Contents lists available at ScienceDirect

ELSEVIER



Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

A hybrid dynamic forecast model for analyzing celebrity endorsement effects on consumer attitudes

Jiuh-Biing Sheu*

Institute of Traffic and Transportation, National Chiao Tung University, 4F, 114 Chung Hsiao W. Rd., Sec. 1, Taipei 100, Taiwan, ROC

ARTICLE INFO

Article history: Received 20 November 2009 Received in revised form 7 June 2010 Accepted 8 June 2010

Keywords: Dynamic forecast model Fuzzy analytical hierarchy process Nonlinear stochastic system Celebrity endorsement

ABSTRACT

This study investigates the time-varying effects of celebrity endorsements on consumer purchase attitudes toward promoted products using a novel dynamic hierarchical multi-attribute attitude forecast model. The induced direct and indirect effects via constructs of product attributes and net product value are then incorporated into the proposed conceptual model, which is formulated with a discrete-time nonlinear stochastic system. An empirical study of product categories of sport shoes and notebook computers demonstrates the feasibility of the proposed methodology. The analytical results demonstrate the capability of the proposed model to forecast consumer attitudes toward a promoted product attributes, price, and endorser performance as perceived by consumers. Furthermore, we infer that celebrity endorsement can significantly influence consumer purchase attitudes via both direct and indirect effects through product attribute via both direct and indirect effects through product attribute such and reveal the other and indirect effects through product attribute such as perceived by consumers.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

Celebrity endorsement is a ubiquitous advertising form in modern marketing. Approximately 25% of American commercials feature a famous spokesperson [1]; such celebrity use is as high as 70% in Japan [2]. Most marketers accept that investing enormous amounts of money in celebrity endorsement is worthwhile as celebrities make advertisements attractive [3], and enhance message recall based on celebrity profiles and attributes, thereby engaging potential consumers [4], and aiding in brand marketing [5,6]. Most importantly, this advertising strategy is particularly suitable for promoting products high in psychological and social risks as celebrity endorsement can increase the likelihood of consumers to purchase endorsed products [7,8].

Despite the popularity of celebrity endorsement as a modern advertising technique, the resulting effect on enterprise profitability and marketing effectiveness has garnered considerable interest. In the case of Tiger Woods, Farrell et al. [9] noted that the activities of a celebrity endorser are observable. However, the associated endorsement effects on company performance and value are seldom investigated in the literature. Walker et al. [10] argued that despite the billions of dollars spent in celebrity endorsement annually, the success and effectiveness of this advertising strategy remain controversial. Agrawal and Kamakura [11] argued that a dynamic assessment of the effectiveness of a celebrity endorsement on a firm's profitability is likely impossible because advertising effects generally change over time, and, furthermore, enterprise profits are not likely based solely on celebrity endorsements.

^{*} Tel.: +886 2 2349 4963; fax: +886 2 2349 4953. *E-mail address:* jbsheu@mail.nctu.edu.tw.

^{0895-7177/\$ –} see front matter s 2010 Elsevier Ltd. All rights reserved. doi:10.1016/j.mcm.2010.06.020

J.-B. Sheu / Mathematical and Computer Modelling 52 (2010) 1554-1569

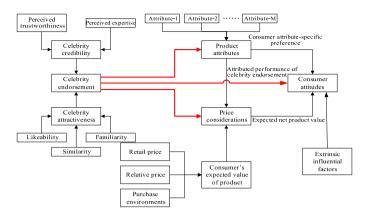


Fig. 1. Conceptual framework of the proposed model.

The problems in determining endorsement effectiveness may stem from the complexity of consumer attitudes toward an endorsed product; these attitudes can change over time due to the changes in consumer perceptions of an endorser (e.g., attractiveness and credibility) and attitudinal changes toward a product [12]. Tom et al. [13] further noted that the use of celebrity endorsers is somewhat risky as endorsers often engage in activities that degrade their image in the eyes of the public. Furthermore, a number of studies have analyzed celebrity endorsement by identifying various source attributes using two model categories: source credibility [8,14] and source attractiveness [7,15].

Although some studies have analyzed the factors influencing celebrity endorsement and the induced endorser–product congruence phenomenon, few have developed quantitative models for investigating the time-varying effects of celebrity endorsements on changes in consumers purchase attitudes, resulting in a lack of a model for promotional consumer demand management. Briefly, considerable research is needed to fill this gap between celebrity endorsement and consumers purchase attitudes.

This study assesses the time-varying effect of endorsement performance on consumer attitudes using a quantitative method. Based on forecasts of consumer attitudes updated during each time interval, analytical results can help marketers assess the promotional performance of an endorser and adopt collaborative marketing and advertising strategies in response to changes in consumer attitudes via promotional demand management. The proposed model is based on the individual decision domain; thus, the primary influential factors are product attributes, price considerations, and celebrity endorsement combined with their sub-factors, which are hierarchically incorporated into a conceptual framework at the individual decision level. The principal contribution of the study is that it provides a dynamic forecasting model to gain insights into the time-varying effects of celebrity endorsement on consumer attitudes toward expected value and attributes of a promoted product. Thus, one can identify consumer purchase decisions and the strategic responses of firms. To the best of our knowledge, this methodology for forecasting the time-varying effects of celebrity endorsement is novel.

2. Conceptual framework

This section introduces a framework for identifying consumer attitudes toward endorsed products. Drawing from concepts in the basic multi-attribute attitude model [16] and balance model [17], we hypothesize that a consumer's instantaneous purchase attitude toward a promoted product endorsed by a celebrity is mainly determined by three constructs: (1) product attributes; (2) price considerations; and, (3) attributed performance of the celebrity (Fig. 1). Attributed performance of an endorser is performance as perceived by consumers. Briefly, the attributed performance of a celebrity is the primary measure of an enterprise's promotional strategies for influencing consumer attitudes toward a promoted product via its direct and indirect effects on consumer assessments of product price and attributes. The rationales for using these three constructs are as follows.

2.1. Celebrity endorsement

In this study, the celebrity endorsement construct is an independent variable controlled by the seller and used to improve customer purchase attitudes toward promoted products. The numerous studies of consumer behavior that used multiattribute attitude models based on weighted beliefs provide significant evidence indicating that consumer attitudes can be changed in one of the three ways—by changing beliefs, changing weights, or adding one or more new weighted beliefs [18]. Thus, purchase attitude can be considered the overall evaluation of a product driven by consumer affective and cognitive responses [12]. Based on previous studies, we infer that through the use of celebrity endorsement as a catalyst, customerperceived performance of a celebrity endorser (called the attributed performance of a celebrity endorsement) contributes to the direct celebrity endorsement effect. Additionally, an endorsement may change consumer affective (e.g., emotions, feelings, and moods) and cognitive responses (e.g., understanding and thinking) toward a product, thus indirectly affecting consumer purchase attitudes.

2.2. Product attributes

Product attributes determine the characteristics of a consumer's decision process. According o conventional theories of buyer behavior, consumer attitudes toward a product are the key input in the consumer decision-making process [19,20]. As noted by Lancaster [19], a consumer derives utility not from the product itself but rather from certain product properties, indicating that consumer utility is a function of product attributes rather than the product itself. Howard [20] focused on the stimuli from product attributes and symbols viewed as the dominant characteristics driving consumer purchase intention. Govoni et al. [21] modeled a product as the total collection of attributes that may include basic physical aspects and special functions, as well as such intangible characteristics as image, reputation, and brand. Accordingly, the product-attribute construct is incorporated into the proposed model.

2.3. Price considerations

Despite the various price perspectives in marketing [22–24], product price must reflect, to a certain extent, consumer perception of the value of product attributes. This study attempts to characterize price factors using two psychological dimensions – (1) price stimuli and (2) price responses – to derive their effects on consumer purchase attitudes. Thus, we argue that retail price combined with relative price [25] and instantaneous purchase environmental effects (e.g., perceived store image and in-store environments) are price stimuli for products targeted to consumer, followed by consumer-expected value of a product in response to perceived price stimuli. Therefore, a consumer may assess net product value by subtracting retail price from expected product price to determine net product value, which is either positive or negative.

3. Methodology

Based on this conceptual framework, we argue that a consumer's (denoted by *i*) instantaneous purchase attitude $(\Theta_{ij}^k(n))$ toward a promoted product (denoted by *j*) advertised by celebrity *k* on a given purchase occasion *n* is composed of deterministic $(\Theta_{ii}^k(n))$ and stochastic $(\varepsilon(n))$ terms; thus,

$$\Theta_{ii}^{k}(n) = \theta_{ii}^{k}(n) + \varepsilon(n) \tag{1}$$

where $\varepsilon(n)$ is independent of $\theta_{ij}^k(n)$, and possibly caused by unknown extrinsic variables. Now, this work rationalizes the structure of $\theta_{ij}^k(n)$ using a multi-attribute attitude form composed of three main constructs, *i.e.*, celebrity endorsement $(E_{ij}^k(n))$, product attributes $(A_{ij}^k(n))$, and price considerations $(P_{ij}^k(n))$, based on the proposed conceptual framework. Thus, $\theta_{ii}^k(n)$ is given by

$$\theta_{ii}^{k}(n) = W_{iA}^{k}(n)A_{ii}^{k}(n) + W_{iP}^{k}(n)P_{ii}^{k}(n) + W_{iF}^{k}(n)E_{ii}^{k}(n)$$
⁽²⁾

where $A_{ij}^k(n)$, $P_{ij}^k(n)$, and $E_{ij}^k(n)$ are the instantaneous evaluations of consumer *i* with respect to the collection of attributes of product *j*, net product value, and celebrity endorser *k* on a given purchase occasion *n*, respectively; and $W_{iA}^k(n)$, $W_{iP}^k(n)$, and $W_{iE}^k(n)$ are the time-varying strengths of consumer beliefs associated with $A_{ij}^k(n)$, $P_{ij}^k(n)$, and $E_{ij}^k(n)$, respectively. As consumer purchase attitudes may change over time [12], these consumer evaluations and belief strengths are defined as dynamic forms to rationalize the time-varying features of these variables and parameters.

According to Lavidge and Steiner [26], $A_{ij}^k(n)$ and $P_{ij}^k(n)$ are likely affected by attributed performance of celebrity endorsement in any one of four phases (awareness, knowledge, liking, and preference) utilized in the consumer purchase decision process. For instance, a consumer may prefer product *A* over *B* due to admiration for the celebrity endorsing product *A*, who thereby increases consumer preference for product *A*. If the celebrity is well known and has considerable experience and expertise associated with the promoted product, consumer-expected product price or quality may increase, leading to a cognitive response that increases consumer belief in a celebrity endorsed product [8]. Accordingly, we propose that $A_{ij}^k(n)$ and $P_{ij}^k(n)$ can be functions of $E_{ij}^k(n)$, which are derived as follows:

3.1. Celebrity endorsement construct

Assessing endorsement performance $(E_{ij}^k(n))$ is a critical step as it is the independent construct influencing other constructs in the proposed conceptual model. Notably, the entire conceptual framework is rooted in individual decision contexts, where $E_{ij}^k(n)$ is assessed based on consumer perceptions. As expected, such an assessment problem can involve consumers scoring multiple criteria, such as credibility and attractiveness, in qualitative domains. Considering the fuzziness

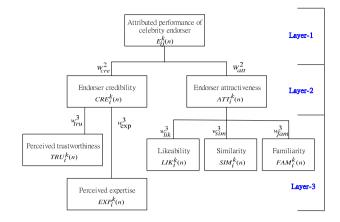


Fig. 2. Hierarchical architecture for assessing celebrity endorser performance.

and vagueness that may be encountered by consumers when assessing celebrity endorser performance, this study constructs the assessment framework for the celebrity endorsement construct using the fuzzy analytical hierarchy process (Fuzzy-AHP¹). According to Bouyssou et al. [27], Fuzzy-AHP is a method particularly suited to dealing with decision-making problems with multiple criteria in qualitative domains, in which a finite number of decision alternatives are evaluated via a finite number of performance criteria.

First, based on the previous work [6,14], this study uses credibility and attractiveness as the two primary criteria for assessing celebrity endorser performance (*i.e.*, $E_{ij}^k(n)$). This study accepts the argument that consumer attitudes may not change significantly without external stimuli, such as promotional and advertising activities, including celebrity endorsement advertising strategies. However, this study considers the direct and indirect effects of celebrity endorsement on consumer attitudes. We argue that such endorsement effects are time-varying as they may rely on instantaneous assessment of $E_{ij}^k(n)$ by consumers in the cognitive and affective domains, *i.e.*, credibility and attractiveness, respectively. This study uses perceived trustworthiness and expertise of an endorser as two sub-criteria to characterize endorser credibility in the cognitive domain. Conversely, attractiveness is considered the main criterion for assessing a celebrity endorser in the affective domain; thus, three related sub-criteria – familiarity, likeability and similarity – are used to assess attributed performance of a celebrity endorser in the affective domain. Accordingly, a three-layer analytical hierarchy (Fig. 2) is constructed; the top layer (layer 1) is the attributed performance of a celebrity endorser (*i.e.*, $E_{ij}^k(n)$); the second layer (layer 2) consists of the two main criteria, *i.e.*, credibility and attractiveness; and the third layer (layer 3) is composed of sub-criteria associated with the main criteria.

Accordingly, the mathematical form of the hierarchical celebrity endorser construct $(E_{ii}^k(n))$ is given by

$$E_{ij}^{k}(n) = W_{cre}^{2} \times CRE_{i}^{k}(n) + W_{att}^{2} \times ATT_{i}^{k}(n)$$

$$= W_{cre}^{2} \times \left[w_{tru}^{3} \times TRU_{i}^{k}(n) + w_{exp}^{3} \times EXP_{i}^{k}(n)\right]$$

$$+ W_{att}^{2} \times \left[w_{lik}^{3} \times LIK_{i}^{k}(n) + w_{sim}^{3} \times SIM_{i}^{k}(n) + w_{fam}^{3} \times FAM_{i}^{k}(n)\right]$$
(3)

where $ATT_i^k(n)$ and $CRE_i^k(n)$ are evaluations by consumer *i* of the attractiveness and credibility of celebrity endorser *k* on a given purchase occasion *n*; W_{att}^2 and W_{cre}^2 are the relative weights for $ATT_i^k(n)$ and $CRE_i^k(n)$ embedded in layer 2 of the proposed endorser assessment hierarchy; $TRU_i^k(n)$ and $EXP_i^k(n)$ are the evaluations of consumer *i* of the credibility of celebrity endorser *k* in terms of trustworthiness and expertise on a given purchase occasion *n*; $LIK_i^k(n)$, $SIM_i^k(n)$ and $FAM_i^k(n)$ are the evaluations of consumer *i* of the attractiveness of celebrity endorser *k* in terms of likeability, similarity, and familiarity on a given purchase occasion *n*; and w_{tru}^3 , w_{exp}^3 , w_{lik}^3 , w_{sim}^3 , and w_{fam}^3 are the relative weights of $TRU_i^k(n)$, $EXP_i^k(n)$, $LIK_i^k(n)$, $SIM_i^k(n)$ and $FAM_i^k(n)$ embedded in layer 3 of the proposed endorser assessment hierarchy, respectively.

Next, the relative weights (*i.e.*, W_{cre}^2 and W_{att}^2) of criteria in layer 2 and sub-criteria (*i.e.*, w_*^3) in layer 3 are estimated using Fuzzy-AHP, as shown in Eq. (3), to assess the performance of a celebrity endorser ($E_{ij}^k(n)$). Details of the procedures for estimating these relative weights are given in Appendix A.

¹ Relative to AHP, Fuzzy-AHP techniques can be viewed as advanced multi-criteria decision-making (MCDM) techniques, which have been extensively investigated in the field of MCDM and related applications such as project assessment, strategic planning, and system evaluation [28,29]. The description of Fuzzy-AHP features can be readily found elsewhere [30–32], and thus is omitted in this article.

3.2. Product-attribute construct

To convert the cognitive and affective effects of celebrity endorsement into an aggregate promotional effect on instantaneous evaluations of product attributes by a consumer, this work incorporates the term $\psi_{E_kA_j}^i(n)$ multiplied by $E_{ij}^k(n)$ into $A_{ij}^k(n)$, where $\psi_{E_kA_j}^i(n)$ is the dynamic association of attributed performance of a celebrity endorsement (*i.e.*, $E_{ij}^k(n)$) for an advertised product. We assume celebrity endorsement has a direct additive effect ($\psi_{E_kA_j}^i(n)E_{ij}^k(n)$) on consumer evaluation of product attributes ($\Psi_{ij}(n)A_{ij}(n)$); thus, consumer attitudes toward promoted product attributes ($A_{ij}^k(n)$) can be formulated as

$$A_{ij}^{k}(n) = \psi_{E_{k}A_{i}}^{i}(n)E_{ij}^{k}(n) + \Psi_{ij}(n)\mathbf{A}_{ij}(n)$$
(4)

where $\mathbf{A}_{ij}(n)$ is an $(M \times 1)$ attribute evaluation vector involving instantaneous evaluations of consumer *i* with respect to considered *M* attributes of product *j* on a given purchase occasion *n*; and $\Psi_{ij}(n)$ is a $(1 \times M)$ belief-strength vector involving the time-varying strengths of consumer beliefs associated with *M* attributes of product *j* considered in $\mathbf{A}_{ij}(n)$. Therefore, $A_{ii}^{k}(n)$ can be further deconstructed as

$$A_{ij}^{k}(n) = \psi_{E_{k}A_{j}}^{i}(n)E_{ij}^{k}(n) + \sum_{m=1}^{M}\psi_{A_{m}^{j}}^{i}(n)a_{A_{m}^{j}}^{i}$$
(5)

where $a_{A_m^i}^i(n)$ is the evaluation by consumer *i* of a given attribute *m* (e.g., brand, color, function, and material) of product *j*; and $\psi_{A_m^j}^i(n)$ is the consumer's time-varying belief strength associated with $a_{A_m^j}^i(n)$. However, based on the consistency of measurement scales between $\psi_{E_kA_j}^i(n)E_{ij}^k(n)$ and $\Psi_{ij}(n)\mathbf{A}_{ij}(n)$, the elements of $\Psi_{ij}(n)$ (*i.e.*, $\psi_{A_m^j}^i(n)$) are further normalized; thus $A_{ii}^k(n)$ derived as

$$A_{ij}^{k}(n) = \psi_{E_{k}A_{j}}^{i}(n)E_{ij}^{k}(n) + \sum_{m=1}^{M} \frac{w_{A_{m}^{i}}^{i}(n)a_{A_{m}^{i}}^{i}(n)}{\sum_{m=1}^{M} w_{A_{m}^{i}}^{i}(n)}$$
(6)

where $\psi_{E_kA_j}^i(n)$ and $w_{A_m^j}^i(n)$ are positive strength-related parameters; in some special cases, some parameters can be zero, depending on the subjective perception of a consumer. For example, consumers who enthusiastically pursue brands likely have extremely high strengths associated with a product brand, such that the resulting purchase decisions can be made without considering other factors, including the celebrity endorser effect. In such a scenario, their evaluations of a promoted product (*i.e.*, $A_{ij}^k(n)$) may depend on their attitudes toward the product brand; conversely, the associated belief strength of celebrity endorsement ($\psi_{E_kA_j}^i(n)$) and the induced promotional effect ($\psi_{E_kA_j}^i(n)E_{ij}^k(n)$) may too weak to be considered in such cases.

3.3. Net product value construct

Although no direct evidence in previous works verifies the ability of the celebrity endorsement effect to increase consumer-expected product price for a promoted product, that celebrity endorsers can make products desirable and glamorous through their unique and persuasive characteristics is generally accepted [33]. Some concluding remarks in early studies of source credibility effectiveness revealed that expert sources may influence consumer perceptions of product quality [34] thereby increasing perceived product value. Furthermore, the identification process, which occurs when the influence of an attractive celebrity endorser is accepted as a result of consumer desire to identify with an endorser, may also help explain why consumers are frequently willing to pay high prices for products endorsed by celebrities [35,36]. Additionally, drawing from the ideas implicit in hedonic price models rooted in associations between product attributes and price [37,38], one can readily infer that celebrity endorsement likely influences consumer-perceived product value because the endorser effect on consumer evaluations of product attributes can be transferred from the cognitive domain to the affective domain, further influencing consumer attitudes toward the perceived product value during the purchase decision process.

Based on the price range concept described in [22,24], we argue that net product value $(P_{ij}^k(n))$, which is contingent on the celebrity endorsement effect, can be formulated as

$$P_{ij}^k(n) = \overline{p}_{ij}^k(n) - \widetilde{p}_{ij}(n) \tag{7}$$

where $\overline{p}_{ij}^k(n)$ is a given consumer's (*i*) expected value for product *j* advertised by a given celebrity endorser *k* on a given purchase occasion *n*; and $\tilde{p}_{ij}(n)$ is the retail price of product *j* perceived by consumer *i* on a given purchase occasion *n*. Notably, all variables in Eq. (7) are measured in monetary units. Here, we postulate that a linear relationship exists between

consumer-expected value of a product $(\overline{p}_{ij}^k(n))$ and its influence factors, which include retail $(\widetilde{p}_{ij}(n))$ and relative $(p_{ij'}(n))$ prices [39], the weighted retail price $(PAST_{ij}(n))$ measured based on previous retail prices [40], consumer-perceived purchase environments denoted by $PC_i(n)$ [23,24], as well as consumer attitudes toward the celebrity endorser $(E_i^k(n))$ and the product itself $(A_{ii}^k(n))$. Thus, $\overline{p}_{ij}^k(n)$ can be mathematically expressed as

$$\overline{p}_{ij}^{k}(n) = \beta_{0} + \beta_{1} \widetilde{p}_{ij}(n) + \beta_{2} PAST_{ij}(n) + \beta_{3} p_{ij'}(n) + \beta_{4} PC_{i}(n) + \beta_{5} \left[\psi_{E_{k}P_{j}}^{i}(n) E_{ij}^{k}(n) \right] + \beta_{6} \left[\psi_{A_{j}P_{j}}^{i}(n) A_{ij}^{k}(n) \right] + \rho_{ij}(n)$$
(8)

where consumer-perceived purchase climate ($PC_i(n)$) can be rated on an assessment scale (e.g., ranging from -3 for "most unfavorable" to 3 for "most favorable"); $\psi_{E_kP_j}^i(n)$ is the time-varying relationship between attributed performance of a celebrity endorsement and consumer-expected product price; $\psi_{A_jP_j}^i(n)$ is the relationship between product attributes and price; $\rho_{ij}(n)$ is an error term denoting the effects of other unobservable factors on expected product price; β_0 is a product-price-specific constant used to capture the uniqueness of a product not captured by other explanatory variables in the linear price model; and β_1 and β_6 are coefficients, each representing the respective sensitivity of consumer-expected product price to change in the associated explanatory variable. Based on the previous literature [23,40], $PAST_{ij}(n)$, which is formulated as an exponential function of a weighted log-mean of prices on the past five purchase occasions, is given by

$$PAST_{ij}(n) = \exp \frac{\sum_{\tau=1}^{5} \varpi^{\tau-1} \ln \tilde{p}_{ij}(n-\tau)}{\sum_{\tau=1}^{5} \varpi^{\tau-1}}$$
(9)

where $\varpi^{\tau-1}$ is the relative weight of retail price on τ previous purchase occasions. Notably, if a customer has fewer purchase experiences than five, we assume retail prices are unobserved and are the same as that observed during the earliest experience; thus, the measure ($PAST_{ij}(n)$) in Eq. (9) remains applicable in this case. Nevertheless, as claimed by Kalwani et al. [23], no theoretical measures may exist for identifying the "optimal" value of ϖ . In the study, ϖ is determined by finding its preset value bounded between 0 and 1 to provide the best overall fit according to the adjusted R^2 of the expected product value model.

3.4. Dynamic estimation

The following stage estimates the time-varying strength-related parameters (W(n)), including consumer belief strengths and perceived associations among assessment constructs in Eqs. (2)–(9) using the proposed structural model. Similar measures are referred to as "structural estimations" by Erdem and Keane [41], who estimates the values of consumer utility function parameters in the dynamic brand choice processes. Based on stochastic system estimation [42,43], this work characterizes the dynamic states of strength-related parameters in a discrete-time nonlinear dynamic and stochastic system, which embeds deterministic and random terms into two groups of equations (*i.e.*, recursive and measurement equations) to reproduce change patterns of these time-varying parameters.

The recursive equations are used to characterize the dynamics of strength-related parameters ($\mathbf{W}(n)$) embedded in the proposed model for forecasting consumer attitudes, where $\mathbf{W}(n)$ is the dynamic state vector involving strength-related parameters to be forecast. Without considering the probability of attitude changes, $\mathbf{W}(n)$ may depend merely on the previous state vector (*i.e.*, $\mathbf{W}(n - 1)$), thereby exhibiting the features of Markov processes in an ideal deterministic system, which may result in time-invariant belief strengths that are consistent with those forecasted by the Ajzen–Fishbein model, which is a well-known static model for estimating consumer attitudes [44]. However, certain influences are unobservable in the duration between two successive purchases, thereby contributing to random errors ($\mathbf{U}(n - 1)$) in determining consumer belief strengths. In other words, a dynamic and stochastic system cannot rely on $\mathbf{W}(n - 1)$ only when forecasting $\mathbf{W}(n)$. Instead, the random error term ($\mathbf{U}(n - 1)$) should also be considered. In this work, these random errors (*i.e.*, the elements of $\mathbf{U}(n - 1)$) are assumed to follow Gaussian white noises associated with recursive and measurement equations may facilitate the approximation of the minimum mean square (MMS) estimates of dynamic and stochastic system states using Kalman filtering techniques. Consequently, the forecasted $\mathbf{W}(n)$ may oscillate around its previous state ($\mathbf{W}(n - 1)$) in a Gaussian-shaped domain, resulting in the random walk phenomenon [45]. Accordingly, the proposed state equations are formulated in a vector form as

$$\mathbf{W}(n) = \mathbf{W}(n-1) + \mathbf{U}(n-1) \tag{10}$$

where $\mathbf{W}(n)$ is an $(M + 6) \times 1$ dynamic state vector including estimates of belief-strength parameters on purchase occasion n; $\mathbf{W}(n - 1)$ is a deterministic term in the recursive equations represented by an $(M + 6) \times 1$ dynamic state vector which contains estimates of belief-strength parameters when estimated on purchase occasion n - 1; and $\mathbf{U}(n - 1)$ is an $(M + 6) \times 1$ noise vector which contains Gaussian white noises associated with elements of $\mathbf{W}(n - 1)$. Notably, $\mathbf{W}(n)$, $\mathbf{W}(n - 1)$, and

 $\mathbf{U}(n-1)$ can be further expressed as

$$\mathbf{W}(n) = [W_{iA}^{k}(n), W_{iP}^{k}(n), W_{iE}^{k}(n)|\psi_{E_{k}A_{j}}^{i}(n), \psi_{I_{k}P_{j}}^{i}(n), \psi_{A_{j}P_{j}}^{i}(n)|w_{A_{m}^{i}}^{i}(n), m = 1, ..., M]^{T}$$

$$\mathbf{W}(n-1) = [W_{iA}^{k}(n-1), W_{iP}^{k}(n-1), W_{iE}^{k}(n-1)|\psi_{E_{k}A_{j}}^{i}(n-1), \psi_{E_{k}P_{j}}^{i}(n-1), \psi_{A_{j}P_{j}}^{i}(n-1)|w_{A_{m}^{i}}^{i}(n-1), m = 1, ..., M]^{T}$$

$$(11)$$

$$m = 1, ..., M]^{T}$$

$$(12)$$

$$\mathbf{U}(n-1) = [u_{iA}^{k}(n-1), u_{iP}^{k}(n-1), u_{iE}^{k}(n-1)]u_{iE_{k}A_{j}}^{i}(n-1), u_{iE_{k}P_{j}}^{i}(n-1), u_{A_{j}P_{j}}^{i}(n-1)]u_{A_{m}^{j}}^{i}(n-1),$$

$$m = 1, \dots, M]^{T}$$
(13)

where $W_{iA}^k(n)$, $W_{iP}^k(n)$, and $W_{iE}^k(n)$, as in Eq. (2), represent the time-varying strengths of consumer beliefs associated with $A_{ij}^k(n)$, $P_{ij}^k(n)$, and $E_{ij}^k(n)$, respectively. Notably, $W_{iA}^k(n)$ and $W_{iP}^k(n)$ can be further expressed as

$$W_{iA}^{k}(n) = \psi_{E_{k}A_{j}}^{i}(n) + \sum_{m=1}^{M} w_{m}^{i}(n)$$
(14)

$$W_{iP}^{k}(n) = \psi_{E_{k}P_{j}}^{i}(n) + \psi_{A_{j}P_{j}}^{i}(n) \left[\psi_{E_{k}A_{j}}^{i}(n) + \sum_{m=1}^{M} w_{m}^{i}(n) \right].$$
(15)

Eqs. (14) and (15) are employed to characterize the magnitudes of associations between endorser-driven consumer attitudes and the constructs of product attributes and net product value. In this study, we propose that consumer belief strengths associated with these two constructs can be enhanced by celebrity endorsement. Particularly, $W_{ip}^k(n)$ may be enhanced directly and the indirectly by celebrity endorsement through the product-attribute construct. Based on these propositions, Eqs. (14) and (15) are used to estimate the values of $W_{iA}^k(n)$ and $W_{ip}^k(n)$.

The measurement equations determine the time-varying causal relationship between measurements of (Z(n)) collected from consumer purchase intention and the specified belief-strength parameters (W(n)). Fishbein and Ajzen [44] defined belief strength as perceived probability of an association between an object and its relevant attributes. Here, we define a measurement datum (*i.e.*, elements of Z(n)) as a consumer's perceived probability of creating an association between purchase intention with any one of the three main constructs—product attributes, price, and celebrity endorser. For example, the perceived relative relationship between purchase intention and celebrity endorsement can be measured by asking a consumer to respond to the following statement on an assessment scale: "*How likely are you to buy the promoted product due to celebrity endorsement, relative to the considerations of product attributes and price*?" Furthermore, as random errors exist in a consumer's judgment process, an error term (V(n)) is included in the model formulation. Accordingly, the vector form of measurement equations (Z(n)) is given by

$$\mathbf{Z}(n) = \mathbf{H}\left(\mathbf{W}(n), n\right) + \mathbf{V}(n) \tag{16}$$

where $\mathbf{Z}(n)$ is a (3 × 1) time-varying measurement vector involving measured consumer-perceived probabilities of a "relative association" existing between purchase attitude and constructs of product attributes, price and celebrity endorser, which are denoted by $Z_{iA}^k(n)$, $Z_{iP}^k(n)$, and $Z_{iE}^k(n)$, respectively. Thus, $\mathbf{Z}(n)$ is given by

$$\mathbf{Z}(n) = \left[Z_{iA}^{k}(n), Z_{iP}^{k}(n), Z_{iE}^{k}(n) \right]^{\mathrm{T}}.$$
(17)

Additionally, $\mathbf{H}(\mathbf{W}(n), n)$ is a (3 × 1) time-varying measurement-component vector that uses three elements to characterize $Z_{iA}^k(n), Z_{iP}^k(n)$, and $Z_{iE}^k(n)$. Using the concept of relative weights, the elements of $\mathbf{H}(\mathbf{W}(n), n)$ can be expressed by using the relative belief strengths associated with these three main constructs (Eqs. (2)–(9)); thus, $\mathbf{H}(\mathbf{W}(n), n)$ is given by

$$\mathbf{H}(\mathbf{W}(n),n) = \left[\frac{W_{iA}^{k}(n)}{W_{iA}^{k}(n) + W_{iP}^{k}(n) + W_{iE}^{k}(n)}, \frac{W_{iP}^{k}(n)}{W_{iA}^{k}(n) + W_{iP}^{k}(n) + W_{iE}^{k}(n)}, \frac{W_{iE}^{k}(n)}{W_{iA}^{k}(n) + W_{iP}^{k}(n) + W_{iE}^{k}(n)}\right]^{\mathrm{T}}.$$
(18)

In addition, $\mathbf{V}(n)$ is a (3×1) error vector given by

$$\mathbf{V}(n) = \left[V_{iA}^{k}(n), V_{iP}^{k}(n), V_{iE}^{k}(n)\right]^{\mathrm{T}}$$
(19)

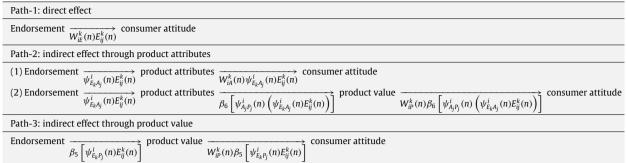
where $V_{iA}^k(n)$, $V_{iP}^k(n)$, and $V_{iE}^k(n)$ are the measurement errors associated with $Z_{iA}^k(n)$, $Z_{iP}^k(n)$, and $Z_{iE}^k(n)$, respectively, and are also assumed to follow Gaussian white noises to facilitate dynamic system estimation. Notably, various extrinsic factors can also affect consumer purchase attitudes. Theoretically, these measurement errors also account for overall affection caused by any extrinsic factor acting on associated measurements.

Given the recursive and measurement equations, Eqs. (10)–(19), as well as the measured probabilities of the relative associations (*i.e.*, $Z_{iA}^k(n)$, $Z_{iP}^k(n)$, and $Z_{iE}^k(n)$), this study approximates the MMS estimates of the dynamic states of strength-related parameters associated with consumer attitudes (*i.e.*, elements of **W**(*n*) in Eq. (11)) using the extended Kalman filtering technique. The details about the proposed Kalman filtering-based algorithm are depicted in Appendix B.

Based on the estimated belief-strength parameters and consumer evaluations of elements associated with the specified three constructs (*i.e.*, $A_{ij}^k(n)$, $P_{ij}^k(n)$, and $E_{ij}^k(n)$), deterministic term ($\theta_{ij}^k(n)$) of a consumer's overall attitude toward an

Table 1

Paths of celebrity endorsement effects on consumer attitude



endorsed product can be predicted by Eq. (2). Furthermore, this study readily quantifies the endorsement effect on consumer overall attitude by elucidating the paths of direct and indirect effects of endorser performance $(E_{ij}^k(n))$ through the other two constructs—product attributes and net product value (Table 1). Thus, the direct effect of celebrity endorsement is $W_{iE}^k(n)E_{ij}^k(n)$, and the corresponding indirect effect is the sum of $W_{iA}^k(n)\psi_{iE_kA_j}^i(n)E_{ij}^k(n)$, $W_{iP}^k(n)\beta_6\left[\psi_{i_kA_j}^i(n)E_{ij}^k(n)\right](i.e., W_{iA}^k(n)\psi_{E_kA_j}^i(n)E_{ij}^k(n) + W_{iP}^k(n)\left[\beta_5\psi_{E_kA_j}^i(n) + \beta_6\psi_{E_kA_j}^i(n)\psi_{A_jP_j}^i(n)\right]E_{ij}^k(n)$, according to the proposed conceptual framework (Fig. 1) and Eqs. (2), (4) and (8).

Accordingly, this work approximates the celebrity endorsement effect by summing the direct and indirect effects derived above to elucidate the correlations between celebrity endorsement and consumer attitude. First, compared with endorsement-free cases, celebrity endorsement has an additional effect on consumer attitude on a given purchase occasion given the same product and purchase context. Therefore, one can easily infer the endorsement-driven consumer attitude by adding the endorsement effect to consumer attitude measured under endorsement-free conditions. Second, the effect induced by a celebrity endorsement can be positive (promotional) or negative (counter promotional), depending on endorser performance ($E_{ij}^k(n)$), which is assessed by consumers using credibility and attractiveness criteria in the cognitive and affective domains, respectively, according to the proposed conceptual framework. For example, when a negative event is associated with an endorser (e.g., a scandal) during the endorsement period, endorser credibility and attractiveness as perceived by a consumer may decrease, thereby resulting in a negative value for $E_{ij}^k(n)$, which contributes to a negative endorsement effect, according to the above forecasting results. Conversely, endorser attractiveness and credibility can be increased by collaborative advertising strategies or continuous good performance by the endorser in his/her field of expertise (e.g., sports and entertainment activities), thereby increasing the value of $E_{ij}^k(n)$ to enhance the induced endorsement effect and consumer attitudes toward an endorsed product.

4. Model estimation

This section estimates the values of parameters embedded in the net product value and celebrity endorsement constructs. To demonstrate the applicability of the proposed model in dynamically forecasting celebrity endorsement effects, an experimental study is conducted using two well-known products—sport shoes (Nike) and notebook computers (Acer). The main reason for selecting these two brands as targeted products is that Nike has used Michael Jordan as its brand endorser, resulting in a very successful celebrity endorsement effect in global markets. Similarly, Acer, which used Wang, a well-known Yankees pitcher, as its brand endorser, reportedly increased its annual sales of notebook computers by 10% in Taiwan [46]. After the demonstration of target products in the interview survey process, survey respondents identified their perceptions and attitudes by responding to questionnaire items, thereby improving the validity of survey data. Additionally, this study adopted the basic multi-attribute attitude model developed by Ajzen and Fishbein [16] by combining Eqs. (2) and (6), and removing the time-varying endorsement influence term ($\psi_{E_kA_j}^i(n)E_{ij}^k(n)$) from Eq. (6), thereby forming a static model called the Ajzen–Fishbein model as a baseline model for determining the relative performance of the proposed model in characterizing consumer attitude changes.

Prior to the questionnaire survey, a pilot survey was conducted, during which sales managers at 12 local sport shoes and consumer electronics stores were interviewed. Based on their sales experience and up-to-date end-customer information, questionnaire items were revised to improve readability and conciseness and reduce any bias in data collection. In total, 357 students attending National Chiao Tung University participated in the pretest to finalize questionnaire items for identifying the principal product attributes and factors considered in expected product value and celebrity endorsement performance through confirmatory factor analysis (CFA).

The following describes the main procedures used and numerical results yielded via the data collection, model estimation and forecasting phases. A discussion of the analytical results follows.

4.1. Data acquisition

The proposed model was calibrated using panel data collected through questionnaire surveys administered through the Internet over a 1-year period from April 2006 to March 2007. The sample population was randomly chosen from students who have shopped for sport shoes and notebook computers in the last 2 years. In total, 1278 of 1492 data samples were valid. Study participants were queried monthly during the 1-year study period about their willingness to participate in the study. The personal information section of the questionnaire collected personal information, including sex, age range, education level, occupation, monthly income, and previous experience shopping for these two product categories. Of the 1278 participants 73.6% were male, and over 81.2% were aged <30 years. Additionally, 82.5% were full-time students, and over 80% of participants had monthly incomes <US\$1000.

The data collected (*i.e.*, model estimation) serve three purposes: to initialize belief-strength-related parameters embedded in the three attitude constructs in the proposed dynamic model (Eq. (2)), to calibrate the coefficients of the proposed linear function for approximating product prices expected by consumers (Eq. (8)), and to estimate the relative weights embedded in the proposed endorser analytical hierarchy (Eq. (3)). The questionnaire has three sections: (1) product attributes; (3) expected product value; and, (4) celebrity endorsement.

To estimate the values of the initial belief-strength parameters in the proposed dynamic attitude forecast model, survey respondents rated the strengths of their beliefs associated with the three attitude constructs on a 10-point integer measurement scale, ranging from 1 for "extremely unlikely" to 10 for "extremely likely" [12]. Notably the initial belief strengths estimated are also applied to the Ajzen–Fishbein model for comparison of model performance.

Coefficients in the proposed expected product value model, namely, $\beta_0-\beta_6$, were calibrated (Eq. (8)) using the data collected from the expected product value section and the least squares regression approach [47]. Given a target product, each survey respondent rated the expected product price and associated independent variables (e.g., purchase environments, perceived relative and previous retail prices) with the exception of the retail price variable ($\tilde{p}_{ij}(n)$), which is the actual price paid for promoted sport shoes when purchased or the retail store price when not purchased.

The relative weights associated with criteria and sub-criteria embedded in layer 2 and layer 3, respectively, of the proposed endorser assessment hierarchical framework were estimated using data collected in the celebrity endorsement section. In layer 2, each survey respondent rated the relative importance of credibility and attractiveness by ranking these criteria; as the ordinal number increases, the importance of the associated component increases. The same procedure was applied to layer 3 to rate the relative importance of sub-criteria. Collected data were the input for estimating the relative weights using the Fuzzy-AHP technique (Appendix A).

4.2. Parameter estimates

As classified by product categories, Table 2 presents parameter estimates associated with the proposed dynamic attitude forecast model, where estimates in the product-attribute construct are mean values of initial belief strength parameters (*i.e.*, $\Psi_{ij}(0)$ (Eq. (4)); the estimates in the expected product value construct are parameters (*i.e.*, β_* (Eq. (8)); and estimates in the celebrity endorsement construct are mean values of relative weights (*i.e.*, W_*^2 and w_*^3 (Eq. (3)) in the endorsement assessment hierarchy.

Overall, the estimated results (Table 2) fit with the expectation that different product categories generate the variety of consumer perception of these three constructs. In terms of sport shoe attributes, "appearance" was rated highest in terms of belief strength, followed by "materials," "function," "fashion," and "personal taste symbolization," indicating that relative to other attributes, appearance was the most important attribute considered by customers when making purchase decisions. Conversely, consumers of the notebook computer cared most about product "functions," followed by "appearance," "style," "materials," and "personal taste symbolization." The sport shoe shoppers rated the criterion of endorser attractiveness higher than endorser credibility; endorser "likeability" was in line with analytical results obtained by Frieden [15]; that is, it is a key factor influencing consumer attitudes toward sport shoes. Conversely, the notebook computer shoppers rated endorser credibility, particularly in terms of perceived trustworthiness, as more important than attractiveness. Such a generalization is also close to expectations as we argue that the success of a celebrity endorsement campaign should focus on the affective communication effect on consumer attitudes, particularly in cases when consumer purchase attitudes toward a target product (e.g., sport shoes) are not related to function. However, for function-oriented products (e.g., notebook computers), the corresponding celebrity endorsement strategies should rely highly on cognitive communication effects to improve consumer purchase attitudes and intentions for a promoted product.

Additionally, all estimated coefficients in the proposed expected product value model are statistically significant by the *t*-test at the $\alpha = 0.01$ significance level, and their signs are in line with initial expectations. Furthermore, the goodness-of-fit index is also consistent with expectations. The values of adjusted R^2 are satisfactory, where the adjusted R^2 , similar to other goodness-of-fit measures, indicates the percentage of total variation explained by the regression.

Nevertheless, one may observe that the function or type of materials is no longer the key determinant when consumers purchase sport shoes (Table 2). Such a finding, in reality, may violate logical thinking. However, customers may no longer purchase a product just for its intrinsic use values (*i.e.*, the purpose in purchasing sport shoes is not just for sports). Rather, additional extended purchase intentions may be synthesized with complex considerations, which are either cognitive or

Table 2

Dynamic model estimates.

1. Product-attribute construct		
	Sport shoes	Notebook computers
Estimates	Mean of initial belief strength	Mean of initial belief strength
Product attribute	$ar{oldsymbol{\Psi}}_{ij}(0)$ (standard deviation)	$ar{oldsymbol{\Psi}}_{ij}(0)$ (standard deviation)
Function	5.03 (1.48)	8.92 (1.74)
Appearance	8.43 (1.64)	6.16 (2.08)
Materials	6.75 (0.99)	3.59 (0.72)
Style	4.20 (1.56)	2.32 (1.89)
Personal taste symbolization	4.64 (1.87)	1.97 (0.63)

2. Net product value construct

		Sport shoes	Notebook computers
Variable	Coefficient	Estimate (t-statistics)	Estimate (t-statistics)
Alternative specific constant	β_0	64.351 (10.756)	1,104.654 (8.790)
Retail price	β_1	0.112 (9.542)	0.069 (6.524)
Previous retail price	β_2	0.067 (3.568)	0.043 (2.779)
Relative price	β_3	0.098 (3.789)	0.024 (4.637)
Purchase climate	β_4	0.033 (3.919)	0.087 (3.441)
Endorsement effect	β_5	0.135 (6.670)	0.195 (5.465)
Product-attribute effect	β_6	0.091 (1.543)	0.068 (2.078)
Adjus	ted R ²	0.670	0.589
Log likelihood (0) Log likelihood (β_0)		-6891.2	-7566.3
		-6234.1	-6834.1
Log likel	ihood (β)	-5633.9	-6159.3
Significa	nce level	$\alpha = 0.01$	$\alpha = 0.01$
3. Celebrity endorsement construct			
Layer 2 (for sport shoes category)			

Criteria Estimate	Endorser credibility		Endorser attracti	iveness	
Relative weight W_*^2	0.326		0.674		
Layer 3					
Sub-criteria Estimate	Perceived trustworthiness	Perceived expertise	Likeability	Similarity	Familiarity
Relative weight w_*^3	0.087	0.239	0.284	0.172	0.218
Layer 2 (for notebook com	puters category)				
Criteria Estimate	Endorser credibility		Endorser attracti	iveness	
Relative weight W_*^2	0.547		0.453		
Sub-criteria Estimate	Perceived trustworthiness	Perceived expertise	Likeability	Similarity	Familiarity
Relative weight w_*^3	0.282	0.265	0.201	0.098	0.154

affective, by consumers that are beyond their purchase behaviors. Furthermore, such an analytical result may also be correlated with characteristics of targeted customers. Most survey respondents were aged <30 years old -38% were teenagers – thus, they may have cared more about the appearance of sport shoes than other product attributes during purchase decision process.

The main parameters of the Ajzen–Fishbein model are mean values of initial belief strengths (*i.e.*, $W_{iA}^k(0)$, $W_{iP}^k(0)$, and $W_{iE}^k(0)$ in Eq. (2)) associated with the three constructs. Initial belief strengths collected by surveying respondents at the start of the survey period were aggregated, and the corresponding mean values were used as Ajzen–Fishbein model estimates (Table 3). As mentioned, the Ajzen–Fishbein model is a static model; thus, the estimated belief-strength parameters for the Ajzen–Fishbein model are constant. Therein, the proposed dynamic model adopts these parameters as initial belief strengths; these belief strengths are updated dynamically during the survey period.

5. Dynamic forecast

The applicability of the proposed model for investigating the time-varying effects of celebrity endorsement on consumer purchase attitudes is demonstrated in this section using the proposed dynamic model and the Ajzen–Fishbein model

Table 3

Ajzen-	FISHDem	model	estimates.

1. Product-attribute construct		
	Sport shoes	Notebook computers
Parameter	Mean of initial belief strength (standard deviation)	Mean of initial belief strength (standard deviation)
Construct-based belief strength $W_{iA}^k(n)$	7.13 (2.25)	7.49 (2.56)
2. Net product value construct		
Construct-based belief strength $W_{ip}^k(n)$	7.38 (1.37)	6.79 (2.45)
3. Celebrity endorsement construct		
Construct-based belief strength $W_{iE}^k(n)$	3.68 (1.89)	4.72 (1.50)

estimated in Section 4. Here, this work addresses the aforementioned "time-varying effect" from two perspectives: (1) the time-varying effects of an endorser's performance on their endorsement performance; and, (2) consumer attitude changes driven by celebrity endorsement during a promotional period. By using the proposed model calibrated through the procedures of data acquisition and parameter estimation described in Sections 4.1 and 4.2, a firm can assess the performance of a celebrity endorser and forecast the time-varying effect of celebrity endorsement on consumer attitudes; only panel data is required at this stage. Relative to conventional multi-attribute models, including the Ajzen–Fishbein model, a significant advantage of the proposed model is that time-varying belief-strength-related parameters can be estimated dynamically to infer changes in consumer attitudes toward product attributes, expected product value, and celebrity endorsement.

To accomplish these purposes, this study used two target products, sport shoes (Nike) and notebook computers (Acer). Additionally, this study selected Michael Jordan and Chien-Ming Wang as the endorsers for the target products. First, both Jordan and Wang are superstars in their respective sports, well known to survey respondents, and thus fit the subject of "celebrity endorsement." Second, based on the differences between Jordan and Wang, one can easily determine how increased performance in their sports affects their endorsement performance, facilitating the determination of the effects on consumer attitude changes during the survey period. Notably, Jordan retired from the NBA several years ago. Thus, Jordan's perceived endorsement performance is no longer affected by his basketball performance. Conversely, Wang currently pitches for the New York Yankees; thus, Wang's perceived endorsement performance may be influenced dynamically by his pitching performance. Through such a comparison, one may readily observe how an endorser's performance dynamically affects endorsement performance and consumer purchase attitudes.

This section (*i.e.*, dynamic forecast) uses collected panel data which includes two parts: (1) input data for measuring the values of explanatory variables of the three constructs; and, (2) the perceived probabilities of the "relative association" (*i.e.*, $Z_{iA}^k(n)$, $Z_{iE}^k(n)$, and $Z_{iE}^k(n)$ in Eq. (17)). For the first section, an 11-point scale ranging from -5 for "extremely unfavorable" to +5 for "extremely favorable" is used by survey respondents to evaluate the respective attributes associated with product-attribute ($A_{ij}^k(n)$) and endorsement assessment ($E_{ij}^k(n)$) constructs. Consumer evaluation of the product value construct can be readily derived using the estimated expected product value model (Eqs. (7) and (8)). The second section has questionnaire items that measure $Z_{iA}^k(n)$, $Z_{iP}^k(n)$, and $Z_{iE}^k(n)$. Given a celebrity endorser (either Jordan or Wang), survey respondents, for instance, were asked to respond the following statement on a 0–1 real-value assessment scale: "*How likely are you to buy the promoted product due to the celebrity endorser, relative to considerations of product attributes and price?*" Data were then collected to estimate the values of the time-varying strength-related parameters using the proposed dynamic-strength estimation model (Eqs. (10)–(19)) combined with the proposed estimation procedures (Appendix B).

The next step is to elucidate the time-varying effects of these two celebrity endorsers on changes in consumer attitudes. The proposed conceptual framework, including the dynamic model, is grounded in the individual decision domain, as mentioned. Therefore, output from either Fuzzy-AHP for assessing celebrity endorser performance $(E_{ij}^k(n))$ or Kalman filtering for estimating the values of time-varying belief-strength parameters is applied merely to the characterize consumer purchase attitudes at the individual decision level. To determine the endorsement effect on changes in consumer attitudes in the demand market, output data must be further aggregated. Three celebrity endorsement scenarios were designed for the two target products, *i.e.*, products without celebrity endorsements (*Scenario* 1), products endorsed by Jordan (*Scenario* 2), and products endorsed by Wang with updates of Wang's performance (*Scenarios* 3). For each scenario, this work forecasts the attitudes of survey respondents toward these two products monthly during the 1-year survey period. The proposed model is still applicable under no endorsement conditions; the only requirement is all endorsement-related parameters and variables must be null.

Figs. 3a and 3b show the aggregate forecast results output from the proposed dynamic model and the Ajzen–Fishbein model for sport shoes and notebook computers, respectively. The forecast results reveal several generalizations, which summarized as follows (Figs. 3a and 3b).

First, for *Scenario* 1, the noticeable changes in consumer attitudes contingent on celebrity endorsement exist; this finding applies more to the sport shoes promotion than the notebook computer promotion. Celebrity endorsement by Jordan (*Scenario* 2) or Wang (*Scenario* 3) significantly affects consumer attitudes toward the promoted product, as the corresponding forecasted consumer attitudes are significantly greater than those obtained under no endorsement (*Scenario* 1) (Fig. 3a).

J.-B. Sheu / Mathematical and Computer Modelling 52 (2010) 1554-1569

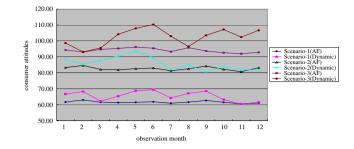


Fig. 3a. Forecasts of consumer purchase attitudes (sport shoes).

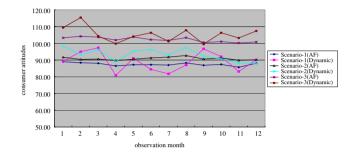


Fig. 3b. Forecasts of consumer purchase attitudes (notebook computer).

According to the analysis of forecast results, celebrity endorsement influences consumer purchase attitudes directly and indirectly. For example, a normal consumer may logically rate the function of sport shoes endorsed by a superstar in that sport with a higher value compared with endorsement-free cases. Further, if a consumer is a fan of the celebrity, the celebrity endorsement effect can also be increased to a certain extent without reason. Drawing from this generalization, both candidates are fit for promoting "sport shoes" because of the high degree of congruence between the endorser and promoted product features.

Second, relative to the static Ajzen–Fishbein model, the proposed dynamic model was more adaptive to changes in consumer purchase attitudes during the 1-year survey period. The variations in consumer attitudes forecasted by the Ajzen–Fishbein model are not as significant as those observed using the proposed model (Figs. 3a and 3b). Thus, we infer that consumer attitudes may be affected dynamically by an endorser's performance, contributing to significant change patterns in purchase attitudes during a promotional period. Therefore, the proposed dynamic model is more promising than the static model in reproducing the time-varying features of consumer purchase attitudes. This is also useful in promotion management, as collaborative promotional and advertising strategies can be designed efficiently in response to the dynamics of consumer attitudes and behavior.

Third, the endorser–product relationship influences consumer purchase attitudes. A comparison of the results of *Scenario* 2 shown in Figs. 3a and 3b indicates that Jordan is less convincing when promoting notebook computers than when promoting sport shoes. This phenomenon may result from endorser–product incongruence. Unlike Wang, who endorsed notebook computers for Acer, Jordan rarely endorses such products and, furthermore, consumers have difficulty linking Jordan to notebook computers. Conversely, Wang's endorsement performance is greater than that of Jordan based on his success endorsing the Acer brand and his overwhelming popularity with local consumers (*i.e.*, survey respondents), thus contributing to the relatively positive effects on consumer purchase attitudes toward the notebook computer in this study.

Fourth, the endorser's recent performance influences corresponding endorsement performance. A comparison of *Scenario* 2 and 3 results (Figs. 3a and 3b) demonstrates that variations in consumer attitudes contingent upon Wang's endorsement are greater than those for Jordan's endorsement. In reality, such a generalization is consistent with our proposition that, unlike Jordan, who is retired, Wang's endorsement performance assessed by survey respondents may also be influenced by his pitching performance, thus leading to relatively significant variations in consumer attitudes during the survey period.

The preceding discussion is based on aggregated forecast results. In reality, the diversity of celebrity endorsement effects can also be caused by customer heterogeneity in demand markets. Fig. 4 shows three typical examples of consumer attitude change patterns exhibited across survey respondents; the three consumer groups are clustered by comparing mean values of consumer time-varying belief strengths with respect to $W_{iA}^k(n)$, $W_{iP}^k(n)$, and $W_{iE}^k(n)$, respectively, with a threshold of 0.5. For example, this study assigns customer *i* to the product-oriented consumer group when the mean value of $W_{iA}^k(n)$ associated with customer *i* exceeds 0.5 during the survey period. The same clustering measure is applied to identify the other two consumer groups. The first example represents the patterns of attitude changes for product-oriented customers, whose purchase decisions are made mainly based on product preference. Notably, celebrity endorsement increases consumer positive attitudes toward product attributes, leading to a significant increase in aggregate purchase attitude. The second

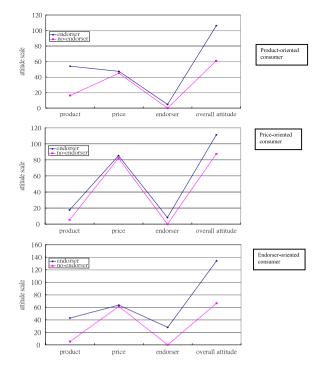


Fig. 4. Selected samples of consumers' attitude changes due to celebrity endorsement.

example is the case in which consumer purchase decisions are mainly based on price. In this case, a celebrity endorsement strategy does not have a significant effect on consumer-expected product value, resulting in a consumer attitude change that in not as significant that in the previous case. Nevertheless, for price-oriented consumers, their evaluations of net product value may be relatively higher than others because of the estimated corresponding belief strengths are high. In contrast with these two cases, a third example illustrates the attitude change of a typical fan due to his/her adoration of an endorser. As exhibited in this study, one can infer that people in the "Jordan club" (or Wang club) may rate the celebrity endorsement construct extremely high. Overall, these analytical results reveal the potential heterogeneity of consumer attitude changes under celebrity endorsement promotion.

6. Discussion

Despite the capability of the proposed model to forecast celebrity endorsement effects on consumer attitudes toward a promoted product and attitude changes, several interesting findings may provide significant implications for further discussion.

First, the celebrity endorsement effect on consumer-expected product value remains unclear. Therein, the socioeconomic characteristics of target customers and product type may be causes. For instance, in the sport shoes case, target consumers may be students positioned within a consumer group that has low income and high consumption intentions. Thus, most of students are affordable at the retail price, and willing to pay for sport shoes when its fits with their demand and preferences. Therefore, "price" may not be their major concern; other the factors related to product attributes and endorser performance may also be important. Additionally, the proposed conceptual framework can be applied to other product categories (e.g., durable goods), as it involves product-attribute construction, which facilitates the incorporation of different product attributes into the framework for characterizing promoted products. Therefore, we infer that different generalizations may be based on study case, which is aimed at different consumer groups (e.g., female consumers) associated with specific product categories (e.g., luxurious dresses and cosmetics products) endorsed by celebrities. Nevertheless, the proposed dynamic forecast model is applicable to diverse cases for determining the time-varying celebrity endorsement effects on consumer attitudes.

Second, the phenomenon of negative celebrity endorsement effects may be noteworthy. The performance of celebrity endorsement may also be influenced by the endorser's performance, which can be positive or negative, particularly when endorsers are superstars in their professionals [9]. Thus, updated performance can be reassessed using the two criteria and five sub-criteria. The resulting direct endorsement effect can then be further applied to estimate consumer attitudes toward product-attribute and net product value constructs, followed by determination of a consumer's overall attitude toward a promoted product. Briefly, the proposed model assesses perceived performance of an endorser, and traces consumer attitude changes to determine the induced impact on demand markets during a promotional period.

Third, celebrity endorsement can influence consumer evaluations of product attributes and the associated belief strengths. Such a phenomenon may be of particular interest under the condition in that certain product attributes are perceived as highly congruent with celebrity endorser characteristics. For example, among the many product attributes, "function" and "style" may be relatively sensitive to the perceived celebrity performance, as consumer evaluations and belief strengths were significantly changed by the comparison of the forecast results with and without celebrity endorsement in the sport shoes case. Thus, we infer that most consumers may agree with the congruence between endorser characteristics (e.g., expertise and likeability of Wang) and the attributes of the sport shoes, thus contributing to a high degree of attitude change in the product-attribute construct, compared to the forecast results obtained under no endorsement.

Fourth, despite the fact that the endorsement by Wang may have a greater promotional effect than that of Michael Jordan in the study cases, some other factors are worthy of consideration. From a strategic perspective, product positioning and segmentation should also be considered when planning a celebrity endorsement strategy. For instance, if the promoted sports shoes are aimed at global markets, an endorsement by Jordan will likely be more successful than an endorsement by Wang. However, Wang may have the advantage when product promoted is aimed mainly at the local market. Furthermore, endorsement cost relative to induced benefit is also noteworthy when assessing alternatives to celebrity endorsements.

An interesting direction for future research is to incorporate diverse promotional strategies into the proposed framework to obtain insights into hybrid promotional effects on consumer behavior and demand markets. Additionally, the proposed model permits the continuous updating of estimates of consumer attitudes as endorser performance is updated. Thus, collaborative advertising strategies can be utilized in response to attitude changes of target consumers during a promotional period. Furthermore, the aggregated output in terms of $E_{ij}^k(n)$ can help marketers assess celebrity endorsement projects, and the proposed model can be furthered by incorporating other novel endorsement assessment approaches into its framework. Extensions of study scope, including the model to domains of purchase intention and behavior, also warrant further research to either measure the overall effect of advertising expenditures on sales or assess the effectiveness of a celebrity endorsement on a firm's profitability; however, these are challenging issues [11]. Study cases for different product categories endorsed by different celebrities may also interest marketers for strategic marketing.

In summary, this study will benefit by adding additional insights into celebrity endorsement effects on consumer purchase attitudes, and by additional studies for promotional strategic planning and management.

Appendix A. Estimation of criteria strengths for endorser performance assessment

Based on the specified three-layer hierarchical architecture of endorser assessment (see Fig. 2), we propose to employ the Fuzzy-AHP technique to determine the strengths of criteria and sub-criteria used to assess the endorser performance in this study. There are two primary procedures executed at this stage: (1) estimation of pair-wise comparison matrices, and (2) approximation of fuzzy weights, which are detailed in the following.

The first procedure is to generate two respective pair-wise comparison matrices associated with layers 2 and 3 of the specified three-layer endorser assessment hierarchy. In the study, we conduct the questionnaire surveys to obtain survey respondents' judgments with respect to the relative importance of the criteria and sub-criteria specified in layers 2 and 3, respectively. For each given layer (*l*), survey respondents are asked to rank the corresponding components (i.e., the criteria in layer 2 and sub-criteria in layer 3) in linear order based on the relative importance among these components, and then associate these components with specific ordinal numbers, where the higher an ordinal number, the more important the associated component is. Then, the element σ_{ab}^l of a given pair-wise comparison matrix associated with components *a* and *b* of layer *l* is given by $\sigma_{ab}^l = r_a^l/r_b^l$, where r_a^l and r_b^l represent the ordinal numbers associated with components *a* and *b* of layer *l* is given by $\sigma_{ab}^l = r_a^l/r_b^l$, where r_a^l and r_b^l represent the ordinal numbers associated with components *a* and *b* of layer *l* is given by $\sigma_{ab}^l = r_a^l/r_b^l$. Accordingly, we have a 2 × 2 pair-wise comparison matrix associated with layer 2 ($\mathbf{D}_{2\times2}^2$) and a 5 × 5 comparison matrix associated with layer 3 ($\mathbf{D}_{5\times5}^3$) in the proposed endorser assessment hierarchy, respectively. Herein, $\mathbf{D}_{2\times2}^2$ and $\mathbf{D}_{5\times5}^3$ are given by

$$\mathbf{D}_{2\times2}^{2} = \begin{bmatrix} 1 & \sigma_{12}^{2} \\ \sigma_{21}^{2} & 1 \end{bmatrix}_{2\times2}$$
(A.1)
$$\mathbf{D}_{5\times5}^{3} = \begin{bmatrix} 1 & \sigma_{12}^{3} & \sigma_{13}^{3} & \sigma_{14}^{3} & \sigma_{15}^{3} \\ \sigma_{21}^{3} & 1 & \sigma_{23}^{3} & \sigma_{24}^{3} & \sigma_{25}^{3} \\ \sigma_{31}^{3} & \sigma_{32}^{3} & 1 & \sigma_{34}^{3} & \sigma_{35}^{3} \\ \sigma_{31}^{3} & \sigma_{32}^{3} & 1 & \sigma_{34}^{3} & \sigma_{35}^{3} \\ \sigma_{31}^{3} & \sigma_{32}^{3} & \sigma_{33}^{3} & 1 & \sigma_{45}^{3} \\ \sigma_{51}^{3} & \sigma_{52}^{3} & \sigma_{53}^{3} & \sigma_{54}^{3} & 1 \end{bmatrix}_{5\times5}$$
(A.2)

The next step is to approximate the fuzzy weights associated with the components in a given layer. Here, we employ the geometric mean technique to facilitate the approximation of the fuzzy weights because this technique is easily extended for computing the weights of fuzzy positive reciprocal matrices, as explicated in the previous literature [48,49]. For instance, given that a fuzzy positive reciprocal matrix is consistent, the geometric mean technique may easily approximate the fuzzy

weights which are the same as those obtained from Saaty's λ – max technique, termed as the largest eigenvalue technique. In addition, the geometric mean technique may also satisfy the condition of the absence of rank reversal [32]. Accordingly, given a $t \times t$ pair-wise comparison matrix associated with a given layer $l(\mathbf{D}_{t\times t}^l)$, we have the fuzzy weight associated with each respective component a of layer $l(w_a^l)$ given by

$$w_a^l = \frac{\sqrt[t]{\left(\prod_{b=1}^t \sigma_{ab}^l\right)}}{\sum\limits_{\tilde{a}=1}^t \left[\sqrt[t]{\left(\prod\limits_{b=1}^t \sigma_{ab}^l\right)}\right]}.$$
(A.3)

Based on Eq. (A.3), the relative weights associated with the criteria of layer 2 and sub-criteria of layer 3 can then be readily approximated, and used in Eq. (3) to estimate $E_{ii}^k(n)$.

Appendix B. Dynamic estimation of strength-related parameters

In order to obtain the minimum mean square estimates of the time-varying strength-related parameters (i.e, $\mathbf{W}(n)$) of the proposed model, the fundamentals of an extended Kalman filter are applied. In reality, Kalman filtering techniques have been investigated for decades, and applied successfully in many areas such as spacecraft navigation, target tracking, and stochastic optimal control [42]. Relative to a basic Kalman filter which is limited to linear stochastic systems, an extended Kalman filter is more advanced as it is derived particularly for the dynamic estimation and prediction of nonlinear stochastic systems. The description in terms of the fundamentals and distinctive features of extended Kalman filtering technologies can also be readily found elsewhere [43], and thus is omitted in this paper. Based on the principles of an extended Kalman filter, four computational steps are mainly involved to dynamically estimate these strength-related parameters, and they are detailed in the following.

Step 0. Initialize the system states and parameters. Let n = 1, and W(0), $Cov_W(0)$, $Cov_U(0)$ be given based on the survey data collected at the beginning of the survey period.

Step 1. Compute the prior prediction of the dynamic state vector ($\mathbf{W}(n \mid n - 1)$) of strength-related parameters and the state prediction error covariance matrix ($\mathbf{Cov}_{\mathbf{W}}(n \mid n - 1)$), respectively, by

$$\mathbf{W}(n \mid n-1) = \mathbf{W}(n-1) \tag{B.1}$$

$$\mathbf{Cov}_{\mathbf{W}}(n \mid n-1) = \mathbf{Cov}_{\mathbf{W}}(n-1) + \mathbf{Cov}_{\mathbf{U}}(n-1) \tag{B.2}$$

where $Cov_U(n-1)$ represents a $(M+6) \times (M+6)$ covariance matrix of U(n-1), and is given based on the collected data associated with a given consumer at purchase occasion n-1.

Step 2. Calculate the dynamic Kalman gain ($\delta(n)$) by

$$\delta(n) = \mathbf{Cov}_{\mathbf{W}}(n \mid n-1)\mathbf{\dot{H}}^{T}(\mathbf{W}(n), n) \left[\mathbf{\dot{H}}(\mathbf{W}(n), n) \mathbf{Cov}_{\mathbf{W}}(n \mid n-1)\mathbf{\dot{H}}^{T}(\mathbf{W}(n), n) + \mathbf{Cov}_{\mathbf{V}}(n)\right]^{-1}$$
(B.3)

where $\mathbf{Cov}_{\mathbf{v}}(n)$ is the covariance matrix of $\mathbf{V}(n)$, and is given based on the collected data associated with a given consumer at purchase occasion n; $\dot{\mathbf{H}}(\mathbf{W}(n), n)$ is a differentiation function of $\mathbf{H}(\mathbf{W}(n), n)$ with respect to $\mathbf{W}(n \mid n - 1)$, and thus is given by

$$\dot{\mathbf{H}}(\mathbf{W}(n),n) = \frac{\partial \mathbf{H}(\mathbf{W}(n),n)}{\partial \mathbf{W}(n\mid n-1)}.$$
(B.4)

Step 3. Estimate the dynamic state vector of strength-related parameters ($\mathbf{W}(n)$) by

$$\mathbf{W}(n) = \mathbf{W}(n \mid n-1) + \delta(n) \Delta \mathbf{Z}(n \mid n-1)$$
(B.5)

where $\Delta \mathbf{Z}(n \mid n-1)$ is given by

$$\Delta \mathbf{Z}(n \mid n-1) = \mathbf{Z}(n) - \mathbf{H}[\mathbf{W}(n \mid n-1), n].$$
(B.6)

In Eq. (B.6), $\mathbf{H}[\mathbf{W}(n \mid n-1), n]$ is the prior measurement-component vector, in which each element is associated with a respective element of $\mathbf{Z}(n)$, and measured using the prior predictions of strength-related parameters involved in $\mathbf{W}(n \mid n-1)$. Furthermore, it is necessary that the estimates of strength-related parameters ($\hat{w}(n)$) embedded in $\mathbf{W}(n)$ should be subject to non-negative values ($\hat{w}(n) > 0, \forall n$). Therefore, a truncation rule is also added here to deal with the above issue, i.e.,

$$IF\hat{w}(n) < 0 \Rightarrow \hat{w}(n) = 0. \tag{B.7}$$

Step 4. Update the state estimation error covariance matrix ($Cov_W(n)$) by

$$\mathbf{Cov}_{\mathbf{W}}(n) = \begin{bmatrix} \mathbf{I} - \boldsymbol{\delta}(n)\dot{\mathbf{H}}(\mathbf{W}(n), n) \end{bmatrix} \mathbf{Cov}_{\mathbf{W}}(n \mid n-1).$$
(B.8)

The estimated strength-related parameters of W(n) (obtained by Eq. (B.5)) then can be fed back for the use of the proposed model. Furthermore, the estimates of W(n) and $Cov_W(n)$ can be utilized for the state estimation at the next purchase occasion (i.e., let n = n + 1, and then go back to *Step* 1 for the recursive estimation of the next iteration).

References

- [1] D.H. Silvera, B. Austad, Factors predicting the effectiveness of celebrity endorsement advertisements, European Journal of Marketing 38 (2004) 1509–1526.
- [2] D. Kilburn, Star power, Adweek 39 (1998) 20-21.
- [3] M.A. Kamins, M.J. Brand, S.A. Hoeke, J.C. Moe, Two-sided versus one-sided celebrity endorsements: the impact on advertising effectiveness and celebrity, Journal of Advertising 18 (1989) 4–10.
- [4] H.H. Friedman, L. Friedman, Endorser effectiveness by product type, Journal of Advertising Research 19 (1979) 63–71.
- [5] R.E. Petty, J.T. Cacioppo, D. Schumann, Central and peripheral routes to advertising effectiveness: the moderating role of involvement, Journal of Consumer Research 10 (1983) 135–146.
- [6] G. McCracken, Who is the celebrity endorser? cultural foundations of the endorsement process, Journal of Consumer Research 16 (1989) 310–321.
 [7] L.B. Kahle, P.M. Homer, physical attractiveness of celebrity endorsers: a social adaptation perspective, Journal of Consumer Research 11 (1985)
- 954–961. [8] R. Ohanian, The impact of celebrity spokespersons' perceived image on consumers' intention to purchase, Journal of Advertising Research 31 (1991)
- 46-54. [9] KA. Farrell, G.V. Karels, K.W. Monfort, C.A. McClatchev, Celebrity performance and endorsement value: the case of Tiger Woods, Managerial Finance
- [9] K.A. Farrell, G.V. Karels, K.W. Monfort, C.A. McClatchey, Celebrity performance and endorsement value: the case of liger woods, Managerial Finance 26 (2000) 1–15.
- [10] M. Walker, L. Langmeyer, D. Langmeyer, Celebrity endorsers: do you get what you pay for?, The Journal of Services Marketing 6 (1992) 35–42.
- [11] J. Agrawal, W.A. Kamakura, The economic worth of celebrity endorsers: an event study analysis, Journal of Marketing 59 (1995) 56-62.
- [12] J.P. Peter, J.C. Olson, Consumer Behavior and Marketing Strategy, McGraw-Hill Companies, Inc., New York, 2001.
 [13] G. Tom, R. Clark, L. Elmer, E. Grech, J. Masetti Jr., H. Sandhar, The use of created versus celebrity spokespersons in advertising, Journal of Consumer Marketing 9 (1992) 45-51.
- [14] R.R. Dholakia, B. Sternthal, Highly credible sources: persuasive facilitators or persuasive liabilities, Journal of Consumer Research 3 (1977) 223–232.
- [15] J. Frieden, Advertising spokesperson effects: an examination of endorser type and gender on two audiences, Journal of Advertising Research 24 (1984)
 33-41.
- [16] I. Ajzen, M. Fishbein, Understanding Attitudes and Predicting Social Behavior, Prentice-Hall, Engewood Cliffs, NJ, 1980.
- [17] J.C. Mowen, S.W. Brown, in: Kent B. Monroe, Ann Arbor (Eds.), On Explaining and Predicting the Effectiveness of Celebrity Endorsers, in: Advances in Consumer Research, vol. 8, Association for Consumer Research, MI, 1980, pp. 437–441.
- [18] R.L. Horton, Buyer Behavior; A Decision-Making Approach, Columbus, Charles E. Merrill Publishing Company, Ohio, 1984.
- [19] K. Lancaster, A new approach to customer's theory, Journal of Political Economy 74 (1966) 132–157.
- [20] J.A. Howard, Consumer Behavior: Application of Theory, McGraw-Hill, New York, 1977.
- [21] N. Govoni, R. Eng, M. Galper, Promotional Management, Prentice-Hall, New Jersey, 1986.
- [22] R. Thaler, Mental accounting and consumer choice, Marketing Science 4 (1985) 199-214.
- [23] M.U. Kalwani, C.K. Yim, H.J. Rinne, Y. Sugita, A price expectations model of consumer brand choice, Journal of Marketing Research 27 (1990) 251–262.
 [24] W.B. Dodds, K.B. Monroe, D. Grewal, Effects of price, brand, and store information on buyers' product evaluation, Journal of Marketing Research 28 (1991) 307–319.
- [25] J. Swait, J.C. Sweeney, Perceived value and its impact on choice behavior in a retail setting, Journal of Retailing and Consumer Services 7 (2000) 77-88.
- [26] R.J. Lavidge, G. Steiner, A model for predictive measurements of advertising effectiveness, Journal of Marketing 25 (1961) 59-62.
- [27] D. Bouyssou, T. Marchant, M. Pirlot, P. Perny, A. Tsoukias, P. Vincke, Evaluation Models: A Critical Perspective, Kluwer, Boston, 2000.
- [28] Y.-M. Wang, K.-S. Chin, A new data envelopment analysis method for priority determination and group decision making in the analytic hierarchy process, European Journal of Operational Research 195 (2009) 239–250.
- [29] C.-H. Yeh, Y.-H. Chang, Modeling subjective evaluation for fuzzy group multicriteria decision making, European Journal of Operational Research 194 (2009) 464–473.
- [30] P.J.M. Laarhoven, W. Pedrycz, A fuzzy extension of Saaty's priority theory, Fuzzy Sets and Systems 11 (1983) 229-241.
- [31] J.J. Buckley, The multiple judge, multiple criteria ranking problem: a fuzzy set approach, Fuzzy Sets and Systems 13 (1984) 25-37.
- [32] F. Lootsma, Fuzzy Logic for Planning and Decision-Making, Kluwer, Dordrecht, 1997.
- [33] H.M. Spielman, The celebrity sell: making it work, Marketing Times 28 (1981) 13-14.
- [34] J. Wiener, J.C. Mowen, Source credibility: On the independent effects of trust and expertise, in: R.J. Lutz (Ed.), in: Advances in Consumer Research, vol. 13, Association for Consumer Research, 1985, pp. 306–310.
- [35] J.B. Cohen, E. Golden, Informational social influence and product evaluation, Journal of Applied Psychology 56 (1972) 54-59.
- [36] H. Assael, Consumer Behavior and Marketing Action, Kent Publishing Company, Boston, 1984.
- [37] S. Rosen, Hedonic prices and implicit markets: production differentiation in pure competition, Journal of Political Economy 82 (1974) 34–55.
- [38] L.P.L. Favero, C.F. De Angelo, A.J. Kos, R.V. Eunni, Modeling consumer preferences: an empirical study of the automobile market in Brazil, Internal Journal of Management 23 (2006) 312–320.
- [39] K.B. Monroe, R. Krishnan, The effect of price on subjective product evaluations, in: Harold H. Kassarjian, Thomas S. Robertson (Eds.), Perspectives in Consumer Behavior, 3rd ed., Scott, Foresman and Company, Glenview, IL, 1985, pp. 43–55.
- [40] H. Helson, Adaptation-Level Theory, Harper & Row Publishers, Inc., New York, 1964.
- [41] T. Erdem, M.P. Keane, Decision-making under uncertainty: capturing dynamic brand choice processes in turbulent consumer goods markets, Marketing Science 15 (1996) 1–20.
- [42] M.S. Santina, A.R. Stubberud, G.H. Hostetter, Stochastic systems and recursive estimation, in: M.S. Santina, et al. (Eds.), Digital Control System Design, Saunders College Publishing, Harcourt Brace College Publishers, San Diego, 1994, pp. 580–679.
- [43] M.S. Grewal, A.P. Andrews, Kalman Filtering: Theory and Practice Using MATLAB, John Wiley and Sons, New York, 2001.
- [44] M. Fishbein, I. Ajzen, Belief, Attitude, Intention, and Behavior: An Introduction to Theory and Research, Addison-Wesley, Reading MA, 1975.
- [45] S.M. Ross, Markov chains, in: Sheldon M. Ross (Ed.), Introduction to Probability Models, Academic Press, San Diego, CA, 1993.
- [46] Freakonomics. A Sports-Star Stimulus Package. The New York Times, 2008. http://freakonomics.blogs.nytimes.com/2008/04/21/a-sports-starstimulus-package/.
- [47] R.A. Johnson, D.W. Wichern, Applied Multivariate Statistical Analysis, Prentice-Hall, New Jersey, 1992.
- [48] J.J. Buckley, Ranking alternatives using fuzzy members, Fuzzy Sets and Systems 15 (1985) 21–31.
- [49] J. Aczel, T.L. Saaty, Procedures for synthesizing ratio judgments, Journal of Mathematical Psychology 27 (1983) 93-102.