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# Subwavelength spatial solitons of TE mode

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**Abstract**

The wave equation of TE subwavelength beam propagations in a nonlinear planar waveguide is derived. This equation contains more higher-order linear and nonlinear terms than the nonlinear Schrödinger equation. The analytic solution of TE subwavelength spatial soliton is found to be the same as the conventional spatial soliton in amplitude but different in phase. The numerical results show that the analytic solution is stable. 

*Keywords:* Nonlinear effect; Spatial soliton; Nonlinear planar waveguide

## 1. Introduction

Spatial solitons in a nonlinear planar waveguide due to the balance of the diffraction and the self-focusing have been studied both theoretically and experimentally [1,2]. The propagation of spatial solitons are commonly described by the nonlinear Schrödinger equation (NSE) which is derived by making the paraxial approximation. However, when a beam width is as narrow as one wavelength or less the validity of the paraxial approximation becomes questionable. To resolve this problem, the full-vector nonlinear Maxwell's equations was solved [3,4], but it is very time consuming. In addition, some researchers suggest considering the additional terms in NSE to enhance the validity of the wave equation [5,6]. It is shown that when the additional terms including a polarization-dependent correction to the soliton propagation constant exists, the dynamics of a narrow spatial soliton with an arbitrary polarization will be influenced [5]. By using the wave equation of TM mode in Ref. [5], they analyze the effects of those addition terms on the shapes of bright and dark solitons of TM mode with a fixed polarization [6].

In this paper, we will derive the propagation equation for a TE subwavelength beam in a nonlinear planar waveguide by the iteration method [7]. The derived equation contains more higher-order linear and nonlinear terms than the NSE. An analytic solution of the subwavelength spatial soliton of TE mode can be found and the numerical results show that this analytic solution is stable.

## 2. Derivation of the wave equation

We now derive the wave equation which can describe the propagations of subwavelength beams of TE mode in a nonlinear planar waveguide. The electric field $E$ of the light obeys the vector wave equation

$$\nabla^2 \boldsymbol{E} - \frac{\omega^2 n_0^2}{c^2} \boldsymbol{E} + \frac{\omega^2}{c^2 \varepsilon_0} \boldsymbol{P}_{\mathrm{NL}} + \frac{1}{n_0^2 \varepsilon_0} \nabla(\nabla \cdot \boldsymbol{P}_{\mathrm{NL}}) = 0, \quad (1)$$

where $\varepsilon_0$ is the vacuum permittivity, $n_0$ is linear refractive index, $\omega$ is the light frequency, $c$ is the velocity of light in vacuum, and $P_{\mathrm{NL}}$ is the third-order nonlinear polarization and

$$(P_{\mathrm{NL}})_i = \frac{3\varepsilon_0}{4} \sum_{j,k,l} \chi^{(3)}_{i,j,k,l}(\omega = \omega_j + \omega_k - \omega_l) E_j E_k E_l^*,$$

where $\chi^{(3)}(\omega)$ is the third-order susceptibility, $i$, $j$, $k$, and $l$ refer to the Cartesian components of the fields.

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For a TE mode of the planar waveguide, we consider the propagation of one-dimensional beam along $z$-direction with a uniform field in the $y$-direction. The electric field of the light can be taken as [5]

$$E(x,z)=\hat{y}A_y(x,z)\exp(ik_0 z), \tag{2}$$

where $A_y(x,z)$ is the envelope and $k_0=(n_0\omega/c)$ is the propagation constant. The total refractive index is given by $n=n_0+n_2|E|^2=n_0+n_2|A_y|^2$, where $n_2$ is the Kerr coefficient and $n_2=(3\chi^{(3)}_{yyyy}/8n_0)$.

Substituting Eq. (2) into Eq. (1), we obtain

$$i\frac{\partial}{\partial z}A_y+\frac{1}{2k_0}\frac{\partial^2}{\partial x^2}A_y+\gamma|A_y|^2A_y=-\frac{1}{2k_0}\frac{\partial^2}{\partial z^2}A_y \tag{3}$$

where $\gamma=k_0n_2/n_0$. To normalize Eq. (3), we make the following transformations:

$$A_y(x,z)=\frac{\sqrt{P_0}}{N}u(\eta,\xi)=\sigma\sqrt{\frac{n_0}{n_2}}\,u(\eta,\xi), \tag{4a}$$

$$x=w_0\eta, \tag{4b}$$

$$z=L_d\,\xi \tag{4c}$$

where the parameter $N=[(n_2P_0)/(k_0^2w_0^2n_0)]^{1/2}$ is the order of the spatial soliton and $N=1$ for the fundamental soliton, $w_0=w_F/1.763$ and $w_F$ is the full width at the half maximum (FWHM) of the beam, $P_0$ is peak power of the incident beam, the parameter $\sigma=1/k_0w_0=0.28(\lambda_0/w_F)$ and $\lambda_0=2\pi/k_0$ is the wavelength of the light in the waveguide, and $L_d=k_0w_0^2$ is the diffraction length. By using Eqs. (4a), (4b) and (4c), Eq. (3) can be normalized to:

$$\frac{\partial}{\partial\xi}u=\frac{i}{2}\frac{\partial^2}{\partial\eta^2}u+i|u|^2u+\frac{i\sigma^2}{2}\frac{\partial^2}{\partial\xi^2}u. \tag{5}$$

## 3. Iterative method

Now we use the iterative method to replace the second derivative $(\partial^2/\partial\xi^2)u$ term by the higher order diffraction (linear) and nonlinear terms, such that the wave equation can be simplified without making the paraxial approximation. First, neglecting the second derivative $(\partial^2/\partial\xi^2)u$ in Eq. (5) and assigning the equation remaining as the wave equation of the zero order approximation:

$$\left(\frac{\partial u}{\partial\xi}\right)_0=H, \tag{6}$$

where

$$H=\frac{i}{2}\frac{\partial^2}{\partial\eta^2}u+i|u|^2u. \tag{7}$$

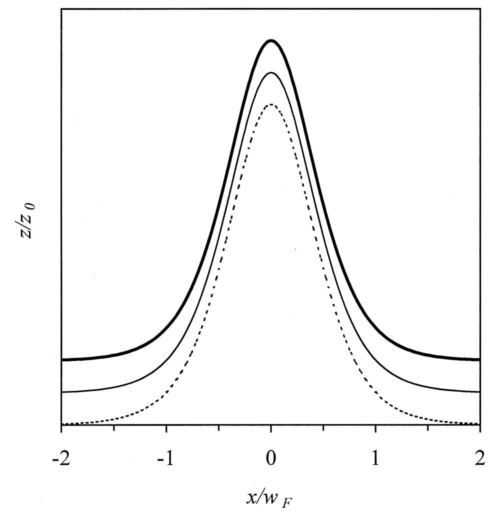


Fig. 1. Power evolution of the beam shape for the TE soliton at $(z/z_0)=0$ (dashed curve), $(z/z_0)=10$ (solid curve), $(z/z_0)=20$ (thick solid curve).

which is the well-known NSE for spatial soliton propagations. By differentiating Eq. (6) with respect to $\xi$, the second derivative $(\partial^2/\partial\xi^2)u$ can be approximated to:

$$\left(\frac{\partial^2}{\partial\xi^2}u\right)_1=\frac{\partial H}{\partial\xi}, \tag{8}$$

which is assigned as the first order approximation. Substituting Eq. (8) into Eq. (5), we obtain the wave equation of the first order approximation:

$$\left(\frac{\partial u}{\partial\xi}\right)_1=H+\frac{i\sigma^2}{2}\left(\frac{\partial^2}{\partial\xi^2}u\right)_1. \tag{9}$$

The process repeats until the expression of the second derivative $(\partial^2/\partial\xi^2)u$ does not change for the required order, so that we can obtain an accurate wave equation that includes only the first distance derivative without making the paraxial approximation.

If we only take into account up to terms of the order of $\sigma^2$, the second derivative $(\partial^2/\partial\xi^2)u$ does not change after one iteration:

$$\left(\frac{\partial^2}{\partial\xi^2}u\right)_1=-\frac{1}{4}\frac{\partial^4}{\partial\eta^4}u-2\left(\frac{\partial^2u}{\partial\eta^2}\right)|u|^2-\left(\frac{\partial u}{\partial\eta}\right)^2u^*$$
$$-2\left|\frac{\partial u}{\partial\eta}\right|^2u-|u|^4u. \tag{10}$$

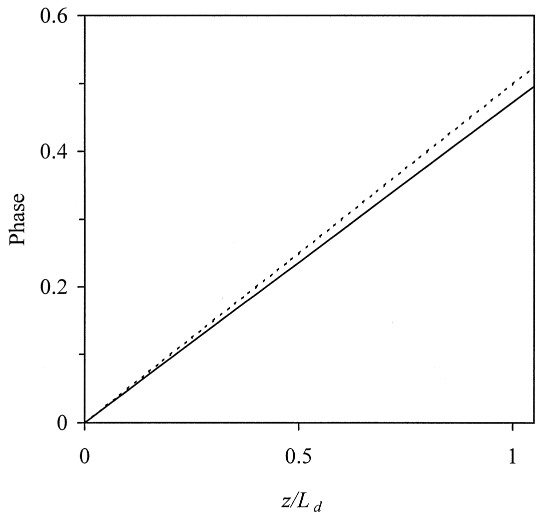


Fig. 2. The phase versus propagation distance for the TE soliton with higher-order terms (solid curve) and without higher-order terms (dashed curve).

Substituting Eq. (10) back into Eq. (5), we obtain the wave equation of the second order approximation:

$$\frac{\partial u}{\partial \xi} = \frac{i}{2}\frac{\partial^2}{\partial \eta^2}u + i|u|^2 u + i\frac{\sigma^2}{2}\left[-\frac{1}{4}\frac{\partial^4}{\partial \eta^4}u - 2\left(\frac{\partial^2 u}{\partial \eta^2}\right)|u|^2 - \left(\frac{\partial u}{\partial \eta}\right)^2 u^* - 2\left|\frac{\partial u}{\partial \eta}\right|^2 u - |u|^4 u\right]. \tag{11}$$

For higher-order approximation, we should have $\sigma^4/4$ terms. These terms are much smaller than $\sigma^2/2$ terms when $\sigma^2/2 \ll 1$. Therefore, Eq. (11) is valid when $\sigma^2/2$ is sufficient smaller than unit.

## 4. Solution and discussion

We have found that the Eq. (11) has a spatial soliton solution and it is:

$$u(\xi,\eta) = \mathrm{sech}(\eta)\exp(i\delta\xi/2), \tag{12}$$

where $\delta = 1 - (\sigma^2/4)$. To numerically simulate the soliton propagation governedly that Eq. (11) is indeed an exact solution for Eq. (11), we take $w_F = 0.6\lambda_0$, i.e., $\sigma^2/2 \approx 0.1$. We use split-step Fourier method to solve Eq. (12). Fig. 1 shows the evolution of the pulse shape for spatial soliton of TE mode in a nonlinear planar waveguide. It is seen that the beam propagates undistortedly over $20z_0$, and $z_0 = (\pi/2)L_d$ is the spatial soliton period. In Fig. 2, we show the phase versus propagation distance. The change of this phase is consistent with the analytic result. Therefore, the analytic soliton with a modified phase in a soliton solution.

## 5. Conclusion

In conclusion, we have derived an accurate wave equation from the Maxwell's equations beyond paraxial approximation by the iterative method for the subwavelength optical beam propagation in a nonlinear planar waveguide. The derived equation of the TE mode contains more higher-order linear and nonlinear terms than the nonlinear Schrödinger equation. We have found an analytic spatial soliton solution from the derived equation, and the amplitude of spatial soliton is the same as that of the conventional spatial soliton but its phase becomes smaller. The numerical results show that this spatial soliton is stable for the subwavelength optical beam propagation in a nonlinear planar waveguide.

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