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A method for measuring Brewster's angle by circularly polarized heterodyne interferomery

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Abstract. A new method for determining Brewster's angle is proposed based on the effect that the phase difference produced by a simple reflection from the test medium, for a two-frequency circularly polarized light source, is exactly equal to zero when the incident angle equals Brewster's angle. Its best resolution is 10^{-3} deg and the validity of the method was demonstrated.

Keywords: Brewster's angle, heterodyne interferometry

Une méthode pour mesure l'angle de Brewster par interférométrie hétérodyne en lumière circulaire

Résumé. Basé sur le fait que la différence de phase produite par une simple réflexion sur le milieu test pour une lumière polarisée circulairement à deux fréquences est égale à zéro quand l'angle d'incidence est exactement zéro à l'incidence de Brewster. La meilleure résolution est de 10^{-3} degré. On démontre ici de l'approche.

Mots clés: Angle de Brewster, interférométrie hétérodyne

1. Introduction

Brewster's angle is an important parameter for optical materials when they are used as a Brewster window [1, 2] or a pile-of-plates polarizer [1, 2]. In general, Brewster's angle is determined according to the intensity variation of the reflected light from the test medium [3, 4]. However, the stability of the light source, the scattering light, the internal reflection and other factors influence the accuracy of measurements and decrease the resolution of the results.

A new method for determining Brewster's angle is proposed. First, a linearly polarized laser, an electro-optic modulator and a quarter-wave plate are used to obtain a circularly polarized heterodyne light source. Next, according to the reflection effect for a two-frequency circularly polarized light source and the Fresnel equations [5], the phase difference produced by the reflection from the test medium is derived to be exactly equal to zero when the incident angle equals Brewster's angle. Then, the experimental relation curve between the phase difference and the incident angle is depicted. According to this curve, Brewster's angle can be estimated. Because the phase difference is measured accurately with heterodyne interferometry, its performance is not affected by the surrounding light noise. This means that it is very stable and has high resolution.

2. Principle

The schematic diagram of this new method is shown in figure 1(a). For convenience, the $+z$-axis is chosen to be along the direction of propagation and the x-axis is along the horizontal direction. Let the light coming from the laser be linearly polarized along the x-axis, then its Jones vector [6] can be written as

$$E = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{1}$$

This linearly polarized light passes through an elecro-optic modulator (EO) and a quarter-wave plate (Q). Let the

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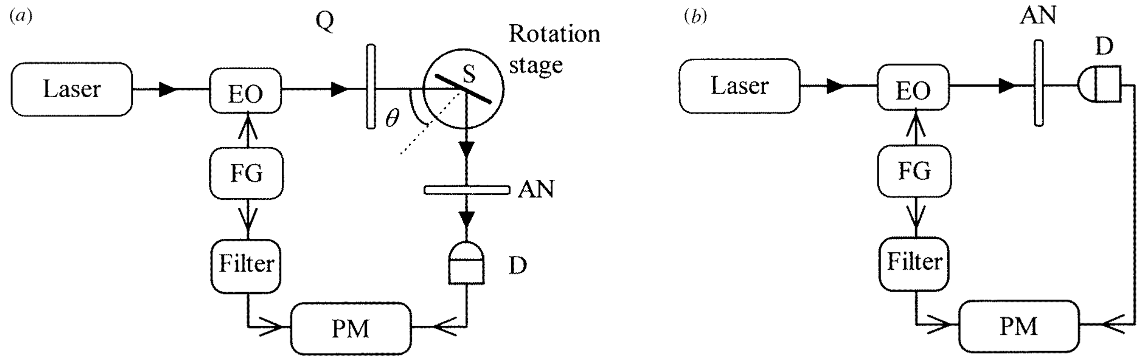

Figure 1. Schematic diagram for measuring (*a*) the phase difference owing to the reflection at S; (*b*) the initial phase of the reference signal; EO, electro-optic modulator; FG, function generator; PM, phase meter; S, test medium; AN, analyser; D, photodetector.

fast axes of EO and Q be at 45° and 0° to the x-axis, respectively. An external sawtooth voltage signal with angular frequency ω and amplitude $V_{\lambda/2}$, the half-voltage of EO, is applied to EO. Then the phase retardation [7] produced by EO can be expressed as ωt, and the Jones vector of the light can be written as

$$\begin{aligned} E' &= Q(0^\circ)\cdot EO(\omega t)\cdot E \\ &= \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{i} \end{pmatrix} \begin{pmatrix} \cos\omega t/2 & \mathrm{i}\sin\omega t/2 \\ \mathrm{i}\sin\omega t/2 & \cos\omega t/2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} \cos\omega t/2 \\ -\sin\omega t/2 \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix} 1 \\ \mathrm{i} \end{pmatrix} \mathrm{e}^{\mathrm{i}\omega t/2} + \frac{1}{2}\begin{pmatrix} 1 \\ -\mathrm{i} \end{pmatrix} \mathrm{e}^{-\mathrm{i}\omega t/2}. \end{aligned} \tag{2}$$

From equation (2), it is obvious that left- and right-circular polarizations have an angular frequency difference ω. The complete set-up for performing the operation of equation (2) consists of a laser, an electro-optic modulator (EO) which is driven by a function generator (FG) and a quarter-wave plate (Q). This set-up acts as a circularly polarized heterodyne light source. Then, the light is incident on the test medium (S). The light reflected from this medium passes through an analyser (AN) with the transmission axis being at α with respect to the x-axis and enters a photodetector (D). Consequently, the Jones vector of the light becomes

$$\begin{aligned} E_\mathrm{t} = AN(\alpha)\cdot \boldsymbol{S}\cdot E' &= \begin{pmatrix} \cos^2\alpha & \sin\alpha\,\cos\alpha \\ \sin\alpha\,\cos\alpha & \sin^2\alpha \end{pmatrix} \\ &\times \begin{pmatrix} r_\mathrm{p} & 0 \\ 0 & r_\mathrm{s} \end{pmatrix} \begin{pmatrix} \cos\omega t/2 \\ -\sin\omega t/2 \end{pmatrix} \\ &= [r_\mathrm{p}\cos\alpha\cos\omega t/2 - r_\mathrm{s}\sin\alpha\,\sin\omega t/2] \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \end{aligned} \tag{3}$$

where $\boldsymbol{S}$ is the Jones matrix of the test medium as the light is reflected from it and r_p and r_s are its reflection coefficients for p- and s-polarizations, respectively. According to the Fresnel's equations [5], we have

$$r_\mathrm{p} = \frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \tag{4}$$

and

$$r_\mathrm{s} = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} \tag{5}$$

where n and θ are the refractive indices of the test medium and the incident angle, respectively. Hence, the intensity measured by D is

$$I_\mathrm{t} = |E_\mathrm{t}|^2 = I_0\,[1 + \cos(\omega t + \phi)]\,. \tag{6}$$

Here, the average intensity I_0 and the phase difference ϕ are given as

$$I_0 = \tfrac{1}{2}(r_\mathrm{p}^2\cos^2\alpha + r_\mathrm{s}^2\sin^2\alpha) \tag{7}$$

and

$$\phi = \tan^{-1}\left(\frac{2\sin\alpha\cos\alpha\, r_\mathrm{p} r_\mathrm{s}}{r_\mathrm{p}^2\cos^2\alpha - r_\mathrm{s}^2\sin^2\alpha}\right). \tag{8}$$

I_t is the test signal. On the other hand, the electrical signal generated by the function generator FG is filtered and becomes the reference signal. So, the reference signal has the form of

$$I_r = I'\,[1 + \cos(\omega t + \phi_0)] \tag{9}$$

where ϕ_0 is the initial phase. These two sinusoidal signals are sent to a phase meter PM as shown in figure 1(*a*). The phase difference between the reference signal and test signal,

$$\phi' = \phi - \phi_0 \tag{10}$$

can be obtained. In the second measurement, the test beam is allowed to enter the photodetector D without passing the quarter-wave plate and test medium as shown in figure 1(*b*). The test signal still has the form of equation (6), but this time with $\phi = 0$. Therefore the phase meter in figure 1(*b*) represents $-\phi_0$. Substituting $-\phi_0$ into equation (10), we can obtain the phase difference ϕ.

It is easily seen from equation (5) that the case $r_\mathrm{s} = 0$ should not exist. And from equation (8) it is obvious that only when α is neither 0° nor 90°, then $\phi = 0^\circ$ as $r_\mathrm{p} = 0$. Under this condition, the incident angle is equivalent to the Brewster's angle θ_B. To understand

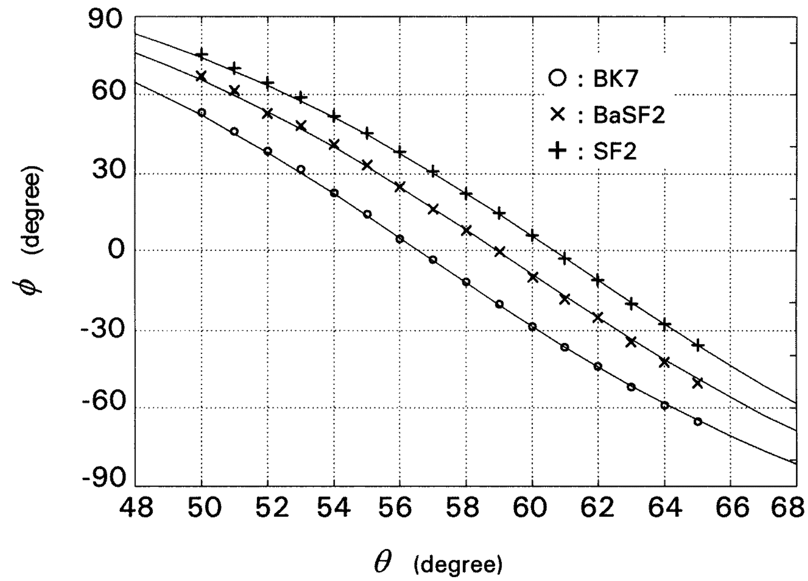

Figure 2. Theoretical and experimental curves of ϕ versus θ, for BK7, $BaSF_2$ and SF_2 as α = 20°.

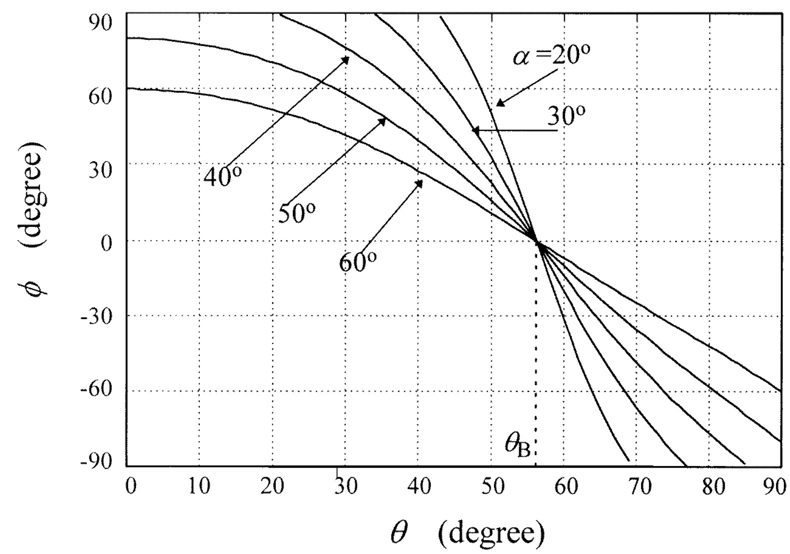

Figure 3. Relation curves of ϕ versus θ, for several different values of α for n = 1.5.

the relation between ϕ and θ, equations (4) and (5) are substituted into equation (8), we obtain

$$\phi = \tan^{-1}\Big[\big(\big(\sin^2\theta - n^2\cos^2\theta\big)\sin 2\alpha\big) \times\big[\big(2\sin^4\theta - \sin^2\theta + n^2\cos^2\theta\big)\cos 2\alpha - 2\sin^2\theta\cos\theta\sqrt{n^2-\sin^2\theta}\,\big]^{-1}\Big]. \quad (11)$$

3. Experiment

In order to show the feasibility of this method, the Brewster's angles of three kinds of glasses BK7, $BaSF_2$ and SF_2 were measured. A He–Ne laser with a wavelength of 632.8 nm and an elecro-optic modulator (Model PC200/2, manufactured by England Electro-Optics Developments Ltd) with a half-wave voltage of 170 V were used in this test. The frequency of the sawtooth signal applied to the electro-optic modulator is 2 kHz. A high-precision rotation stage (PS-θ-90) with an angular resolution of 0.005° (manufactured by Japan Chuo Precision Industrial Company Ltd) was used to mount the testing glass. For easier operation, $\alpha = 20°$ was chosen. In these conditions, the Brewster's angles of BK7, $BaSF_2$ and SF_2 were measured to be 56.574°, 58.942° and 60.634°, respectively. The theoretical and experimental curves of ϕ versus θ for these medium are shown in figure 2. In this figure, the full curves represent the theoretical values which are obtained by introducing their reference refractive indices [8] into equation (11), and the symbols ○, × and + represent the direct readouts of the division mark of the rotation stage

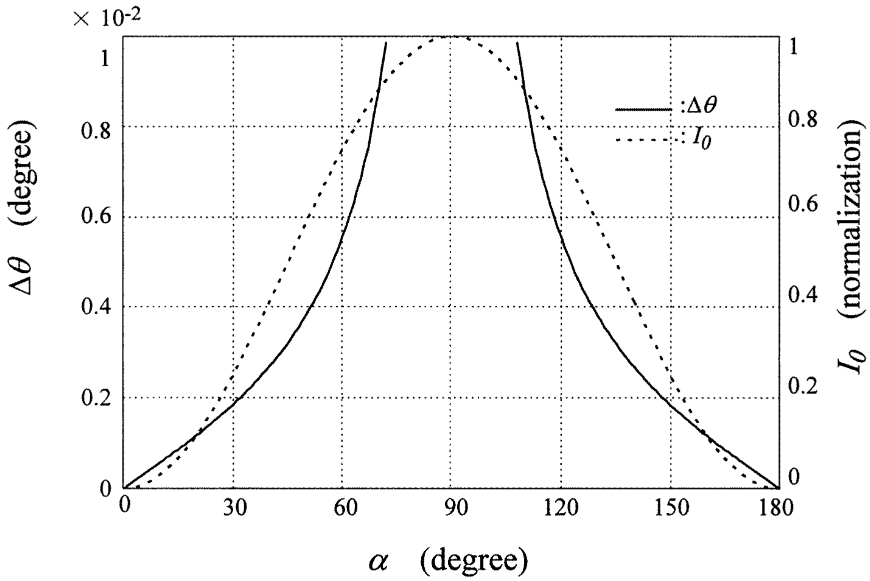

Figure 4. Theoretical curves of $\Delta\theta$ and I_0 versus α for BK7, as $\theta = \theta_B$.

for BK7, $BaSF_2$ and SF_2, respectively. It is clear that these three curves show good correspondence.

4. Discussion

From equation (11), it is obvious that the phase difference ϕ is a function of θ, α and n. To understand their relations, $n = 1.5$ is substituted into equation (11) and the relation curves of ϕ versus θ for several different α are obtained and shown in figure 3. Although these relation curves intersect one another at the same point which represents the fact that $\phi = 0°$ at Brewster's angle, they have different slopes $(d\phi/d\theta)$. It means that the angular resolution of the measurement is dependent on α. From equation (11), we have

$$\Delta\theta \cong |\sin^3\theta_B \cos\theta_B \tan\alpha|\Delta\phi \tag{12}$$

where $\Delta\theta$ and $\Delta\phi$ are the errors in the incident angle and the phase meter, respectively.

In our experiment, the angular resolution of the phase meter is 0.01°. To understand the relation between $\Delta\theta$, α and I_0, the curves of $\Delta\theta$ and I_0 versus α for BK7 have been obtained by substituting $\theta = \theta_B = 56.574°$, $n \cong 1.515$ (which is estimated by the formula $n = \tan\theta_B$), and $\Delta\phi = 0.01°$ into (4), (5), (7) and (12), as shown in figure 4. Obviously, the average intensity I_0 and the measurement error $\Delta\theta$ will increase as α increases in the range 0° to 90°. In our experiment, $\alpha = 20°$ is chosen so that the measurable accuracy and the intensity of the test signal were optimized. Therefore the measurable accuracy is almost 0.001°.

The rotating-analyser ellipsometer (RAE) [9, 10] can also measure the refractive index or Brewster's angle of the test medium. However, it is easily influenced by systematic errors produced by misalignment, if the ellipsometric parameters are derived directly from electronic signals and it needs tedious processes [10, 11] to correct these errors. Moreover, it often suffers from mechanical vibrations so that its resolution decreases. Generally, there is a second harmonic error $\Delta\phi_1$ [12] and the polarization mixing error $\Delta\phi_2$ [13, 14] in a polarization heterodyne light source, and they can be written as

$$\Delta\phi_1 = \tan^{-1}\left[\frac{\tan\phi\,(\sec 2\theta_R - 1)}{1 + \sec 2\theta_R \tan^2\phi}\right] \tag{13}$$

and

$$\Delta\phi_2 \cong \beta\sin\phi, \tag{14}$$

where θ_R is the deviation angle between the direction of the p polarization of the incident beam and the incidence plane and β is the deviation angle of the two polarization directions from orthogonality. Because the phase difference ϕ becomes zero as the incident angle equals Brewster's angle, so both $\Delta\phi_1$ and $\Delta\phi_2$ become zero. Consequently, this method is almost free from the second harmonic error and the polarization mixing error. Hence, this method is better than the rotating-analyser ellipsometer for measuring Brewster's angle.

This method is not related to the measurement of light intensity variations, it is free from the stability of a light source. In addition, because of its common-path interferometric structure, it is very stable and has a high resolution.

5. Conclusion

A new method for determining Brewster's angle is proposed based on the effect that the phase difference produced by a simple reflection from the test medium for a two-frequency circularly polarized light source only equals zero when the incident angle equals Brewster's angle. Because the phase difference is measured accurately with heterodyne interferometry, its performance is not affected by surrounding light noise. So, it is very stable and has a high resolution. Its validity has been demonstrated.

Acknowledgments

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