layered infinite regions for which multilayer PMLs are needed. It turns out that it can be used to solve scattering problems due to buried objects, with very good numerical performances.

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SMALL PERTURBATION ANALYSIS OF OBLIQUE INCIDENCE IN DIELECTRIC GRATINGS

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ABSTRACT: *In this paper, the method of first-order perturbation is proposed to investigate the plane-wave scattering by dielectric gratings with a small modulation index at oblique incidence. Under the assumption of small perturbation, the cross coupling between the fields of the polarizations (TE and TM) due to the oblique incidence can be represented by the TE and TM transmission lines fed by distributed current and (or) voltage sources, respectively. The results show good agreement with that obtained by a rigorous formulation.* © 1998 John Wiley & Sons, Inc. Microwave Opt Technol Lett 19: 434–437, 1998.

Key words: *dielectric gratings; periodic structure; small perturbation*

1. INTRODUCTION

The analysis of plane-wave scattering by dielectric gratings using a rigorous formulation has been studied well [1] for the last two decades. Recently, several researchers have employed the approximate methods successfully. An approximate solution in terms of boundary diffraction coefficients was published [2], and the generalization of the boundary diffraction method for volume gratings was studied [3].

Extensive numerical calculations and complicated physical pictures are used in rigorous theory. Thus, using the first-order approximation may make the problem easier to comprehend. Although much research has been devoted to exploring the approximate method for the scattering of a plane wave by dielectric gratings, little attention has been paid to the condition of oblique plane-wave incidence.

This paper extends our previous work [4], plane-wave scattering by unslanted holographic gratings at principal-plane incidence, to that of the general condition of oblique plane-wave incidence. Thus, it becomes necessary to consider the three-dimensional (3-D) boundary-value problem which requires the simultaneous presence of mixed polarization.

The transmission-line network is used to model the original structure. The cross coupling between the fields of the two polarizations (TE and TM) due to the oblique incidence can be decoupled and represented by the TE and TM transmission-line networks, under the assumption of small perturbation. We observed that it approaches the results obtained from rigorous treatments while retaining the simplicity of perturbation methods. Moreover, this approach provides a description of the electromagnetic fields in terms of two transverse transmission-line networks, which bring considerable physical insight into the overall behavior concerning the dielectric gratings.

2. STATEMENT OF PROBLEM AND BACKGROUND INFORMATION

The scattering of a plane wave by a periodic dielectric layer is depicted in Figure 1. A plane wave is obliquely incident at an arbitrary elevation angle of θ_{inc} and at an azimuthal angle ϕ_{inc}, upon a dielectric grating, which is shown in Figure 2.

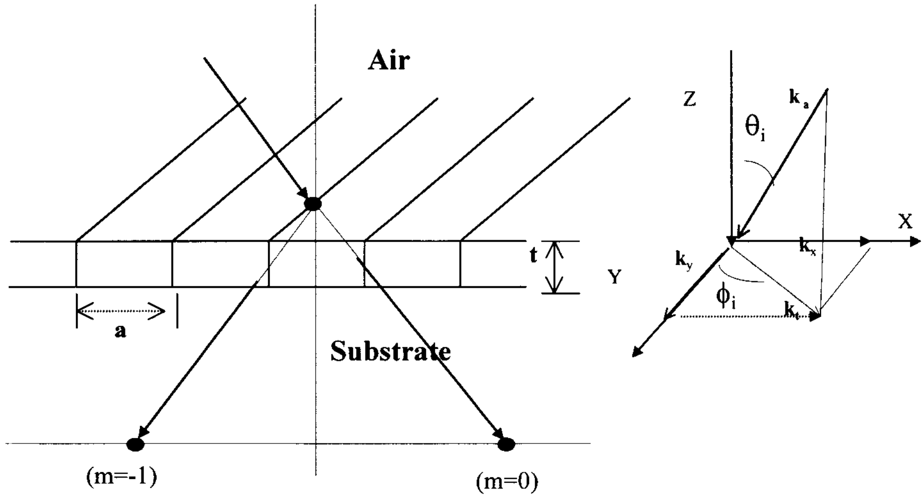

Figure 1 Structure configuration of a dielectric grating and propagation vector in spherical and rectangular coordinate system

The grating vector is in the x-direction, with period a. And the grating layer is bound by two different media with ε_a and ε_s.

In the grating region ($0 \leq z \leq t$), the periodic relative dielectric constant is expandable in a Fourier series of the form

$$\varepsilon(x) = \varepsilon_{\text{ave}} + \sum_{n=1}^{\infty} \varepsilon_n \cos\left(n \frac{2\pi x}{a}\right) \tag{1}$$

where ε_{ave} is the average dielectric constant of the periodic media.

In the spherical coordinate system, the incident plane wave is characterized by the propagation constant k_a, the elevation angle θ_{inc}, and the azimuth angle ϕ_{inc}. The x- and y-components of the propagation constants must be the same everywhere, but not the z-component.

Due to the spatial periodicity in the x-direction, all of the space harmonics are generally excited in the structure. The propagation constant of the nth space harmonic in the x-direction is related to that of the incident wave by

$$k_{xn} = k_x + n\frac{2\pi}{a}, \qquad \text{for } n = 0, \pm 1, \pm 2, \pm 3, \ldots . \tag{2}$$

In addition, in the uniform region (ε_a and ε_s), each space harmonic propagates independently as a plane wave. The propagation characteristics of each plane wave can be investigated readily through the dispersion relation

$$k_z^{(i)} = \left(k_0^2 \varepsilon_i - k_y^2 - k_{xn}^2\right)^{1/2} \tag{3}$$

where $i = a$ or s denotes the air or substrate region.

As soon as the parameters of the incident plane wave and geometric structure of the grating are specified, the transverse propagation vector of each space harmonic should be determined readily. In each uniform region, the propagated direction can be easily obtained, but the amplitude must be determined by a 3-D boundary-value problem, to be explained in the next section.

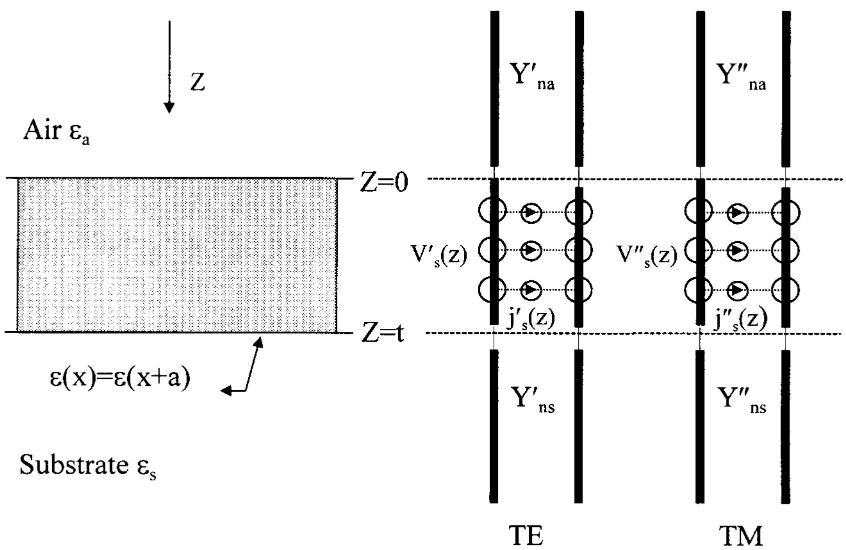

Figure 2 Equivalent transmission line with current and voltage source

3. METHOD OF ANALYSIS

A. Representations of Fields. The relative dielectric constants can be represented by the summation of unperturbed and perturbed terms in the form

$$\varepsilon(x) = \varepsilon_u + \varepsilon_p(x) \tag{4}$$

where ε_u characterizes a uniform unperturbed structure, and $\varepsilon_p(x)$ characterizes a periodic perturbation:

$$\varepsilon_u = \begin{cases} \varepsilon_a, & 0 \geq z \\ \varepsilon_{\text{ave}}, & t \geq z \geq 0 \\ \varepsilon_s, & z \geq t. \end{cases} \tag{5}$$

Substitution of (4) into the Maxwell equations leads to

$$\nabla \times \mathbf{H} = j\omega\varepsilon_o\varepsilon_u\mathbf{E} + \mathbf{J} \tag{6}$$

where

$$\mathbf{J} = j\omega\varepsilon_o\varepsilon_p\mathbf{E}. \tag{7}$$

We could regard the term $\mathbf{J}$ as the equivalent periodic current source, which is immersed in the uniform medium ε_u. Also, it may be decomposed into the transverse and longitudinal components represented by t and z, respectively.

As the perturbed quantities are expected to be small, the equivalent periodic current source $\mathbf{J}_t$ and J_z may be approximated by:

$$\mathbf{J}_t \approx j\omega\varepsilon_0\varepsilon_p(x)\mathbf{E}_{tu} \tag{8}$$

and

$$J_z \approx j\omega\varepsilon_o\varepsilon_p(x)E_{zu.} \tag{9}$$

Because of periodicity, the electromagnetic fields can be written as a superposition of the Fourier components with orthogonally polarized fields ($E^{(z)}$-type and $H^{(z)}$-type) which characterize the variation of the electric fields V_n' and V_n'' and the magnetic fields I_n' and I_n'' with TE and TM polarization, respectively.

$$j'_{n,s}(z) = \begin{cases} -j\dfrac{k_0\varepsilon_n}{2\eta_0}V_0(z)\cdot a_n'', & \text{for } t \geq z \geq 0 \\ 0, & \text{elsewhere} \end{cases} \tag{10}$$

and

$$j''_{n,s}(z) = \begin{cases} j\dfrac{k_0\varepsilon_n}{2\eta_0}V_0(z)\cdot a_n', & \text{for } t \geq z \geq 0 \\ 0, & \text{elsewhere} \end{cases} \tag{11}$$

$$v''_{n,s}(z) = \begin{cases} j\dfrac{\eta_0\varepsilon_n k_{t0}k_{tn}}{2\varepsilon_u^2 k_0}I_0''(z), & \text{for } t \geq z \geq 0 \\ 0, & \text{elsewhere} \end{cases} \tag{12}$$

for $n = \pm 1, \pm 2, \pm 3, \ldots$.

The single and double primes denote the TE and TM polarizations, respectively. $V_0(z)$ may be obtained by replacing the grating layer with the uniform layer, which has a relative dielectric constant ε_{ave}. Under the assumption of small perturbation, every nth space harmonic is independent of the other harmonics.

B. Boundary-Value Problem. The transverse electric and magnetic fields across the interface of grating–air and grating–substrate must satisfy the boundary conditions at $z = 0$ and $z = t$. After the transmission-line equation and boundary conditions are given, the entire problem is then reduced to the solution of a straightforward problem involving transmission lines, as shown in Figure 3. Every V_n' (V_n'') and I_n' (I_n'') in the transmission line can be found by solving the TE and TM transmission equations fed by the distributed current and voltage sources via the techniques of a Green's function for transmission-line equations [5], and can be written as

$$\begin{aligned} V_n^{\prime(\prime\prime)}(z) = &-\int_0^z dz'\, v_{n,s}^{\prime(\prime\prime)}(z')\frac{\overrightarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)}}{\overrightarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)} + \overleftarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)}} \\ &\times\left[\cos k_{z,n}^{(\text{ave})}(z-z') - j\overrightarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)}\sin k_{z,n}^{(\text{ave})}(z-z')\right] \\ &+\int_z^t dz'\, v_{n,s}^{\prime(\prime\prime)}(z')\frac{\overleftarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)}}{\overrightarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)} + \overleftarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)}} \\ &\times\left[\cos k_{z,n}^{(\text{ave})}(z-z') - j\overleftarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)}\sin k_{z,n}^{(\text{ave})}(z-z')\right] \\ &-\int_0^z dz'\, j_{n,s}^{\prime(\prime\prime)}(z')\frac{1}{\overrightarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)} + \overleftarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)}} \\ &\times\left[\cos k_{z,n}^{(\text{ave})}(z-z') - j\overrightarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)}\sin k_{z,n}^{(\text{ave})}(z-z')\right] \\ &-\int_z^t dz'\, j_{n,s}^{\prime(\prime\prime)}(z')\frac{1}{\overrightarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)} + \overleftarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)}} \\ &\times\left[\cos k_{z,n}^{(\text{ave})}(z-z') - J\overleftarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)}\sin k_{z,n}^{(\text{ave})}(z-z')\right] \end{aligned} \tag{13}$$

$$\begin{aligned} I_n^{\prime(\prime\prime)}(z) = &-\int_0^z dz'\, v_{n,s}^{\prime(\prime\prime)}(z')\frac{1}{\overrightarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)} + \overleftarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)}} \\ &\times\left[\cos k_{z,n}^{(\text{ave})}(z-z') - j\overrightarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)}\sin k_{z,n}^{(\text{ave})}(z-z')\right] \\ &-\int_z^t dz'\, v_{n,s}^{\prime(\prime\prime)}(z')\frac{1}{\overrightarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)} + \overleftarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)}} \\ &\times\left[\cos k_{z,n}^{(\text{ave})}(z-z') - j\overleftarrow{Z}_{n,\text{ave}}^{\prime(\prime\prime)}\sin k_{z,n}^{(\text{ave})}(z-z')\right] \\ &-\int_0^z dz'\, j_{n,s}^{\prime(\prime\prime)}(z')\frac{\overrightarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)}}{\overrightarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)} + \overleftarrow{Y}_{n,\text{ave}}^{\prime(\prime\prime)}} \end{aligned}$$

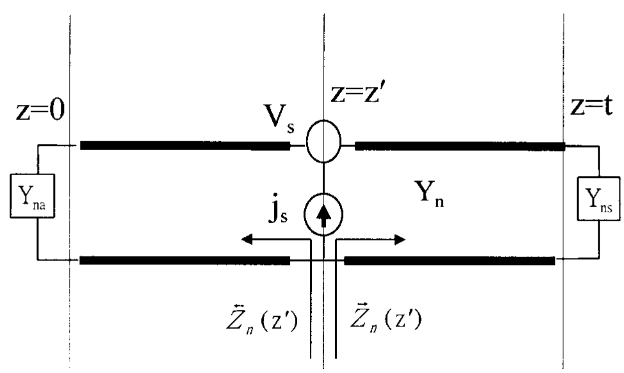

Figure 3 Network representation for point-source-excited transmission line

$$\times\left[\cos k_{z,n}^{(\mathrm{ave})}(z-z') - j\vec{Z}_{n,\mathrm{ave}}'^{(\prime\prime)} \sin k_{z,n}^{(\mathrm{ave})}(z-z')\right]$$
$$-\int_z^t dz'\, j_{n,s}'^{(\prime\prime)}(z') \frac{\overleftarrow{Y}_{n,\mathrm{ave}}'^{(\prime\prime)}}{\vec{Y}_{n,\mathrm{ave}}'^{(\prime\prime)} + \overleftarrow{Y}_{n,\mathrm{ave}}'^{(\prime\prime)}}$$
$$\times\left[\cos k_{z,n}^{(\mathrm{ave})}(z-z') - j\overleftarrow{Z}_{n,\mathrm{ave}}'^{(\prime\prime)} \sin k_{z,n}^{(\mathrm{ave})}(z-z')\right] \tag{14}$$

where

$$k_{z,n}^{(\mathrm{ave})} = \left(k_0^2 \varepsilon_{\mathrm{ave}} - k_{x,n}^2 - k_y^2\right)^{1/2}. \tag{15}$$

In (13) and (14), $j_{n,s}'^{(\prime\prime)}$ and $\nu_{n,s}'^{(\prime\prime)}$ are the distributed current and voltage sources given in (10)–(12). In addition, $\vec{Z}_{n,\mathrm{ave}}$ ($\overleftarrow{Z}_{n,\mathrm{ave}}$) are the input impedance looking to the right (looking to the left), at any location z' within the line, which is shown in Figure 3.

The voltage and current solutions can be easily and quickly obtained by evaluating (13) and (14) with numerical integration.

4. NUMERICAL RESULTS

In general, the relative dielectric constant of gratings may vary spatially arbitrarily. For the present paper, we will consider a sinusoidally modulated medium, with the relative dielectric constant given by

$$\varepsilon(x) = \varepsilon_u\left[1 + 2\delta\cos\left(\frac{2\pi x}{a}\right)\right] \tag{16}$$

where ε_u, δ, and a are the average dielectric constant, modulation index, and period of the grating. The average relative dielectric is set to be $\varepsilon_u = 1.44$, and outside the grating layer is the air region with the dielectric constant designated by $\varepsilon_a = \varepsilon_s = 1$.

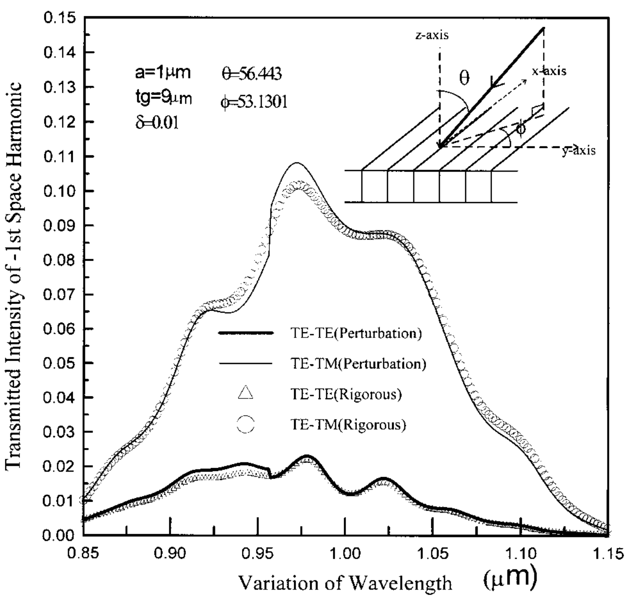

Figure 4 Effect of wavelength on the diffraction efficiency

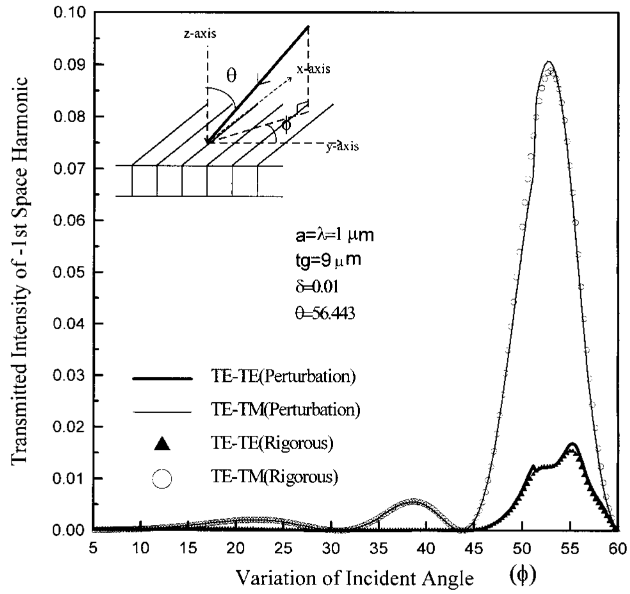

Figure 5 Effect of incident azimuth angle on the diffraction efficiency

Figure 4 shows the effect of the wavelength on the diffraction efficiencies. Figure 5 presents the effect of the azimuth angle on the diffraction efficiencies. All results in Figures 4 and 5, obtained easily by the perturbation method, show good agreement with that obtained by a rigorous formulation.

We have also investigated the effects of incidence with a TM polarized plane wave.

5. CONCLUSION

A perturbation analysis has been presented for the scattering of a plane wave by a dielectric grating at oblique incidence. The transmission-line network representation simplifies the field analysis considerably, and provides a physical picture of the polarization conversion. We have shown the approximation to be valid for a large range of parameters by comparing them with numerical results obtained by a proposed rigorous formulation [2].

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