ELSEVIER Discrete Applied Mathematics 87 (1998) 245–253

DISCRETE APPLIED MATHEMATICS

Weighted connected domination and Steiner trees in distance-hereditary graphs ☆

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Received 20 September 1994 received in revised form 3 March 1998; accepted 9 March 1998

Abstract

Distance-hereditary graphs are graphs in which every two vertices have the same distance in every connected induced subgraph containing them. This paper studies distance-hereditary graphs from an algorithmic viewpoint. In particular, we present linear-time algorithms for finding a minimum weighted connected dominating set and a minimum vertex-weighted Steiner tree in a distance-hereditary graph. Both problems are $\mathcal{NP}$-complete in general graphs. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Distance-hereditary graph; Connected domination; Steiner tree; Algorithm; Cograph

1. Introduction

The concept of domination can be used to model many location problems in operations research. In a graph $G = (V,E)$, a *dominating set* is a subset D of vertices such that every vertex in $V - D$ is adjacent to some vertex in D. A dominating set of G is *connected* if the subgraph $G[D]$ induced by D is connected. The *connected domination problem* is to find a minimum-sized connected dominating set of a graph. Suppose, moreover, that each vertex v in G is associated with a weight $w(v)$ that is a real number. The *weighted connected domination problem* is to find a connected dominating set D such that $w(D) \equiv \sum_{v \in D} w(v)$ is as small as possible.

The concept of Steiner trees originally concerned points in Euclidean spaces, but it is also closely related to connected domination in graphs. Suppose T is a subset of vertices in a graph $G = (V,E)$. The *Steiner tree problem* is to find a minimal subset S of $V - T$ such that $G[S \cup T]$ is connected. S and T are called the *Steiner set* and *target set*, respectively. We can also consider the *vertex-weighted version* of the Steiner tree problem, which was originally introduced by Segev [27]. The vertex weight of a

☆ Supported in part by the National Science Council under grant NSC84-2121-M009-023.

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PII S0166-218X(98)00060-2

Steiner vertex can be interpreted as the cost of adding this vertex when forming the tree. Traditionally, the problem of finding the Steiner tree for a set of points in a graph has been studied for edge-weighted graphs (see [21]). However, Johnson [22, p. 445, line 9], pointed out that the edge-weighted Steiner tree problem is NP-complete for *any* classes that contains all complete graphs. In particular, the edge-weighted Steiner tree problem is $\mathcal{NP}$-complete for the edge-weighted distance-hereditary graphs as these contain the edge-weighted complete graphs. So, we only consider the vertex-weighted Steiner tree problem for distance-hereditary graphs in this paper.

The connected domination and Steiner tree problems have the same complexity for many classes of graphs. For instance, they are both polynomially solvable for strongly chordal graphs [30], permutation graphs [7], cographs [9, 22], series–parallel graphs [12, 26, 29, 30], and distance-hereditary graphs [3, 13]; and they are $\mathcal{NP}$-complete for bipartite graphs [18, 25], split graphs [23, 30], chordal graphs [23, 30], and chordal bipartite graphs [24]. It is also known that the connected domination problem is polynomially solvable for k-trees (fixed k) [1] and 1-CUBs [11] and $\mathcal{NP}$-complete for k-CUBs ($k \geqslant 2$) [11]. The Steiner tree problem is polynomially solvable in homogeneous graphs [14].

For many location problems, the corresponding domination problems may have different constraints or objective functions. Typical examples are r-domination and weighted versions. Results for these variant domination problems are relatively fewer than the usual version. Some well-known results of this kind are polynomial algorithms for the weighted domination and the weighted independent domination problems in strongly chordal graphs [17], the weighted perfect domination problem in co-comparability graphs [6], the r-domination problems in trees [28] and strongly chordal graphs [5], the connected r-domination problem in strongly chordal graphs [5] and distance-hereditary graphs [3]. The purpose of this paper is to present linear-time algorithms for the weighted connected domination problem with arbitrary weights and the vertex-weighted Steiner tree problem with non-negative weights in distance-hereditary graphs.

In the rest of this section, we give a brief survey of distance-hereditary graphs. A graph is *distance-hereditary* if every two vertices have the same distance in every connected-induced subgraph. Distance-hereditary graphs were introduced by Howorka [20]. The characterization and recognition of distance-hereditary graphs have been studied in [2, 13, 15, 19, 20]. Note that the class of distance-hereditary graphs is a subclass of all parity graphs [4] and a superclass of all cographs [8, 10].

Suppose A and B are two sets of vertices in a graph $G=(V,E)$. The *neighborhood* $N_A(B)$ of B in A is the set of vertices in A that are adjacent to some vertex in B. The *closed neighborhood* $N_A[B]$ of B in A is $N_A[B] \cup B$. For simplicity, $N_A(v)$, $N_A[v]$, $N(B)$, and $N[B]$ stand for $N_A(\{v\})$, $N_A[\{v\}]$, $N_V(B)$, and $N_V[B]$, respectively. The *distance* $d_G(x,y)$ or $d(x,y)$ between two vertices x and y in G is the minimum length of an x–y path in G. The *hanging* h_u of a connected graph $G=(V,E)$ at a vertex $u \in V$ is the collection of sets $L_0(u)$, $L_1(u), \ldots, L_t(u)$ (or L_0, $L_1, \ldots, L_t$ if there is no ambiguity), where $t = \max_{v \in V} d_G(u,v)$ and $L_i(u) = \{v \in V : d_G(u,v) = i\}$ for $0 \leqslant i \leqslant t$. For any $1 \leqslant i \leqslant t$ and any vertex $v \in L_i$, let $N'(v) = N(v) \cap L_{i-1}$. For any $U \subseteq V$, a vertex

$v \in U \cap L_i$ with $1 \leqslant i \leqslant t$ has a *minimal neighborhood* in L_{i-1} *with respect to* U if $N'(w)$ is not a proper subset of $N'(v)$ for any $w \in U \cap L_i$. When $U = V$ we omit the term U in the above definition.

Theorem 1 (Bandelt and Mulder [2], D'Atri and Moscarini [13], and Day et al. [15]). *For a connected graph $G = (V,E)$ the following statements are equivalent*:
(1) *G is a distance-hereditary graph.*
(2) *Every cycle of length at least five in G has two crossing chords.*
(3) *For every hanging $h_u = (L_0, L_1, \ldots, L_t)$ of G and every pair of vertices $x, y \in L_i$ $(1 \leqslant i \leqslant t)$ that are in the same component of $G[V - L_{i-1}]$, we have $N'(x) = N'(y)$.*

Theorem 2 (Bandelt and Mulder [2]). *Suppose $h_u = (L_0, L_1, \ldots, L_t)$ is a hanging of a connected distance-hereditary graph at u. For any two vertices $x, y \in L_i$ with $i \geqslant 1$, $N'(x)$ and $N'(y)$ are either disjoint, or one of the two sets is contained in the other.*

Theorem 3 (Fact 3.4 in Hammer and Maffray [19]). *Suppose $h_u = (L_0, L_1, \ldots, L_t)$ is a hanging of a connected distance-hereditary graph at u. For each $1 \leqslant i \leqslant t$, there exists a vertex $v \in L_i$ such that v has a minimal neighborhood in L_{i-1}. In addition, if v satisfies the above condition then for every pair of vertices x and y in $N'(v)$, we have $N_{V-N'(v)}(x) = N_{V-N'(v)}(y)$.*

2. Weighted connected domination

This section presents a linear-time algorithm for finding a minimum weighted connected dominating set of a connected distance-hereditary graph $G = (V,E)$ in which each vertex v has a weight $w(v)$ that is a real number.

Lemma 4. *Suppose $G = (V,E)$ is a connected graph with a weight function w on V. Let V' be the set of vertices v with $w(v) < 0$ and w' be defined by $w'(v) = \max\{w(v), 0\}$ for all $v \in V$. If D is a minimum w'-weighted connected dominating set of G, then $D \cup V'$ is a minimum w-weighted connected dominating set of G.*

Proof. First of all, since D is a connected dominating set of G, $D \cup V'$ is also. Next, suppose M is a minimum w-weighted connected dominating set of G. Since M is a connected dominating set of G and D is a minimum w'-weighted connected dominating set of G, $w'(M) \geqslant w'(D)$, i.e., $w(M - V') = w'(M) \geqslant w'(D) = w(D - V')$, and so

$$w(M) = w(M - V') + w(M \cap V') \geqslant w(D - V') + w(V') = w(D \cup V').$$

This completes the proof of the lemma. □

Lemma 4 suggests that it suffices to consider the weighted connected domination problem with a non-negative weight function.

Lemma 5. *Suppose $h_u = \{L_0, L_1, \ldots, L_t\}$ is a hanging of a connected distance-hereditary graph at u. For any connected dominating set D and $v \in L_i$ with $2 \leqslant i \leqslant t$, $D \cap N'(v) \neq \emptyset$.*

Proof. Choose a vertex y in D that dominates v. Then $y \in L_{i-1} \cup L_i \cup L_{i+1}$. If $y \in L_{i-1}$, then $y \in D \cap N'(v)$. So we may assume that $y \in L_i \cup L_{i+1}$. Choose a vertex $x \in D \cap (L_0 \cup L_1)$ and an x–y path P:

$$x = v_1, v_2, \ldots, v_m = y$$

using vertices only in D. Let j be the smallest index such that $\{v_j, v_{j+1}, \ldots, v_m\} \subseteq L_i \cup L_{i+1} \cup \cdots \cup L_t$. Then $v_j \in L_i$, $v_{j-1} \in N'(v_j)$, and v and v_j are in the same component of $G[V - L_{i-1}]$. By Theorem 1 (3), $N'(v) = N'(v_j)$ and so $v_{j-1} \in D \cap N'(v)$. In any case, $D \cap N'(v) \neq \emptyset$. □

Theorem 6. *Suppose $G = (V, E)$ is a connected distance-hereditary graph with a non-negative weight function w on its vertices. Let $h_u = \{L_0, L_1, \ldots, L_t\}$ be a hanging at a vertex u of minimum weight. Consider the set $\mathscr{A} = \{N'(v) : v \in L_i$ with $2 \leqslant i \leqslant t$ and v has a minimal neighborhood in $L_{i-1}\}$. For each $N'(v)$ in $\mathscr{A}$, choose one vertex v^* in $N'(v)$ of minimum weight, and let D be the set of all such v^*. Then D or $D \cup \{u\}$ or some $\{v\}$ with $v \in V$ is a minimum weighted connected dominating set of G.*

Proof. For any $x \in L_i$ with $2 \leqslant i \leqslant t$, by Theorem 2, $N'(x)$ includes some $N'(v)$ in $\mathscr{A}$. Thus we have Claim 1.

Claim 1. *For any $x \in L_i$ with $2 \leqslant i \leqslant t$, x is adjacent to some vertex in $L_{i-1} \cap D$.*

Claim 2. *$D \cup \{u\}$ is a connected dominating set of G.*

Proof of Claim 2. By Claim 1 and $N[u] = L_1 \cup \{u\}$, $D \cup \{u\}$ is a dominating set of G. Also, by Claim 1, for any vertex x in $D \cup \{u\}$ there exists an x–u path using vertices only in $D \cup \{u\}$, i.e., $G[D \cup \{u\}]$ is connected. □

Suppose M is a minimum weighted connected dominating set of G. By Lemma 5, $M \cap N'(v) \neq \emptyset$ for each $N'(v) \in \mathscr{A}$, say $v^{**} \in M \cap N'(v)$. Note that any two sets in $\mathscr{A}$ are disjoint, so $|M| \geqslant |\mathscr{A}| = |D|$.

Case 1: $|M| = 1$. The theorem is obvious in this case.

Case 2: $|M| > |D|$. In this case, there is at least one vertex x in M that is not a v^{**}. So

$$w(M) \geqslant \sum_{v^{**}} w(v^{**}) + w(x) \geqslant \sum_{v^*} w(v^*) + w(u) = w(D \cup \{u\}).$$

This together with Claim 2 proves that $D \cup \{u\}$ is a minimum weighted connected dominating set of G.

Case 3: $|M| = |D| \geqslant 2$. Since $\mathscr{A}$ contains pairwise disjoint sets, $M = \{v^{**}: N'(v) \in \mathscr{A}\}$. So $w(M) = \sum_{v^{**}} w(v^{**}) \geqslant \sum_{v^*} w(v^*) = w(D)$.

For any two vertices x^* and y^* in D, x^{**} and y^{**} are in M. Since $G[M]$ is connected, there is an x^{**}–y^{**} path in $G[M]$:

$$x^{**} = v_0^{**}, v_1^{**}, \ldots, v_n^{**} = y^{**}.$$

For any $1 \leqslant i \leqslant n$, since v_i^* and v_i^{**} are both in $N'(v_i) \in \mathscr{A}$, by Theorem 3, $N_{V-N'(v_i)}(v_i^*) = N_{V-N'(v_i)}(v_i^{**})$. But $v_{i-1}^{**} \in N_{V-N'(v_i)}(v_i^{**})$. So $v_{i-1}^{**} \in N_{V-N'(v_i)}(v_i^*)$ and $v_i^* \in N_{V-N'_{(v_{i-1})}}(v_{i-1}^{**})$. Also, that v_{i-1}^* and v_{i-1}^{**} are both in $N'(v_{i-1}) \in \mathscr{A}$ implies that $N_{V-N'(v_{i-1})}(v_{i-1}^*) = N_{V-N'(v_{i-1})}(v_{i-1}^{**})$. Then $v_i^* \in N_{V-N'(v_{i-1})}(v_{i-1}^*)$. This proves that v_{i-1}^* is adjacent to v_i^* for $1 \leqslant i \leqslant n$ and then

$$x^* = v_0^*, v_1^*, \ldots, v_n^* = y^*$$

is an x^*–y^* path in $G[D]$, i.e., $G[D]$ is connected.

For any x in V, since M is a dominating set, $x \in N[v^{**}]$ for some $N'(v) \in \mathscr{A}$. Note that v^{**} and v^* are both in $N'(v)$. By Theorem 3, $N_{V-N'(v)}(v^{**}) = N_{V-N'(v)}(v^*)$. In the case of $x \notin N'(v)$, $x \in N[v^{**}]$ implies $x \in N[v^*]$, i.e., D dominates x. In the case of $x \in N'(v)$, $N_{V-N'(v)}(v^*) = N_{V-N'(v)}(x)$. Since $G[D]$ is connected and $|D| \geqslant 2$, v^* is adjacent to some $y^* \in D - N'(v)$. Then x is also adjacent to y^*, i.e., D dominates x. In any case, D is a dominating set. Therefore, D is a minimum weighted connected dominating set of G. □

By Lemma 4 and Theorem 6, we can design an efficient algorithm for the weighted connected domination problem in distance-hereditary graphs. To implement the algorithm efficiently, we do not actually find the set $\mathscr{A}$. Instead, we perform the following step for each $2 \leqslant i \leqslant t$. Sort the vertices in L_i such that

$$|N'(x_1)| \leqslant |N'(x_2)| \leqslant \cdots \leqslant |N'(x_j)|.$$

We then process $N'(x_k)$ for k from 1 to j. At iteration k, if $N'(x_k) \cap D = \emptyset$, then $N'(x_k)$ is in $\mathscr{A}$ and we choose a vertex of minimum weight to put it into D; otherwise, $N'(x_k) \notin \mathscr{A}$ and we do nothing.

Algorithm WCD-dh. Find a minimum weighted connected dominating set of a connected distance-hereditary graph.
Input: A connected distance-hereditary graph $G = (V, E)$ and a weight $w(v)$ of real number for each $v \in V$.
Output: A minimum weighted connected dominating set D of graph G.
begin
$D \longleftarrow \emptyset$;
let $V' = \{v \in V : w(v) < 0\}$;
$w(v) \longleftarrow 0$ for each $v \in V'$;
let u be a vertex of minimum weight in V;
determine the hanging $h_u = (L_0, L_1, \ldots, L_t)$ of G at u;

for $i = 2$ **to** t **do**
begin
  let $L_i = \{x_1, \ldots, x_j\}$;
  sort L_i such that $|N'(x_{i_1})| \leqslant |N'(x_{i_2})| \leqslant \cdots \leqslant |N'(x_{i_j})|$;
  for $k = 1$ **to** j **do**
    if $N'(x_{i_k}) \cap D = \emptyset$ **then** $D \longleftarrow D \cup \{y\}$ where y is a vertex of minimum weight in $N'(x_{i_k})$
end
if not ($L_1 \subseteq N[D]$ and $G[D]$ is connected) **then** $D \longleftarrow D \cup \{u\}$;
for $v \in V$ that dominates V: **if** $w(v) < w(D)$ **then** $D \longleftarrow \{v\}$;
$D \longleftarrow D \cup V'$
end

Theorem 7. *Algorithm WCD-dh gives a minimum weighted connected dominating set of a connected distance-hereditary graph in linear time.*

Proof. The correctness of the algorithm follows from Lemma 4 and Theorem 6. For each i, we can sort L_i by using a bucket sort. So the algorithm is linear to $|V| + |E|$. □

3. Vertex-weighted Steiner tree

This section presents a linear-time algorithm for finding a minimum vertex-weighted Steiner tree with respect to a target set $T \subseteq V$ in a connected distance-hereditary graph $G = (V,E)$ with a non-negative weight $w(v)$ for each $v \in V$. In this section $h_u = \{L_0, \ldots, L_t\}$ denotes a hanging of G at a vertex u in the target set T. The key to our algorithm for the vertex-weighted Steiner tree problem is the following theorem. The theorem is similar to Theorem 6, but even simpler.

Theorem 8. *Suppose $U = \{x \in V : x$ lies on a shortest u–v path for some v in $T\}$ and $\mathscr{B} = \{N'(x) : x \in U \cap L_i$ has a minimal neighborhood in L_{i-1} relative to U and $N'(x) \cap T = \emptyset\}$. Then the set S formed by choosing a vertex x^* of minimum weight in each $N'(x) \in \mathscr{B}$ is a minimum vertex-weighted Steiner set with respect to T in G.*

Proof. We first note that for each u–v path P with $v \in L_i$, P is a shortest path if and only if P is of the form

$$u = v_0, v_1, \ldots, v_i = v,$$

where $v_j \in L_j$ for $0 \leqslant j \leqslant i$. Therefore $N'(x) \subseteq U$ for each $x \in U$. Consequently, $S \cup T \subseteq U$. For each $x \in S \cup T$, $x \in U$. Either $N'(x) \in \mathscr{B}$ or $N'(x) \cap T \neq \emptyset$ or $N'(x) \supseteq N'(y)$ for some $N'(y) \in \mathscr{B}$. Then there exists some vertex $z \in N'(x) \cap (S \cup T)$. The same argument can be applied repeatedly to show that there exists a shortest x–u path using vertices only in $S \cup T$. This proves that $G[S \cup T]$ is connected.

Next, suppose M is a minimum vertex-weighted Steiner set with respect to T in G. For each $N'(x) \in \mathscr{B}$, by the definition of U, x lies on a shortest u–v path

$$u = v_0, v_1, \ldots, x = v_i, \ldots, v_j = v$$

for some v in T, where each $v_k \in L_k$. Since $G[M \cup T]$ is connected, there is a u–v path

$$u = u_0, u_1, \ldots, u_r, \ldots, u_s = v$$

in $G[M \cup T]$, where $u_{r-1} \in L_{i-1}$, $u_r \in L_i$, and $u_{r+1}, \ldots, u_s$ are in L_k's with $k \geqslant i$. By Theorem 1 (3), $N'(x) = N'(u_r)$. Therefore $N'(x)$ contains $u_{r-1} \in M \cup T$. Since $T \cap N'(x) = \emptyset$, $M \cap N'(x) \neq \emptyset$. Since any two sets in $\mathscr{B}$ are disjoint and x^* is a vertex of minimum vertex-weight in $N'(x)$, we conclude that $w(M) \geqslant w(S)$. This proves that S is a minimum vertex-weighted Steiner set with respect to T in G. □

Theorem 8 provides the basic idea for designing a good algorithm for the vertex-weighted Steiner tree problem. Similar to the implementation of WCD-dh, we do not actually find U and $\mathscr{B}$. Instead, at any L_i we sort $|N'(x)|$ for all $x \in S \cup T$ to find $\mathscr{B}$.

Algorithm WST-dh. Find a minimum vertex-weighted Steiner set of a distance-hereditary graph with non-negative weights on its vertices.
Input: A connected distance-hereditary graph $G = (V, E)$ with non-negative weight $w(v)$ for each $v \in V$ and a subset $T \subseteq V$.
Output: A subset $S \subseteq V - T$ of minimum weight such that $G[S \cup T]$ is connected.

```
begin
   S ⟵ ∅;
   let u be a vertex of T;
   determine the hanging h_u = (L_0, L_1, ..., L_t) of G at u;
   for i = t to 2 step -1 do
   begin
      let (S ∪ T) ∩ L_i = {x_1, ..., x_p};
      if p ≠ 0 then
      begin
        sort x_1, x_2, ..., x_p such that |N'(x_{j_1})| ≤ ... ≤ |N'(x_{j_p})|;
        for k = 1 to p do
          if N'(x_{j_k}) has no vertices in S ∪ T
          then S ⟵ S ∪ {y} where y is a vertex of minimum weight in N'(x_{j_k})
      end
   end
end
```

Theorem 9. *Algorithm WST-dh solves the vertex-weighted Steiner tree problem for a connected distance-hereditary graph with a non-negative weight function in linear time.*

Proof. Similar to the proof of Theorem 7. □

The vertex-weighted Steiner tree problem with arbitrary real weights on its vertices remains open.

Acknowledgements

The authors thank referees for many constructive suggestions on the revision of this paper.

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