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Knowledge sharing in communities of practice: A game theoretic analysis

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ABSTRACT

This research applies game theory to analyze the incentives of knowledge-sharing activities in various types of communities of practice (COPs), characterized by individual profiles and decision structures. Indeed, individual decision making results in the under-provision of knowledge; however, the benefit of knowledge sharing may be raised by IT investment and suitable incentive mechanisms we study here. In general conditions, improving communication and collaboration technologies should be prior to developing data mining technologies. However, when the number of community members is sufficiently small and the heterogeneity of the expected value of knowledge among community members is sufficiently large, developing data mining technologies should be considered more important than the other if most community members are low-type ones. On the other hand, based on a screening technique, we find that the benefit of knowledge sharing in the incomplete information setting can be the same as that in the complete information setting if the cost of more efficient community member is smaller than that of less efficient one.

1. Introduction

Due to rapid growth in information technology and increase in knowledge-based work the concept of knowledge management has gained momentum in recent years. To help employees share and integrate their knowledge, a large number of companies put themselves to the great expense of hiring consultants to set up information systems, gathering and retrieving their useful knowledge. According to a survey conducted by McKinsey & Company, many of executives from 40 companies in Europe, Japan and the US think that knowledge sharing is only about information technology and how to procure the right knowledge management tool (Kluge et al., 2001). As summarized in Shin (2004), costs for establishing KMS range from hundreds of thousands of dollars to many millions. However, industry reports and surveys reveal that the performance of KM in many organizations is not obvious even if they have spent substantial budgets on developing tools for storage and distribution of explicit knowledge (Alavi and Leidner, 1999). Some researchers anticipate that the failure rate of knowledge management projects is about 50%. But Daniel Morehead, director of organizational research at British Telecommunications PLC in Reston, Va., considers that the rate is closer to 70% (Ambrosio, 2000). Nevo and Chan (2007) empirically explore the roles and scope of KMS in organizations. One of the panel members in their research stated that "there has to be some incentive for employees to collaborate and contribute to the quality of information and to go more into the depth and breadth of contents in the system".

Prior empirical study supports the viewpoint that correct incentives in a compensation system can produce significant positive results (Fey and Furu, 2008). McKinsey spends at least 10% of its annual revenues on managing and sharing knowledge, creating an incentive program by establishing special promotion tracks for their employees to ensure that theses experts want to develop and share their expertise (Kluge et al., 2001). Today, McKinsey is an eight-time Global MAKE (Most Admired Knowledge Enterprises) Winner, including two-time overall Global MAKE Winner; in addition, it is also the first consulting firm to be recognized as the overall Global MAKE Winner (Global MAKE Report, 2008). Other successful Global MAKE Winners, such as Ernst & Young and IBM, also reward people for knowledge sharing through resembling mechanisms (Bartol and Srivastava, 2002; Global MAKE Report, 2008). All of these practical examples and empirical study show the importance of incentive programs on knowledge sharing. Without appropriate incentive programs, most knowledge management projects would not achieve their stated goals and objectives.

Because tacit knowledge usually takes place through the means of social and interpersonal interactions, organizations need other more effective social approaches to facilitate the sharing of tacit understanding (Turban et al., 2006). Communities of practice (COPs), types of practice-related social networks, are tightly knit groups where people can work together to share thoughts, find solutions, and pursue

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Decision / Profile (Type)	Decentralized (Individual)	Centralized (Social planner)
Observable	(I)	(II)
Unobservable	(III)	(IV)

Fig. 1. Analytic structure in knowledge sharing.

innovation (Wang et al., 2008). The practice approach is mainly for the sharing of tacit knowledge. Before building up knowledge-sharing environments for COPs, there are two critical factors that should be considered. One is whether each community member's profile can be transparent for all, whereas the other is whether the decision structure is centralized (i.e., a social planner) or decentralized (i.e., individual decision making). A transparent community member's profile means that each community member can observe all other members' background information such as work experience, education levels, and individual beliefs, whereas a centralized decision structure means that each member's effort on knowledge sharing could be allocated by a top supervisor. Whether the decision structure is centralized or decentralized would significantly influence the efficacy of knowledge sharing. For example, the phenomenon known as ''Free-riding'' is usually the result of selfish behavior. When exerting effort without monitoring or negotiating, each community member may choose to stop exerting effort and enjoy the fruits of others' work.

The term ''Information'' studied in this paper refers to each community member's profile. Depending on whether the decision structure is centralized or decentralized and whether each community member's profile is observable, there are four representative forms of COPs in knowledge sharing, as illustrated in Fig. 1: (I), (II), (III), and (IV). Furthermore, whether the decision structure is centralized or decentralized and whether each member's profile is observable in COPs are associated with the property of community structure. For example, a tight community can offer an environment whose information is more easily observed than a loose community. In addition, information technology also plays an important role in the activities of knowledge sharing. For example, organizations can improve their level of communication and collaboration technologies to disseminate each community member's information to all and help them behave cooperatively. To date, E-mail, instant messaging, corporate intranets, and other Web-based tools can provide communication capabilities, whereas collaboration technologies provide the means to perform group work (Turban et al., 2006). On the other hand, community members' profiles can be screened and filtered by further data analysis such as data mining techniques. By collecting a large amount of data associated with community members, each member's profile can be discovered from personal work and education history, and current project and learning records.

1.1. Problems and motivation

In order to ensure the success of KM efforts, all the KMS users should each be in at least one COP (Turban et al., 2006). In multinational enterprises, such as Microsoft and Google, employees' actions are not easily observed by others due to geographical limits. In addition, when organizations have great personnel fluidity within their divisions, they encounter the problem of how to identify their employees' profiles. When information cannot be correctly observed by community members and organizations, the performance of knowledge sharing would decrease. The reason for the low performance of knowledge sharing is as follows. First, when individual actions cannot be coordinated, community members may choose to stay idle and enjoy the fruits of others' contributions. Second, when profile information is not observable, the real benefit of knowledge sharing cannot be identified so as to discourage community members' motivations for sharing. Since low quality information would decrease the benefit of knowledge sharing, organizations should propose appropriate mechanisms to improve the performance of knowledge sharing according to available structures.

For instance, multinational enterprises often invest in communication and collaboration technologies to eliminate the limits of geography, whereas establishing human resources databases can help disseminate information regarding profiles. Another solution is to utilize incentive mechanisms to encourage organization members to share their tacit knowledge in COPs. However, these solutions would significantly increase enterprises' operating costs. Therefore, the executives of enterprises who initiate knowledge management should examine their cost structures and choose the best approaches to their solutions in order to correct the defects of incomplete information and eliminate the effect of selfish behavior. In this research, we aim to address the following two questions. First, what is the impact of information on a knowledge-sharing activity in COPs? Furthermore, when budget limitations exist, how do the executives of enterprises prioritize the relative importance of the IT investment influencing knowledge sharing? Second, instead of IT investment, may we establish an incentive mechanism to guide community members to contribute their knowledge when information is not observable?

1.2. Findings and contribution

Investigating the four representative forms of COPs, we find that the benefit of knowledge sharing in COPs would be the highest when the decision structure is centralized and all information is observable. In form (I), the community members with smaller knowledge-cost ratios will not exert the anticipated level of effort on the activities of knowledge sharing. The shirking effect is a classical free-riding problem. In order to improve the performance of knowledge sharing, IT investment or incentive mechanisms are necessities. Furthermore, compared with form (I), we find that the free-riding problem still exists in form (III), but the community members with smaller knowledge-cost ratios may exert more effort on knowledge sharing. On the other hand, comparing form (I) with form (III), we find that the benefit of knowledge sharing in form (I) is higher than that in form (III); however, the benefit of knowledge sharing in form (III) can increase with the community scale, whereas the outcome of form (I) will not be affected by the number of members in COPs. In this research, we propose a useful suggestion for the executives of enterprises when budget limitations exist. In general, coordinating individual actions is more important than distributing individual profiles. However, we also point out that they should focus on community members' profiles when certain conditions hold.

This research also compares the performance of knowledge sharing in the four representative forms of COPs. Actually, COPs with complete information are more efficient than those with incomplete information; however, community members in COPs with incomplete information can behave cooperatively like community members in COPs with complete information when incentive mechanisms are in place (Malone, 2004). Our economic solutions can at least improve the benefit of knowledge sharing from the level of form (I) and form (III) to the level of form (II) and form (IV), respectively. Based on our research results, we may partly explain why Wikipedia is so successful. On the other hand, the economic mechanisms studied in this research can be regarded as a prototype which can be adopted by enterprises to boost organization members' motivations for sharing knowledge. For example, CRAMSTER.COM[1] is a global study community for homework help in mathematically-based subjects and provides various types of study material created by the company or contributed by its members. The incentive-compatible mechanism this community adopts for collecting exercise answers from its community members is as follows. Not only can its free members view step-by-step solutions to all odd textbook problems, but they can also ask or search questions on its answer board. Moreover, all members can get karma points, a virtual currency to be redeemed for rewards, by submitting their own solution to the problems without answers or solve the questions asked by others on the board. This example illustrates the value of our research in practical perspectives. The rest of the paper is organized as follows. The literature review is given in Section 2. Our model and parameters are given in Section 3. Sections 4 and 5 analyze four representative types of COPs in knowledge sharing. Finally, management implications, the limitation of this research, and future directions are given in Sections 6 and 7.

2. Related literature

Management's reminders of the importance of the goals can encourage knowledge sharing in organizations (Marks et al., 2008). However, the quality of provided knowledge is dependent from the willingness to cooperate (Barachini, 2009). Trust in management reduces the fear of losing one's unique value in the knowledge-sharing process and encourages individuals' cooperative behavior (Renzl, 2008). Ba et al. (2001) claim that building KMS from software engineering and user acceptance perspectives is not enough.

2.1. Communities of practice

A practice-related social network is one in which individuals undertake or engage fully in a common task, job, or profession. Learning theorists have argued that knowledge cannot be isolated from practice and developed a view of learning as social participation (Wenger, 1998; Brown and Duguid, 1991). That is, knowledge is inseparable from practice because it is impossible to know without doing (Sierhuis and Clancey, 1997). Moreover, if people share a practice, then they will share know-how, or tacit knowledge (Brown and Duguid, 2001). As information technology developed, Blog has been recognized as a successful community of practice and used by individuals as a discussion space (Yang, 2009). Hara (2007) investigates what roles IT can play in supporting different types of communities of practice and explores the use of IT to achieve work efficiency and to foster professional identities. Turoff and Hiltz (2009) suggest that social tagging and social recommender system features are increasingly important for professional communities. By examining the evolution of the COP concept, Li et al. (2009) argue that improving the understanding of the process of negotiating boundaries of emerging COPs can enhance organization's competitiveness.

2.2. Knowledge sharing and game theory

Recently, game theory has been applied to the issues of knowledge sharing. Statistical data shows that the individual's perceived payoff of sharing knowledge in a group can be characterized by a multi-person game (Samieh and Wahba, 2007). Firms that have a higher degree of knowledge sharing will have a better business performance and a higher level of innovation than firms that do not (Shih et al., 2006a,b). Shih et al. (2006a,b) claim that cooperative learning in a design studio relies not only on information technology increasing communication efficiency, but also on how studio participants are motivated to cooperate. Cai and Kock (2009) study the strategic interaction between players as they decide whether and how much to collaborate with each other. They show that the social punishment should be large enough to enforce a full cooperation in symmetric discrete-strategy e-collaboration games. Jolly and Wakeland (2008) examine the result of interactions among individuals in an organization with different preferences regarding knowledge sharing, arguing that organizations should actively encourage knowledge sharing. For knowledge-sharing behavior to be engendered and sustained effectively among participants accessing a common knowledge base, Chua (2003) points out that participation rate has to exceed a minimum threshold. Samaddar and Kadiyala (2006) use the game theoretic framework to model the collaboration for knowledge creation as a Stackelberg leader–follower game. They identify the importance of maintaining an optimal ratio between the leader's and follower's marginal gains for the formation and continuation of the collaboration. Bandyopadhyay and Pathak (2007) model the interaction between two employees serving in different firms, who have to share their knowledge to work effectively as a team. The result shows that the top management should enforce cooperation between the employees so that better payoffs can be achieved when the degree of complementarity of knowledge between the employees is high enough. Song et al. (2008) analyze the sharing of mutually-complementary knowledge resources across organizations in a competitive strategic alliance and examine the conditions of maintaining their cooperation. Their conclusion indicates that the shared knowledge resource makes every alliance member's expected income increase so that this becomes the source of the power in establishing an alliance.

2.3. Incentive mechanisms for knowledge sharing

Because sharing advantage added knowledge significantly contributing to a firm's competitive advantage may diminish the individual's competitiveness, rewards are necessary for the individuals to share (Ho et al., 2006). Effective incentive design is affected by asymmetric information and the variability of the intangible nature of knowledge itself (Nan, 2008). Yang and Wu (2008) find that rewarding each knowledge-sharing action is more effective than the periodic organizational incentives to encourage members' knowledge-sharing behaviors. Wang et al. (2009) confirm that "Ratchet Effects" exist in knowledge-sharing activities and waken the validity of incentive mecha-

[1] http://www.cramster.com.

nisms, suggesting that a more objective evaluation standard and long-term incentive contract can eliminate such effects. Lee and Ahn (2007) develop and analyze reward systems for intra-organizational knowledge sharing. By comparing group-based reward with individual-based reward, they find that the former is not only less efficient than the latter, but the former also subjects to a potential productivity problem, in which workers with more productive knowledge do not exert effort in knowledge sharing.

The goal of this research is to better understand knowledge-sharing activities by investigating how organization employees in COPs spend their time and effort on knowledge sharing. Furthermore, we design appropriate incentive mechanisms to improve the performance of knowledge sharing and compare the performances of knowledge sharing among different forms to offer useful managerial insights for the executives of enterprises.

3. The model

Consider a knowledge aggregation and sharing model in a community of practice for n participants. There are four settings which are discussed in this research: form (I), (II), (III), and (IV), as shown in Fig. 1. These community members want to aggregate knowledge and share the benefit, such as homework answers, creative ideas, or technology innovation. In Lin et al. (2005), the expected value of a participant's knowledge is denoted as either of a high type, K_H, or a low type, K_L, where $K_H > K_L > 0$ (see Table 1 for a complete list of notations). Here, we adopt the same notations to denote knowledge profiles. The number of high-type community members is denoted as η. Each community member in the knowledge-sharing group decides on x_i, the quantity of effort she exerts, where $x_i \geqslant 0$. Thus, individual knowledge contribution in the community can be assumed to be $K_i x_i$, where the notation catches the key feature that a high-type community member could make a more valuable contribution than a low-type one when their efforts are the same. Then, all contributions can be aggregated into a KMS. Intuitively, the amount of knowledge aggregation can be defined as $Q = \sum_i K_i x_i$, where $K_i \in \{K_L, K_H\}$. Although there have been some concerns about the difficulty in identifying the contribution of knowledge sharing to the community benefit, attempts have been made to formulate the benefit in knowledge sharing (Chang and Ahn, 2005; Samaddar and Kadiyala, 2006; Lee and Ahn, 2007). In this research, the performance of knowledge sharing is measured by the amount of knowledge aggregation directly. Actually, the amount of knowledge aggregation could be defined by other more complicated forms, such as multiplication forms. However, multiplication forms are intractable in the same approach. Because we aim to derive analytical solutions to investigate the causal relationship between information and knowledge sharing, we adopt the additive form in this research.

Finally, the KMS would process and transform all members' contributions into various forms for sharing. Consequently, all members in the community can access all others' knowledge by proper media such as browsers. Therefore, the benefit of knowledge sharing a community member receives can be defined as $f(Q)$, where the benefit function $f(Q) \geqslant 0$is assumed to be continuously differentiable, increasing, and concave in its argument. In addition, we define c_H and c_L as the cost to a high-type community member and a low-type one, respectively. The payoff a community member receives can be written as $f(Q) - c_i x_i$, where $c_i \in \{c_L, c_H\}$. For the sake of convenience, we define some notations to simplify mathematical expressions. $G(\cdot)$ is denoted as the inverse function of the derivate of $f(Q)$. $KC_{\max}$ and $KC_{\min}$ are denoted as the highest and the lowest knowledge-cost ratios, respectively. Formally, $KC_{\max} = \max(K_H/c_H, K_L/c_L)$ and $KC_{\min} = Min(K_H/c_H, K_L/c_L)$. If $K_i/c_i > K_j/c_j$ holds, we call it that the type i community member is more efficient than the type j one.

4. COPs with complete type information

In this section, we consider a knowledge-sharing setting where individual profiles are available. Under the setting, we first examine the performance of knowledge sharing in form (I) and form (II). Finally, we design an incentive mechanism to enforce the amount of knowledge aggregation in form (I) to become that in form (II).

4.1. Knowledge contributions with individual decision making: Form (I)

When exerting effort without the social planner's coordination, each community member would choose x_i to maximize individual benefit of knowledge sharing; that is, $\underset{x_i}{Max} f(Q) - c_i x_i$. Nash equilibrium is given by simultaneously solving $\partial f(Q)/\partial x_i - c_i \leqslant 0$ for each community member. Suppose that the number of community members with the highest knowledge-cost ratio is m. Then, there are infinite

Table 1
Notation.

Notation	Description
x_i	The quantity of effort member i exerts in a COP
K_H	The expected value of a member's knowledge whose type is high
K_L	The expected value of a member's knowledge whose type is low
c_H	A high-type member's unit cost of knowledge contribution
c_L	A low-type member's unit cost of knowledge contribution
Q	The amount of knowledge aggregation
$f(\cdot)$	The benefit of knowledge sharing
$G(\cdot)$	The inverse function of the derivate of $f(\cdot)$
n	The total number of community members in a COP
m	The number of community members with the highest knowledge-cost ratio in a COP
η	The number of high-type community members in a COP
p_H	The amount of compensation paid to a high-type member
p_L	The amount of compensation paid to a low-type member
θ	The probability that a community member owns the highest knowledge-cost ratio
$KC_{\max}$	The highest knowledge-cost ratio
$KC_{\min}$	The smallest knowledge-cost ratio

equilibria in the activity of knowledge sharing as long as $m \geqslant 2$. These equilibria can be derived from $\sum_{1 \leqslant i \leqslant n, K_i/c_i = KC_{\max}} K_i x_i = G\left(\frac{1}{KC_{\max}}\right)$. In order to examine the impact of information and decision structures on the performance of knowledge sharing, we focus on symmetric equilibrium and the general case $K_H/c_H \neq K_L/c_L$ in this research. The analysis of the special case $K_H/c_H = K_L/c_L$ and the multiple equilibrium solutions can be found in Appendices A,B,C,D,E,F,G.

Proposition 1.

1. *In form (I), the community members with smaller knowledge-cost ratios always free ride on the others with the highest knowledge-cost ratio. Formally,* $x_i^* = \frac{1}{mK_i} G\left(\frac{1}{KC_{\max}}\right)$ *where* $K_i/c_i = KC_{max}$ *and* $x_j^* = 0$ *where* $K_j/c_j = KC_{min}$*. All the proofs can be found in Appendices A,B,C,D,E,F,G.*
2. *For any community size, the amount of knowledge aggregation* Q_I^* *is fixed. Formally,* $Q_I^* = G\left(\frac{1}{KC_{\max}}\right)$.

This result is similar to the work of Bhattacharya et al. (1992) which shows that the most intelligent agent's knowledge is the only useful input for efficient development effort when individual knowledge levels are revealed. Proposition 1 shows that the free-riding problem always exists in general cases when each community member exerts effort without a social planner. Furthermore, the benefit of knowledge sharing cannot be improved by increasing the number of community members. If the probability that a community member owns the highest knowledge-cost ratio is θ, the expected amount of knowledge aggregation is calculated as

$$E\left[Q_I^*\right] = \left(1 - (1-\theta)^n\right) G\left(\frac{1}{KC_{\max}}\right) + (1-\theta)^n G\left(\frac{1}{KC_{\min}}\right). \tag{4.1}$$

4.2. Knowledge contributions with the social planner's coordination: Form (II)

Although individual decision making results in under-provision of individual knowledge, it may be remedied by offering better communication technologies and/or incentive mechanisms. In the following, we analyze the performance of knowledge sharing in form (II) where a social planner coordinates each community member's action. Under social planning, the performance of knowledge sharing is efficient when all members' efforts can be allocated by a supervisor. The knowledge-sharing problem can be formulated as

$$\underset{x_1, x_2, \ldots, x_n}{\text{Maximize}} \quad W = \sum_{i=1}^{n} f(Q) - c_i x_i. \tag{4.2}$$

Solving $\partial W / \partial x_i \leqslant 0, \ \forall i$ simultaneously, we have the following results.

Proposition 2.

1. *In form (II), the activity of knowledge sharing would be supported by only the community members with the highest knowledge-cost ratio. Formally,* $x_i^* = \frac{1}{mK_i} G\left(\frac{1}{n \cdot KC_{\max}}\right)$ *where* $K_i/c_i = KC_{max}$ *and* $x_j^* = 0$ *where* $K_j/c_j = KC_{min}$.
2. *The amount of knowledge aggregation* Q_{II}^* *increases with community size. Formally,* $Q_{II}^* = G\left(\frac{1}{n \cdot KC_{\max}}\right)$.

Proposition 2 states that the most efficient knowledge contributors would be the only knowledge source. Thus, the expected amount of knowledge aggregation can be calculated as

$$E\left[Q_{II}^*\right] = \left(1 - (1-\theta)^n\right) G\left(\frac{1}{n \cdot KC_{\max}}\right) + (1-\theta)^n G\left(\frac{1}{n \cdot KC_{\min}}\right). \tag{4.3}$$

An empirical study suggests that practitioners in some kinds of organizations should not neglect supervisory control mechanisms for motivating knowledge sharing (King and Marks, 2008).

4.3. Incentive mechanism

Here, we design an incentive mechanism to induce efficient individual knowledge sharing in COPs. Under the incentive mechanism, each community member would choose x_i to maximize individual payoff of knowledge sharing. Formally, solving $\underset{x_i}{Max} f(Q) - c_i x_i + p_i x_i$ yields $x_i^*(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$, where $p_i x_i$ is compensation for community member i.

Proposition 3. *In form (I), if all community members are paid* $p_i x_i$ *based on their types, the benefit of knowledge sharing in form (I) is the same as that in form (II). Formally,* $p_i = (n-1)c_i/n$ *where* $K_i/c_i = KC_{max}$ *and* $p_j = 0$ *where* $K_j/c_j = KC_{min}$.

From Proposition 3 we find that the subsidy is associated with the cost of exerting effort and the number of community members. Actually, when the number of community members increases, each community member finds it hard to gain more benefit by exerting individual effort. Therefore, the amount of subsidy increases with the number of community members and is bounded by individual cost. In general, most businesses either adopt reward mechanisms, such as rewards based on relative importance of contributions to a knowledge-sharing activity, or consider knowledge-sharing records when evaluating employees' performances at the end of the year.

5. COPs with incomplete type information

In this section, we consider a knowledge-sharing setting where type information is not observable. Each community member only has a correct expectation of the value of her own knowledge, which the others do not know, and knows $Prob(K_i = K_H) = \theta$. We follow the same approach as Section 4 to examine the benefit of knowledge sharing and design incentive mechanisms to boost the expected amount of

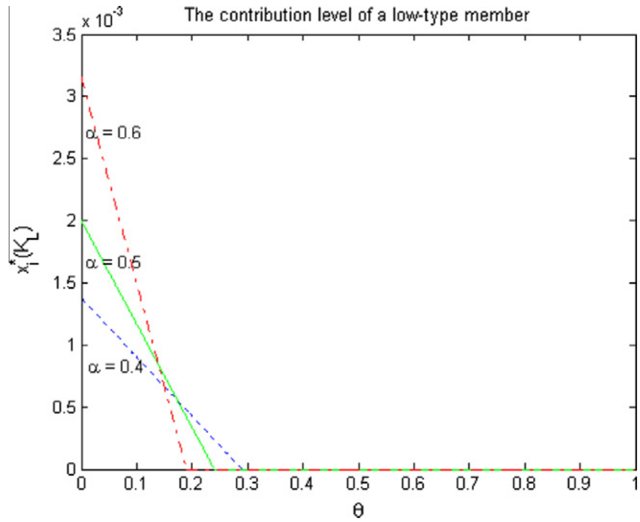

Fig. 2. The contribution level of a low-type member.

knowledge aggregation. In general, we may improve the loss resulting from individual decision making. However, under certain conditions, the benefit of knowledge sharing in form (III) can be the same as that in form (II).

5.1. Knowledge contributions with individual decision making: Form (III)

We first examine form (III) where type information is not available and each community member exerts effort without the social planner's coordination. For simplicity, we assume that all community members' types are independent and denote $x_i^*(K_H)$ and $x_i^*(K_L)$ as community member *i*'s effort based on individual knowledge type, which implies that each member's expected level of effort is given by $\theta K_H x_j^*(K_H) + (1-\theta)K_L x_j^*(K_L)$. Thus, based on individual knowledge type, community member *i* chooses $x_i^*(K_i)$ to maximize her payoff as follows:

$$\max_{x_i(K_i)} U_i(K_i) = f\left(K_i x_i + \sum_{j\neq i} \theta K_H x_j^*(K_H) + (1-\theta)K_L x_j^*(K_L)\right) - c_i x_i. \tag{5.1}$$

We first consider the case where a high-type community member works more efficiently than a low-type one, i.e., $K_H/c_H > K_L/c_L$. In this case, high-type community members always contribute their knowledge. On the other hand, assuming that all low-type community members do not contribute at all, i.e., $x_i^*(K_L) = 0$, we could derive the indifference point $\hat{\theta}$ where $\hat{\theta} = G\left(\frac{1}{KC_{\min}}\right)\Big/\left((n-1)\left(G\left(\frac{1}{KC_{\max}}\right) - G\left(\frac{1}{KC_{\min}}\right)\right)\right)$. Consequently, we know that low-type community members prefer to contribute their effort rather than shrink when most community members are low-type ones, i.e., $\theta < \hat{\theta}$. In contrast, free-riding effect still occurs when $\theta \geqslant \hat{\theta}$ holds. An illustrated example[2] can be found in Figs. 2 and 3. By the same approach, we can derive each community member's effort in the case where $K_H/c_H < K_L/c_L$ holds.

Proposition 4. *In form (III), each community member's effort is given by*

$$x_i^*(K_H) = \begin{cases} \frac{1}{nK_H}\left(G\left(\frac{c_H}{K_H}\right) + (n-1)(1-\theta)\left(G\left(\frac{c_H}{K_H}\right) - G\left(\frac{c_L}{K_L}\right)\right)\right), & \text{if } \sigma < \hat{\theta}, \\ G\left(\frac{c_H}{K_H}\right)((1+(n-1)\theta)K_H)^{-1}, & \text{if } \sigma > \hat{\theta} \text{ and } \frac{K_H}{c_H} > \frac{K_L}{c_L}, \\ 0, & \text{if } \sigma > \hat{\theta} \text{ and } \frac{K_H}{c_H} < \frac{K_L}{c_L}, \end{cases}$$

$$x_i^*(K_L) = \begin{cases} \frac{1}{nK_L}\left(G\left(\frac{c_L}{K_L}\right) - (n-1)\theta\left(G\left(\frac{c_H}{K_H}\right) - G\left(\frac{c_L}{K_L}\right)\right)\right), & \text{if } \sigma < \hat{\theta}, \\ 0, & \text{if } \sigma > \hat{\theta} \text{ and } \frac{K_H}{c_H} > \frac{K_L}{c_L}, \\ G\left(\frac{c_L}{K_L}\right)((1+(n-1)(1-\theta))K_L)^{-1}, & \text{if } \sigma > \hat{\theta} \text{ and } \frac{K_H}{c_H} < \frac{K_L}{c_L}, \end{cases}$$

where $\sigma = \begin{cases} \theta, & \text{if } K_H/c_H > K_L/c_L, \\ 1-\theta, & \text{if } K_H/c_H < K_L/c_L. \end{cases}$

Furthermore, the expected amount of knowledge aggregation in form (III) is given by $E[Q_{III}^*]$, as shown in Fig. 4, where

[2] We apply a specific form, $f(x) = x^\alpha$ (where $0 < \alpha < 1$), to examine the behaviors of community members in form (III), the degree of concavity, can be viewed as the efficiency of knowledge sharing, which means that the more α, the more benefit of knowledge sharing. The parameters of Figs. 2–4 are given by $\alpha = \{0.4, 0.5, 0.6\}$, $n = 50$, $c_H = 6$, $K_H = 12.5$, $c_L = 5$, and $K_L = 10$.

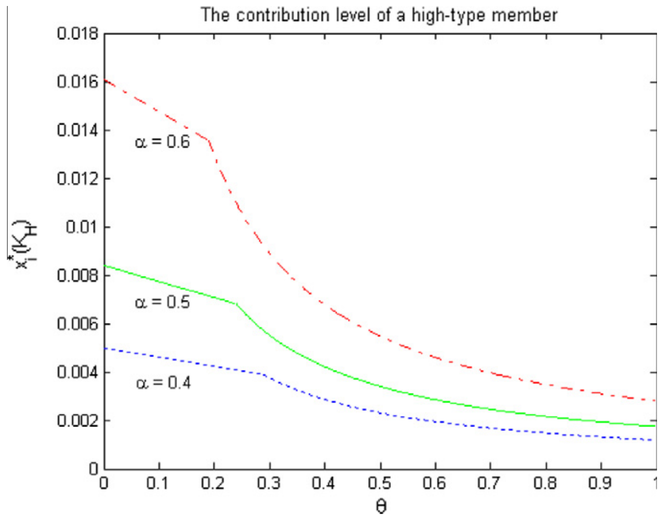

Fig. 3. The contribution level of a high-type member.

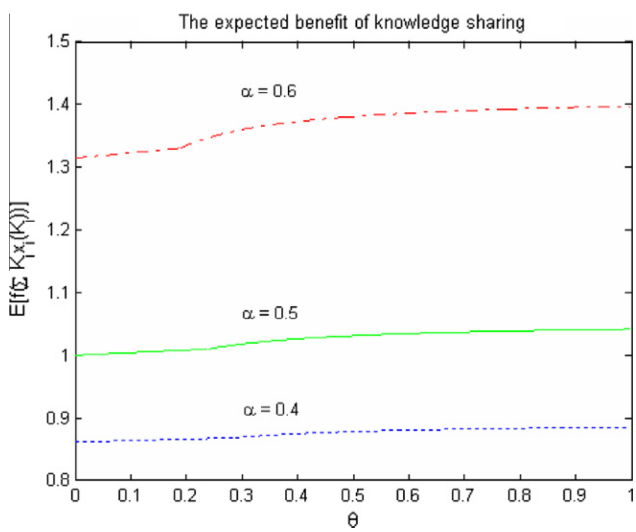

Fig. 4. The expected benefit of knowledge sharing.

Table 2
The variation of critical mass dependent with different parameters.

Critical mass	Parameters		
	$\frac{1}{KC_{\min}}$	$\frac{1}{KC_{\max}}$	σ
$\tilde{n}$	+	–	–

$$E[Q_{III}^*] = \begin{cases} \theta G(c_H/K_H) + (1-\theta)G(c_L/K_L), & \text{if } \sigma < \hat{\theta}, \\ \frac{n\sigma}{1+(n-1)\sigma} G\left(\frac{1}{KC_{\max}}\right), & \text{if } \sigma \geqslant \hat{\theta}. \end{cases} \quad (5.2)$$

From (5.2), we find that the community members with smaller knowledge-cost ratios prefer to shrink rather than exert effort when the expected amount of knowledge aggregation is greater than $\frac{n}{n-1}G\left(\frac{1}{KC_{\min}}\right)$. Subsequently, because $\hat{\theta}$ changes with n, critical mass $\tilde{n} = \left\{G\left(\frac{1}{KC_{\min}}\right)\sigma^{-1} \Big/ \left(G\left(\frac{1}{KC_{\max}}\right) - G\left(\frac{1}{KC_{\min}}\right)\right)\right\} + 1$ can be derived from $\hat{\theta}\left(1-\hat{\theta}\right)$ immediately when $K_H/c_H > K_L/c_L (K_H/c_H < K_L/c_L)$ holds. As a result, given the value of θ, the expected amount of knowledge aggregation is a linear combination in a small-scale community ($n < \tilde{n}$), which results from the fact that the community members with smaller knowledge-cost ratios believe that their contribution can enhance individual payoff. However, when the scale of the community is sufficiently large ($n > \tilde{n}$), such as Wikipedia, $E[Q_{III}^*]$ is an increasing function which grows with the number of community members and is bounded by $G(1/KC_{\max})$ due to a decrease in the growth rate. The relation between critical mass and the other parameters is listed in Table 2.

Proposition 5.

1. *In form (III), the expected benefit of knowledge sharing is fixed when the community scale is sufficiently small. However, it grows with the number of community members when the community scale is beyond critical mass. Formally,* $\partial E[Q^*_{III}]/\partial n = 0$ *when* $n < \tilde{n}$ and $\partial E[Q^*_{III}]/\partial n > 0$ *when* $n > \tilde{n}$.
2. *Comparing with form (I), incomplete type information results in the community members with smaller knowledge-cost ratios in form (III) have more incentive to share knowledge.*

Interestingly, even if people never know each other, they can still aggregate a great deal of knowledge in COPs. Unlike form (I), because each community member in form (III) makes individual decisions under incomplete type information, the free-riding effect may disappear when $\sigma < \hat{\theta}$ holds.

5.2. Knowledge contributions with the social planner's coordination: Form (IV)

Subsequently, we consider form (IV) where type information is not available but there is a social planner coordinating each community member's action. In this case, because individual profiles are not available, each community member's effort is given by

$$\underset{x_1,x_2,\ldots,x_n}{\text{Maximize}} \quad W = \sum_{i=1}^{n} f\left(\sum_{i=1}^{n}(\theta K_H + (1-\theta)K_L)x_i\right) - (\theta c_H + (1-\theta)c_L)x_i. \tag{5.3}$$

Proposition 6. *In form (IV), each community member's effort is given by* $x_i^* = \frac{1}{n\overline{K}}G\left(\frac{\bar{c}}{n\overline{K}}\right)$ *where* $\overline{K} = \theta K_H + (1-\theta)K_L$ and $\bar{c} = \theta c_H + (1-\theta)c_L$. *Moreover,* $E\left[Q^*_{IV}\right] = G\left(\frac{\bar{c}}{n\overline{K}}\right)$.

With social planning, the community members with the highest knowledge-cost ratio would prefer hiding their abilities because transparent type information results in overwork for them. In contrast, community members with smaller knowledge-cost ratios would prefer to show their hands because they could free ride on the others.

5.3. Incentive mechanism

The simplest incentive mechanism in form (III) is to provide an offer that the subsidy and workload are the same for all community members. Since we provide only one option for community members within the knowledge-sharing group, the minimal subsidy will be the minimum amount of reward such that each community member has motivation to accept the offer.

Proposition 7. *If the workload and reward are given by* $x = \frac{1}{n\overline{K}}G\left(\frac{\bar{c}}{n\overline{K}}\right)$ and $p = \max\left\{0, c_H\bar{x} - f\left(G\left(\frac{\bar{c}}{n\overline{K}}\right)\frac{K_H+(n-1)\overline{K}}{n\overline{K}}\right), c_L\bar{x} - f\left(G\left(\frac{\bar{c}}{n\overline{K}}\right)\frac{K_L+(n-1)\overline{K}}{n\overline{K}}\right)\right\}$, *respectively, all community members accept the offer and the expected benefit of knowledge sharing in form (III) is the same as that in form (IV).*

In fact, under certain conditions, we may refine the offer into a more exquisite one. Since community members' types are unknown in form (III), the best way is to design a payment mechanism, or a contract, such that each member, based on maximizing individual payoff, truthfully reveals her type to achieve the maximal benefit of knowledge sharing. This contract[3] can be described as $[P_H, x_H]$ and $[P_L, x_L]$; that is, each community member feels free to pick one of the two options, and then achieves the stated workload and receives the deserved subsidy. Under this architecture, P_H and P_L are the subsidy paid to the community members who exert x_H and x_L, respectively. Hence, under this mechanism, also known as truth revelation or screening, we must have two incentive-rationality (*IR*) constraints; that is; no matter what contract each community member executes, she earns a nonnegative payoff after ending the activity. We denote these two constraints as (*IRH*) and (*IRL*) where '*H*' and '*L*' represent a member's type. In order to entice each community member to truthfully reveal her type, we must have another two incentive-compatibility (*IC*) constraints; that is, because each member cannot earn more payoff by mimicking the behavior of the other type, they choose workloads based on their individual types. These two constraints are denoted as (*ICH*) and (*ICL*). Thus, this framework can be described as

$$\max_{x_H,x_L} \quad W = nf\left(n\widehat{K}\right) - n\hat{c},$$

subject to

$$f\left(K_H x_H + (n-1)\widehat{K}\right) - c_H x_H + P_H \geqslant f\left(K_H x_L + (n-1)\widehat{K}\right) - c_H x_L + P_L \qquad (ICH),$$

$$f\left(K_L x_L + (n-1)\widehat{K}\right) - c_L x_L + P_L \geqslant f\left(K_L x_H + (n-1)\widehat{K}\right) - c_L x_H + P_H \qquad (ICL),$$

$$f\left(K_H x_H + (n-1)\widehat{K}\right) - c_H x_H + P_H \geqslant 0 \qquad (IRH),$$

[3] In fact, "output-contingent contracts are likely to be imperfect (rather than completely infeasible), but there will remain a role for the repeated-game incentives studied here" (Gibbons, 1992). However, designing the repeated-game incentives with asymmetric information is beyond the scope of this paper, enabling us to omit the issue.

$$f\left(K_L x_L + (n-1)\widehat{K}\right) - c_L x_L + P_L \geqslant 0 \qquad (IRL),$$

where $\widehat{K} = \theta K_H x_H + (1-\theta) K_L x_L$ and $\hat{c} = \theta c_H x_H + (1-\theta) c_L x_L$.

Our approach to this problem is to relax it by deleting all constraints, solving the relaxed problem, and checking whether there exist P_H and P_L to satisfy these omitted incentive constraints. If certain conditions hold, the compensation can be derived from our approach directly. The following proposition shows that there exists a payment mechanism based on the expected number of community members with the highest knowledge-cost ratio to maximize the program.

Proposition 8. *Assume* $0 < \theta < 1$. *Let the contract be:* $\left[P_T = c_T x_T^*, x_T^* = \frac{1}{n\sigma K_T} G\left(\frac{1}{nKC_{\max}}\right)\right]$, $\left[P_{\overline{T}} = f\left(K_T x_T^* + (n-1)\overline{K}\right) - f\left((n-1)\overline{K}\right), x_{\overline{T}}^* = 0\right]$ where $(T, \overline{T}, \sigma) = \begin{cases} (H, L, \theta), & \text{if } K_H/c_H > K_L/c_L \text{ and } c_H \leqslant c_L \\ (L, H, 1-\theta), & \text{if } K_H/c_H < K_L/c_L \text{ and } c_H \geqslant c_L \end{cases}$. Then, all community members in form (III) complete their works based on individual type.

Thus, the expected benefit of knowledge sharing in form (III) under screening is the same as that in form (II). Actually, the contract works well when $c_H \leqslant c_L (c_H \geqslant c_L)$ holds if $K_H/c_H > K_L/c_L$ $(K_H/c_H < K_L/c_L)$; however, because all community members are sensitive to the difference of the options, the subsidy cannot be evaluated directly by our approach mentioned above when $c_H > c_L$ $(c_H < c_L)$ holds. Therefore, this investigation shows that cost factors will dominate community members' behaviors in form (III) if the difference in the benefit of knowledge sharing between these two options is light.

6. Managerial implications

Social planning maximizes public interest; however, it may be infeasible in today's business environment because it violates individual interest. On the other hand, although individual decision making results in under-provision in knowledge sharing, the community members can behave cooperatively if these community members can communicate with each other and make binding commitments (Chen and Komorita, 1994; Brosig et al., 2003; Bischoff, 2007). All of the functions rely on premium IT infrastructure offering a complete information environment in which each community member's action can be coordinated by themselves rather than such a role like a social planner. Thus, one of the popular solutions for improving the performance of knowledge sharing is to invest in IT such as communication and collaboration technologies to construct an environment in which each community's action can be self-coordinated. In addition, KMS can involve data mining techniques to evaluate and disseminate each community member's profile in organizations.

Considering the performance of knowledge aggregation in COPs, we find that $E[Q_{II}^*] > E[Q_I^*]$, $E[Q_{IV}^*] > E[Q_{III}^*]$, $E[Q_{II}^*] > E[Q_{IV}^*]$, and $E[Q_I^*] > E[Q_{III}^*]$. These results identify the impact of information and decision structures on knowledge sharing. By examining the increasing benefit of knowledge sharing resulted from IT investment, executives of enterprises can consider whether they should improve organization environment in attempts to boost the performance of knowledge sharing. Sometimes, because of the limitations of budgets, they may be compelled to make a decision: which investment they should focus onto maximize the benefit of knowledge sharing? In addition to cost consideration, we propose a useful suggestion from the analytical results. In general conditions, improving communication and collaboration technologies should be prior to developing data mining technologies due to $E[Q_{IV}^*] > E[Q_I^*]$. However, when the number of community members is sufficiently small and the difference in the expected value of knowledge between high-type and low-type ones are sufficiently large, developing data mining technologies may be prior to improving communication and collaboration technologies if most community members are low-type ones, as shown in Fig. 5. The parameters of Fig. 5 are given by $\alpha = 0.5$, $n = 25$, $c_H = 2$, $K_H = 200$, $c_L = 3$, $K_L = 1$, and $0 < \theta < 0.1$. The example shows that $E[Q_{IV}^*] < E[Q_I^*]$ when most community members are low-type ones.

Offering suitable rewards to encourage sharing knowledge is also a popular approach. Although selfish behavior and incomplete information results in the difficulty of knowledge sharing in COPs, it still works well when the right incentives are in place. In this research, we design incentive mechanisms to enhance the performance of knowledge sharing without requiring the assistance in IT. Both IT investment and incentive mechanisms raise the operating cost in the activity of knowledge sharing. The executives of enterprises need to consider the

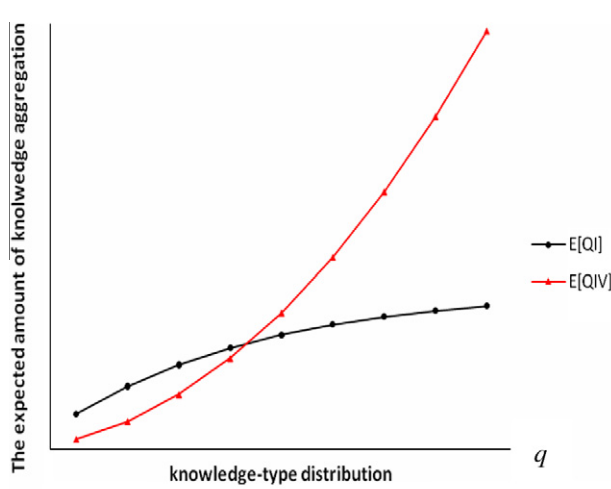

Fig. 5. The comparison between $E[Q_I]$ and $E[Q_{IV}]$.

advantage of these approaches carefully and make final decisions. It is worth noting that the expected benefit of knowledge sharing in form (III) is approximately the same as that in form (I) when the number of community members is sufficiently large. That is, the former still has a chance to reach the best performance as approximately the same as the latter due to the characteristic of the Internet.

7. Concluding remarks

This research builds upon four representative types of community of practice, investigating how community members in COPs face knowledge-sharing activities and how organizations design incentive mechanisms to encourage sharing knowledge. In addition, because a centralized decision structure and complete type information can lead to the best performance of knowledge sharing, the organizations should consider IT investment in attempts to observe and disseminate the information. However, when budget limitation exists, we also offer useful suggestions to support the investment decision made by the executives of enterprises. In this research, we assume that the quality of knowledge is homogeneous; in fact, in real life each community member may decide knowledge quality to adjust his/her effort cost. Therefore, in order to keep the level of knowledge quality, the existence of reputation systems is necessary. On the other hand, we also omit the address of altruistic participants in COPs when our model is established. In fact, the existence of these altruistic participants just speeds the effect of free-riding; therefore, we only consider that all members in the community of practice are self-interested.

When type information is observable, the architecture of the incentive mechanism implies that organizations should offer different incentive programs to members according to their abilities. McKinsey does this by evaluating all consultants based on their roles and increases visibility through various mechanisms such as its Practice Development Network (PDNet), conferences, and word of mouth (Kluge et al., 2001). When type information is not observable, we can raise the performance of knowledge sharing by applying a screening technique into knowledge sharing. For example, in Cramster.com, people can decide to register for free membership or become a premium member by paying a membership fee. Moreover, they can even submit their own knowledge to its answer board to earn virtual currency. Through the self-selected options, the knowledge content in Cramster.com can become ample in a short time.

This research also provides insights for understanding why the number of participants is an eventful key to the performance of form (III). We find that when the number of community members is beyond critical mass, the benefit of knowledge sharing is a bounded function and increases with the community scale. Although the performance of knowledge sharing in form (III) is less than that in form (I), the Internet can make the world a smaller place and reduce the threshold of knowledge sharing, making form (III) become one of the most important knowledge sources. Recently, Yahoo! Answers has amassed millions of resolved questions which are answered by its users without payment. Others, such as Microsoft's Live QnA and Amazon's Askville, also follow the same basic interaction model. The present study shows the importance of the number of participants because it can serve as a basic criterion for measuring whether these emerging social knowledge bases are successful.

In this study, when community members' efforts can be allocated by a supervisor, we find that the community members with the highest knowledge-cost ratio may prefer incomplete type information than complete type information, whereas the others are opposite to the former. Moreover, in form (I), we find that the benefit of knowledge sharing is fixed. That is, if individuals in decentralized organizations contribute their knowledge without external forces, such as monitoring mechanisms or reward policies, the benefit of knowledge sharing will not increase with the community scale. Indeed, social planning may be infeasible in most of business environments; however, those employees serving in some kinds of organizations, such as US federal agencies or military units, may be used to supervisory control mechanisms. Therefore, these kinds of organizations should not neglect supervisory control mechanisms for motivating knowledge sharing (King and Marks, 2008).

If supervisory control is not a suitable approach for motivating knowledge sharing, in addition to incentive programs, enterprises should also offer an efficient communication platform to induce cooperation among community members, offering the opportunity of self-coordinated behavior. For example, SAP, one of the market and technology leaders in business software solutions, facilitates knowledge sharing by offering a number of coordination mechanisms. By deploying various technologies, such as Internet and Web-technologies, SAP helps to ensure that all teams from any remote locations can access to the same, most up-to-date information (Kotlarsky et al., 2008). In addition, transparent information is beneficial to motivate the behavior of knowledge sharing because it helps the members in COPs build right cognitions. For example, firms may publish periodical reports recording the statistics results of knowledge contribution that happened over a certain period, which forces their employees to contribute due to peer pressure. In VIA technologies, one of the world's largest supplier of power efficient x86 processor platforms, department managers can know an individual's effort on knowledge sharing through the report generated by the company's knowledge management system so that the employees in the company do more than what their superior asks of them (Yeh et al., 2006).

One of future research issues is how to efficiently improve the credibility of knowledge in open architectures based on the spirit of Web 2.0. Actually, peer production establishes a new production model; however, it also generates many new questions and challenges we have to overcome in the future. On the other hand, because knowledge can be treated as a good and people can trade their own knowledge in a market, one of the future directions is to conduct an economic analysis on knowledge trading platform, such as Cramster.com. Therefore, many interesting issues are worthy of further study such as how to price the membership fee, how to establish a reputation system to keep the quality of knowledge provided by members, and how to determine the DRM level of online knowledge.

Appendix A. Proof of Proposition 1

By FONC, solving $\underset{x_i}{Max} f(Q) - c_i x_i$ yields $x_i^* = \max\left\{\frac{1}{K_i} G\left(\frac{c_i}{K_i}\right) - \frac{1}{K_i}\sum_{j\neq i} K_j x_j^*, 0\right\}$. Therefore, when $K_H/c_H \neq K_L/c_L$ holds, each community member with smaller knowledge-cost ratios always free rides on others with the highest knowledge-cost ratio. In other words, all equilibrium solutions must satisfy the equation $\sum_{1\leqslant i\leqslant n, K_i/c_i = KC_{\max}} K_i x_i = G\left(\frac{1}{KC_{\max}}\right)$. Thus, the symmetric equilibrium in this case is given by $x_i^* = \frac{1}{mK_i} G\left(\frac{1}{KC_{\max}}\right)$ and $x_j^* = 0$ where $K_i/c_i = KC_{\max}$ and $K_j/c_j = KC_{\min}$. When $K_H/c_H = K_L/c_L$ holds, all equilibrium solutions must satisfy the equation $\sum_{1\leqslant i\leqslant n} K_i x_i = G\left(\frac{1}{KC_{\max}}\right)$. The symmetric equilibrium in this case is given by $x_i^* = \frac{1}{\eta K_H + (n-\eta)K_L} G\left(\frac{1}{KC_{\max}}\right)$.

Appendix B. Proof of Proposition 2

By FONC, solving $\underset{x_1,x_2,\ldots,x_n}{\text{Maximize}}\ W = \sum_{i=1}^{n} f(Q) - c_i x_i$ yields $Q - G\left(\frac{c_i}{nK_i}\right) \leqslant 0$ for each i. Thus, when $K_H/c_H \neq K_L/c_L$ holds, each community member's symmetric effort is given by $x_i^* = \frac{1}{mK_i} G\left(\frac{1}{n \cdot KC_{\max}}\right)$ where $K_i/c_i = KC_{\max}$ and $x_j^* = 0$ where $K_j/c_j = KC_{\min}$. When $K_H/c_H = K_L/c_L$ holds, each community member's symmetric effort is given by $x_i^* = \frac{1}{\eta K_H + (n-\eta)K_L} G\left(\frac{1}{n \cdot KC_{\max}}\right)$.

Appendix C. Proof of Proposition 3

Because $W = \sum_{i=1}^{n} (f(Q) - c_i x_i)$ can be decomposed into $f(Q) - c_i x_i$ and $\sum_{j \neq i}^{n} (f(Q) - c_j x_j)$, by first-order condition we derive $\frac{\partial W}{\partial x_i} = \frac{\partial}{\partial x_i}(f(Q) - c_i x_i) + \frac{\partial}{\partial x_i}\sum_{j \neq i}^{n}(f(Q))$. Hence, the externalities can be measured by $\frac{\partial}{\partial x_i}\sum_{j \neq i}^{n}(f(Q))$. When $K_H/c_H \neq K_L/c_L$ holds, the community members with the highest knowledge-cost ratio are more efficient than others. Therefore, comparing $\frac{\partial W}{\partial x_i}$ with $\frac{\partial}{\partial x_i}\sum_{j \neq i}^{n}(f(Q)) = (n-1)f'(Q)K_i$, only the community members with the highest knowledge-cost ratio would receive compensation which is given by $p_i = (n-1)c_i/n$ where $K_i/c_i = KC_{\max}$. The special case, $K_H/c_H = K_L/c_L$, is a degenerate case where all community members can receive the compensation.

Appendix D. Proof of Proposition 4

First, we consider the case where $K_H/c_H > K_L/c_L$. In a separating strategy, we let $x_i^*(K_H) = x_j^*(K_H)$ and $x_i^*(K_L) = x_j^*(K_L)$ where $i \neq j$; thus, solving first-order condition yields:

$$x_i^*(K_H) = \frac{\frac{1}{K_H} G\left(\frac{c_H}{K_H}\right) - (n-1)(1-\theta)\frac{K_L}{K_H} x_j(K_L)}{1 + (n-1)\theta}, \tag{D1}$$

$$x_i^*(K_L) = \frac{\frac{1}{K_L} G\left(\frac{c_L}{K_L}\right) - (n-1)\theta\frac{K_H}{K_L} x_j(K_H)}{1 + (n-1)(1-\theta)}. \tag{D2}$$

On the other hand, assuming $x_i^*(K_L) = 0$, the necessary and sufficient condition for low-type community members to free ride on the others is $G\left(\frac{c_L}{K_L}\right) \leqslant \frac{(n-1)\theta}{1+(n-1)\theta} G\left(\frac{c_H}{K_H}\right)$, which implies that the indifference point $\hat{\theta} = G\left(\frac{c_L}{K_L}\right) \Big/ \left((n-1)\left(G\left(\frac{c_H}{K_H}\right) - G\left(\frac{c_L}{K_L}\right)\right)\right)$. Since all low-type community members will not share tacit knowledge when $\theta > \hat{\theta}$ holds, each based her private type has two strategies relying on whether θ is less than $\hat{\theta}$ or not. Hence, when $\theta < \hat{\theta}$ holds, solving (D1) and (D2) simultaneously yields $x_i^*(K_H)$ and $x_i^*(K_L)$ given by

$$x_i^*(K_H) = \frac{1}{nK_H}\left(G\left(\frac{c_H}{K_H}\right) + (n-1)(1-\theta)\left(G\left(\frac{c_H}{K_H}\right) - G\left(\frac{c_L}{K_L}\right)\right)\right), \tag{D3}$$

$$x_i^*(K_L) = \frac{1}{nK_L}\left(G\left(\frac{c_L}{K_L}\right) - (n-1)\theta\left(G\left(\frac{c_H}{K_H}\right) - G\left(\frac{c_L}{K_L}\right)\right)\right). \tag{D4}$$

When $\theta \geqslant \hat{\theta}$ holds, plugging $x_i^*(K_L) = 0$ into (D1) yields new $x_i^*(K_H) : G\left(\frac{c_H}{K_H}\right)((1 + (n-1)\theta)K_H)^{-1}$. By the same approach, we can show the case where $K_H/c_H < K_L/c_L$. When $K_H/c_H = K_L/c_L$ holds, because there is no equilibrium solution when $x_i^*(K_H) = 0$ or $x_i^*(K_L) = 0, x_i^*(K_H)$ and $x_i^*(K_L)$ are given by (D3) and (D4) directly. Finally, we derive that

$$E[Q_{IV}] = \sum_{i=0}^{n}\binom{n}{i}\theta^i(1-\theta)^{n-i}\left(i \cdot K_H x_i^*(K_H) + (n-i)\cdot K_L x_i^*(K_L)\right) = n\left(\theta K_H x_i^*(K_H) + (1-\theta)K_L x_i^*(K_L)\right).$$

Appendix E. Proof of Propositions 5 and 6

By first-order condition, we complete the proof.

Appendix F. Proof of Proposition 7

First, we rewrite the program as follows:

$$\begin{aligned}
&\max_x \quad W = nf\left(n\overline{K}x\right) - n\bar{c}x \quad \text{s.t.} \\
&f\left(K_H x + (n-1)\overline{K}x\right) - c_H x + P \geqslant 0 \quad (IRH), \\
&f\left(K_L x_L + (n-1)\overline{K}x\right) - c_L x + P \geqslant 0 \quad (IRL),
\end{aligned}$$

where $\overline{K} = \theta K_H + (1-\theta)K_L$ and $\bar{c} = \theta c_H + (1-\theta)c_L$. By first-order condition, the equation $n\overline{K}f'\left(n\overline{K}x\right) - \bar{c} = 0$ must be satisfied, which implies $x^* = \frac{1}{n\overline{K}} G\left(\frac{\bar{c}}{n\overline{K}}\right)$. Hence, the subsidy can be calculated immediately by plugging x^* into (IRH) and (IRL).

Appendix G. Proof of Proposition 8

W.L.O.G., we assume that $K_H/c_H > K_L/c_L$ holds. Given the unconstrained mathematical program, by first-order condition, the following inequalities must be satisfied:

$$nf'\left(n\widehat{K}\right)n\theta K_H \leqslant n\theta c_H, \tag{G1}$$

$$nf'\left(n\widehat{K}\right)n(1-\theta)K_L \leqslant n(1-\theta)c_L. \tag{G2}$$

Because of $K_H/c_H > K_L/c_L$, (G1) will bind at the optimum so as to $\widehat{K} = \frac{1}{n}G\left(\frac{c_H}{nK_H}\right)$. Thus, $x_H^* = \frac{1}{n\theta K_H}G\left(\frac{c_H}{nK_H}\right)$ and $x_L^* = 0$ will satisfy $\widehat{K} = \frac{1}{n}G\left(\frac{c_H}{nK_H}\right)$ if given $\theta > 0$. Furthermore, let $P_H = c_H x_H^*,\ P_L = f\left(K_H x_H^* + (n-1)\widehat{K}\right) - f\left((n-1)\widehat{K}\right)$ and check all incentive constraints as follows:

$$\text{(IRH)}\qquad f\left(K_H x_H^* + (n-1)\widehat{K}\right) - c_H x_H^* + P_H = f\left(K_H x_H^* + (n-1)\widehat{K}\right) \geqslant 0,$$

$$(IRL)\qquad f\left(K_L x_L^* + (n-1)\widehat{K}\right) - c_L x_L^* + P_L = f\left(K_H x_H^* + (n-1)\widehat{K}\right) \geqslant 0,$$

$$(ICH)\qquad f\left(K_H x_L^* + (n-1)\widehat{K}\right) - c_H x_L^* + P_L = f\left(K_H x_H^* + (n-1)\widehat{K}\right) = f\left(K_H x_H^* + (n-1)\widehat{K}\right) - c_H x_H^* + P_H,$$

$$(ICL)\qquad f\left(K_L x_H^* + (n-1)\widehat{K}\right) - c_L x_H^* + P_H = f\left(K_L x_H^* + (n-1)\widehat{K}\right) + x_H^*(c_H - c_L) \leqslant f\left(K_L x_L^* + (n-1)\widehat{K}\right) - c_L x_L^* + P_L.$$

References

Alavi, M., Leidner, D., 1999. Knowledge management systems: Issues, challenges and benefits. Communication of the Association for Information Systems 1, 1–28.
Ambrosio, J. 2000. Knowledge Management Mistakes. Computerworld. <http://www.computerworld.com/s/article/46693/Knowledge_Management_Mistakes> (accessed 12.10.09).
Ba, S., Stallaert, J., Whinston, A.B., 2001. Research commentary: Introducing a third dimension in information system design – The case for incentive alignment. Information System Research 12 (3), 225–239.
Bandyopadhyay, S., Pathak, P., 2007. Knowledge sharing and cooperation in outsourcing projects – A game theoretic analysis. Decision Support Systems 43 (2), 349–358.
Barachini, F., 2009. Cultural and social issues for knowledge sharing. Journal of Knowledge Management 13 (1), 98–110.
Bartol, K.M., Srivastava, A., 2002. Encouraging knowledge sharing: The role of organizational reward systems. Journal of Leadership and Organization Studies 9 (1), 64–76.
Bhattacharya, S., Glazer, J., Sappington, D., 1992. Licensing and the sharing of knowledge in research joint ventures. Journal of Economic Theory 56 (1), 43–69.
Bischoff, I., 2007. Institutional choice versus communication in social dilemmas – An experimental approach. Journal of Economic Behavior & Organization 62 (1), 20–36.
Brosig, J., Ockenfels, A., Weimann, J., 2003. The effect of communication media on cooperation. German Economic Review 4 (2), 217–241.
Brown, J.S., Duguid, P., 1991. Organizational learning and communities-of-practice: Toward a unified view of working, learning, and innovation. Organization Science 2 (1), 40–57.
Brown, J.S., Duguid, P., 2001. Knowledge and organization: A social-practice perspective. Organization Science 12 (2), 198–213.
Cai, G., Kock, N., 2009. An evolutionary game theoretic perspective on e-collaboration: The collaboration effort and media relativeness. European Journal of Operational Research 194 (3), 821–833.
Chang, S.-G., Ahn, J.-H., 2005. Product and process knowledge in the performance-oriented knowledge management approach. Journal of knowledge Management 9 (4), 114–132.
Chen, X.-P., Komorita, S.S., 1994. The effects of communication and commitment in a public goods social dilemma. Organizational Behavior and Human Decision Processes 60 (3), 367–386.
Chua, A., 2003. Knowledge sharing: A game people play. Aslib Proceedings 55 (3), 117–129.
Fey, C.F., Furu, P., 2008. Top management incentive compensation and knowledge sharing in multinational corporations. Strategic Management Journal 29 (12), 130–1323.
Gibbons, R., 1992. Game Theory for Applied Economists. Princeton University Press, Princeton, NJ.
Global Most Admired Knowledge Enterprises (MAKE) Report, 2008. <http://arquivos.premiomake.com.br/ftp/MAKESUMMARY2008.pdf> (accessed 12.10.09).
Hara, N., 2007. Information technology support for communities of practice: How public defenders learn about winning and losing in court? Journal of the American Society for Information Science and Technology 58 (1), 76–87.
Ho, S.P., Wu, P.-C., Hsu, Y., 2006. Knowledge sharing model and its implication on knowledge categorization and management. In: Proceedings of the 23rd International Symposium on Automation and Robotics in Construction (ISARC).
Jolly, R., Wakeland, W., 2008. Using agent based simulation and game theory analysis to study information sharing in organizations – The InfoScape. Proceedings of the 41st Annual Hawaii International Conference on System Sciences.
King, W.R., Marks Jr., P.V., 2008. Motivating knowledge sharing through a knowledge management system. OMEGA 36 (1), 131–146.
Kluge, J., Stein, W., Licht, T., 2001. Knowledge Unplugged: The McKinsey & Company Global Survey on Knowledge Management, Palgrave, New York.
Kotlarsky, J., Fenema, P.C., Willcocks, L.P., 2008. Developing a knowledge-based perspective on coordination: The case of global software projects. Information & Management 45 (2), 96–108.
Lee, D.-J., Ahn, J.-H., 2007. Reward systems for intra-organizational knowledge sharing. European Journal of Operational Research 180 (2), 938–956.
Li, L.C., Grimshaw, J.M., Nielsen, C., Judd, M., Coyte, P.C., Graham, I.D., 2009. Evolution of Wenger's concept of community of practice. Implementation Science 4 (11), 1–8.
Lin, L., Geng, X., Whinston, A.B., 2005. A sender–receiver framework for knowledge transfer. MIS Quarterly 29 (2), 197–219.
Malone, T.W., 2004. The Future of Work: How the New Order of Business Will Shape Your Organization, Your Management Style, and Your Life. Harvard Business School Press.
Marks, P., Polak, P., Mccoy, S., Galletta, D., 2008. Sharing knowledge. Communications of the ACM 51 (2), 60–65.
Nan, N., 2008. A principal-agent model for incentive design in knowledge sharing. Journal of Knowledge Management 12 (3), 101–113.
Nevo, D., Chan, Y.E., 2007. A Delphi study of knowledge management systems: Scope and requirements. Information and Management 44 (6), 583–597.
Renzl, B., 2008. Trust in management and knowledge sharing: The mediating effects of fear and knowledge documentation. OMEGA 36 (2), 206–220.
Samaddar, S., Kadiyala, S.S., 2006. An analysis of interorganizational resource sharing decisions in collaborative knowledge creation. European Journal of Operational Research 170 (1), 192–210.
Samieh, H.M., Wahba, K., 2007. Knowledge sharing behavior from game theory and socio-psychology perspectives. In: Proceedings of the 40th Annual Hawaii International Conference on System Sciences.
Shih, M.-H., Tsai, H.-T., Wu, C.-C., 2006a. A holistic knowledge sharing framework in high-tech firms: Game and co-opetition perspectives. International Journal of Technology Management 36 (4), 354–367.
Shih, S.-G., Hu, T.-P., Chen, C.-N., 2006b. A game theory-based approach to the analysis of cooperative learning in design studios. Design Studies 27 (6), 711–722.
Shin, M., 2004. A framework for evaluating economics of knowledge management systems. Information and Management 42 (1), 179–196.

Sierhuis, M., Clancey, W.J., 1997. Knowledge, practice, activities and people. In: Gaines, B. (Ed.), Proceedings of AAAI Spring Symposium on Artificial Intelligence in Knowledge Management, Stanford University.
Song, X., Li, J., Xu, N., 2008. Dynamic game analysis on knowledge-sharing and knowledge-spillover in competitive alliances. In: Proceedings of the 2008 International Conference on Computer Science and Software Engineering.
Turban, E., Leidner, D., Mclean, E., Wetherbe, J., 2006. Information Technology for Management: Transforming Organizations in the Digital Economy. Wiley, New York.
Turoff, M., Hiltz, S.R., 2009. The future of professional communities of practice. Lecture Notes in Business Information Processing 22, 1348–1865.
Wang, C.-Y., Yang, H.-Y., Chou, S.T., 2008. Using peer-to-peer technology for knowledge sharing in communities of practices. Decision Support Systems 45 (3), 528–540.
Wang, Y., Zhang, L., Zheng, X., Shi, Y., 2009. Research on ratchet effects in enterprises' knowledge sharing based on game models. In: Proceedings of the 20th International Conference on Multiple Criteria Decision Making.
Wenger, E., 1998. Communities of Practice. Cambridge University Press, New York.
Yang, S.-H., 2009. Using blogs to enhance critical reflection and community of practice. Educational Technology and Society 12 (2), 11–21.
Yang, H.-L., Wu, T.C.T., 2008. Knowledge sharing in an organization. Technological Forecasting and Social Change 75 (8), 1128–1156.
Yeh, Y.-J., Lai, S.-Q., Ho, C.-T., 2006. Knowledge management enablers: A case study. Industrial Management & Data Systems 106 (6), 793–810.