

Early-Pruned K-Best Sphere Decoding Algorithm Based on Radius Constraints

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Abstract—A technique to prune the paths for K -best sphere decoding algorithm (SDA) based on radius constraint is presented. The proposed scheme preserves breadth-first searching nature, and the distinct radii for each decoding layer are theoretically derived from the system model with the noise statistics. In addition, based on the data range provided by the radius, a low complexity sorting strategy is proposed. The proposed method can apply to SDA with various path cost functions. Euclidean norm and sum of absolute difference are demonstrated in this paper. With SNR degradation less than 0.2dB, more than 47% and 90% computation complexity can be reduced in 16-QAM and 64-QAM 4×4 MIMO detection, respectively.

I. INTRODUCTION

Multiple input multiple output (MIMO) technology has been widely applied in many wireless communications for better transmission efficiency and signal quality due to the inherent diversity gain [1]. For maximum likelihood (ML) detection of the received signals, sphere decoding algorithm (SDA) can be applied as an efficient means to searching for the sequence with the minimum path metric [2]–[5]. Instead of exhaustively search, only the signals within the radius will be searched in SDA. However, the computation complexity depends on the channel conditions and the noise variance, and the non-constant decoder throughput results to difficulties in hardware implementation. Thus, K -best SDA [6]–[9], is often used as an alternative approximation. K -best SDA maintains a breadth-first searching strategy. Thus, constant and predictable complexity guarantees manageable hardware realization. For a signal vector of size $N_T \times 1$, the signals are detected from layer N_T to layer 1, and the term layer stands for the transmitted antenna index. In the i -th decoding layer of K -best SDA, only the K best candidates are kept and used for detecting signals of $(i - 1)$ -th layer, and sorting is required to select the K best candidates. It is obvious that K dominates the performance and the complexity. In this paper, a pruning technique is proposed for the K -best SDA based on the radius constrained. Unlike the conventional SDA, distinct radius is set for each layer, and therefore, the proposed K -best SDA preserves the breadth-first searching nature. Moreover, a low complexity sorting strategy is proposed, and the computation complexity can be greatly reduced. According to our simulation, for 16-QAM and 64-QAM 4×4 MIMO detection, with

SNR degradation less than 0.2dB, more than 47% and 90% computation complexity can be reduced, respectively. The rest of this paper is organized as follows. In Section II, SDA and K -best SDA are briefly introduced. The proposed schemes, the radius-constraint K -best SDA and the roughly sorting strategy, are presented in Section III. Theoretical derivation for the radii for every decoding layer is described in this section as well. The simulation results are shown in Section IV. The bit error probabilities and computation complexities of conventional K -best SDA and the proposed K -best SDA are compared. Finally, a summary concluding our work is given in Section V.

II. K -BEST SPHERE DECODING ALGORITHM

A MIMO system of N_T transmit antennas and N_R receive antennas can be represented by

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{s}} + \tilde{\mathbf{n}}, \quad (1)$$

where $\tilde{\mathbf{s}}$ is the $N_T \times 1$ transmitted signal vector, $\tilde{\mathbf{H}}$ is an $N_T \times N_R$ channel matrix of independent and identical distributed (i.i.d.) complex Gaussian elements, and $\tilde{\mathbf{n}}$ is an $N_R \times 1$ i.i.d. complex Gaussian noise vector, and $\tilde{\mathbf{y}}$ is the $N_R \times 1$ received signal. Note that an independent and flat-fading channel is assumed in (1). For convenience, (1) is often represented by an equivalent real-valued form as

$$\begin{aligned} \mathbf{y} &= \begin{bmatrix} \text{Re}\{\tilde{\mathbf{y}}\} \\ \text{Im}\{\tilde{\mathbf{y}}\} \end{bmatrix} \\ &= \begin{bmatrix} \text{Re}\{\tilde{\mathbf{H}}\} & -\text{Im}\{\tilde{\mathbf{H}}\} \\ \text{Im}\{\tilde{\mathbf{H}}\} & \text{Re}\{\tilde{\mathbf{H}}\} \end{bmatrix} \begin{bmatrix} \text{Re}\{\tilde{\mathbf{s}}\} \\ \text{Im}\{\tilde{\mathbf{s}}\} \end{bmatrix} + \begin{bmatrix} \text{Re}\{\tilde{\mathbf{n}}\} \\ \text{Im}\{\tilde{\mathbf{n}}\} \end{bmatrix} \\ &= \mathbf{H}\mathbf{s} + \mathbf{n}. \end{aligned} \quad (2)$$

Maximum likelihood (ML) detection is to find a vector $\hat{\mathbf{s}}$ that minimizes the Euclidean norm $\|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2$, that is,

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \Omega} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2, \quad (3)$$

Exhaustively searching for the minimizer in (2) becomes infeasible when the dimension of $\hat{\mathbf{s}}$ increases. However, (3) equivalent to finding a closest point in a lattice, and *sphere decoding algorithm*(SDA) can be applied. SDA reduces the computation by setting a radius r , and it only examines the $\hat{\mathbf{s}}$

vectors that satisfy the radius constraint $\|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 < r$. Moreover, SDA transforms the closest-point searching problem into a tree-search problem by factorizing the channel matrix as $\mathbf{H} = \mathbf{Q}\mathbf{R}$, where \mathbf{Q} is a $2N_R \times 2N_T$ unitary matrix and \mathbf{R} is an upper triangular matrix of size $2N_T \times 2N_T$. Therefore, (3) can be rewritten as

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \Omega, \|\hat{\mathbf{y}} - \mathbf{H}\mathbf{s}\|^2 < r} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2, \quad (4)$$

with $\hat{\mathbf{y}} = \mathbf{Q}^T \mathbf{y}$. Due to the triangular form of matrix \mathbf{R} , we can express the vector form in (4) by

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \Omega, \|\hat{\mathbf{y}} - \mathbf{R}\mathbf{s}\|^2 < r} \sum_{i=1}^{2N_T} \left(\hat{y}_i - \sum_{j=i}^{2N_T} R_{ij} s_j^{(i)} \right)^2 \quad (5)$$

The detection process starting from the $2N_T$ -th layer of the tree to the first layer, and each survived candidate of the i -th layer is represented by $\mathbf{s}^{(i)} = [s_{i+1}^{(i)} s_{i+2}^{(i)} \cdots s_{2N_T}^{(i)}]^T$. The *partial Euclidean distance (PED)* of $\mathbf{s}^{(i)}$ is defined as

$$T(\mathbf{s}^{(i+1)}) = \sum_{i'=i}^{2N_T} \left(\hat{y}_{i'} - \sum_{j=i'}^{2N_T} R_{i'j} s_j^{(i')} \right)^2 \quad (6)$$

and it can be derived recursively from

$$T(\mathbf{s}^{(i)}) = T(\mathbf{s}^{(i+1)}) + e(\mathbf{s}^{(i)}) \quad (7)$$

with

$$e(\mathbf{s}^{(i)}) = \left(\hat{y}_i - \sum_{j=i}^{2N_T} R_{ij} s_j^{(i)} \right)^2 \quad (8)$$

Obviously the initial radius selection affects the computation complexity. Moreover, the radius is updated by $r^l = \|\hat{\mathbf{y}} - \mathbf{R}\mathbf{s}\|^2 < r$ when an $\hat{\mathbf{s}}$ satisfying $\|\hat{\mathbf{y}} - \mathbf{R}\mathbf{s}\|^2 < r$ is found.

However, the depth-first tree-search nature restricts the decoding throughput, and the resulting non-constant computation limits the decoding efficiency. In order to achieve constant decoding speed, K -best SDA is often used, and simpler hardware design can be deduced. Since K -best SDA only keeps the paths corresponding to the first K smallest PEDs in every layer, constant computation is promised. But K -best SDA can not guarantee the same performance as SDA. When K is not large enough, the ML solution may be eliminated in PED computation process. Thus, there is a tradeoff between complexity and bit error rate.

III. K -BEST SDA WITH RADIUS CONSTRAINTS

Although K -best SDA promises a constant decoding throughput and can approach similar error performance of the conventional SDA as long as K is sufficiently large, computation, especially the sorting operation in finding the K best PSDs, will also grow with K . Thus, a K -best sphere decoder with large K and low sorting complexity is desired.

In this section, a technique is proposed to prune the less reliable paths in advance. Unlike the conventional SDA which only restrains the overall Euclidean norm, a distinct radius constraint is applied to the PEDs of each layer. The radii are independent of the channel matrix \mathbf{H} and can be determined at design time according to the noise statistics in (2). Similar to K -best SDA, the proposed scheme keeps at most K paths to guarantee a predictable and manageable computation complexity. Due to the radius constraints, the ranges of the data to be sorted in each decoding layer are known in advance. A roughly sorting strategy can be applied. The proposed sorting can be realized by few comparing operations, resulting to significant reduction in sorting complexity.

A. Radius constraints derivation

According to (2), $\hat{\mathbf{n}} = \hat{\mathbf{y}} - \mathbf{R}\mathbf{s}$ is still a vector of i.i.d. Gaussian random variables, and the PED

$$T(\mathbf{s}^{(i)}) = \sum_{i'=i}^{2N_T} \left(\hat{y}_{i'} - \sum_{j=i'}^{2N_T} R_{i'j} s_j^{(i')} \right)^2 = \sum_{i'=i}^{2N_T} \hat{n}_{i'}^2 \quad (9)$$

is a Chi-square random variable of degree $2N_T - i + 1$. Therefore, we can always find a value $A^{(i)}$ such that

$$Pr \left(\sum_{i'=i}^{2N_T} \hat{n}_{i'}^2 < A^{(i)} \right) = P. \quad (10)$$

P is a design parameter, a larger P value, 0.9999 for instance, means a more conservative pruning constraints: 99.99% of PED of the ML solution is smaller than $A^{(i)}$. Hence, the value $A^{(i)}$, which is determined by P , can be used as the radius of the PEDs in the i -th layer.

Fig. 1 illustrates a two-dimensional examples for conventional SDA, conventional 4-best SDA, and the proposed 4-best SDA. The points labeled in color gray are the candidates generated by the detector; the one corresponds to minimum cost will be the detector output. Since Fig. 1(a) only consider the initial radius, more points are left at layer 1. For Fig. 1(b), all points are kept in Layer-2, and then the 4 points having the smallest $T(\mathbf{s}^{(1)})$ will be the candidates of layer-1. For Fig. 1(c), 8 points are first pruned by the layer-2 radius constraint, then 4 points with the smallest $T(\mathbf{s}^{(1)})$ are kept.

So far, the i -th layer radius $A^{(i)}$ is acquired when P and the chi-square distribution (as well as the corresponding noise variance) are known. To avoid estimating noise variance in decoding time, the radius $A^{(i)}$ should be assigned according to the *minimum working SNR*, which stands for the minimum signal to noise ratio such that the conventional SDA can provide an acceptable bit error rate, such as 10^{-3} for example. Note that the selection of minimum working SNR affects computation complexity and bit error probabilities. A lower minimum working SNR deduces to a larger $A^{(i)}$'s. Thus more PEDs satisfy the radius constraints and the computation increases. On the other hand, the resulted large $A^{(i)}$'s promises a higher probability that the PED of the ML solution always fall into the radii for all layers, and a better error performance

can be achieved. In words, the proposed K -best SDA can be expressed by the following steps:

Initialization (at design time):

- i) Determine the value P and the minimum working SNR.
- ii) Compute the radius $A^{(1)}, A^{(2)}, \dots, A^{(2N_T)}$

Decoding:

Step 1: set $i = 2N_T$

Step 2: Compute the PEDs according to (6) - (8)

Step 3: Prune the paths with PEDs computed in *Step 2* that are greater than $A^{(i)}$.

Step 4: If more than K paths are preserved in *Step 3*, choose the K paths associated to the K smallest PEDs by sorting.

Step 5: set $i = i - 1$. Go back to *Step 2*.

Step 6: $i = 1$, output the path with minimum Euclidean norm and the decoding process is completed.

It is perceived in *Step 4* that a sorter is still needed in the proposed algorithm. However, the sorting can be realized by a few comparators, and the sorting strategy will be presented subsequently.

B. Low complexity sorting strategy

The goal of K -best SDA is to search for the closest point in a lattice. As long as the closest point is kept, the rest $K-1$ PEDs do not need to be the "smallest $K-1$ " PEDs. Therefore, instead of precisely sorted, the data can be sorted in a *rough order* and was presented as fast sequential decoding [10]. The data is first separated into L groups, and only the orders among the groups will be concerned. Since we have the knowledge of the maximum value among all the data to be sorted, which is $A^{(i)}$, we can divide $A^{(i)}$ into L uniform regions, $[\frac{l-1}{L}A^{(i)}, \frac{l}{L}A^{(i)}], l = 1, 2, \dots, L$, and the data to be sorted can be classified into L groups by the regions they fall into. Afterwards, randomly pick K data from the group with $l = 1$. If there are less than K data in this group, we pick the rest data (randomly) from the group with $l = 2$. Then the selecting process proceeds for $l = 3, 4, \dots$, until K paths are selected. Consequently, the data in the groups of smaller l index are chosen first, we can always choose the K paths with smaller PEDs. According to the simulation results $L = 16$ is large enough a choice to provide the similar bit-error probability compared to conventional K -best SDA.

C. Radius derivation for other cost functions

The above chi-square distributed radius constraint can apply only when the path cost function is computed by the Euclidean norm as (3) to (8). When the cost function is replaced by other methods, such as sum of absolute difference, the radius constraint can still be applied according to the system model (2) and the knowledge of the noise statistics.

Let $g(\bullet)$ denote the cost function, for sum of absolute difference, $g(\hat{n}_i) = |\hat{n}_i|$, and the overall path metric is

$$\sum_{i'=i}^{2N_T} \left| \hat{y}_{i'} - \sum_{j=i'}^{2N_T} R_{i'j} s_j^{(i')} \right| = \sum_{i'=i}^{2N_T} |\hat{n}_{i'}| \quad (11)$$

In the following, we will use (11) as example for deriving the radius constraints. The procedure of finding the probability

density function (PDF) of $\sum_{i'=i}^{2N_T} |\hat{n}_{i'}|$ for deriving the radius $A^{(i)}$ will be described as follows.

First, the PDF of $|\hat{n}_{2N_T}|$, represented by $f_{|\hat{n}_{2N_T}|}(x)$, can be easily found because \hat{n}_{2N_T} is known to be Gaussian distribution. Thus,

$$f_{|\hat{n}_{2N_T}|}(x) = f_{2N_T}(x) = 2f_{\hat{n}_{2N_T}}(x) = \frac{2}{\sqrt{2\pi\sigma}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

where σ^2 is the noise variance. Moreover, for two independent random variables X and Y that have PDFs f_X and f_Y , respectively, the PDF of $X+Y$ is the convolution of f_X and f_Y . Therefore, the PDF of $\sum_{i'=i}^{2N_T} |\hat{n}_{i'}|$, which is the summation of $(2N_T - i + 1)$ i.i.d random variables, can be derived by

$$f_i(x) = f_{2N_T}(x) \otimes f_{2N_T-1}(x) \otimes \dots \otimes f_{i+1}(x)$$

where the notation \otimes represents convolution. As a result, there is always a probability P and $A^{(i)}$ such that

$$Pr \left(\sum_{i'=i}^{2N_T} |\hat{n}_{i'}| < A^{(i)} \right) = \int_0^{A^{(i)}} f_{i'}(x) dx = P \quad (12)$$

IV. SIMULATION RESULTS

The proposed K -best SDA are compared with the conventional K -best SDA by the bit error rate (BER) and the computation complexity. Information with 16-QAM and 64-QAM signal mapping are simulated in a 4×4 Rayleigh flat fading plus additive white Gaussian noise (AWGN) MIMO channel. The minimum working SNR of the proposed K -best SDA are chosen as 16dB and 25dB for 16-QAM and 64-QAM symbols, respectively. Both Euclidean norm and sum of absolute difference are used in calculating the PEDs. Furthermore, the bit error probabilities of ML detection, which is realized by conventional SDA, are also provided as a performance baseline. $K=16$ and $K=64$ are for 16-QAM and 64-QAM, respectively, which are chosen to provide similar error performance of ML detection.

A. Bit error rate

Fig. 2 to Fig. 5 illustrate the impacts on BER of different L . As the figures show, larger L provides finer resolution for our proposed roughly sorting strategy. Therefore better error performance can be achieved. The figures show that $L=16$ is sufficient to provide error performances very similar to that of the ML detection. Moreover, different L values shown here all have the form of 2^m in order to reduce the sorting computation. For each PED satisfying the radius constraint, it takes m comparing operations to examine to which region of $[\frac{l-1}{L}A^{(i)}, \frac{l}{L}A^{(i)}], l=1,2,\dots,L$, the PED belongs. When L is too small, the performance will have significant degradation. This is more clear in Euclidean norm than sum of absolute difference. Because the PEDs have larger variance by squaring operation, the same L conducts to a poorer signal resolution, as compared to that of the sum of absolute difference. The results show that the SNR degradation for 16-QAM and 64-QAM constellation is below 0.2dB and 0.1dB respectively.

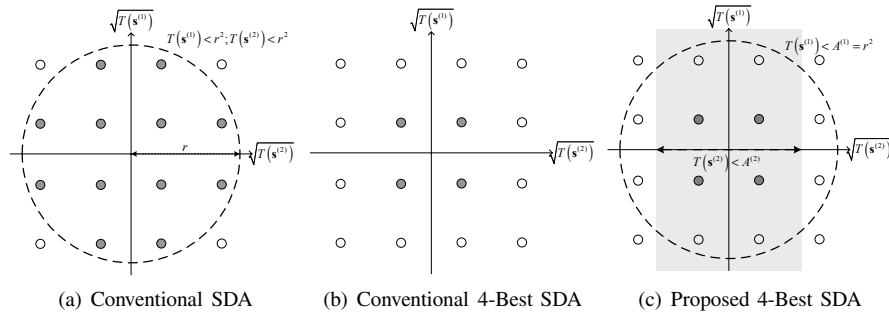


Fig. 1. Geometric two dimensional example for conventional SDA, conventional K -best SDA, and the proposed K -best SDA ($K = 4$). The gray point that contributes to minimum cost will be the detector output.

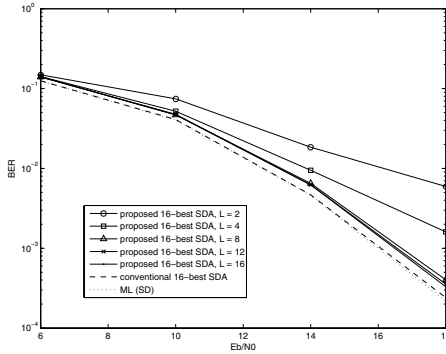


Fig. 2. Simulation result of 16-QAM with cost computed by sum of absolute difference.

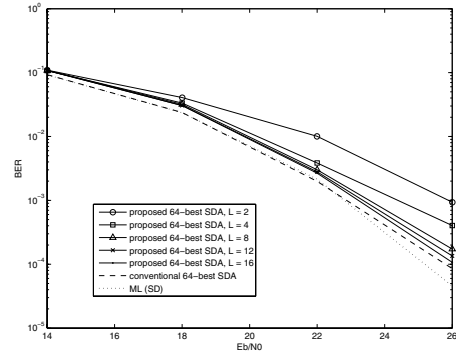


Fig. 4. Simulation result of 64-QAM with cost computed by sum of absolute difference.

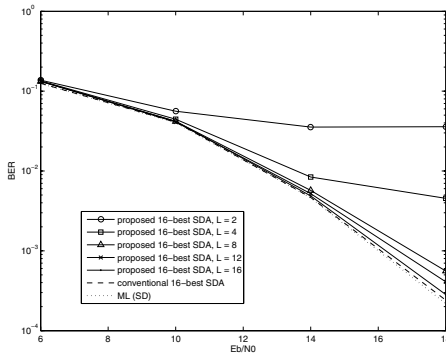


Fig. 3. Simulation result of 16-QAM with cost computed by Euclidean norm.

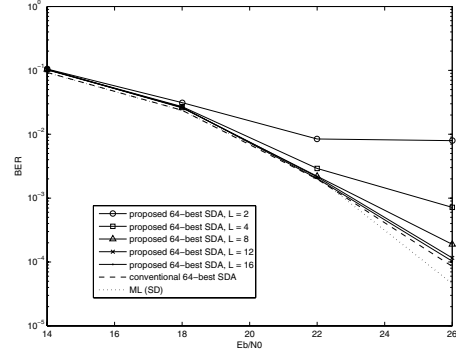


Fig. 5. Simulation result of 64-QAM with cost computed by Euclidean norm.

B. Computation complexity

TABLE I and TABLE II show the averaged number of multiplications, additions and comparisons in sorting per transmitted MIMO signal block. Quick sort algorithm [11] is used for conventional K -best SDA. The computation are reduced by pruning the paths with the proposed radius constraints of K -best SDA. Especially for the system of 64-QAM signals that operates in higher SNR, larger K value ($K=64$) and the smaller radii conducted to a significant computation reduction. Compared to K -best SDA, more than 90% computation can be reduced.

TABLE III and TABLE IV present the average number of paths survived at the first layer of the proposed K -best SDA

for 16-QAM and 64-QAM mapping. Because the radii for the proposed K -best SDA are derived in advance according to the minimum working SNR, not only the computation, but the number of survived paths also increases with SNR. Nonetheless, at least 43% and 93% of the paths are pruned for 16-QAM and 64-QAM respectively.

V. CONCLUSION

In this paper, a technique to prune the paths for K -best SDA based on radius constraints is proposed. The proposed scheme effectively reduces the number of paths, leading to significant reduction in computations. For each decoding layer, the radius can be derived according to the path cost function, system model, and the noise statistics at design time. Besides, the

TABLE I

AVERAGE NUMBER OF COMPARISONS(CMP) IN SORTING, MULTIPLICATIONS (MUL), AND ADDITIONS (ADD)FOR 4x4-16QAM

Method	Cost	SNR(dB)	6	10	14	18
CMP	EN	Conventional	6605	6603	6614	6615
		16-Best SDA	100%	100%	100%	100%
		Proposed	2544	2990	3163	3227
	SAD	16-Best SDA	38.46%	45.20%	47.82%	48.78%
		Conventional	6512	6533	6568	6605
		16-Best SDA	100%	100%	100%	100%
MUL	EN	Conventional	1256	1256	1256	1256
		16-Best SDA	100%	100%	100%	100%
		Proposed	441	520	555	570
	SAD	16-Best SDA	35.11%	41.40%	44.19%	45.38%
		Conventional	852	852	852	852
		16-Best SDA	35.11%	41.40%	44.19%	45.38%
ADD	EN	Conventional	3304	3304	3304	3304
		16-Best SDA	100%	100%	100%	100%
		Proposed	1221	1430	1516	1555
	SAD	16-Best SDA	36.96%	43.28%	45.88%	47.06%
		Conventional	3320	3320	3320	3320
		16-Best SDA	61.09%	61.09%	61.09%	61.09%
SAD	Proposed	1054	1321	1534	1684	
	16-Best SDA	31.90%	39.98%	46.43%	50.97%	

* EN and SAD stand for Euclidean norm and sum of absolute difference, respectively.

TABLE II

AVERAGE NUMBER OF COMPARISONS(CMP) IN SORTING, MULTIPLICATIONS (MUL), AND ADDITIONS (ADD)FOR 4x4-64QAM

Method	Cost	SNR(dB)	14	18	22	26
CMP	EN	Conventional	83413	83419	83574	83592
		64-Best SDA	99.79%	99.88%	99.98%	100%
		Proposed	3958	4852	4725	4765
	SAD	64-Best SDA	4.73%	5.48%	5.65%	5.70%
		Conventional	82359	82443	82563	82713
		64-Best SDA	98.52%	98.63%	98.77%	98.95%
MUL	EN	Conventional	8080	8080	8080	8080
		64-Best SDA	100%	100%	100%	100%
		Proposed	577	650	676	690
	SAD	64-Best SDA	7.14%	8.04%	8.37%	8.54%
		Conventional	4936	4936	4936	4936
		64-Best SDA	61.09%	61.09%	61.09%	61.09%
ADD	EN	Conventional	17740	17740	17740	17740
		64-Best SDA	100%	100%	100%	100%
		Proposed	1438	1626	1686	1717
	SAD	64-Best SDA	8.11%	9.17%	9.50%	9.68%
		Conventional	17740	17740	17740	17740
		64-Best SDA	100%	100%	100%	100%
SAD	Proposed	1204	1434	1605	1776	
	64-Best SDA	6.79%	8.08%	9.05%	10.01%	

* EN and SAD stand for Euclidean norm and sum of absolute difference, respectively.

TABLE III

AVERAGE PATH NUMBER FOR 4 × 416-QAM

Method	Cost/SNR(dB)	6	10	14	18
Conventional	EN	16	16	16	16
	SAD	16	16	16	16
Proposed	EN	4	5	6	6
	SAD	3	5	7	9

TABLE IV

AVERAGE PATH NUMBER FOR 4 × 4 64-QAM

Method	Cost/SNR(dB)	14	18	22	26
Conventional	EN	64	64	64	64
	SAD	64	64	64	64
Proposed	EN	2	2	2	2
	SAD	1	2	3	4

radius is independent of the channel matrix, and the required sorting operation can be realized by few comparators since the range of the sorted data are known in advance. Moreover, the reduction computation complexity is more obvious for larger K. Higher-order-modulation usually requires larger K to derive better performance. Thus the proposed K-best SDA is suitable for higher-order modulated signals. According to our simulation of 16-QAM and 64-QAM 4 × 4 MIMO detection, more than 47% and 90% computation complexity can be reduced with SNR degradation less than 0.2dB.

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