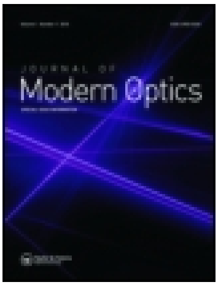

Journal of Modern Optics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tmop20

Direct focusing modifications of elliptical gaussian beam by non-circular apertures

Sheng-Hsiung Chan [a] , Shin-Gwo Shiue [a] & Mao-Hong Lu [a]
[a] Institute of Electro-Optical Engineering, National Chiao Tung University , 1001 Ta Hsueh Road, Hsinchu, 30050, Taiwan
Published online: 03 Jul 2009.

JOURNAL OF MODERN OPTICS, 1998, VOL. 45, NO. 11, 2379–2387

Direct focusing modifications of elliptical Gaussian beam by non-circular apertures

SHENG-HSIUNG CHAN, SHIN-GWO SHIUE and MAO-HONG LU

Institute of Electro-Optical Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 30050, Taiwan

(*Received 9 October 1997; revision received 9 March 1998*)

Abstract. For focusing the elliptical Gaussian beam directly, the effects of a non-circular aperture on the focusing properties are studied. The focusing properties for different shapes of apertures, which include a circle, an ellipse and a rectangle, are calculated and compared. Moreover, for different elliptical Gaussian beams, an empirical aperture selection rule that can be used to circularize the focusing spot is proposed. The energy transmission ratios are also considered in this paper.

1. Introduction

The advent of the laser diode has made it necessary to deal with elliptical Gaussian beams. In practical applications, there are two methods of focusing elliptical Gaussian beams. The first method, which is most often used, is to use prisms or cylindrical lenses [1] to obtain a circular beam before focusing. Because of misalignment and defects of the optical elements this approach may cause unavoidable throughput losses and increase the system cost. The other method is the so-called direct focusing method, which utilizes the aperture to truncate the incident beam to get the desired characteristics of the focusing spot. Marchant [2] studied the direct focusing properties of the elliptical Gaussian beams by using the central irradiance and the line spread function. With considerations of system cost and power increase of the laser diode, Kondo [3] has used this approach for the optical heads. However, when the elliptical beam can be focused through a circular objective lens, the focusing spot will also be non-circular, and the ellipticity of the focusing spot depends on the beam ellipticity and the apodization values [2]. Kondo [3] showed that the orientation of the major or minor axis of the elliptical Gaussian beam can affect the signal level, the spatial frequency characteristics and the cross-talk level. In some other applications, for instance the laser printer, a non-circular focusing spot is an annoying problem.

According to the diffraction theory, the shape of the aperture can affect the diffraction pattern in the image plane of an optical system [4–6]. The diffraction of an elliptical Gaussian beam truncated by an elliptical aperture has been calculated numerically by Kathuria [7], who found that the beam ellipticity, which is not equal to the ellipticity of the aperture, can be changed in the far field. In this paper, the aperture diffraction method is put forward to modify the characteristics of the elliptical focusing spot. The effects of rectangular and elliptical apertures on the focusing spot are calculated in the canonical coordinates used by Chung and Hopkins [8], and some focusing properties of the optical systems with a circular, a specific elliptical and a specific rectangular aperture are compared. For different elliptical Gaussian beams, an empirical rule to select a specific aperture for circularizing focusing spots is also proposed.

2. Analysis of the focusing properties

For elliptical Gaussian beams, the equations, which are the focusing spot profile and the energy transmission ratio through the optical system with the non-circular aperture, are introduced. Moreover, for convenience of discussion, the canonical coordinates used by Chung and Hopkins are employed in this paper.

2.1. *Focusing spot profile*

2.1.1. *Elliptical aperture*

According to the diffraction theory, the complex amplitude in the image plane of an aberration-free system with an elliptical aperture and elliptical Gaussian beam incidence can be written as

$$F(u',v')=\frac{1}{A}\iint_{\text{ellipse}} \exp\left[-\left(\frac{x'}{\sigma_x}\right)^2-\left(\frac{y'}{\sigma_y}\right)^2\right] \exp\left[\mathrm{i}2\pi(u'x'+v'y')\right]\mathrm{d}x'\,\mathrm{d}y', \qquad (1)$$

where A is introduced as a normalizing factor, and σ_x and σ_y are the Gaussian constants for the x' and y' axes respectively.

The diameters along the x' and y' axes of an elliptical aperture are defined as 2α and 2β respectively (figure 1), where $0<\alpha,\beta\leqslant 1$, and the axial ratio ε of the aperture is defined as β/α. Defining the geometry of the aperture in polar

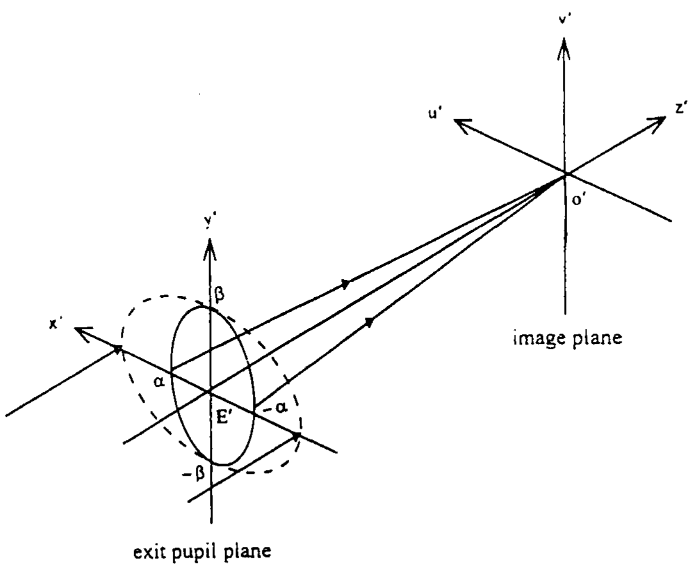

Figure 1. The canonical coordinate diagram of an elliptical aperture.

coordinates [7] as

$$x' = r\cos\theta,$$

$$y' = r\sin\theta,$$

$$\mathrm{d}x'\,\mathrm{d}y' = \varepsilon r\,\mathrm{d}r\,\mathrm{d}\theta,$$

equation (1) can be rewritten as

$$F(u',v') = \frac{1}{A}\int_0^1\int_0^{2\pi} \exp\left[-\left(\frac{r\cos\theta}{\sigma_x}\right)^2 - \left(\frac{\varepsilon r\sin\theta}{\sigma_y}\right)^2\right] \times \exp\left[\mathrm{i}2\pi(u' r\cos\theta + v'\varepsilon r\sin\theta)\right]\varepsilon r\,\mathrm{d}r\,\mathrm{d}\theta.$$

The algorithm in [7, 8] is used for numerical calculation of this equation. Finally, the relative intensity distribution in the image plane can be given by

$$I(u',v') = |F(u',v')|^2. \tag{2}$$

The unit intensity at the point $u' = v' = 0$ of a diffraction-limited system is ensured by the normalizing factor A.

2.1.2. *Rectangular aperture*

The diameters along the x' and y' axes of a rectangular aperture are defined as $2L_x$ and $2L_y$ respectively (figure 2), where $0 < L_x, L_y \leqslant 1$, and, for this kind of aperture, the complex amplitude in image plane can be written as

$$F(u',v') = \frac{1}{A}\int_{-L_x}^{L_x}\int_{-L_y}^{L_y} \exp\left[-\left(\frac{x'}{\sigma_x}\right)^2 - \left(\frac{y'}{\sigma_y}\right)^2\right] \exp\left[\mathrm{i}2\pi(u'x' + v'y')\right]\mathrm{d}x'\,\mathrm{d}y'.$$

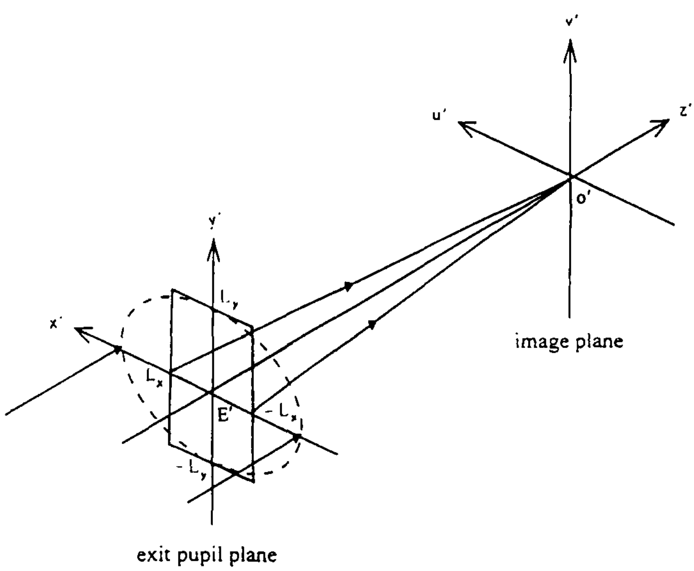

Figure 2. The canonical coordinate diagram of a rectangular aperture.

Using the error function defined as $\mathrm{erf}(r) = (2/\pi^{1/2}) \int_0^r \exp(-\mu^2)\,\mathrm{d}\mu$, and changing the variables of integration, we have

$$F(u', v') = \exp(-\sigma_x^2\pi^2u'^2)\exp(-\sigma_y^2\pi^2v'^2)\left[4\,\mathrm{erf}\left(\frac{L_x}{\sigma_x}\right)\mathrm{erf}\left(\frac{L_y}{\sigma_y}\right)\right]^{-1}$$
$$\times\left[\mathrm{erf}\left(\frac{L_x - \mathrm{i}\sigma_x^2\pi u'}{\sigma_x}\right) - \mathrm{erf}\left(\frac{-L_x - \mathrm{i}\sigma_x^2\pi u'}{\sigma_x}\right)\right]$$
$$\times\left[\mathrm{erf}\left(\frac{L_y - \mathrm{i}\sigma_y^2\pi v'}{\sigma_y}\right) - \mathrm{erf}\left(\frac{-L_y - \mathrm{i}\sigma_y^2\pi v'}{\sigma_y}\right)\right] \tag{3}$$

Finally, substituting equation (3) into equation (2), we thus get the relative intensity distribution in the image plane for a rectangular pupil.

2.2. *Energy transmission*

In the direct focusing method, the truncation of the elliptical Gaussian beam can be adjusted to get desired characteristics of the focusing spot. However, the size and the ellipticity of the beam can affect the efficiency. The energy transmission ratio is defined as [3]

$$T = \frac{1}{A'}\iint_s \exp\left\{-2\left[\left(\frac{x'}{\sigma_x}\right)^2 + \left(\frac{y'}{\sigma_y}\right)^2\right]\right\}\mathrm{d}x'\,\mathrm{d}y', \tag{4}$$

where s denotes the effective region of the pupil, and A' is a normalizing factor defined as the total power of the elliptical Gaussian beam.

2.2.1. *Elliptical aperture*

For this case, the integration region s of equation (4) is an ellipse. Using polar coordinates, equation (4) can be rewritten as

$$F(u', v') = \frac{1}{A}\int_0^1\int_0^{2\pi} \exp\left[-\left(\frac{r\cos\theta}{\sigma_x}\right)^2 - \left(\frac{\varepsilon r\sin\theta}{\sigma_y}\right)^2\right]\varepsilon r\,\mathrm{d}r\,\mathrm{d}\theta. \tag{5}$$

Again, the Hopkins numerical algorithm is used in this integration.

2.2.2. *Rectangular aperture*

For a rectangular aperture, equation (4) can be rewritten as

$$T = \frac{1}{A'}\int_{-L_x}^{L_x}\int_{-L_y}^{L_y} \exp\left\{-2\left[\left(\frac{x'}{\sigma_x}\right)^2 + \left(\frac{y'}{\sigma_y}\right)^2\right]\right\}\mathrm{d}x'\,\mathrm{d}y'$$
$$= \frac{\pi\sigma_x\sigma_y}{2A'}\mathrm{erf}\left(\frac{2^{1/2}L_x}{\sigma_x}\right)\mathrm{erf}\left(\frac{2^{1/2}L_y}{\sigma_y}\right),$$

where $A' = 2/\pi\sigma_x\sigma_y$; so we have

$$T = \mathrm{erf}\left(\frac{2^{1/2}L_x}{\sigma_x}\right)\mathrm{erf}\left(\frac{2^{1/2}L_y}{\sigma_y}\right). \tag{6}$$

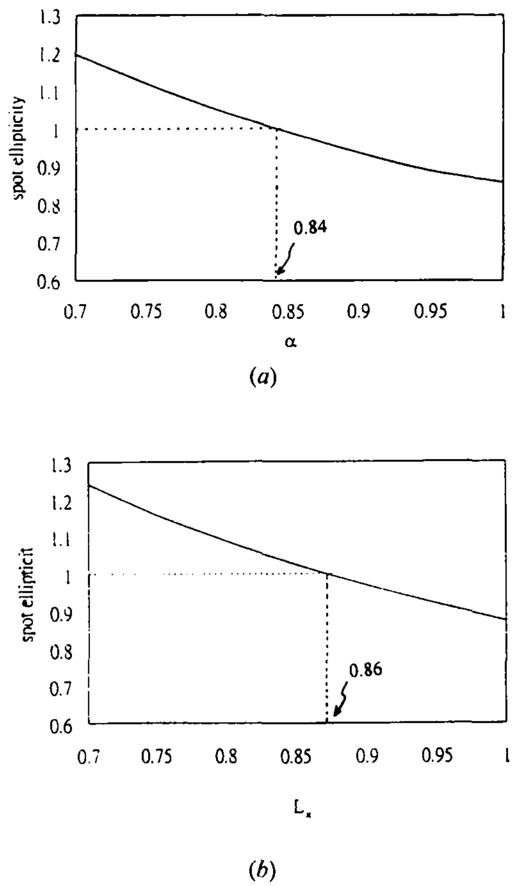

Figure 3. The beam ellipticity ε' is set to 3 and $\sigma_y = 1$. (*a*) Focusing spot ellipticity against the diameter α of the elliptical aperture in x' direction. The diameter β of the elliptical aperture in y' direction is kept at unity. (*b*) Focusing spot ellipticity against the diameter L_x of the rectangular aperture in x' direction. The diameter L_y of the elliptical aperture in y' is kept at unity.

3. Results of calculation and discussion

In this paper, $\sigma_x \geqslant \sigma_y$ is always assumed, that is σ_x and σ_y are the Gaussian constants in the major and minor axes respectively of the elliptical Gaussian beam. The beam ellipticity ε' is defined as σ_x/σ_y. The spot diameters in u' and v' directions (e^{-2} power point) are expressed as ρ_u and ρ_v respectively, and the spot ellipticity P is defined as ρ_u/ρ_v.

For different elliptical or rectangular apertures, the shape variation of the focusing spot can be calculated and is demonstrated in figures 3 (*a*) and (*b*). For convenience of discussions, $\beta = 1$ in figure 3 (*a*), and $L_y = 1$ in figure 3 (*b*). $\varepsilon'(= \sigma_x/\sigma_y) = 3$ and $\sigma_y = 1$ are set in both the figures. According to these two figures, the specified sizes of apertures, which can circularize the focusing spots, can be found for these two kinds of aperture.

For the elliptical Gaussian beam, $\varepsilon' = 3$ and $\sigma_y = 1$, the effects of a unit circular, a specific elliptical and a specific rectangular aperture on the focusing

Table 1. Energy transmission ratios, ρ_u and ρ_v, the focusing spot ellipticities P and the secondary maxima in the u' and v' directions of the three kinds of aperture which are a unit circular aperture, a specific elliptical aperture and a specific rectangular aperture are listed. The beam ellipticity $\varepsilon' = 3$ and $\sigma_y = 1$.

	Energy transmission ratio	ρ_u	ρ_v	Focusing spot ellipticity $P = \rho_u/\rho_v$	Secondary maximum value in u' direction	Secondary maximum value in v' direction
Circular aperture (radius, 1.0)	0.423	0.401	0.474	0.846	0.022	0.003
Elliptical aperature ($\alpha = 0.84, \beta = 1.0$)	0.362	0.474	0.474	1.000	0.023	0.003
Rectangular aperture ($L_x = 0.86, L_y = 1.0$)	0.414	0.412	0.412	1.000	0.042	0.001

properties are calculated and listed in table 1 for comparison. The profiles of the focusing spot for these three kinds of aperture are also shown in figure 4. We give a summary for the calculations as follows.

(1) The transmitted energy through the specific elliptical aperture is the smallest. The transmitted energies through the specific rectangular and the circular aperture are very close.
(2) Among the three apertures, the focusing spot size of the specific elliptical aperture is the largest, and that of the specific rectangular aperture is the smallest.
(3) Both the elliptical and the rectangular apertures are able to circularize the focusing spot, and the focusing spot of the specific elliptical aperture is more circular than that of the specific rectangular aperture (see figures 4 (*b*) and (*c*)). For the specific rectangular aperture, the spot profile in the $u' = v'$ direction is not the same as those in u' and v' directions.
(4) The secondary maxima of the focusing spot profiles for the specific elliptical aperture in the u' and v' directions are almost the same as those for the circular aperture. For the specific rectangular aperture, the secondary maxima in the u' direction is almost twice the others, but the secondary maximum in the v' direction is the smallest.

To proceed further in an analysis of these two non-circular apertures, for different σ_y the specific diameters α and L_x of the elliptical and the rectangular apertures as functions of the beam ellipticity ε' are calculated and shown in figures 5 (*a*) and (*b*). From these two figures, we find that the diameter α or L_x is almost independent of the beam ellipticity ε' when it is larger than 2. An empirical rule is that, for elliptical Gaussian beams with different beam ellipticities, the same specific aperture can be used to circularize the focusing spot if the Gaussian constant in the minor axis undergoes no change. However, the circular focusing spot can be obtained using the same aperture, but the energy transmission is changed for the different beam ellipticities. According to equations (5) and (6), the energy transmission ratio can be calculated for these two kinds of aperture. In figure 6, the energy transmission ratio can be calculated for these two kinds of

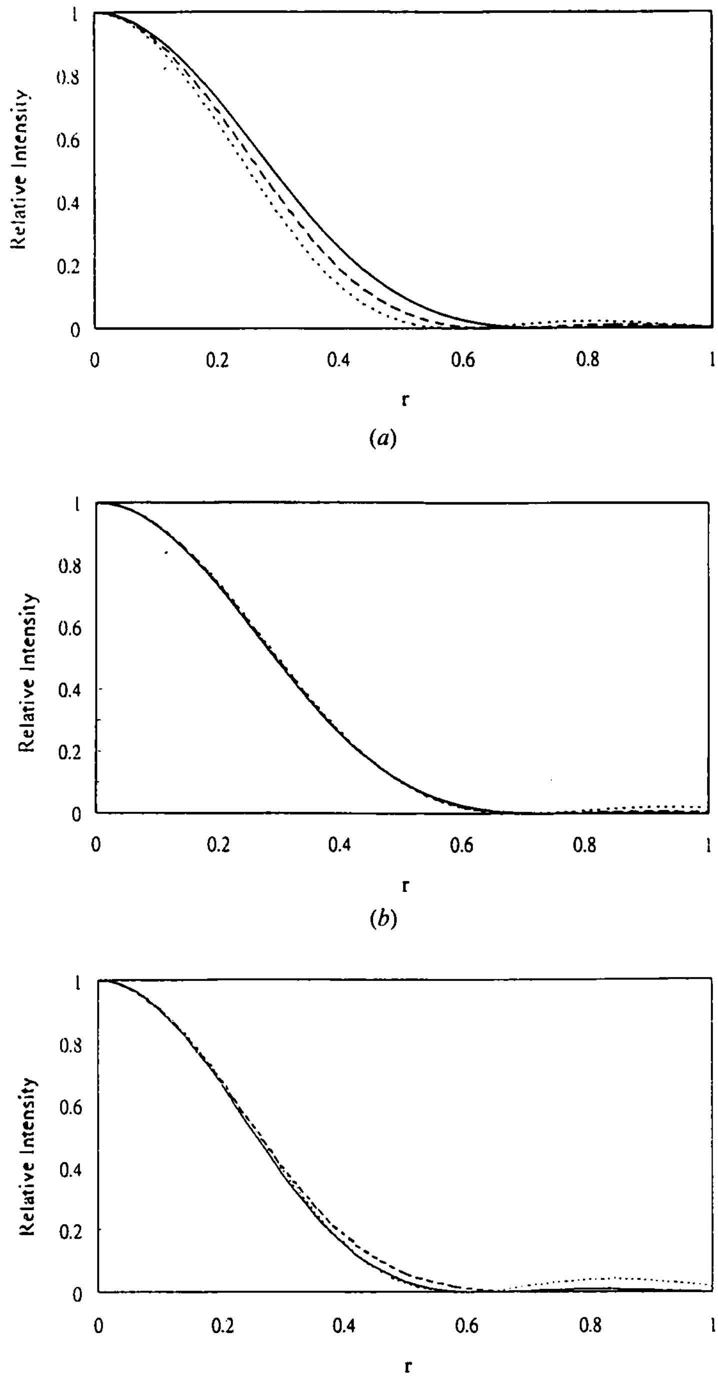

Figure 4. The focusing profiles of the elliptical Gaussian beam with $\varepsilon' = 3$ and $\sigma_y = 1$ for (*a*) a unit circular aperture, (*b*) a specific elliptical aperture ($\alpha = 0.84$ and $\beta = 1$) and (*c*) a specific rectangular aperture ($L_x = 0.86$ and $L_y = 1$). In each figure, there are three profiles in three different directions of the focal plane (——), $u' = 0$; (······), $v' = 0$; (– – –), $u' = v'$.

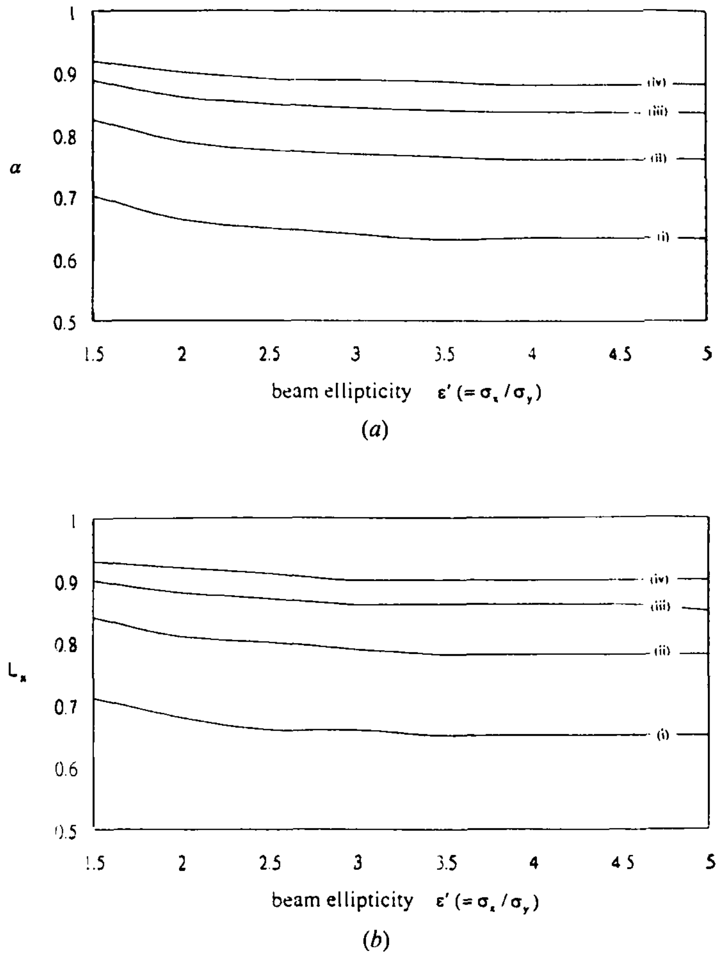

Figure 5. For different Gaussian constants σ_y for the minor axis, the aperture size (*a*) α and (*b*) L_x, which can circularize the focusing spot, against beam ellipticity ε' are shown: curves (i), $\sigma_y = 0.6$; curves (ii), $\sigma_y = 0.8$; curves (iii), $\sigma_y = 1.0$; curves (iv), $\sigma_y = 1.2$. Both β and L_y are kept at unity.

aperture. In figure 6, the energy transmission ratios of the specific apertures as a function of the beam ellipticity ε' are shown when $\sigma_y = 1$. For comparison, the energy transmission ratio of a unit circular aperture is also shown in this figure. In this case of circularizing the focusing spot, the energy transmission ratio decreases monotonically as the beam ellipticity increases for these three kinds of aperture.

4. Conclusions

In this paper, the aperture diffraction method is demonstrated to modify the non-circular focusing spot of the elliptical Gaussian beam. The circular focusing spot has been obtained by using a specific rectangular aperture or a specific elliptical aperture. Moreover, as a result of calculations, an empirical aperture

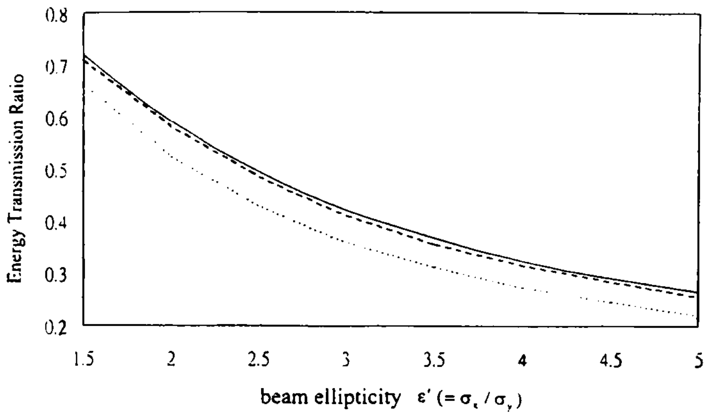

Figure 6. The energy transmissin ratio through the aperture against the beam ellipticity ε', for the unit circular aperture (——), the specific elliptical aperture (······) and the specific rectangular aperture (– – –). The Gaussian constant σ_y for the minor axis is set to unity.

selection rule for different elliptical Gaussian beams is proposed, and the energy transmission ratios are also considered.

According to these results in this paper, the applications of the direct focusing of the elliptical Gaussian beam can be expanded.

Acknowledgment

This project was supported by the National Science Council of Taiwan, ROC under grant NSC-86-2215-E-009-015.

References

[1] Smith, W. J., 1996, *Modern Optical Engineering* (New York: McGraw-Hill).
[2] Marchant, A. B., 1984, *Appl. Optics*, **23,** 670.
[3] Kondo, M., 1992, *Jpn. J. appl. Phys.*, **31,** 2452.
[4] Kathuria, Y. P., 1985, *J. opt. Soc. Am.* A, **2,** 852.
[5] Smith, R. C., and Marsh, J. S., 1987, *J. opt. Soc. Am.*, **64,** 798.
[6] Tanaka, T., Taguchi, M., and Tanaka, K., 1992, *Optics Commun.*, **90,** 207.
[7] Kathuria, Y. P., 1987, *J. mod. Optics*, **34,** 1085.
[8] Chung, C. S., and Hopkins, H. H., 1988, *J. mod. Optics*, **35,** 1485.