Flavor transition mechanisms of propagating astrophysical neutrinos: A model independent parametrization

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One of the important goals for future neutrino telescopes is to identify the flavors of astrophysical neutrinos and therefore determine the flavor ratio. The flavor ratio of astrophysical neutrinos observed on the Earth depends on both the initial flavor ratio at the source and flavor transitions taking place during propagations of these neutrinos. We propose a model independent parametrization for describing the above flavor transitions. A few flavor transition models are employed to test our parametrization. The observational test for flavor transition mechanisms through our parametrization is discussed.

I. INTRODUCTION

Recent developments of neutrino telescopes [1–5] have inspired numerous efforts of studying neutrino flavor transitions utilizing astrophysical neutrinos as the beam source [6–25]. Given the same neutrino flavor ratio at the source, some flavor transition models predict rather different neutrino flavor ratios on the Earth compared to those predicted by the standard neutrino oscillations [10]. In this article, we propose a scheme to parametrize flavor transition mechanisms of astrophysical neutrinos propagating from the source to the Earth. As will be shown later, such a parametrization is very convenient for classifying flavor transition models which can be tested by future neutrino telescopes.

To test flavor transition mechanisms, it is necessary to measure the flavor ratio of astrophysical neutrinos reaching to the Earth. The possibility for such a measurement in IceCube has been discussed in Ref. [8]. It is demonstrated that the ν_e fraction can be extracted from the measurement of the muon track to shower ratio by assuming flavor independence of the neutrino spectrum and the equality of ν_μ and ν_τ fluxes on the Earth due to the approximate $\nu_\mu - \nu_\tau$ symmetry [26,27]. Taking a neutrino source with fluxes of ν_e and ν_μ given by $E_{\nu_e}^2 dN_{\nu_e}/dE_{\nu_e} = 0.5 E_{\nu_\mu}^2 dN_{\nu_\mu}/dE_{\nu_\mu} = 10^{-7}$ GeV cm^{-2} s^{-1}, which is roughly the order of the Waxman-Bahcall bound [28], and thresholds for muon and shower energies taken to be 100 GeV and 1 TeV, respectively, the ν_e fraction can be determined to an accuracy of 25% at IceCube for 1 yr of data taking, or equivalently to an accuracy of 8% for a decade of data taking. Such an accuracy is obtained for a ν_e fraction in the vicinity of 1/3. The accuracies corresponding to other central values of the ν_e fraction are also presented in Ref. [8]. The ν_μ to ν_τ event ratio can also be measured in IceCube. However, the accuracy of this measurement is limited by the low statistics of ν_τ events.

II. NEUTRINO FLAVOR RATIO REPRESENTED BY THE TERNARY PLOT

To study neutrino flavor transitions, we describe the neutrino flavor composition at the source by a normalized flux $\Phi_0 = (\phi_{0,e}, \phi_{0,\mu}, \phi_{0,\tau})^T$ satisfying the condition [29]

$$\phi_{0,e} + \phi_{0,\mu} + \phi_{0,\tau} = 1,$$
$$\phi_{0,\alpha} \geq 0, \quad \text{for } \alpha = e, \mu, \tau, \tag{1}$$

where each $\phi_{0,\alpha}$ is the sum of neutrino and antineutrino fluxes. Any point on or inside the triangle shown in Fig. 1 represents a specific flavor ratio characterizing the source. The triangular region bounded by vertices $(1, 0, 0)^T$, $(0, 1, 0)^T$, and $(0, 0, 1)^T$ contains all possible source types in terms of flavor ratios. The pion source and the muon-damped source with flavor compositions $\Phi_{0,\pi} = (1/3, 2/3, 0)^T$ and $\Phi_{0,\mu} = (0, 1, 0)^T$, respectively, are explicitly marked on the figure [30].

The net effect of flavor transition processes occurring between the source and the Earth is represented by the matrix P such that

$$\Phi = P\Phi_0, \tag{2}$$

where $\Phi = (\phi_e, \phi_\mu, \phi_\tau)^T$ is the flux of neutrinos reaching to the Earth. We note that our convention implies $P_{\alpha\beta} \equiv P(\nu_\beta \to \nu_\alpha)$.

III. Q MATRIX PARAMETRIZATION FOR FLAVOR TRANSITIONS OF ASTROPHYSICAL NEUTRINOS

Since the triangular region in Fig. 1 represents all possible neutrino flavor composition at the source, it is convenient to parametrize Φ_0 by [31]

$$\Phi_0 = \tfrac{1}{3}\mathbf{V}_1 + a\mathbf{V}_2 + b\mathbf{V}_3, \tag{3}$$

where $\mathbf{V}_1 = (1, 1, 1)^T$, $\mathbf{V}_2 = (0, -1, 1)^T$, and $\mathbf{V}_3 = (2, -1, -1)^T$. Mathematically, $\mathbf{V}_1/3$ represents the center of the triangle, while $a\mathbf{V}_2$ and $b\mathbf{V}_3$ represent horizontal and
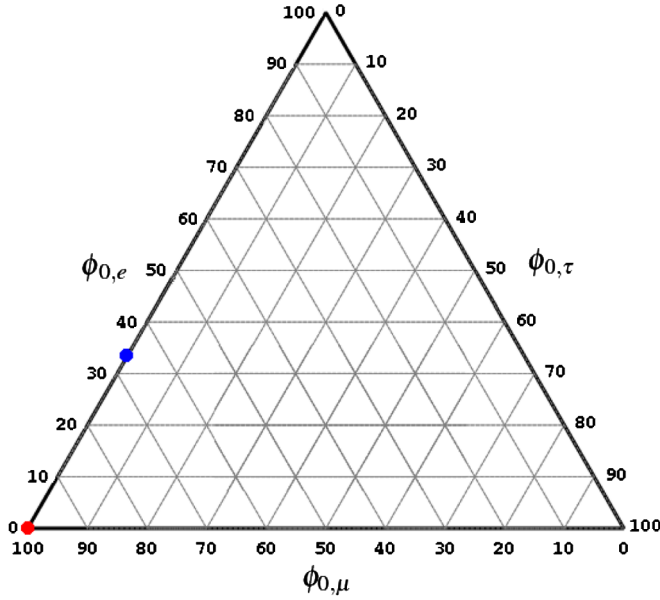
FIG. 1 (color online). The ternary plot for representing the flavor ratio of astrophysical neutrinos. The numbers on each side of the triangle denote the flux fraction of a specific flavor of neutrino. The blue point, situated on the left side of the triangle, marks the pion source $\Phi_{0,\pi} = (1/3, 2/3, 0)^T$ and the red point, situated at the lower-left corner of the triangle, marks the muon-damped source $\Phi_{0,\mu} = (0, 1, 0)^T$.

vertical displacements within the triangle, respectively. The ranges for a and b are $-1/3 + b \leq a \leq 1/3 - b$ and $-1/6 \leq b \leq 1/3$ such that Eq. (3) covers all points of the triangular region. The pion source and the muon-damped source mentioned in Fig. 1 correspond to $(a, b) = (-1/3, 0)$ and $(a, b) = (-1/2, -1/6)$, respectively. In general, a source with a negligible ν_τ flux corresponds to $a = -1/3 + b$.

The above parametrization for Φ_0 is also physically motivated. The vector $\mathbf{V}_1$ gives the normalization for the neutrino flux since the sum of components in $\mathbf{V}_1/3$ is equal to unity, while the sums of components in $\mathbf{V}_2$ and that in $\mathbf{V}_3$ are both equal to zero. The vector $a\mathbf{V}_2$ determines the difference between ν_μ and ν_τ flux, $\phi_{0,\mu} - \phi_{0,\tau}$, while preserving their sum, $\phi_{0,\mu} + \phi_{0,\tau}$. Finally the vector $b\mathbf{V}_3$ determines the sum of ν_μ and ν_τ flux, $\phi_{0,\mu} + \phi_{0,\tau}$, while preserving their difference $\phi_{0,\mu} - \phi_{0,\tau}$.

Following the same parametrization, we write the neutrino flux reaching to the Earth as

$$\Phi = \kappa\mathbf{V}_1 + \rho\mathbf{V}_2 + \lambda\mathbf{V}_3. \tag{4}$$

It is easy to show that

$$\begin{pmatrix} \kappa \\ \rho \\ \lambda \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{pmatrix} \begin{pmatrix} 1/3 \\ a \\ b \end{pmatrix}, \tag{5}$$

where $Q = \mathbf{A}^{-1}P\mathbf{A}$ with

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{pmatrix}. \tag{6}$$

In other words, Q is related to P by a similarity transformation where columns of the transformation matrix $\mathbf{A}$ correspond to vectors $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$, respectively.

The parameters κ, ρ, and λ are related to the flux of each neutrino flavor by

$$\phi_e = \kappa + 2\lambda, \qquad \phi_\mu = \kappa - \rho - \lambda, \qquad \phi_\tau = \kappa + \rho - \lambda, \tag{7}$$

with the normalization $\phi_e + \phi_\mu + \phi_\tau = 3\kappa$. Since we have chosen the normalization $\phi_{0,e} + \phi_{0,\mu} + \phi_{0,\tau} = 1$ for the neutrino flux at the source, the conservation of total neutrino flux during propagations corresponds to $\kappa = 1/3$. In general flavor transition models, κ could be less than $1/3$ as a consequence of (ordinary) neutrino decaying into invisible states or oscillating into sterile neutrinos. To continue our discussions, it is helpful to rewrite Eq. (7) as

$$\rho = (\phi_\tau - \phi_\mu)/2, \qquad \lambda = \phi_e/3 - (\phi_\mu + \phi_\tau)/6. \tag{8}$$

It is then clear from Eqs. (5) and (8) that, for fixed a and b, the first row of matrix Q determines the normalization for the total neutrino flux reaching to the Earth, the second row of Q determines the breaking of $\nu_\mu - \nu_\tau$ symmetry in the arrival neutrino flux, and the third row of Q determines the flux difference $\phi_e - (\phi_\mu + \phi_\tau)/2$.

Compared to P, matrix Q is very convenient for classifying flavor transition models. First of all, those models which preserve the total neutrino flux are characterized by the condition $\sum_{\alpha=e,\mu,\tau} P_{\alpha\beta} = 1$ in the P matrix parametrization. On the other hand, these flux-conserving models must give $\kappa = 1/3$ in the Q matrix parametrization, irrespective of the initial flavor composition characterized by parameters a and b. This implies $Q_{11} = 1$ and $Q_{12} = Q_{13} = 0$ from Eq. (5). Clearly the flux-conservation condition in the Q matrix parametrization is much simpler. Second, for those models which do not seriously break the $\nu_\mu - \nu_\tau$ symmetry, the second and third rows of P are almost identical, i.e., $(P_{\mu e}, P_{\mu\mu}, P_{\mu\tau}) \approx (P_{\tau e}, P_{\tau\mu}, P_{\tau\tau})$, and the second and third columns of P are also almost identical, i.e., $(P_{e\mu}, P_{\mu\mu}, P_{\tau\mu})^T \approx (P_{e\tau}, P_{\mu\tau}, P_{\tau\tau})^T$. Using these conditions and the relation $Q = \mathbf{A}^{-1}P\mathbf{A}$, one can show that $(Q_{21}, Q_{22}, Q_{23}) \approx (0, 0, 0)$ and $(Q_{12}, Q_{22}, Q_{32})^T \approx (0, 0, 0)^T$. Obviously, the approximate $\nu_\mu - \nu_\tau$ symmetry is realized by a much simpler condition in the Q matrix parametrization. In summary, we have seen that the first and second rows of Q as well as the matrix element Q_{32} are already constrained in a simple way by assuming the conservation of total neutrino flux and the validity of approximate $\nu_\mu - \nu_\tau$ symmetry. Hence, under these two assumptions, one can simply use the values for Q_{31} and Q_{33} to classify flavor transition models. This is the

most important advantage of Q matrix parametrization. In fact, as will be elaborated later, this parametrization is also very useful for discussing the effect of flux nonconservation, i.e., the case with $\kappa \neq 1/3$.

It was first discussed in Ref. [7] that the flavor measurement in neutrino telescopes is useful for studying neutrino flavor composition at the astrophysical source (for recent studies, see Refs. [31,32]) and neutrino flavor transition properties during its propagation from the source to the Earth (see also Ref. [6]). For probing flavor transition properties of astrophysical neutrinos, the authors of Ref. [7] considered typical astrophysical sources and applied the flavor transition matrix P (denoted by χ in the original paper) derived from the standard neutrino oscillation model or flavor transition models involving new physics for obtaining possible flavor ratios to be measured by terrestrial neutrino telescopes. It was pointed out that there are some flavor transition models which can produce rather distinctive neutrino flavor ratios on the Earth compared to those produced by the standard neutrino oscillation model, even with uncertainties of neutrino mixing parameters taken into account. Hence these flavor transition models can be tested on the basis of their flavor-ratio predictions for astrophysical neutrinos arriving on the Earth. In our approach, we test the fundamental structure of a given flavor transition model, namely, the Q matrix of the model. As noted earlier, possible neutrino flavor transition models which conserve the total neutrino flux are encompassed by possible values of Q_{2i} and Q_{3i} with $i = 1$, 2, and 3. In the $\nu_\mu - \nu_\tau$ symmetry limit, only the values of Q_{31} and Q_{33} are relevant. The matrix elements of Q can be determined by performing fittings to the flavor-ratio measurements in the neutrino telescopes, as will be demonstrated later. The obtained ranges for these matrix elements can be used as the basis for testing any flavor transition model.

IV. EXAMPLES

In the previous section, we have discussed the properties of the Q matrix and their advantages. In this section, we shall confirm such properties using a few flavor transition models as examples. We begin by considering the standard three-flavor neutrino oscillations. It is well known that

$$P^{\mathrm{osc}}_{\alpha\beta} = \sum_{i=1}^{3} |U_{\beta i}|^2 |U_{\alpha i}|^2, \qquad (9)$$

for astrophysical neutrinos traversing a vast distance where $U_{\alpha i}$ and $U_{\beta i}$ are elements of the neutrino mixing matrix. It is easily seen that $P^{\mathrm{osc}}_{\alpha\beta} = P^{\mathrm{osc}}_{\beta\alpha}$. Because of the probability conservation, the flux of neutrinos on the Earth also satisfies the normalization condition given by Eq. (1). We first compute Q^{osc} in the tribimaximal limit [33] of neutrino mixing angles, i.e., $\sin^2\theta_{23} = 1/2$, $\sin^2\theta_{12} = 1/3$, and $\sin^2\theta_{13} = 0$. In this limit, the

$\nu_\mu - \nu_\tau$ symmetry is exact. In fact, an exact $\nu_\mu - \nu_\tau$ symmetry amounts to the condition $|U_{\mu i}| = |U_{\tau i}|$ for $i = 1$, 2, 3 [26,27]. This condition can be realized by having both $\sin^2\theta_{23} = 1/2$ and $\sin\theta_{13}\cos\delta = 0$, which are respected by the above tribimaximal limit of neutrino mixing angles. Denoting P^{osc} in this limit as P^{osc}_0, we have

$$P^{\mathrm{osc}}_0 = \begin{pmatrix} 5/9 & 2/9 & 2/9 \\ 2/9 & 7/18 & 7/18 \\ 2/9 & 7/18 & 7/18 \end{pmatrix}. \qquad (10)$$

Since $\nu_\mu - \nu_\tau$ symmetry is exact in this case, one can see that the second and the third rows of P^{osc}_0 are identical, so are the second and third columns of P^{osc}_0. The corresponding Q matrix in this limit is given by

$$Q^{\mathrm{osc}}_0 \equiv \mathbf{A}^{-1} P^{\mathrm{osc}}_0 \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}. \qquad (11)$$

As expected, $Q^{\mathrm{osc}}_{0,11} = 1$ and $Q^{\mathrm{osc}}_{0,12} = Q^{\mathrm{osc}}_{0,13} = 0$. Furthermore, any element in either the second row or the second column of Q^{osc}_0 vanishes.

We can compute the correction to Q^{osc}_0 as neutrino mixing parameters deviate from the tribimaximal limit. We consider such deviations for θ_{13} and θ_{23} while keeping $\sin^2\theta_{12} = 1/3$. In this case $P^{\mathrm{osc}} = P^{\mathrm{osc}}_0 + P^{\mathrm{osc}}_1 + \cdots$ where P^{osc}_1 is the leading order correction in powers of $\cos2\theta_{23}$ and $\sin\theta_{13}$. One has [34]

$$P^{\mathrm{osc}}_1 = \begin{pmatrix} 0 & \epsilon & -\epsilon \\ \epsilon & -\epsilon & 0 \\ -\epsilon & 0 & \epsilon \end{pmatrix}, \qquad (12)$$

where $\epsilon = 2\cos2\theta_{23}/9 + \sqrt{2}\sin\theta_{13}\cos\delta/9$ with δ the CP phase. Taking into account P^{osc}_1, we obtain $Q^{\mathrm{osc}} = Q^{\mathrm{osc}}_0 + Q^{\mathrm{osc}}_1$ with

$$Q^{\mathrm{osc}}_1 = \mathbf{A}^{-1} P^{\mathrm{osc}}_1 \mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -3\epsilon \\ 0 & -\epsilon & 0 \end{pmatrix}. \qquad (13)$$

Therefore Q^{osc} is given by

$$Q^{\mathrm{osc}} = \mathbf{A}^{-1} P^{\mathrm{osc}} \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -3\epsilon \\ 0 & -\epsilon & 1/3 \end{pmatrix}. \qquad (14)$$

Because of the correction term Q^{osc}_1, one can see from Eq. (14) that the $\nu_\mu - \nu_\tau$ symmetry is broken since Q^{osc}_{23} and Q^{osc}_{32} are nonvanishing. Focusing on the third row of Q^{osc}, we obtain $\lambda = b/3 - a\epsilon$ from Eqs. (5) and (14).

We next consider models of neutrino decays. Flavor transitions of astrophysical neutrinos due to effects of neutrino decays were discussed in Ref. [6]. The simplest case of neutrino decays is that both the heaviest and the middle mass eigenstates decay to the lightest mass eigenstate. We first assume the branching ratios for the above two decays are both 100%. Under this condition, the

transition matrix is given by $P_{\alpha\beta}^{dec} = |U_{\alpha 1}|^2$ for the normal mass hierarchy and $P_{\alpha\beta}^{dec} = |U_{\alpha 3}|^2$ for the inverted mass hierarchy. The corresponding matrix Q^{dec} then reads

$$Q^{dec} = \begin{pmatrix} 1 & 0 & 0 \\ 3(|U_{\tau j}|^2 - |U_{\mu j}|^2)/2 & 0 & 0 \\ |U_{ej}|^2 - (|U_{\mu j}|^2 + |U_{\tau j}|^2)/2 & 0 & 0 \end{pmatrix}, \quad (15)$$

where $j = 1$ for the normal mass hierarchy and $j = 3$ for the inverted mass hierarchy. One can see that $Q_{11}^{dec} = 1$ and $Q_{12}^{dec} = Q_{13}^{dec} = 0$. Furthermore, in the limit of exact $\nu_\mu - \nu_\tau$ symmetry, one has $|U_{\tau j}| = |U_{\mu j}|$ such that the elements in both the second row and the second column of Q^{dec} vanish. If branching ratios for the above decays are not 100%, the resulting Q^{dec} matrix would be different but nevertheless gives rise to the same neutrino flavor ratio on the Earth. It is interesting to see that all the nonvanishing elements of Q^{dec} are located in the first column. Hence, following Eq. (5), the neutrino flavor ratio on the Earth is independent of the neutrino flavor ratio at the source in this scenario.

Let us consider another neutrino decay scenario where only the heaviest neutrino decays. Following earlier treatments, we set $\sin^2\theta_{12} = 1/3$ while allowing θ_{23} and θ_{13} to deviate from $\pi/4$ and 0, respectively. For the normal mass hierarchy, we write $Q'^{dec} = Q_0'^{dec} + Q_1'^{dec}$ where $Q_0'^{dec}$ is the leading term obtained in the limit $\sin^2\theta_{23} = 1/2$ and $\sin\theta_{13} = 0$, while $Q_1'^{dec}$ is the first-order correction which is linear in $\cos 2\theta_{23}$ and $\sin\theta_{13}$. We find

$$Q_0'^{dec} = \frac{1}{6}\begin{pmatrix} 4 + 2(r + s) & 0 & 2 - 2(r + s) \\ 0 & 0 & 0 \\ 1 + s & 0 & 1 - s \end{pmatrix}, \quad (16)$$

and

$$Q_1'^{dec} = \begin{pmatrix} 0 & (Q_1'^{dec})_{12} & 0 \\ (Q_1'^{dec})_{21} & 0 & (Q_1'^{dec})_{23} \\ 0 & (Q_1'^{dec})_{32} & 0 \end{pmatrix}, \quad (17)$$

where r and s are the branching ratios for the decay modes $\nu_3 \to \nu_2$ and $\nu_3 \to \nu_1$, respectively. The nonzero elements of $Q_1'^{dec}$ are given in Table I.

If ν_3 exclusively decays into either ν_2 or ν_1, one has $r + s = 1$. In this limit, $Q_{11}'^{dec} = 1$, $Q_{12}'^{dec} = Q_{13}'^{dec} = 0$ as expected. One also observes that the elements in the second row and the second column of the leading matrix $Q_0'^{dec}$

vanish due to $\nu_\mu - \nu_\tau$ symmetry. Finally, the third row of Q'^{dec} gives rise to $\lambda = [(1 + 3b) + (1 - 3b)s]/18 + a[s(\epsilon_1 + \epsilon_2) - \epsilon_2]/3$. Focusing on the leading order contributions, one has $\lambda \geq 0$ since $b \leq 1/3$; i.e., ϕ_e is either equal or larger than $(\phi_\mu + \phi_\tau)/2$ irrespective of the flavor ratio at the source. For comparison, the standard oscillation scenario gives $\lambda = b/3$ at the leading order, which is either positive or negative depending on the sign of b.

For the inverted mass hierarchy, we denote r and s as branching ratios for the decay modes $\nu_2 \to \nu_1$ and $\nu_2 \to \nu_3$, respectively. We obtain $Q''^{dec} = Q_0''^{dec} + Q_1''^{dec}$ with

$$Q_0''^{dec} = \frac{1}{6}\begin{pmatrix} 4 + 2(r + s) & 0 & 0 \\ 0 & 0 & 0 \\ r - s & 0 & 2 \end{pmatrix}, \quad (18)$$

and

$$Q_1''^{dec} = \begin{pmatrix} 0 & (Q_1''^{dec})_{12} & 0 \\ (Q_1''^{dec})_{21} & 0 & (Q_1''^{dec})_{23} \\ 0 & (Q_1''^{dec})_{32} & 0 \end{pmatrix}. \quad (19)$$

The nonzero matrix elements of $Q_1''^{dec}$ are given in Table I. In the limit $r + s = 1$, $Q_{11}''^{dec} = 1$, $Q_{12}''^{dec} = Q_{13}''^{dec} = 0$ as expected. It is also observed that the elements in the second row and the second column of the leading matrix $Q_0''^{dec}$ vanish. Finally, the third row of Q''^{dec} gives rise to $\lambda = (r - s + 6b)/18 - a[(1 + r - s)\epsilon_1 + 2\epsilon_2]/3$.

As the last example, we discuss neutrino flavor transitions affected by the decoherence effect from the Planck-scale physics [35]. In a three-flavor framework, it has been shown that [36–38]

$$P_{\alpha\beta}^{dc} = \frac{1}{3} + [\frac{1}{2}e^{-\gamma_3 d}(U_{\beta 1}^2 - U_{\beta 2}^2)(U_{\alpha 1}^2 - U_{\alpha 2}^2)$$
$$+ \frac{1}{6}e^{-\gamma_8 d}(U_{\beta 1}^2 + U_{\beta 2}^2 - 2U_{\beta 3}^2)$$
$$\times (U_{\alpha 1}^2 + U_{\alpha 2}^2 - 2U_{\alpha 3}^2)], \quad (20)$$

where γ_3 and γ_8 are eigenvalues of the decoherence matrix, and d is the neutrino propagating distance from the source. The CP phase in the neutrino mixing matrix U has been set to zero. Taking $\gamma_3 = \gamma_8 = \gamma$, we obtain $Q^{dc} \equiv \mathbf{A}^{-1}P^{dc}\mathbf{A} = Q_0^{dc} + Q_1^{dc}$ where

TABLE I. Nonzero elements for subleading matrices $Q_1'^{dec}$ and $Q_1''^{dec}$. The indices 12, 21, 23, and 32 in the heading of the table denote the positions of matrix elements. r and s denote branching ratios for the decays $\nu_3 \to \nu_2$ and $\nu_3 \to \nu_1$, respectively, in the case of normal mass hierarchy, and branching ratios for the decays $\nu_2 \to \nu_1$ and $\nu_2 \to \nu_3$, respectively, in the case of inverted mass hierarchy. $\epsilon_1 \equiv (\cos 2\theta_{23} - \sqrt{2}\sin\theta_{13}\cos\delta)/3$ and $\epsilon_2 \equiv \cos 2\theta_{23}/2 - \epsilon_1$. To obtain these expressions, we have taken $\sin^2\theta_{12} = 1/3$.

| | Elements of subleading matrices $Q_1'^{dec}$ and $Q_1''^{dec}$ | | | |
	12	21	23	32
$Q_1'^{dec}$	$-2(1 - r - s)(\epsilon_1 + \epsilon_2)/3$	$-(1 + r)\epsilon_1 - (1 + s)\epsilon_2$	$r\epsilon_1 - (1 - s)\epsilon_2$	$[s(\epsilon_1 + \epsilon_2) - \epsilon_2]/3$
$Q_1''^{dec}$	$2(1 - r - s)\epsilon_1/3$	$(1 + s)\epsilon_1 - (r - s)\epsilon_2$	$-\epsilon_1 - 2\epsilon_2$	$-[(1 + r - s)\epsilon_1 + 2\epsilon_2]/3$

$$Q_0^{dc} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & e^{-\gamma d}/3 \end{pmatrix}, \qquad (21)$$

and

$$Q_1^{dc} = e^{-\gamma d} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -3\epsilon_0 \\ 0 & -\epsilon_0 & 0 \end{pmatrix}, \qquad (22)$$

with $\epsilon_0 = 2\cos 2\theta_{23}/9 + \sqrt{2}\sin\theta_{13}/9$. From the definition right below Eq. (12), we note that $\epsilon_0 = \epsilon(\delta = 0)$. One can see that $Q_{11}^{dc} = 1$, and $Q_{12}^{dc} = Q_{13}^{dc} = 0$. Furthermore the elements in the second row and the second column of the leading matrix Q_0^{dc} vanish. In the absence of the decoherence effect, i.e., $\gamma \to 0$, it is seen that Q^{dc} reduces to Q^{osc}. In the full decoherence case, i.e., $e^{-\gamma d} \to 0$, we have $\kappa = 1/3$, $\rho = \lambda = 0$ such that $\phi_e:\phi_\mu:\phi_\tau = 1:1:1$.

V. PROBING Q BY MEASURING FLAVOR RATIOS OF ASTROPHYSICAL NEUTRINOS

We have shown that the flavor transitions of astrophysical neutrinos can be parametrized by the matrix Q. As we have argued earlier, the Q matrix is very convenient for classifying flavor transition models. One could determine the matrix elements Q_{ij} by measuring flavor ratios of astrophysical neutrinos arriving on the Earth. In this regard, we derive from Eqs. (5) and (8) that

$$3(f_\tau(a, b) - f_\mu(a, b))/2 = (\tfrac{1}{3}Q_{21} + aQ_{22} + bQ_{23})/\kappa(a, b), \qquad (23)$$

$$f_e(a, b) - (f_\mu(a, b) + f_\tau(a, b))/2 = (\tfrac{1}{3}Q_{31} + aQ_{32} + bQ_{33})/\kappa(a, b), \qquad (24)$$

where f_α is the fraction of ν_α, i.e., $f_\alpha \equiv \phi_\alpha/(\phi_e + \phi_\mu + \phi_\tau) = \phi_\alpha/3\kappa$. In the above equations, we have explicitly denoted the dependence of f_α on the source parameters a and b. Furthermore we also indicated that κ is generally a function of source parameters since the total neutrino flux is not necessarily conserved during neutrino propagations.

In the flux-conservation case, $Q_{11} = 1$ and $Q_{12} = Q_{13} = 0$, which gives $\kappa = 1/3$. In principle, the matrix elements Q_{2i} in the second row of Q can be solved from Eq. (23) by inputting three sets of $f_\alpha(a, b)$ measured from three different astrophysical sources. Here we assume precise knowledge of parameters a and b from each source. The matrix elements Q_{3i} in the third row of Q can be solved from Eq. (24) in a similar way. In the case that $\nu_\mu - \nu_\tau$ symmetry is not significantly broken, one expects Q_{21}, Q_{22}, and Q_{23} are all suppressed. Therefore, it is more involved to probe the second row of Q than to probe the third one. To probe the latter, we have

$$f_e(a, b)/3 - (f_\mu(a, b) + f_\tau(a, b))/6 \approx \tfrac{1}{3}Q_{31} + bQ_{33}, \qquad (25)$$

since Q_{32} is also suppressed due to the approximate $\nu_\mu - \nu_\tau$ symmetry. We note that $f_\alpha(a, b)$ on the left-hand side of Eq. (25) only depends on b. It is possible to solve for Q_{31} and Q_{33} if the measurement on $f_e - (f_\mu + f_\tau)/2$ can be performed with respect to two different astrophysical sources where the value of the b parameter in each source is known.

In the case of flux nonconservation, the function $\kappa(a, b)$ is not known since it is difficult to determine the absolute flux of astrophysical neutrinos at the source. Hence one cannot directly solve for Q_{2i} and Q_{3i} from Eqs. (23) and (24) by inputting $f_\alpha(a, b)$ from measurements. On the other hand, the signature for $\kappa \neq 1/3$ could still be detected by the following consistency analysis. We recall from Eq. (24) that the third row of Q is related to the measurement by

$$f_e(a, b) - (f_\mu(a, b) + f_\tau(a, b))/2$$
$$\approx (\tfrac{1}{3}Q_{31} + bQ_{33})/\kappa(a, b). \qquad (26)$$

As it was just argued, one could set $\kappa = 1/3$ in the above equation and invoke two astrophysical sources to solve for Q_{31} and Q_{33}. However, taking this set of Q_{31} and Q_{33} as an input, one expects that the right-hand side of Eq. (26) is likely to be inconsistent with the left-hand side obtained from the third astrophysical source.

It is clear that the knowledge of the neutrino flavor ratio at the source is crucial for probing the matrix Q. Previous studies [39,40] pointed out that this ratio is energy dependent for a general astrophysical source. For parent pions with an E^{-2} energy spectrum, the flavor ratio of neutrinos arising from the decays of these pions and the subsequent muon decays is $\phi_{0,e}:\phi_{0,\mu}:\phi_{0,\tau} = 1:1.86:0$ at low energies [41] where energy losses of pions and muons in the source are negligible. The ratio $\phi_{0,e}/\phi_{0,\mu}$ however decreases with the increase of muon (pion) energy and eventually approaches zero. This behavior results from the above-mentioned energy losses which are important at higher energies. Recently, a systematic study on possible neutrino flavor ratios from cosmic accelerators listed on the Hillas plot was initiated [42]. The neutrino flavor ratio at the source depends on the spectrum index of injecting protons, the size of the acceleration region, and the magnetic field strength at the source. In some regions of the above-mentioned parameters, the neutrino flavor ratios are energy dependent, while in some other parameter regions they could behave as those of a pion source or those of a muon-damped source, which are both energy independent. In the following, we illustrate the determination of Q_{31} and Q_{33} by measuring flavor ratios of astrophysical neutrinos arriving on the Earth from a pion source and a muon-damped source, respectively.

To determine Q_{31} and Q_{33}, we assume an exact $\nu_\mu - \nu_\tau$ symmetry so that $\phi_\mu = \phi_\tau$. The measurement of muon track to shower ratio [8] in a neutrino telescope such as

IceCube can be used to extract the flux ratio $R \equiv \phi_\mu/(\phi_e + \phi_\tau)$. Clearly R depends on the source parameter b and the matrix elements Q_{31} and Q_{33} as can be seen from Eq. (25). One can in principle disentangle Q_{31} and Q_{33} by measuring R from two sources with different b values, say, a pion source with $b = 0$ and a muon-damped source with $b = -1/6$. Taking into account experimental errors in determining R, the ranges for Q_{31} and Q_{33} in a given confidence level can be determined by the formula

$$\chi^2 = \left(\frac{R_{\pi,\text{th}} - R_{\pi,\text{exp}}}{\sigma_{R_{\pi,\text{exp}}}}\right)^2 + \left(\frac{R_{\mu,\text{th}} - R_{\mu,\text{exp}}}{\sigma_{R_{\mu,\text{exp}}}}\right)^2, \quad (27)$$

where $R_{\pi,\text{exp}}$ and $R_{\mu,\text{exp}}$ are experimentally measured flux ratios for neutrinos coming from a pion source and muon-damped source, respectively, while $R_{\pi,\text{th}}$ and $R_{\mu,\text{th}}$, which depend on Q_{31} and Q_{33}, are theoretically predicted values for R_π and R_μ, respectively. Furthermore, $\sigma_{R_{\pi,\text{exp}}} = (\Delta R_\pi/R_\pi)R_{\pi,\text{exp}}$ and $\sigma_{R_{\mu,\text{exp}}} = (\Delta R_\mu/R_\mu)R_{\mu,\text{exp}}$ with ΔR_π and ΔR_μ the experimental errors in determining R for neutrinos coming from a pion source and muon-damped source, respectively. One does not need to include uncertainties of neutrino mixing angles θ_{ij} and CP phase δ in Eq. (27) since their effects are already embedded in Q_{31} and Q_{33}.

Let us first take the input (true) flavor transition mechanism to be a standard neutrino oscillation model with neutrino mixing parameters taking the tribimaximal values. One expects that $R_{\pi,\text{exp}}$ and $R_{\mu,\text{exp}}$ are around 0.50 and 0.64, respectively. Applying the χ^2 analysis, Eq. (27), with given accuracies $\sigma_{R_{\pi,\text{exp}}}$ and $\sigma_{R_{\mu,\text{exp}}}$, the fitted 1σ and 3σ ranges for Q_{31} and Q_{33} are presented in Fig. 2. We note that the left panel is obtained with $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 10\%$ while the right panel is the result of taking $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 20\%$. For both measurement accuracies, the neutrino decay scenario given by Eq. (15) can be ruled out at the 3σ level for both mass hierarchies. We stress that the confidence ranges in Fig. 2 can be used to test any model with specific values for Q_{31} and Q_{33}.

We next consider the case where the input flavor transition model is the neutrino decay scenario given by Eq. (15) with normal mass hierarchy. This model corresponds to $(Q_{31}, Q_{33}) = (0.5, 0)$ for neutrino mixing parameters taking the tribimaximal values. Hence one expects that $R_{\pi,\text{exp}}$ and $R_{\mu,\text{exp}}$ are both around 0.2. Applying the χ^2 analysis, we obtain the fitted 1σ and 3σ ranges for Q_{31} and Q_{33} as shown in Fig. 3. Once more, the left panel is obtained with $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 10\%$, while the right panel results from $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 20\%$. For both cases, it is seen that the standard neutrino oscillation model and the neutrino decay model with $(Q_{31}, Q_{33}) = (-0.5, 0)$ (inverted mass hierarchy) can be ruled out at the 3σ level.
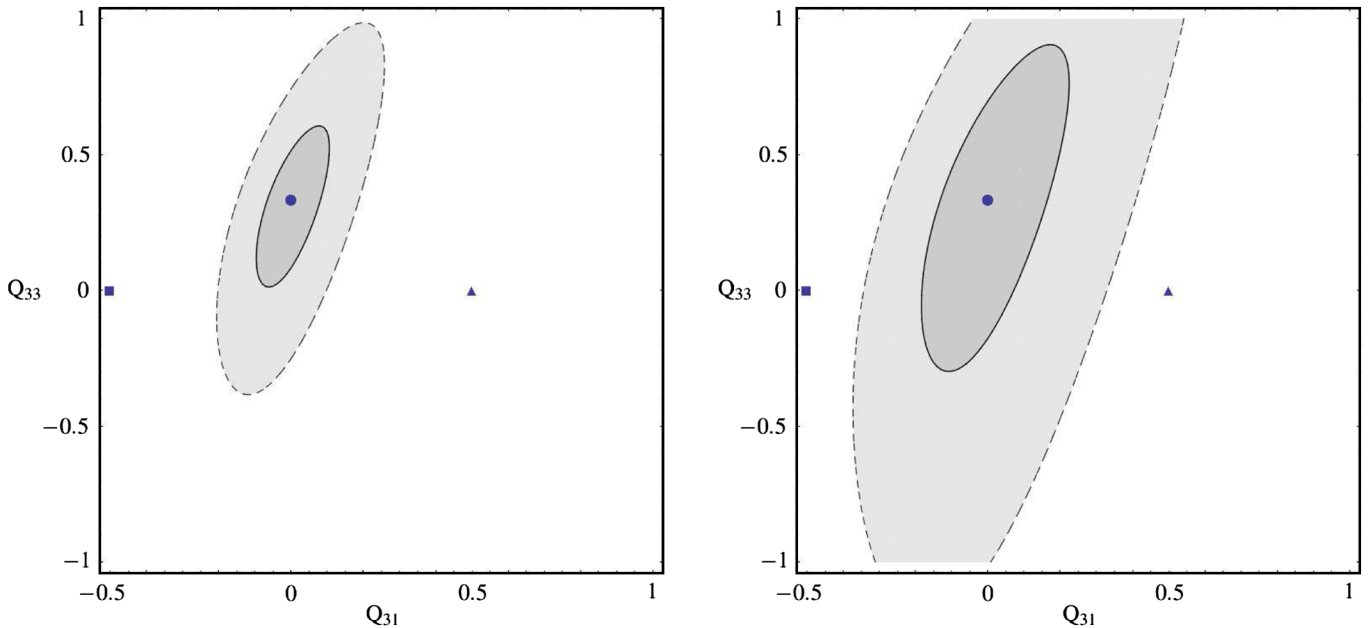

FIG. 2 (color online). The fitted 1σ (solid line) and 3σ (dashed line) ranges for Q_{31} and Q_{33}. The left panel is obtained with measurement accuracies $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 10\%$, while the right panel is obtained with $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 20\%$. The circle describes the best-fit parameter values $Q_{31} = 0$ and $Q_{33} = 0.33$, corresponding to the input flavor transition model. For reference, the parameter values for the neutrino decay scenario given by Eq. (15) are denoted by the triangle and the square, respectively, for normal and inverted mass hierarchies. The former corresponds to $(Q_{31}, Q_{33}) = (0.5, 0)$, while the latter corresponds to $(Q_{31}, Q_{33}) = (-0.5, 0)$ for neutrino mixing parameters taking the tribimaximal values.
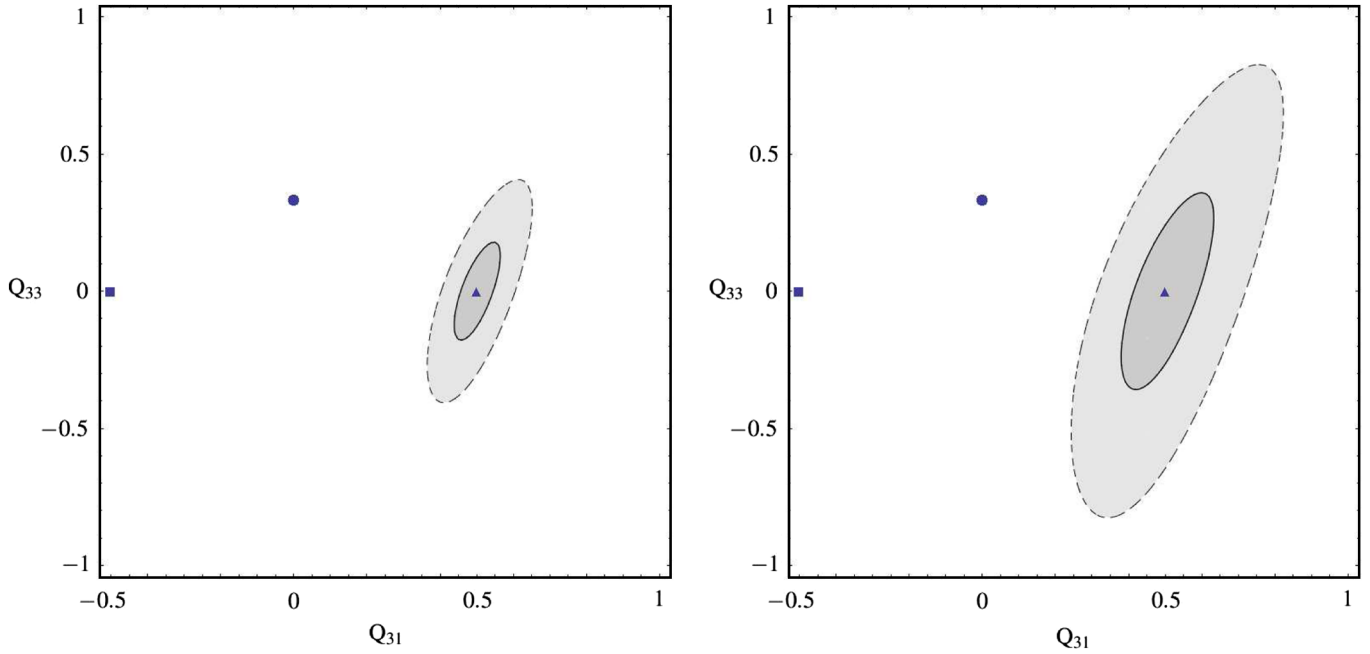
FIG. 3 (color online).   The fitted 1σ (solid line) and 3σ (dashed line) ranges for Q_{31} and Q_{33}. The left panel is obtained with measurement accuracies $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 10\%$, while the right panel is obtained with $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 20\%$. The triangle describes the best-fit parameter values, $(Q_{31}, Q_{33}) = (0.5, 0)$, corresponding to the input flavor transition model. The circle corresponds to the standard neutrino oscillation model, while the square corresponds to the neutrino decay scenario given by Eq. (15) with inverted mass hierarchy.
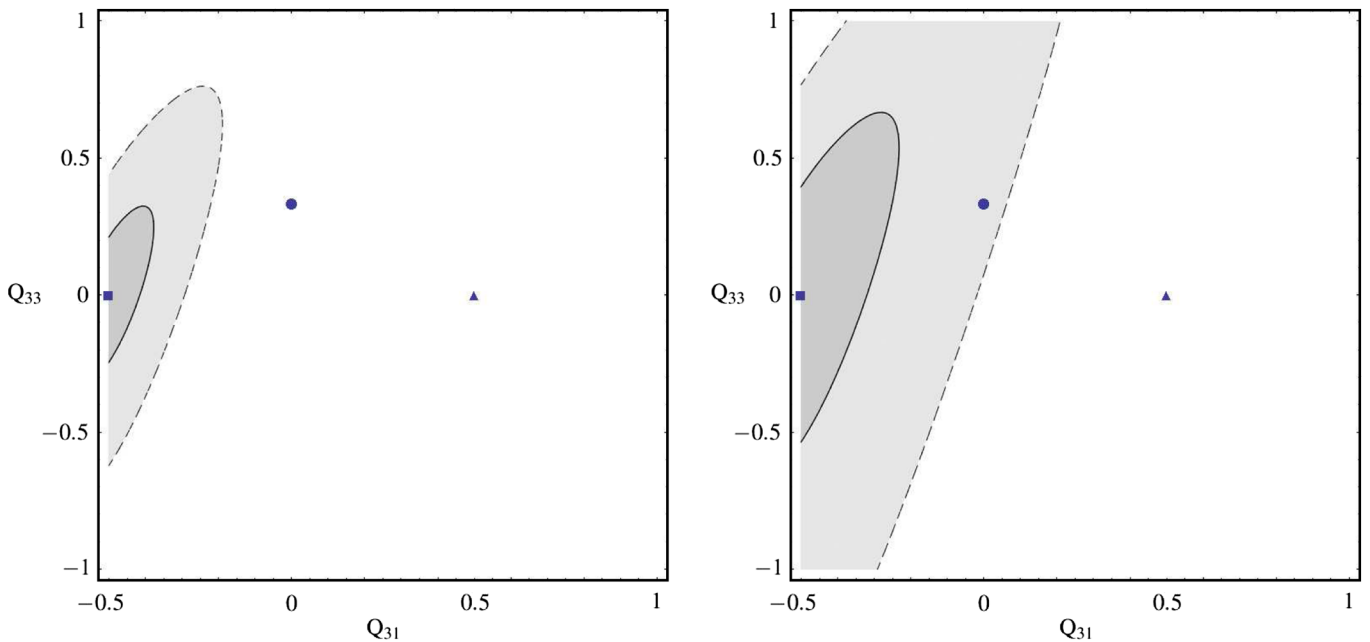

FIG. 4 (color online).   The fitted 1σ (solid line) and 3σ (dashed line) ranges for Q_{31} and Q_{33}. The left panel is obtained with measurement accuracies $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 10\%$, while the right panel is obtained with $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 20\%$. The square describes the best-fit parameter values, $(Q_{31}, Q_{33}) = (-0.5, 0)$, corresponding to the input flavor transition model. The circle corresponds to the standard neutrino oscillation model, while the triangle corresponds to the neutrino decay scenario given by Eq. (15) with normal mass hierarchy.

Finally, if the input flavor transition model is the neutrino decay scenario given by Eq. (15) with inverted mass hierarchy, i.e., $(Q_{31}, Q_{33}) = (-0.5, 0)$, one expects that $R_{\pi,\exp}$ and $R_{\mu,\exp}$ are both around 1.0. Applying the χ^2 analysis, we obtain the fitted 1σ and 3σ ranges for Q_{31} and Q_{33} as shown in Fig. 4. For $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 10\%$ (left panel), it is seen that the other two models displayed on the figure can be ruled out at the 3σ level. However, for $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 20\%$ (right panel), the standard neutrino oscillation model cannot be ruled out at the same confidence level.

VI. CONCLUSION

In summary, we have proposed to parametrize the flavor transitions of propagating astrophysical neutrinos by the matrix Q, which is related to the usual flavor transition matrix P by $Q = \mathbf{A}^{-1}P\mathbf{A}$ where $\mathbf{A}$ is given by Eq. (6). We have argued that it is much easier to classify flavor transition models by the Q matrix parametrization, where each row of Q carries a clear physical meaning as illustrated by Eq. (5). We have also argued that the signature for flux nonconservation might be detectable if it is possible to observe sufficient numbers of astrophysical neutrino sources with different flavor ratios. For the case of flux conservation, the above observations can probe the second and the third rows of matrix Q in a model independent fashion.

For illustration, we considered the determination of the Q matrix in the exact $\nu_\mu - \nu_\tau$ symmetry limit. The relevant matrix elements in this case are Q_{31} and Q_{33}. We proposed to determine them by measuring the flux ratio $R \equiv \phi_\mu/(\phi_e + \phi_\tau)$ for astrophysical neutrinos coming from a pion source and those coming from a muon-damped source, respectively. We fitted Q_{31} and Q_{33} to the measured flux ratios $R_{\pi,\exp}$ and $R_{\mu,\exp}$ using Eq. (27). The ranges for Q_{31} and Q_{33} are presented up to the 3σ confidence level for three different input models for neutrino flavor transitions.

We have found that the measurement accuracies $\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 10\%$ are sufficient to discriminate among the standard neutrino oscillation model and neutrino decay scenario given by Eq. (15) for normal and inverted mass hierarchies. We reiterate that the confidence ranges in Figs. 2–4 can be used to test any flavor transition model with specific values for Q_{31} and Q_{33}.

Taking a neutrino source flux $E_{\nu_e}^2 dN_{\nu_e}/dE_{\nu_e} = 0.5E_{\nu_\mu}^2 dN_{\nu_\mu}/dE_{\nu_\mu} = 10^{-7}$ GeV cm^{-2} s^{-1}, which is roughly the order of the Waxman-Bahcall bound [28], the accuracy $\Delta R/R = 10\%$ is reachable by a decade of data taking in Icecube [8], as stated in the beginning of this article. However, we stress that the Waxman-Bahcall bound is for diffuse neutrino flux. The flux from an individual point source is smaller. Hence it could take more than a decade to reach a 10% accurate measurement on R arising from a point source. The radio extension of IceCube [5] is expected to accumulate neutrino events at a much faster pace. It is crucial to study the efficiency of flavor identification in this type of detector.

In this work, the Q matrix is probed by assuming an exact $\nu_\mu - \nu_\tau$ symmetry and a precise knowledge of the neutrino flavor ratio at the source. Away from the $\nu_\mu - \nu_\tau$ symmetry limit, the second row of Q and Q_{32} shall become relevant in addition to Q_{31} and Q_{33}. Furthermore, the statistical analysis outlined by Eq. (27) should be refined once the uncertainty of the neutrino flavor ratio at the source is taken into account. We shall address these issues in a future publication.

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