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Economic design of $\bar{x}$-control charts for continuous flow process with multiple assignable causes

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ABSTRACT

Extending Koo and Case's model (1990) for the case of a single assignable cause to allow for the second occurrence of an assignable cause following the first occurrence and the probability of the assignable causes following exponential distribution, the process-failure mechanism having a fixed hazard rate. We present a cost model and determine the optimal values of the design parameters, the sample size, the sampling intervals and control limit coefficient by minimizing the expected cost per unit time with respect to exponential parameters change. Finally, suggesting the performance of the loss-cost and sensitivity analyses of the design parameters and loss-cost which depend on the model parameters and shift amounts closely are also presented.

1. Introduction

The $\overline{X}$-charts investigated in this paper are those used to maintain current control of a process. The initial model was designated by Duncan. Duncan (1956) proposed a single assignable cause model for the determination of the optimal economic design of $\bar{x}$-control charts and assumed that the occurrence time of the assignable cause is an exponentially distributed random variable, a constant failure rate is implied and the number of periods for the processes remain in control has memory less property associated with the Poisson process. That is, the process is a uniform inspection scheme. The majority of the research in the last few years focuses on the subject according as the assumption. Excellent reviews of these subjects can be found in, for example, Goel et al. (1968), Barker (1971), Gibra (1971, 1975), Chiu and Wetherill (1974), Saniga (1977), Chiu and Cheung (1977), Collani (1986), Chung (1990), Alexander et al. (1995), Chen and Tirupati (1997), Bai and Lee (1998), Chen and Yang (2002), Chen (2004), Lin and Chou (2005), Chen et al. (2007) and many other reviews.

However, these reviews are centered on piece part manufacturing. Koo and Case (1990) first proposed an economic design of $\bar{x}$-control charts for using in monitoring continuous flow process, where the amount of time the process remains in control can be formulated as exponential distribution. A sampling scheme in a continuous flow process is to take one sample from the process at each sampling time and then combine n analytical results into a subgroup. That is considerably different from pulling n samples at one time as in a discrete piece-part process.

Many production processes are affected by several assignable causes. In such situations, a single assignable cause model would seem inappropriate. Duncan (1971) has generalized his single assignable model to a situation in which there are s assignable causes, however, where different special causes will shift the process mean by different amounts. Duncan's multiple causes model is divided into two types: Model I presents "a single occurrence" model. It is assumed that once assignable cause A_i occurs, the process remains in that other assignable causes occur no longer till assignable cause A_i is detected; Model II presents "double occurrence". It is assumed that the model allows for the second occurrence of an assignable cause following the first occurrence and the joint effect of the two assignable causes is always to produce a shift of constant magnitude regardless of what two assignable causes occur jointly. The occurrence time of the assignable causes is assumed to be independent exponential random variable. Many papers were presented according to Duncan's model (1971), for example, George and Lee (1988), Jaraiedi and Zhuang (1991) and Chung (1994).

This paper adopts Duncan's multiple causes model (1971), Koo and Case's sampling scheme (1990) and cost structure of Koo et al. (1994) developing a new economic design of $\bar{x}$-control chart for continuous flow process which subjects to a multiplicity of special causes (denoted by A_i, $i=1,2,\ldots,s$). The definitions and assumptions are presented in Section 2. The model of average time and loss-cost is derived in Sections 3 and 4. Then, the selection of model parameters and sensitivity analysis are suggested in Section 5. Finally, the conclusions are presented in Section 6.

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2. Definition and assumption

The definitions and assumptions considered in our model are as follows:

(1) The time of assignable cause A_i that the process is in-control state follows an exponential distribution, the probability density function is given by $f(t)=\lambda e^{-\lambda t}$ for $t>0$, $\lambda>0$, then the probability is $\exp(-\lambda t)$ if no assignable cause has occurred at the end of time t.
(2) The time at which the process goes out of control is distributed as the minimum of n independent exponentials with means $1/\lambda_1, 1/\lambda_2, \ldots, 1/\lambda_n$ and thus has an exponential distribution with mean $1/\lambda$, where

$$\lambda=\sum_{i=1}^{n}\lambda_i. \tag{1}$$

(3) The process is normally distributed and characterized by an in-control state u_0, because of the occurrence of an assignable cause A_i which occurs at random, resulting in a shift in the mean from u_0 to either $u_0+\delta_i\sigma$ or $u_0-\delta_i\sigma$. Where u_0, σ and δ_i are, respectively, the process mean, the process standard deviation and shift parameter.
(4) The occurrence of an assignable cause A_i does not affect the process variability, that is, the process mean and the process variability are independent.
(5) The process mean is not shifting slowly, but instantaneously.
(6) The time to sample and draw control point is negligible and production ceases during the searches and repair.

In this paper, we propose two models. Model I, having been presented by Koo et al. (1994), assumes that the process is at any time in one of two states. Either it is in control or it has been distributed by the occurrence of an assignable cause A_i which produces a shift of $\delta_i\sigma$ in the process mean. The probability of δ_i is followed three prior distribution, respectively, negative-exponential $((1/2)\exp(-\delta_i/2))$, uniform and half-normal $((1/\sqrt{2\pi})\exp(-((0.5\delta_i)^2/2))$. Model II is to ascertain the effect of Model I that allows for the second occurrence of the assignable causes following the first cause A_i occurrence in the next subsequent subgroup. To simplify the analysis we assume the joint occurrence of the any two assignable causes in Model II always results in the same shift of $\Delta\delta\sigma$ in the process mean. Therefore, there is no need in the model to consider the prior distribution of second causes.

3. Formulation of the expected loss-cost of Model I

The paper by Koo et al. (1994) considered economic design of $\bar{x}$-control charts for using in continuous flow process—when there is a multiplicity of assignable cause.

3.1. Average cycle length

Koo et al. (1994) assume that there are s assignable causes in Model I. The occurrence times of the s $(s\to\infty)$ causes are assumed to be independently exponential distributed. After being disturbed by cause A_i, the process is assumed not to be affected by any other assignable causes. The probability of non-occurrence of multiple assignable causes is

$$\begin{aligned}P(T>t)&=P(A_1>t,A_2>t,\ldots,A_s>t)\\&=P(A_1>t)\cdot P(A_2>t),\ldots,P(A_s>t)=e^{-\lambda t},\end{aligned} \tag{2}$$

where $\lambda=\sum\lambda_i$ for $i=1,2,\ldots,s$.

Therefore, the occurrence average time of any various assignable causes is denoted as $AVGT$, and is

$$AVGT=\int_0^\infty t\sum_{i=1}^{s}\lambda_i\exp\left(-\sum_{i=1}^{s}\lambda_i t\right)dt=\frac{1}{\sum_{i=1}^{s}\lambda_i}=\frac{1}{\lambda}. \tag{3}$$

Define p_{ij} $(i=1,2,\ldots,s)$ as the probability that the assignable cause A_i will occur during the sampling interval jth and $(j+1)^{st}$, that is

$$P_{ij}=\frac{\int_{jh}^{(j+1)h}\lambda_i e^{-\lambda_i t}\,dt}{\int_0^{nh}\lambda_i e^{-\lambda_i t}\,dt}=\frac{e^{-\lambda_i(j+1)h}+e^{-\lambda_i jh}}{1-e^{-\lambda_i nh}}=\frac{e^{-\lambda_i jh}(1-e^{-\lambda_i h})}{1-e^{-\lambda_i nh}}\quad\text{for } i=1,2,\ldots,s. \tag{4}$$

When the process is out-of-control, the mean of the process will shift to $u_0+\delta_i\sigma$. If the shift occurs during the sampling interval jth and $(j+1)^{st}$, then the mean of the process in this subgroup will be $u=u_0+(n-j/n)\delta_i\sigma$. Let the probabilities of detecting an assignable cause in a shift occurring subgroup and the next subsequent subgroups after the occurrence of assignable cause A_i be P_i' and P_i, respectively, P_i' and P_i are formulated as follows:

$$P'_i=\sum_{j}^{n-1}P_{ij}\left[1-\Phi\left(k-\frac{n-j}{\sqrt{n}\delta_i}\right)+\Phi\left(-k-\frac{n-j}{\sqrt{n}\delta_i}\right)\right], \tag{5}$$

$$P_i=1-\Phi(k-\delta_i\sqrt{n})+\Phi(-k-\delta_i\sqrt{n}). \tag{6}$$

$\Phi(\cdot)$ is the cumulative density function of the standard normal distribution.

The average cycle length ($ACL1$) is expressed as follows:

$ACL1$=(a) average process of in-control state+(b) average time of out-of-control state.

These two average length components will be derived in turn:

(a) Since the average time for the occurrence of multiple assignable causes in exponential distribution with mean time $1/\lambda$, this is the average process in-control time.
(b) The average time that the process is out-of control until assignable cause A_i is detected and discovered is derived as follows:

Let the probability on the first sampling interval of a point falling outside the control limits after the occurrence of assignable cause A_i be P_i' and the probability on the other sampling interval of a point falling outside the control limits after the occurrence of assignable cause A_i be $(1-P_i')(1-P_i)^{r-2}P_i$ for $r=2,3,\ldots$.

Let τ_i be the expected time between the samples taken just prior to the occurrence of assignable cause A_i and the occurrence itself. That is

$$\begin{aligned}\tau_i&=\frac{\int_{jnh}^{(j+1)nh}e^{-\lambda_i t}\lambda_i(t-jnh)\,dt}{\int_{jnh}^{(j+1)nh}e^{-\lambda_i t}\lambda_i\,dt}=\frac{e^{-\lambda_i jnh}\int_0^{nh}te^{-\lambda_i t}\lambda_i\,dt}{e^{-\lambda_i jnh}\int_0^{nh}e^{-\lambda_i t}\lambda_i\,dt}\\&=\frac{1-(1+\lambda_i nh)e^{-\lambda_i nh}}{\lambda_i(1-e^{-\lambda_i nh})}.\end{aligned} \tag{7}$$

We obtain the expected time which is to detect the assignable cause after the process is shift, and the expected time is denoted as $AVGOOCT_i$, that is

$$\begin{aligned}AVGOOCT_i&=nh\left(P'_i+\sum_{r=2}^{\infty}r(1-P'_i)(1-P_i)^{r-2}P_i\right)-\tau_i\\&=P'_i+\frac{(1-P'_i)(1+P_i)}{P_i}=\frac{1-P'_i+P_i}{P_i}.\end{aligned} \tag{8}$$

Let D_i denote average time taken to find assignable cause A_i after it has caused a point to fall outside the control limits, let e denote the time of sampling, inspecting, evaluating and plotting. Then let B_i denote the average time that between the occurrence of cause A_i and its removal, that is, in out of control state, can be denoted by

$$B_i = AVGOOCT_i + e + D_i. \tag{9}$$

Under the condition, the occurrence rate of assignable cause A_i for s assignable causes is λ_i/λ, average time for a cycle will be denoted as $ACL1$, that is

$$ACL1 = \frac{1}{\lambda} + \frac{\sum \lambda_i}{\lambda} B_i \quad \text{for } i = 1,2,\ldots,s. \tag{10}$$

3.2. Loss-cost generated in average cycle length

Based upon the above derivation of average cycle length, formulation of average cost $E_1(C)$ is derived as follows:

1. Let M_i denote the increased loss per hour of operation due to the presence of assignable cause A_i. The expected additional loss per cycle arising from out-of-control conditions will be summary of $\lambda_i B_i M_i/\lambda$ and the expected additional loss per hour of operations will be

$$L_1 = \frac{(\sum \lambda_i/\lambda) B_i M_i}{ACL1} \quad \text{for } i = 1,2,\ldots,s. \tag{11}$$

2. Let ENF be the expected number of false alarm. ENF depends on type I error α and the number of sampled subgroups before the assignable cause occurred. That is

$$ENF = \alpha \sum_{j=0}^{\infty} j \int_{jnh}^{(j+1)nh} \lambda e^{-\lambda t}\, dt$$
$$= \alpha(1-e^{-\lambda nh}) \sum_{j=0}^{\infty} j e^{-\lambda jnh} = \alpha \frac{e^{-\lambda nh}}{1-e^{-\lambda nh}}, \tag{12}$$

where $\alpha = 2[1-\Phi(k)]$.
Let T be the average cost of looking for an assignable cause when a false alarm occurs and the average time to search false alarm is $T\,ENF$. The expected cost per hour of operations on this account will be

$$L_2 = \frac{ENF\,T}{ACL1}. \tag{13}$$

3. Let W_i denote the average cost of finding assignable cause A_i when it occurs. Then the expected cost per cycle of discovering assignable causes will be $\sum \lambda_i W_i/\lambda$ and the expected cost per hour of operations on this account will be

$$L_3 = \frac{(\sum \lambda_i/\lambda) W_i}{\text{ACL1}} \quad \text{for } i = 1,2,\ldots,s. \tag{14}$$

4. Let b denote the fixed cost per sampling of sampling, inspecting, evaluating and plotting and let c denote the variable cost per item of sampling, inspecting, evaluating and plotting. The average hourly cost of maintaining the control chart is

$$L_4 = \frac{b+cn}{nh} = \frac{b}{nh} + \frac{c}{h}. \tag{15}$$

Hence for Model I we have the expected loss-cost equal to

$$E_1(C) = L_1 + L_2 + L_3 + L_4.$$

Our objective is to find the optimal design parameter. The parameters, n, h and k, are given by time, cost and shift parameters of s assignable causes. The optimal solution of the loss-cost function $E_1(C)$ is minimized.

4. Formulation of expected loss-cost of Model II

4.1. Average cycle length

To ascertain of effect of this assumption, a study is made in this section of a model that allows for the second occurrence of an assignable cause in a later intervals following the first occurrence. The process is assumed to be in one of the three states. It is (1) in a state of in-control or (2) it has been disturbed by the occurrence of an assignable cause A_i which produces a shift of $\delta_i\sigma$ in the process mean or (3) it has been disturbed by the occurrence of a second assignable cause following the first, the joint effect of which in every case is arbitrarily assumed to produce a shift of $\Delta\delta\sigma$ in the process mean. As in Model I, the occurrence times of the various assignable cause are assumed to be independently exponentially distributed with mean time $1/\lambda_i$ for $i = 1,2,\ldots,s$. The occurrence time of the first assignable cause has an exponential distribution with mean time $1/\lambda$ where λ is the summation of λ_i and the occurrence time of a second assignable cause has an exponential distribution with mean time $1/\lambda'$ where λ' is a function of λ. The average time for cycle that can be derived for the each state is as follows: The process is in a state of in-control and the average time that the assignable cause will occur is $1/\lambda$.

(1) *State* 1: The process is in a state of in-control and the average time that the assignable cause will occur is $1/\lambda$.

(1) *State* 2: The process has been disturbed by the occurrence of the first assignable cause A_i and produces a shift of $\delta_i\sigma$ in the process mean. The process can be classified into two situations.

Situation 1: Consider the situation in Fig. 1. The process is the period that a second assignable cause will be not to occur until assignable cause A_i detected. Based upon the above Eqs. (5) and (6), the probability that a point falls outside the control limits at the first sampling interval or at the other sampling interval after the occurrence of the first assignable cause is P_i' or $(1-P'_i)e^{-\lambda' nh}[(1-P_i)e^{-\lambda' nh}]^{r-2}P_i, r = 2,3,\ldots$.

Let τ_i be the average time between the sample taken just prior to the occurrence of assignable cause A_i and the occurrence itself. Define τ_i is the same as Eq. (7).

Then let the average time in situation 1 be E_1. The E_1 is

$$E_1 = nh\left\{P'_i + \sum_{r=2}^{\infty} r\left[(1-P'_i)e^{-\lambda' nh}((1-P_i)e^{-\lambda' nh})^{r-2}P_i\right]\right\} - \tau_i + e + D_i$$

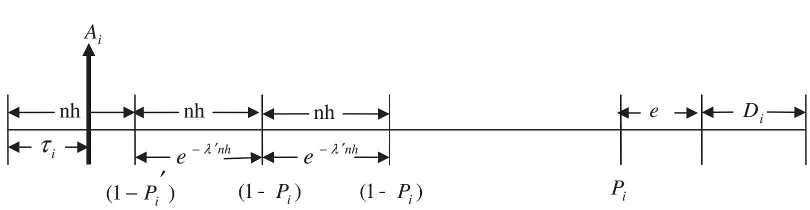

Fig. 1. The process of situation 1.

$$=nh\left[P'_i+r'_iP_i\frac{2-r_i}{(1-r_i)^2}\right]-\tau_i+e+D_i, \tag{16}$$

where $r'_i=(1-P'_i)e^{-\lambda' nh}$ and $r_i=(1-P_i)e^{-\lambda' nh}$.

Situation 2: Consider the situation in Fig. 2. The process is from the occurrence of the first assignable A_i to the occurrence of a second assignable cause, during the period that the cause A_i is never to be detected. The conditional probability that a second assignable cause will occur between the first and the second subgroup is $(1-P_i')(1-e^{-\lambda' nh})$, then will occur between the $(r+1)$ and $(r+2)$ subgroup is $(1-P_i')(1-e^{-\lambda' nh})[(1-P_i)e^{-\lambda' nh}]^{r-1}$ for $r=1,2,\ldots$.

Let the average time in situation 2 be E_2. The E_2 is

$$E_2=nh\left\{\sum_{r=1}^{\infty}r(1-P'_i)(1-e^{-\lambda' nh})\left[(1-P_i)e^{-\lambda' nh}\right]^{r-1}\right\}-\tau_i+\tau'$$

$$=nh\left[\frac{1-P'_i-r'_i}{(1-r_i)^2}\right]-\tau_i+\tau', \tag{17}$$

where τ_i is given by Eq. (7), τ' is the same formula and definition with λ' replacing λ_i.

It follows from what has been derived that the average time of the *State* 2, respectively, is $E[State2]$:

$$E[State2]=E_1+E_2$$

$$\left\{nh\left[P'_i+r'_iP_i\frac{2-r_i}{(1-r_i)^2}\right]-\tau_i+e+D_i\right\}+\left\{nh\left[\frac{1-P'_i-r'_i}{(1-r_i)^2}\right]-\tau_i+\tau'\right\}. \tag{18}$$

(3) *State* 3: Consider the situation in Fig. 3. The process is from the occurrence of the first assignable A_i to the joint assignable cause detected. The joint effect is to produce a shift of $\Delta\delta\sigma$ in the process mean. Define $\beta_0=\Phi(L-\Delta\delta\sqrt{n})-\Phi(-L-\Delta\delta\sqrt{n})$ is the probability that a point falls inside the control limits after the occurrence of a second assignable cause, then the probability that the joint assignable cause detected is $1-\beta_0$.

Define $\hat{p}_{ij}$ $(i=1,2,\ldots,s)$ as the probability that a second assignable cause will occur during the sampling interval jth and $(j+1)^{st}$ after the occurrence of assignable cause A_i, and p_{ij} is given by (4), $\hat{p}_{ij}$ is the same formula with λ' replacing λ_i.

Let the probabilities of detecting joint assignable causes in a shift occurring subgroup and the next subsequent subgroups be

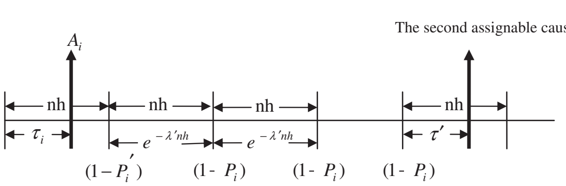

Fig. 2. The process of situation 2.

$\hat{P}'_i$ and $\hat{P}_i$, respectively, $\hat{P}'_i$ and $\hat{P}_i$ are formulated as follows:

$$\hat{P}'_i=\sum_{j=0}^{n-1}\hat{P}_{ij}(1-\beta_0)=\sum_{j=0}^{n-1}\left[\frac{(1-e^{-\lambda' h})e^{-\lambda' jh}}{(1-e^{-\lambda' nh})}\right]\left[1-\Phi\left(k-\frac{n\Delta\delta+j(\delta_i-\Delta\delta)}{\sqrt{n}}\right)\right.$$
$$\left.+\Phi\left(-k-\frac{n\Delta\delta+j(\delta_i-\Delta\delta)}{\sqrt{n}}\right)\right], \tag{19}$$

$$\hat{P}_i=1-\Phi(k-\Delta\delta\sqrt{n})+\Phi(-k-\Delta\delta\sqrt{n}). \tag{20}$$

Let the probability on the first sampling interval of a point falling outside the control limits after the occurrence of joint effect be $\hat{P}'_i$ and the probability on the other sampling interval of a point falling outside the control limits after the occurrence of joint effect be $(1-\hat{P}_i')(1-\hat{P}_i)^{r-2}\hat{P}_i$ for $r=2,3,\ldots$.

Let D' denote the average time taken to find the combined assignable causes after a point has fallen outside the control limits when the process is in *State* 3, assumed to be independent of the assignable causes. The D' is not changed by the joint effect of cause A_i and a second assignable cause, then the average time of *State* 3 is given by $E(State3)$, will be

$$E(State3)=\left\{nh\left[\hat{P}'_i+\sum_{r=2}^{\infty}r(1-\hat{P}'_i)(1-\hat{P}_i)^{r-2}\hat{P}_i\right]-\tau'+e+D'\right\}\left(\frac{1-P'_i-r'_i}{(1-r_i)}\right)$$

$$=\left[nh\left(\frac{1+\hat{P}_i-\hat{P}_i'}{\hat{P}_i}\right)-\tau'+e+D'\right]\left(\frac{1-P'_i-r'_i}{(1-r_i)}\right). \tag{21}$$

Summing up the various average time of *States* 1–3 and finding the average cycle length of Model II will be

$$ACL2=\frac{1}{\lambda}+\frac{\sum_{i=1}^{s}\lambda_i}{\lambda}(E(State2)+E(State3)). \tag{22}$$

4.2. Loss-cost generated in average cycle length

Based upon the above derivation of average cycle length, formulation of average cost $E_2(C)$ is derived as follows:

1. Let M_i denote the increased loss per hour of operation due to the presence of assignable cause A_i in *State* 2 and M' denote the additional loss per hour of operations when the process is in *State* 3. The average hourly loss when out of control is

$$L_1=\frac{(\sum_{i=1}^{s}\lambda_i/\lambda)(E(State2)M_i)+(\sum_{i=1}^{s}\lambda_i/\lambda)(E(State3)M')}{\mathrm{ACL2}}. \tag{23}$$

2. The expected number of false alarms before the process goes out-of-control will be the probability of a false alarm (α) times the expected number of subgroups taken in an ;in-control period. Hence, the expected number (ENF) of false alarms per hour of operation will be $ENF=\alpha[\exp(-\lambda nh)/(1-\exp(-\lambda nh))$. Thus, the average hourly false-alarm cost is

$$L_2=\frac{ENF\,T}{ACL2}, \tag{24}$$

where T is the average cost of looking for an assignable cause when a false alarm occurs.

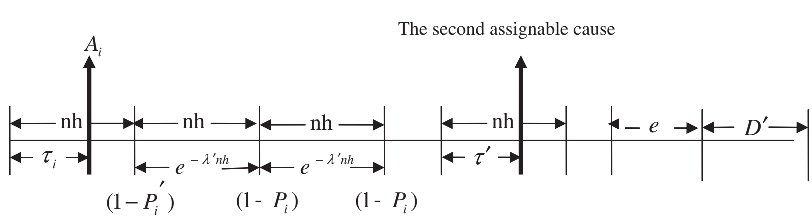

Fig. 3. The process of state 3.

3. Let W_i denote the average cost of finding assignable cause A_i when it occurs and W' denote the cost of finding the combined assignable causes, assumed to be independent of the assignable causes. The average hourly cost of finding and repairing the assignable cause is

$$L_3 = \frac{(\sum_{i=1}^{s} \lambda_i/\lambda)\left[W_i\left(1-\frac{1-P'_i-r'_i}{1-r_i}\right)\right]+(\sum_{i=1}^{s} \lambda_i/\lambda)\left[W'\left(1-\frac{1-P'_i-r'_i}{1-r_i}\right)\right]}{ACL2}. \tag{25}$$

4. Let b denote the fixed cost per sampling of sampling, inspecting, evaluating and plotting and c denote the variable cost per item of sampling, inspecting, evaluating and plotting. The average hourly cost of maintaining the control chart is

$$L_4 = \frac{b+cn}{nh} = \frac{b}{nh} + \frac{c}{h}. \tag{26}$$

Hence for Model II, we have the expected loss-cost equal to

$E_2(C) = L_1 + L_2 + L_3 + L_4.$

The optimal design parameters n, h and k for Model II is to minimize the loss-cost function $E_2(C)$.

5. Determination of optimal design parameters

5.1. Selection of model parameters

In the expression for the total expected loss-cost of operations ($E_1(C)$ and $E_2(C)$) certain quantities can be classified into five kinds of parameters. There are cost parameters T, b, c, M_i, W_i; time parameters e, D_i; shift parameters δ_i; exponential distribution parameters λ_i and design parameters n, h and k for Model I. And addition cost parameters M', W'; time parameters D'; shift parameters $\Delta\delta$ and exponential distribution parameters λ' for Model II. A numerical example will be used to illustrate of Koo and Case Model (1990), value for $T=2000$, $b=20$, $c=20$, $M'=4000$, $W'=1000$, $\Delta\delta=2$, $\lambda'=0.02$, $D'=2$, $e=1.25$ are not changed by assignable cause A_i. The M_i, W_i, D_i and λ_i are taken to be a function of δ_i, the rule of selection is as follows:

(1) The λ_i is a non-increasing function of δ_i. When the cause A_i occurs, μ_0 shift to $\mu_0+\delta_i\sigma$, M_i is proportional to the resulting increase in the percent of product outside specification $(1-\beta_i)$, where $\beta_i=\Phi(3-\delta_i)$. As δ_i varies above and below 2, the percent beyond specifications increases and decreases to cause corresponding variations in M_i.
(2) Assume the process exists seven assignable causes (A_i,$i=1,2,...,7$), those causes will produce 1σ, 1.5σ, 1.8σ, 2σ, 2.2σ, 2.5σ and 3σ shift , the occurrence of each assignable cause randomly and indecently produce single shift.
(3) Like D_i, M_i and W_i are function of δ_i. For $\delta_i=2$, M_i, W_i and D_i is equal to 4000, 1000 and 2.
(4) When the parameters, T, b, c and e, are kept fixed, the numerical examples used. For example, the parameter $\Delta\delta$ is varied from 1–1.5 to 2–2.5. The parameter λ' is varied from 0.005–0.01 to 0.02–0.04. Then D', M' and W' are also the same as D_i, M_i and W_i of $\delta_i=2$ for Model I. Owing to obtain λ_i, assume $\sum \lambda_i M_i = \lambda M' = 80$ for $i=1,2,\ldots,s$.
(5) When the parameters, $\Delta\delta$, λ', D', M' and W', are kept fixed, the numerical examples used. For example, the parameter T is varied from 1000–2000 to 3000. The parameter b is varied from 10–20 to 30. The parameter c is varied from 10–20 to 30. The parameter e is varied from 0.625–1.25 to 1.875.

Let PD_i denoted prior distribution of $\delta_i(\delta_1=1,\ \delta_2=1.5,\ \delta_3=1.8,\ldots,\delta_7=3)$. In this study, the negative-exponential, uniform and half-normal are considered for PD_i. Prior distribution of $\delta_4=2$ is PD_4, we set up the time and cost values for $\delta_4=2$ as "base case", and in one set, λ_i are chosen as proportional to PD_i. According to the discussion of above (1), (2), (3), (4) and (5) rules, we have $W_i=(PD_i/PD_4)1000$, $D_i=((PD_iPD_4)2$, $M_i=(PD_i/PD_4)4000$ and $\lambda_i=(PD_i/PD_1)\lambda_1$. The values of W_i, D_i and λ_i for different prior distribution and the values of M_i for different δ_i are listed in Table 1.

5.2. Effects of changes in the cost parameters of Model II

We used search technique which is developed by Rahim (1993) to determine the optimal design parameters. The code was considered to minimize loss-cost, and provides economically optimal values of n, h and k. The effects of changes in the cost parameters on the minimum loss-cost design are listed in Table 2 along with other data. Table 2 suggests the following general conclusions:

(a) For $\lambda'=0$, the loss-cost function $E_2(C)$ of Model II is equal to loss-cost function $E_1(C)$ of Model I. Therefore, one result from economic design for Model II stood out clearly. It was noted that if λ' is decreased, the loss-cost of Model II approaches to the loss-cost of Model I.
(b) With the same value of the parameters in both models, the loss-cost of Model II is larger than the loss-cost of Model I, but smaller than the loss-cost of the Koo and Case model. If the conservative designing point of view is applying, the multiplicity-cause model can replace the single-cause model.
(c) T, b, c and e are kept fixed at the reference values listed in Table 2. Variation in $\Delta\delta$, D' and W' have little effect on the loss-cost, but variation in λ' and M' have their dominant effect on the loss-cost.
(d) Among the economic design for three prior distributions, the negative exponential prior distribution is the best while

Table 1
The reference set of cost and probability parameters for Model II.[a]

A_i	δ_i	$1-\Phi(3-\delta_i)=1-\beta_i$	Prior distribution (PD_i)			M_i	D_i			W_i			$\lambda_i\ (\times 10^{-3})$		
			NE	*Un*	*HN*		*NE*	*Un*	*HN*	*NE*	*Un*	*HN*	*NE*	*Un*	*HN*$_i$
1	1	0.0228	0.303	0.143	0.352	575	3.293	2	2.909	1647	1000	1454	4.566	2.294	4.220
2	1.5	0.0668	0.236	0.143	0.301	1684	2.565	2	2.488	1283	1000	1244	3.557	2.294	3.608
3	1.8	0.1151	0.203	0.143	0.266	2901	1.103	2	2.198	1103	1000	1099	3.059	2.294	3.190
4	2	0.1587	0.184	0.143	0.242	4000	2	2	2	1000	1000	1000	2.772	2.294	2.901
5	2.2	0.2119	0.166	0.143	0.218	5341	1.804	2	1.802	902	1000	901	2.502	2.294	2.612
6	2.5	0.3085	0.143	0.143	0.183	7776	1.554	2	1.512	777	1000	756	2.155	2.294	2.194
7	3	0.5	0.112	0.143	0.130	12602	1.217	2	1.074	609	1000	537	1.689	2.294	1.557

[a] *NE*, negative-exponential; *Un*, uniform; *HN*, half-normal.

Table 2
Optimum design parameters for Model II at three different prior distributions.[a]

T	b	c	e	$\Delta\delta$	λ'	M'	D'	W'	Prior distribution	Optimal design			loss-cost
										n	h	k	
2000	20	20	1.25	1	0.02	4000	2	1000	*NE*	3	0.354	2.465	427.712
									Un	3	0.352	2.568	442.129
									HN	3	0.365	2.495	431.020
				1.5					*NE*	3	0.363	2.595	424.329
									Un	2	0.451	2.587	438.291
									HN	3	0.346	2.592	427.627
				2					*NE*	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
				2.5					*NE*	2	0.401	2.569	429.645
									Un	2	0.415	2.617	440.229
									HN	2	0.428	2.524	433.159
				2	0.005	4000	2	1000	*NE*	2	0.483	2.512	417.650
									Un	2	0.442	2.637	433.239
									HN	2	0.464	2.504	421.480
					0.01				*NE*	2	0.438	2.545	420.378
									Un	2	0.510	2.507	435.130
									HN	2	0.442	2.529	424.079
					0.02				*NE*	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
					0.04				*NE*	2	0.423	2.481	433.107
									Un	2	0.457	2.565	442.146
									HN	2	0.407	2.502	436.311
				2	0.02	575	2	1000	*NE*	2	0.444	2.585	410.458
									Un	2	0.505	2.556	427.322
									HN	2	0.444	2.585	414.505
						1684			*NE*	2	0.459	2.514	415.286
									Un	2	0.460	2.545	430.957
									HN	2	0.496	2.500	419.230
						4000			*NE*	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
						7776			*NE*	2	0.461	2.407	440.185
									Un	2	0.455	2.518	447.608
									HN	2	0.434	2.418	442.769
2000	20	20	1.25	2	0.02	4000	3.3	1000	*NE*	2	0.400	2.555	428.969
							0.5		*Un*	2	0.416	2.619	435.097
							2.9		*HN*	2	0.400	2.555	431.435
							2.6		*NE*	2	0.400	2.555	427.119
							1		*Un*	2	0.483	2.565	435.798
							2.5		*HN*	2	0.487	2.445	430.322
							2		*NE*	2	0.487	2.445	425.596
							2		*Un*	2	0.433	2.621	437.589
							2		*HN*	2	0.464	2.498	428.724
							1.6		*NE*	2	0.429	2.527	423.969
							3		*Un*	2	0.433	2.621	439.453
							1.5		*HN*	2	0.429	2.527	427.420
				2	0.02	4000	2	1647	*NE*	2	0.487	2.445	426.058
								250	*Un*	2	0.472	2.512	437.242
								1454	*HN*	2	0.487	2.445	429.413
								1283	*NE*	2	0.487	2.445	425.798
								500	*Un*	2	0.472	2.512	437.362

Table 2 (*continued*)

T	b	c	e	$\Delta\delta$	λ'	M'	D'	W'	Prior distribution	Optimal design			loss-cost
										n	h	k	
								1244	*HN*	2	0.464	2.498	428.898
								1000	*NE*	2	0.487	2.445	425.595
								1000	*Un*	2	0.433	2.621	437.589
								1000	*HN*	2	0.464	2.498	428.724
								777	*NE*	2	0.487	2.445	425.437
								1500	*Un*	2	0.483	2.565	436.061
								756	*HN*	2	0.464	2.498	428.551
1000	20	20	1.25	2	0.02	4000	2	1000	*NE*	2	0.436	2.250	410.201
									Un	2	0.493	2.389	424.369
									HN	2	0.436	2.250	413.827
2000									*NE*	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
3000									*NE*	3	0.315	2.756	433.948
									Un	2	0.432	2.697	446.118
									HN	3	0.324	2.761	437.141
2000	10	20	1.25						*NE*	2	0.411	2.541	413.229
									Un	2	0.390	2.645	425.844
									HN	2	0.411	2.541	416.752
	20								*NE*	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
	30								*NE*	2	0.465	2.486	435.951
									Un	2	0.538	2.482	448.340
									HN	2	0.445	2.506	439.800
2000	20	10	1.25						*NE*	3	0.271	2.751	393.807
									Un	3	0.288	2.722	408.808
									HN	3	0.257	2.743	397.111
		20							*NE*	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
		30							*NE*	2	0.482	2.454	446.222
									Un	2	0.569	2.486	458.027
									HN	2	0.544	2.389	449.364
2000	20	20	0.625						*NE*	2	0.432	2.522	382.936
									Un	2	0.467	2.563	393.987
									HN	2	0.432	2.498	428.724
			1.25						*NE*	2	0.487	2.522	386.522
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	449.364
			1.875						*NE*	2	0.464	2.498	466.341
									Un	2	0.472	2.512	480.262
									HN	2	0.464	2.498	469.766

[a] *NE*, negative-exponential; *Un*, uniform; *HN*, half-normal.

uniform prior distribution is the worse. But the difference between the loss-cost for half-normal and negative- exponential prior distribution is less than 0.7%.

(e) For the negative-exponential prior distribution, variation in λ' has its primary effect upon the optimal value of k. The sample size and the frequency of sampling are affected moderately. Thus for large λ' we should use charts with 2.5 sigma limits.

(f) For the negative-exponential prior distribution, variation in M' has its dominant effect on the optimal value of k. When M' is relatively large, k should be small; when M' is relatively small, k should be large. Variation in M' has little effect on the optimal values of n and h.

(g) $\Delta\delta$, λ', M', D' and W' are kept fixed at the reference values listed in Table 2. Variation in T and b has little effect on the loss-cost, but variation in c and e has more effect on the loss-cost.

(h) For the negative-exponential prior distribution, variation in c affects all three of the elements of design. For high values of c, the optimal design calls for taking small samples, possibly only samples of 2, at large intervals between samples and with control limits at low multiples of sigma.

(i) For the negative-exponential prior distribution, variation in e affects primarily the optimal value of k, possibly we should use charts with 2.5 sigma limits. It also has a moderate affect on the frequency of sampling.

6. Conclusions

In practice, multiple assignable causes are more realistic than the single ones. From an economic viewpoint, a study is conducted in this paper that allows for the second occurrence of an assignable cause following the first occurrence. We depict the detailed development of an economic model for the optimal design of $\bar{x}$-control chart for continuous flow process. The process-failure mechanism is assumed with multiple assignable causes and each assignable cause follows an exponential distribution. Solutions of the optimal design parameters, n, h and k, have been obtained according to the different values of the model parameters. The optimal economic design of Model II is listed in Table 2. Overall, this paper advances economically-based $\bar{x}$-control chart to the important area of multiple assignable causes process in continuous flow process.

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