simulation results for $P_{\text{oEx}}, P_{\text{oIn}}, P_{\text{ncEx}}$ and P_{ncIn}. For P_{f} (the '$\circ$' curve in figure 4(a)), there is some discrepancy between analysis and simulation. However, the trends of the two P_{f} curves are consistent.

Figure 4 indicates that when C is small, increasing C significantly decreases P_{oEx} (the '$*$' curve in (a)) and P_{ncEx} (the '$*$' curve in (b)), but slightly increases P_{oIn} (the '$\bullet$' curve in (a)) and P_{ncIn} (the '$\bullet$' curve in (b)). Intuition suggests that as C is increased, the probability that external calls are blocked due to insufficient external lines decreases,



(a) P_f, P_oEx and P_oIn



(b) P_ncEx and P_ncIn

Figure 4. The effect of C ($\lambda = 0.7\mu$, $\lambda^* = 0.2\mu$, $\eta = 0.1\mu$, $c = 4$ and $k = 4$).

and P_oEx decreases. Since the number of accepted external calls increases, the number of radio channels available to internal call requests decreases, and P_oIn increases. We observe that there exists a threshold point C^* such that beyond this point, increasing C does not improve the P_oEx (or P_ncEx) performance. In this particular experiment, we observe that $C^* = 9$. When C is large, increasing C only has insignificant effect on all measurements. Thus, C is no longer the bottleneck resource, and the performance is affected by other factors such as the number of radio channels.

3.2. The effect of mobility

The effect of mobility in a WPBX system with intra-BS traffic was investigated in [2]. This subsection compares the performance of WPBX systems with and without intra-BS traffic. To simplify our discussion, we consider output measures P_o (the new call blocking probability for an arbitrary call), P_nc (the incompletion probability for an arbitrary call) and P_f defined by (16) in appendix A.

For mobility rate η, define the incompletion probability ratio as

$$\Theta(\eta) = \frac{P_\mathrm{nc}(\eta) - P_\mathrm{nc}(0)}{P_\mathrm{nc}(0)},$$

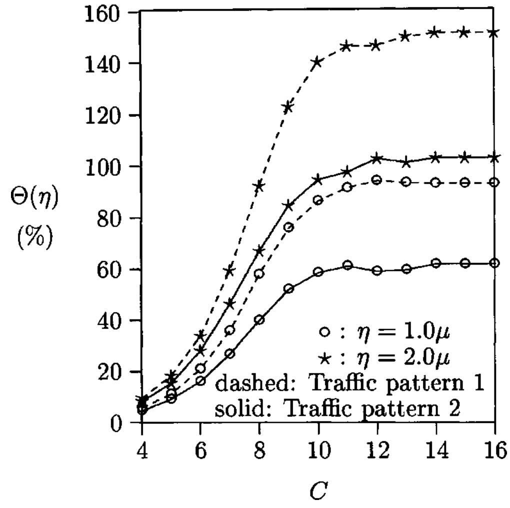
where $P_\mathrm{nc}(\eta)$ denotes the call incompletion probability when the user mobility rate is η. We consider two traffic patterns for a 4-BS WPBX:

Traffic pattern 1 (without intra-BS traffic). The traffic to the system includes the external traffic with arrival rate $\lambda = \mu$ and the internal (inter-BS) traffic with arrival rate $\lambda^* = 0.2666\mu$.

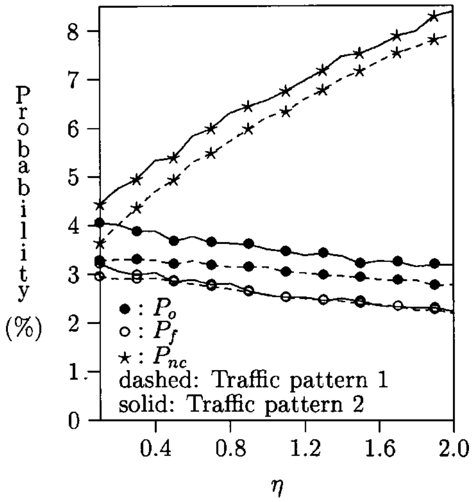
Traffic pattern 2 (with intra-BS traffic). The traffic to the system includes $\lambda = \mu$, $\lambda^* = 0.2\mu$, and the additional intra-BS traffic with arrival rate 0.1μ for each BS.

Note that in a 4-BS WPBX, both traffic patterns 1 and 2 have the same internal traffic (inter-BS traffic plus intra-BS traffic). Figure 5(a) plots $\Theta(\eta = \mu)$ and $\Theta(\eta = 2\mu)$ as functions of C for traffic patterns 1 and 2, where $c_i = 5$ for $1 \leqslant i \leqslant 4$ and $4 \leqslant C \leqslant 16$. Figure 5(b) plots $P_\mathrm{o}, P_\mathrm{f}$ and P_nc as functions of η for the two traffic patterns, where $C = 12$ and $c_i = 5$ for $1 \leqslant i \leqslant 4$. The dashed curves are for traffic pattern 1 and the solid curves are for traffic pattern 2. Figure 5(a) indicates that $\Theta(\eta)$ increases as C is increased and the differences between the ratios for $\eta = \mu$ and $\eta = 2\mu$ at $C = 4$ are smaller than those at $C = 16$. The phenomena suggest that the mobility has more significant effect on the incompletion probability for larger C (i.e., when the radio channels become the bottleneck resource of the system). The figure also indicates that the performance of the WPBX is more sensitive to the mobility for traffic pattern 1 (without intra-BS traffic) than for traffic pattern 2 (with intra-BS traffic). The new call blocking probabilities are large for the WPBX with heavy intra-BS traffic [8]. On the contrary, more new calls are accepted and have opportunities to hand off (and thus be force-terminated) for WPBX without intra-BS traffic (that is, mobility has more impact on this case).

Figure 5(b) indicates that P_nc (the '$*$' curve) increases rapidly as η increases. On the other hand, P_o (the '•' curve) and P_f (the '$\circ$' curve) decrease slowly as η increases. Note that external lines are no longer the bottleneck resource when $C = 12$. We observe two phenomena. Firstly, as η increases, the channel occupation time of a call (either new or handoff) at a BS decreases. Secondly, as η increases, the handoff rate increases and the number of requests for radio channels at a BS increases. The interaction between these two conflicting phenomena is subtle. The net effect to P_o and P_f is that both probabilities decrease slowly as η increases. On the other hand, when η increases, a connecting call experiences more handoffs and has a greater probability of being force-terminated (although P_f decreases).



(a) The incompletion probability ratio



(b) P_o, P_f and P_{nc} ($C = 12$)

Figure 5. The effects of η and $\Theta(\eta)$ on a WPBX system with 4 BSs ($c_i = 5$ for $1 \leqslant i \leqslant 4$, $\lambda = \mu$ and $\lambda^* = 0.2\mu$).

Figure 5(b) indicates that mobility rate η has significant effect on P_{nc}. This figure also indicates that P_{o} and P_{nc} for the WPBX with traffic pattern 2 are larger than those with traffic pattern 1. On the other hand, P_{f} (the 'o' curve) for traffic pattern 2 is almost identical to that for traffic pattern 1. Since every intra-BS call consumes two radio channels in one BS at the same time, it is more difficult to setup a new intra-BS call than to setup a new inter-BS call. These phenomena result in larger P_{o} and P_{nc} in traffic pattern 2. Since only one idle radio channel is required for every link transfer during handoff, both traffic patterns 1 and 2 have the same effect on P_{f}.

3.3. The effect of the variance of the residence times

This subsection studies the effect of the handset residence time distribution. Arbitrary cell residence time distributions can be studied by using our analytic model. For the demonstration purposes, we use Gamma residence time distribution. The Gamma distribution is selected because this distribution has been widely used in the PCS studies [9].

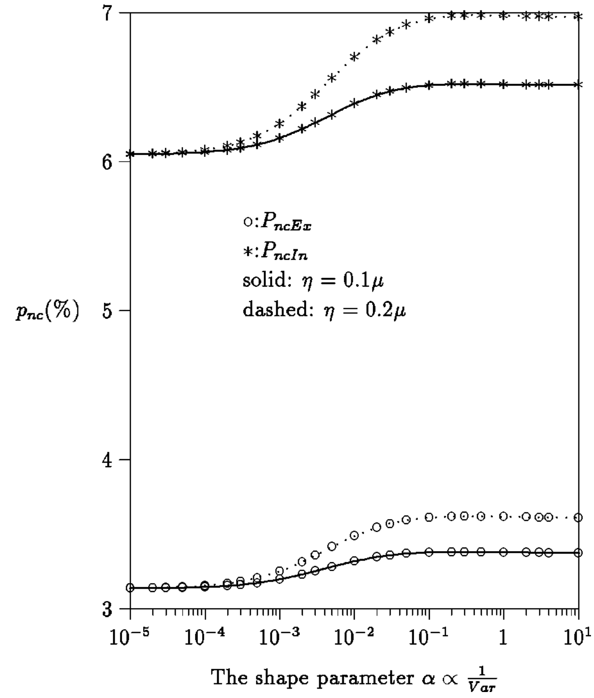


Figure 6. The effects of α ($C = 10$, $c = 4$, $k = 4$, $\eta = 0.1\mu$, $\lambda = 0.7\mu$ and $\lambda^* = 0.2\mu$).

A Gamma distribution has the density function

$$f_{\mathrm{G}}(t) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} t^{\alpha-1} \mathrm{e}^{-\beta t} \quad \text{for } t > 0,$$

where $\alpha > 0$ is the shape parameter, $\beta > 0$ is the scale parameter and $\Gamma(p) = \int_{x=0}^{\infty} x^{p-1} \mathrm{e}^{-x}\,\mathrm{d}x$. The Laplace Transform of the Gamma distribution is

$$f_{\mathrm{G}}^*(s) = \left(\frac{\beta}{\beta + s}\right)^{\alpha}.$$

For the same mean residence times, we observe the effect of the variance Var of the Gamma residence time distribution. Figure 6 illustrates P_{ncEx} and P_{ncIn} as functions of the scale parameter $\alpha = 1/(\eta^2\mathrm{Var})$. In this figure, $C = 10$, $c = 4$, $k = 4$, $\eta = 0.1\mu$, $\lambda = 0.7\mu$ and $\lambda^* = 0.2\mu$. We observe the following:

- For $\alpha \geqslant 1$ (i.e., $\mathrm{Var} \leqslant 1/\eta^2$), P_{ncEx} and P_{ncIn} are insensitive to the variance of the residence time distribution, and are only affected by η.
- For $10^{-3} \leqslant \alpha < 1$ (i.e., $10^3/\eta^2 \geqslant \mathrm{Var} > 1/\eta^2$), P_{ncEx} and P_{ncIn} are decreasing functions of Var.
- For $\alpha < 10^{-3}$ (i.e., $\mathrm{Var} \geqslant 10^3/\eta^2$), P_{ncEx} and P_{ncIn} are insensitive to the variance of the residence time distribution and η.

Although there is no intuitive explanation to these phenomena, the above observations indicate that better WPBX performance (smaller P_{nc}) is expected for larger variance of the cell residence times.

4. Conclusion

This paper studied WPBX resource planning issues. We investigated how the external line capacity, the radio channel capacities, and the offered loads affect the WPBX performance. Several results are observed:

- There exists a threshold point C^* such that beyond this point, increasing the number of external lines C does not improve system performance. For any WPBX system with specific call traffic, the performance models proposed in this paper can be used to determine the threshold C^*.
- When the external lines are the bottleneck resource, mobility only has an insignificant effect on the system. On the other hand, when the radio channels are the bottleneck resource, mobility has a significant effect on the system. For a WPBX system with high (low) mobility, investing more radio channels (external lines) is essential.
- The performance of the WPBX is more sensitive to the mobility for the case without intra-BS traffic than for the case with intra-BS traffic.
- When the variance of residence time distribution is large, the system performance is not affected by the mobility. On the other hand, when the variance of the residence time distribution is small, the system performance is only affected by the mobility rate. In both cases, the incompletion probability is insensitive to the variance of the residence time distribution.

Appendix A. The simulation model

A discrete event simulation model was developed to validate the analytic model in section 2. We consider WPBX systems with 4 BSs and 25 BSs, respectively. Similar performance results are observed for both cases. Thus, it suffices to illustrate the results for the 4-BS case. Let *ExLine* be the number of available external lines and initially, *ExLine* $= C$. We consider an $m \times m$ mesh cell topology, where $2 \leqslant m \leqslant 5$. For Cell i, let BS[i] be the number of available radio channels, and initially, BS[i] $= c_i$ for $1 \leqslant i \leqslant m^2$. A handset resides in a cell for a period, then it moves to one of the neighboring cells with the same routing probability. In fact, the cell residence periods can be generated from any random number generator. For the demonstration purposes, we use exponential residence time distribution in this section. The external (internal) call arrivals to each cell form a Poisson process with arrival rate λ (λ^*). The call holding times are generated from an exponential random variable with mean $1/\mu$.

In the simulation model, an event occurs when the external lines or the radio channels are allocated or released. Each event consists of the following attributes:

- The *Type* attribute indicates the event type. Three types of events are considered in the simulation: an ARRIVAL event represents a new call arrival, a HANDOFF event represents a handoff request from one cell to another, and a COMPLETION event represents a call completion.
- The (i_1, j_1) attribute indicates the current locations of the calling/called parties. For an internal call (i.e., a call between two WPBX users), i_1 and j_1 represent the cells of the calling/called parties. For an external call, i_1 indicates the handset's location and j_1 is not used.
- The (i_2, j_2) attribute is used in a HANDOFF event, which specifies the new locations of the calling/called parties after a handoff request occurs.
- The t_s attribute indicates the timestamp when the event occurs.
- The $(t_\mathrm{m1}, t_\mathrm{m2})$ attribute is a residence-time pair, where t_m1 and t_m2 indicate the calling/called parties' residual residence times at the current cells. For an external call, t_m2 is not used.
- The t_c^* attribute indicates the residual call life, i.e., the period between when the event occurs and when the call is complete. In figure 3, $t_\mathrm{c}^* = t_{\mathrm{c},i}$ for Cell i, where $1 \leqslant i \leqslant n$.

Several counters are used to measure the output statistics in the simulation:

- N_Ex counts the number of external call arrivals during the observation period.
- N_In counts the number of internal call arrivals during the observation period.
- $N = N_\mathrm{Ex} + N_\mathrm{In}$ is the total number of call arrivals during the observation period.
- N_oEx counts the number of external blocked calls during the observation period.
- N_oIn counts the number of internal blocked calls during the observation period.
- $N_\mathrm{o} = N_\mathrm{oEx} + N_\mathrm{oIn}$ is the total number of blocked calls during the observation period.
- N_H counts the number of handoff requests during the observation period.
- N_F counts the number of forced terminations during the observation period.
- N_cEx counts the number of external completion calls during the observation period.
- N_cIn counts the number of internal completion calls during the observation period.
- $N_\mathrm{c} = N_\mathrm{cEx} + N_\mathrm{cIn}$ is the total number of completion calls during the observation period.

The above output measures are used to compute P_oEx, P_oIn, P_f, P_ncEx, P_ncIn and P_o (the new call blocking proba-

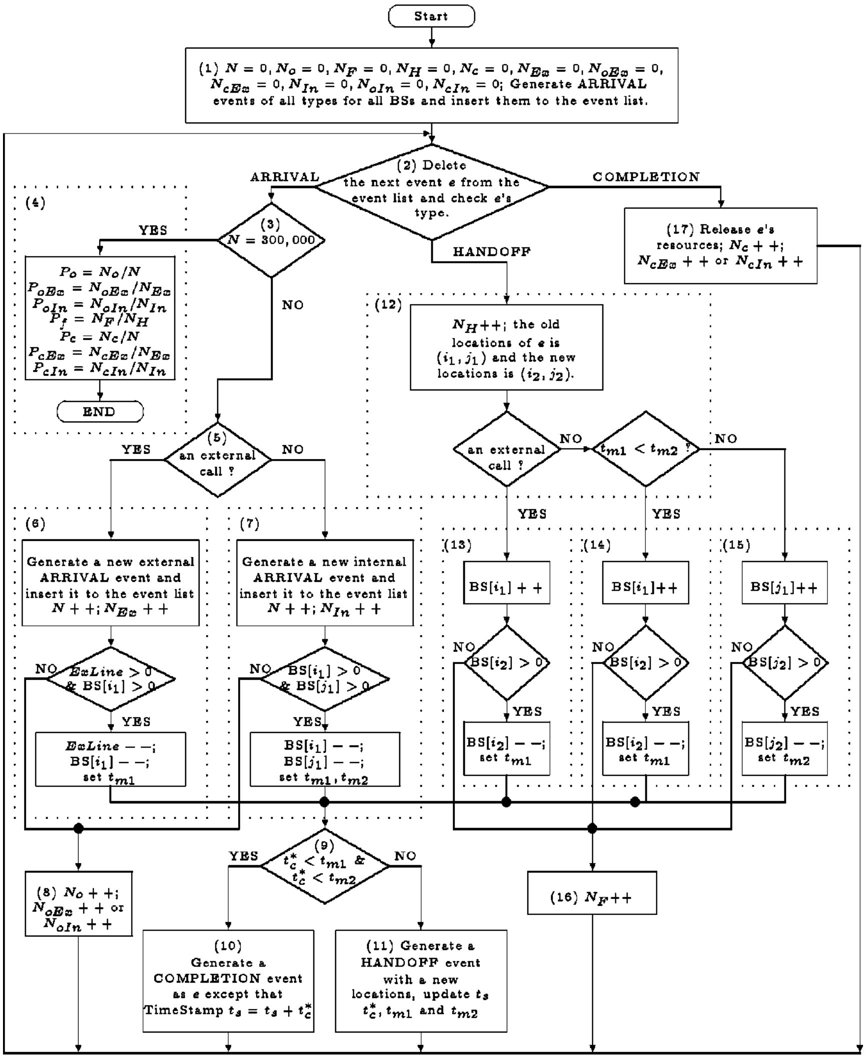


Figure 7. Flowchart of the simulation model.

bility for an arbitrary call) and P_{nc} (the incompletion probability for an arbitrary call) as follows:

$$\begin{aligned} &P_{\mathrm{o}} = \frac{N_{\mathrm{o}}}{N}, && P_{\mathrm{oEx}} = \frac{N_{\mathrm{oEx}}}{N_{\mathrm{Ex}}}, && P_{\mathrm{oIn}} = \frac{N_{\mathrm{oIn}}}{N_{\mathrm{In}}}, \\ &P_{\mathrm{f}} = \frac{N_{\mathrm{F}}}{N_{\mathrm{H}}}, && P_{\mathrm{nc}} = 1 - \frac{N_{\mathrm{c}}}{N}, && \\ &P_{\mathrm{ncEx}} = 1 - \frac{N_{\mathrm{cEx}}}{N_{\mathrm{Ex}}}, && P_{\mathrm{ncIn}} = 1 - \frac{N_{\mathrm{cIn}}}{N_{\mathrm{In}}}. && \end{aligned} \tag{16}$$

In our experiment, 300,000 ARRIVAL events were simulated to ensure that the simulation results are stable. Figure 7 illustrates the flowchart of the simulation:

Step 1. An external ARRIVAL event for each BS and an internal ARRIVAL event for each BS pair are generated (with attribute t_{s} drawn from an Exponential inter-arrival time distribution with mean $1/\lambda$ or $1/\lambda^*$, and t_{c}^* drawn from an Exponential distribution with mean $1/\mu$). Then these events are inserted into an event list in the non-decreasing order.

Step 2. The next event e is deleted from the event list, and is processed based on its event type. The system clock is set to t_{s} (the timestamp of e). For an ARRIVAL event (a new call arrival), go to step 3. For a HANDOFF event (a handoff request), go to step 12. For a COMPLETION event (a call completion), go to step 17.

Steps 3 and 4. If $N = 300,000$, then the simulation terminates and the performance measures $P_{\mathrm{o}}, P_{\mathrm{oEx}}, P_{\mathrm{oIn}}, P_{\mathrm{f}}, P_{\mathrm{nc}}, P_{\mathrm{ncEx}}$ and P_{ncIn} are calculated based on (16). If $N < 300,000$, go to step 5.

Step 5. Based on the location attribute (i_1, j_1), external and internal calls are distinguished. If the event e represents an external call, go to step 6. Otherwise, go to step 7.

Step 6. Generate a new external ARRIVAL event for Cell i_1, where the t_{s} attribute is set to the current clock plus the inter-arrival time that is drawn from an Exponential distribution with mean $1/\lambda$, and the t_{c}^* attribute is generated from an Exponential distribution with mean $1/\mu$. Insert the new event into the event list. Check if an idle external line exists in the PBX (i.e., *ExLine* > 0) and an idle radio channel exists in Cell i (i.e., BS$[i_1] > 0$).

If not, the call is blocked and the control flow switches to step 8. If the resources are available, generate t_{m1} from the cell residence time distribution with the mobility rate η. Go to step 9.

Step 7. Generate a new internal ARRIVAL event from Cell i_1 to Cell j_1, where the t_s attribute is set to the current clock plus the inter-arrival time drawn from an Exponential distribution with mean $1/\lambda^*$, and t_{c}^* is drawn from an Exponential distribution with mean $1/\mu$. Insert the new event into the event list. Check if idle radio channels in BS i_1 and BS j_1 exist (i.e., BS$[i_1] > 0$ and BS$[j_1] > 0$). If not, the call is blocked and the control flow switches to step 8. If the resources are available, generate the cell residence times t_{m1} and t_{m2}. Go to step 9.

Step 9. Compare the residual call holding time t_{c}^*, the residual cell residence times t_{m1} and t_{m2}. If t_{c}^* is the smallest, go to step 10. Otherwise, go to step 11.

Step 10. Generate a COMPLETION event where t_{s} is set to $t_{\mathrm{s}} + t_{\mathrm{c}}^*$, the cell residence time pair $(t_{\mathrm{m1}}, t_{\mathrm{m2}})$ is set to $(t_{\mathrm{m1}} - t_{\mathrm{c}}^*, t_{\mathrm{m2}} - t_{\mathrm{c}}^*)$ and t_{c}^* is set to 0. Insert the COMPLETION event into the event list. Go to step 2.

Step 11. If any one of the WPBX call parties moves to a new cell before call completion, generate a HANDOFF event. For an external call, the HANDOFF event has the t_{s} attribute with the value $t_{\mathrm{s}} + t_{\mathrm{m1}}$, the $(t_{\mathrm{m1}}, t_{\mathrm{m2}})$ attribute with the value $(0, -)$ and the t_{c}^* attribute with the value $t_{\mathrm{c}}^* - t_{\mathrm{m1}}$. For an internal call, the HANDOFF event is set to the following values: $t_{\mathrm{s}} \leftarrow t_{\mathrm{s}} + \min(t_{\mathrm{m1}}, t_{\mathrm{m2}})$ and $t_{\mathrm{c}}^* \leftarrow t_{\mathrm{c}}^* - \min(t_{\mathrm{m1}}, t_{\mathrm{m2}})$. If $t_{\mathrm{m1}} > t_{\mathrm{m2}}$, $(t_{\mathrm{m1}}, t_{\mathrm{m2}}) \leftarrow (t_{\mathrm{m1}} - t_{\mathrm{m2}}, 0)$. Otherwise, $(t_{\mathrm{m1}}, t_{\mathrm{m2}}) \leftarrow (0, t_{\mathrm{m2}} - t_{\mathrm{m1}})$. Go to step 2.

Step 12. A HANDOFF event occurs. If e is for an external call, go to step 13. Otherwise, determine which user moves. If $t_{\mathrm{m1}} < t_{\mathrm{m2}}$, go to step 14. If $t_{\mathrm{m1}} \geqslant t_{\mathrm{m2}}$, go to step 15.

Steps 13, 14 and 15. The actions for these three steps are similar. We only describe step 13. The radio channel of the old BS is released. If an idle channel exists in the new BS, then allocate the channel to the call and go to step 9. Otherwise, the call is force-terminated and N_{F} is incremented by 1 (**Step 16**).

Step 17. A COMPLETION event occurs. Release the system resource (radio channels and/or the external line), increment N_{c}, N_{cEx} or N_{cIn}. Go to step 2.

B. Notation

The notations used in this paper are listed below.

- C: the number of external lines,
- c_i: the number of radio channels of BS i,
- λ_i: the new call arrival rate for external calls at BS i,
- $\lambda_{i,j}$: the new call arrival rate for internal calls between BS i and BS j,
- λ: the new call arrival rate for external calls,
- λ^*: the new call arrival rate for internal calls,
- γ: the handoff arrival rate for external calls,
- γ^*: the handoff arrival rate for internal calls,
- $1/\mu_i$: the mean of call holding time for external calls at BS i,
- $1/\mu_{i,j}$: the mean of call holding time for internal calls between BS i and BS j,
- $1/\mu$: the mean of call holding time for an arbitrary call, i.e., $E[t_{\mathrm{c}}]$,
- $1/\eta$: the mean of residence time for an arbitrary portable, i.e., $E[t_i]$,
- ρ: the offered load of external calls between a BS and the PSTN,
- ρ^*: the offered load of internal calls between two BS,
- t_i: the residence time for the portable at Cell i,
- τ: the period from the time when a call arrives to the time when the portable enters another coverage area,
- t_{c}: the call holding time,
- $f_{\mathrm{c}}(t_{\mathrm{c}})$: the Exponential density function of the call holding times,
- $f(t)$: an arbitrary density function of the portable residence times,
- $f^*(s) = \int_{t=0}^{\infty} f(t)\mathrm{e}^{-st}\,\mathrm{d}t$: the Laplace Transform of $f(t)$'s distribution,
- $t_{\mathrm{c},i}$: the period from the time when the portable enters the coverage i to the time when the call is complete,
- t_{cn}: the channel occupation time for a new call,
- t_{ch}: the channel occupation time of a handoff call,
- P_{o}: the new call blocking probability,
- P_{f}: the forced termination probability,
- P_{nc}: the probability that a call is blocked or is force-terminated,
- P_{oEx}: the new call blocking probability of an external call,
- P_{oIn}: the new call blocking probability of an internal call,
- P_{ncEx}: the probability that an external call is blocked or force-terminated,
- P_{ncIn}: the probability that an internal call is blocked or force-terminated.

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W.-R. Lai is a graduate student in the Department of Computer Science and Information Engineering, National Chiao Tung University. Her current research interests include design and analysis of personal communications services network.
E-mail: wrlai@csie.nctu.edu.tw

Yi-Bing Lin received his BSEE degree from National Cheng Kung University in 1983, and his Ph.D. degree in computer science from the University of Washington in 1990. From 1990 to 1995, he was with the Applied Research Area at Bell Communications Research (Bellcore), Morristown, NJ. In 1995, he was appointed as a professor of Department of Computer Science and Information Engineering (CSIE), National Chiao Tung University (NCTU). In 1996, he was appointed as Deputy Director of Microelectronics and Information Systems Research Center, NCTU. In 1997, he was elected as Chairman of CSIE, NCTU. His current research interests include design and analysis of personal communications services network, mobile computing, distributed simulation, and performance modeling. Dr. Lin is a subject area editor of the Journal of Parallel and Distributed Computing, an associate editor of the International Journal in Computer Simulation, an associate editor of IEEE Network, an editor of the ACM/Baltzer/URSI WINET, an associate editor of SIMULATION magazine, an area editor of ACM Mobile Computing and Communication Review, a columnist of ACM Simulation Digest, a member of the editorial board of International Journal of Communications Systems, a member of the editorial board of Computer Simulation Modeling and Analysis, Program Chair for the 8th Workshop on Distributed and Parallel Simulation, General Chair for the 9th Workshop on Distributed and Parallel Simulation. Program Chair for the 2nd International Mobile Computing Conference, the publicity chair of ACM Sigmobile, Guest Editor for the ACM/Baltzer MONET special issue on Personal Communication Services, and Guest Editor for IEEE Transactions on Computers special issue on Mobile Computing. Dr. Lin is a senior member of IEEE.
E-mail: liny@csie.nctu.edu.tw