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A dynamic programming algorithm based on expected revenue approximation for the network revenue management problem

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ABSTRACT

Since American Airlines successfully applied revenue management (RM) to raise its revenue, RM has become a common technique in the airline industry. Due to the current hub-and-spoke operation of the airline industry, the focus of RM research has shifted from the traditional single-leg problem to the network-type problem. The mainstream approaches, bid price and virtual nesting, are faced with some limitations such as inaccuracy due to their suboptimal nature and operation interruption caused by the required updates. This study developed an algorithm to generate a seat control policy by approximating the expected revenue function in a dynamic programming (DP) model. In order to deal with the issue of dimensionality for the DP model in a network context, this study used a suitable parameterized function and a sampling concept to achieve the approximation. In the numerical experiment, the objective function value of the developed algorithm was very close to the one achieved by the optimal control. We believe that this approach can serve as an alternative to the current mainstream approaches for the network RM problem for airlines and will provide an inspiring concept for other types of multi-resource RM problems.

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1. Introduction

Revenue management (RM) has become common practice in the airline industry ever since American Airlines successfully applied several RM techniques to raise its revenue. For example, it has been estimated that RM practices generated an additional revenue of US\$1.4 billion for American Airlines over a 3-year period around 1988 (Smith et al., 1992). In today's market it is very difficult for any major airline to operate profitably without RM, since according to most estimates the revenue gained by applying RM is about 4–5%, which is comparable to many airlines' total profitability in a good year (Talluri and van Ryzin, 2004). Nonetheless, how to realize the basic concept of RM, selling the right seat to the right customer at the right price, remains a challenge.

Due to the current hub-and-spoke operation of the airline industry, the focus of RM research has shifted from the traditional single-leg problem to the network problem. With multiple types of products and resources, the decision of how to sell one type of product is complicated by its impact on the future sales of the product types sharing the same resource(s). The problem complexity and the associated computational load make it impossible to derive the optimal control for a problem of practical size. The mainstream approaches, bid price and virtual nesting, have some limitations such as the inaccuracy due to their suboptimal nature and the operation interruption caused by the required updates. This study developed an algorithm

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to generate a seat control policy by approximating the expected revenue function in a dynamic programming (DP) model. In order to deal with the issue of dimensionality in a network context, this study used a suitable parameterized function and a sampling concept to achieve the approximation.

The remainder of this paper is organized as follows. Section 2 provides the background of the problem and reviews the related literature. The DP model for the network RM problem and the algorithm based on a parameterized function are presented in Section 3. The numerical experiment is described in Section 4. Finally, the findings of this study are summarized and conclusions are drawn in Section 5.

2. Background and prior research

Most early seat-inventory control researches relied on the following six assumptions: (1) sequential booking classes, (2) low-before-high-fare booking arrival pattern, (3) statistical independence of demands between booking classes, (4) no cancellation or no-shows, (5) single flight leg, and (6) no batch booking (McGill and Van Ryzin, 1999). For example, Belobaba (1989) developed the Expected Marginal Seat Revenue (EMSR) heuristic. The key concept of the EMSR approach is to compare the marginal value of the seat with the ticket price of the fare class when making the accept/deny decision. Although it only provides the optimal solution for the two-fare case, one advantage of the EMSR approach is that its implementation is relatively easy. In addition, the generated solution appears to be very close to the optimal solution. Nonetheless, Curry (1990), Wollmer (1992), Brumelle and McGill (1993) further developed the method in order to find the global optimal solution.

These above researches are often referred to as the static models, since the demand for each fare class is modeled by a random variable, based on the above first and second assumptions. These two assumptions greatly simplify the complexity of the RM problems, but inevitably leave some demand characteristics overlooked. In order to incorporate the time-dependent characteristic of demand, Lee and Hersh (1993) developed a DP model in which the request probability based on the Poisson arrival process is used to represent the demand pattern. Thus, the assumption of the sequential arrival for booking classes is relaxed and the booking patterns for different classes, characterized by the request probabilities indexed by booking periods, are allowed to overlap in time. Lee and Hersh (1993) further generalized the sixth assumption from single-seat booking to batch booking, and thus the request probability turns out to be dependent upon the booking size as well. Finally, Subramanian et al. (1999) developed a DP model to take into account cancellation and no-show in the fourth assumption and considered the penalty due to overbooking, a classic method to counter cancellation and no-show. The network RM problem relaxes the fifth assumption to handle a problem of multiple products with shared resources. Although the features of batch booking and cancellation/no-show are not considered in the formulated problem for this study, they can be incorporated by the techniques developed by Lee and Hersh (1993) and Subramanian et al. (1999).

The most popular approach for the network RM problem is the bid-price control (Williamson, 1992). A bid price is attached to each leg, and a booking request for a fare class of an origin–destination pair (later referred to as an ODF) is accepted if its revenue is greater than the sum of the bid prices of the used legs. The key issue for most bid-price based algorithms is finding a suitable set of bid prices, supposedly depending on the available leg seats and the time periods left before departure. Williamson (1992) set the bid prices as the dual prices of the leg capacity constraints in a linear programming (LP) model, which was basically a static model. This generally required frequent updates of bid prices during the booking process. The other issue associated with Williamson (1992) was that the stochastic feature of the demand was overlooked in the deterministic LP model. Several researches (e.g., Adelman, 2007; Bertsimas and Popescu, 2003; Klein, 2007; Talluri and van Ryzin, 1999; Topaloglu, 2008; Kunnumkal and Topaloglu, 2010) have developed sophisticated algorithms to generate better bid prices for addressing the dynamic and/or stochastic feature of the demand. In addition, Talluri and van Ryzin (1998) and Topaloglu (2009) discussed the characteristics of the asymptotic optimality of the bid-price control.

The other popular approach for the network RM problem is virtual nesting, which is more suitable for being integrated with the pre-existing leg-based RM systems of many airlines. The network RM problem is decomposed into subproblems for the individual legs through mapping the multi-leg ODFs to the fare groups in the single-leg problems, which are handled by the control of booking limits or protection levels. The associated mapping (or called as indexing) process and the resulted control scheme are more complicated when compared with the simple decision rule of the bid-price control. In addition, the estimation of the seat value, which should be time- and state-dependent, remains an essential task for this type of control. Several researchers, such as Bertsimas and de Boer (2005), van Ryzin and Vulcano (2008), and Erdelyi and Topaloglu (2009), developed sophisticated virtual-nesting algorithms to handle the dynamic feature of the demand and the interrelationship among resources and products inherited in the network RM problem.

The discussion of the advantages and disadvantages of the bid-price control and the virtual nesting can be found in the chapter dedicated to the network RM problem in Talluri and van Ryzin (2004) and Phillips (2005). However, the present study does not use these two indirect control mechanisms and directly works on approximating the expected value to generate the seat-inventory control policy.

3. Mathematic models and seat control algorithms

In an airline network, an ODF can utilize the seats of multiple legs, and a seat on a leg is usually shared by multiple ODFs. The network RM problem incorporating this network feature and the dynamic characteristics of the demand can be

formulated as a DP model, as described in the first sub-section. The second sub-section presents the control policy based on a parameterized function for expected revenue approximation. The third sub-section provides the DP model used to compute the objective function value (*i.e.*, the expected revenue at the end of the booking process) associated with a control policy.

3.1. DP model and optimal control of the network RM problem

Following a similar DP formulation in Lee and Hersh (1993) for the dynamic single-leg RM problem, the network RM problem considered in this study is as follows (Talluri and van Ryzin, 2004):

$$V_t(\mathbf{x}) = P_t^0 V_{t-1}(\mathbf{x}) + \sum_{j=1}^J P_t^j \max(V_{t-1}(\mathbf{x} - \mathbf{S}^j) + F_j, V_{t-1}(\mathbf{x})) \tag{1}$$

where $P_t^0 = 1 - \sum_{j=1}^J P_t^j$.

- t : indices of decision periods ($t = 0 \dots T$, assuming $t = 0$ is the period of flight departure, and $t = T$ is the beginning of the booking process).
- i : indices of legs ($i = 1 \dots I$).
- j : indices of ODFs ($j = 1 \dots J$).
- P_t^j : probability of the booking request for ODF j at period t .
- F_j : revenue of ODF j .
- S : an incident matrix ($I \times J$), representing the relation between the ODFs and the legs. Its entry s_{ij} is equal to 1 if ODF j uses leg i ; otherwise, it is 0.
- S^j : the j th column vector of S , representing the legs used by ODF j .
- x_i : the number of available seats on leg i , and the vector x represents the available seats on all legs.
- $V_t(x)$: expected revenue given the available seats on the legs x at period t .

The Bellman equation of the DP model (1) shows how to evaluate the expected revenue given the arrival information of the demands in a recursive manner. With the boundary condition $V_0(x) = 0$ at the end of the booking process (flight departure), the objective is to maximize the expected revenue $V_T(C)$ given C seats available on the legs (*i.e.*, the system capacity) at the beginning of the booking process. With a two-leg system as an example, Fig. 1 illustrates the structure of this DP problem.

The optimal control policy that results in the maximum expected revenue can be generated by (2) based on the two terms inside the max function of (1). For each period t given the available seats on the legs x , a booking request of ODF j should be accepted if its revenue is larger than the expected revenue decrease due to the state change (*i.e.*, the opportunity cost) at period $(t - 1)$.

$$F_j \geq V_{t-1}(\mathbf{x}) - V_{t-1}(\mathbf{x} - \mathbf{S}^j) \tag{2}$$

The computational load to evaluate the expected revenue for (1) and then generate the optimal control policy based on (2) for the entire state space is intractable for most practical problems. Even for the illustrative example with a two-dimensional state space, the computation time using a personal computer is non-negligible. Thus, an approximate algorithm with manageable computational load and acceptable solution quality is needed. The present study uses a suitable type of parameterized function that approximates the expected revenue $V_t(x)$ in (1), for generating the control policy. This new approach shares some advantages with the classic bid-price control, such as the simplicity of the control scheme and the nominal requirement for data storage. However, the parameters of the approximate functions for all time periods can be estimated

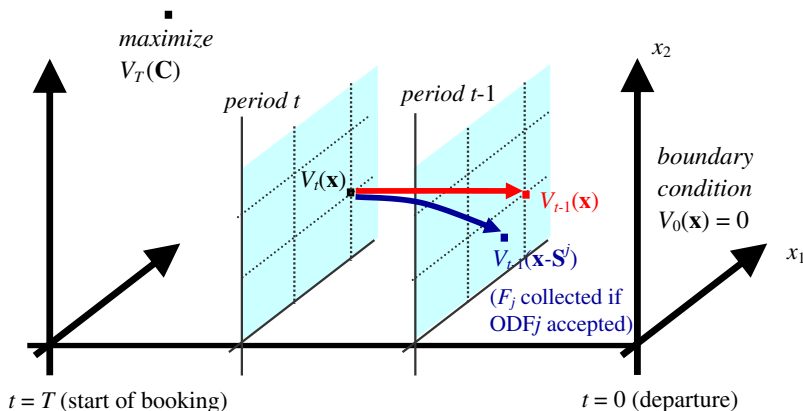


Fig. 1. Illustration of the DP model for the network RM problem.

by standard curve fitting techniques, and the control policy is thus available for all time periods and resource states. Therefore, unlike many bid-price and virtual-nesting algorithms, no update is required during the booking process if the initial estimation of the demand remains unchanged. Finally, in order to alleviate the computational load, only a limited number of points in the state space are evaluated for curve fitting and parameter estimation.

Approximate DP has been an important branch of DP research, as the curse of dimensionality is a serious problem for DP. Bertsekas (2005) and Powell (2007) serve as excellent references for the associated techniques. Working on a reduced set of the state space is one of the techniques to overcome the computational challenge. As an example for the RM researches applying a similar technique, Huang and Chang (2010) evaluate a limited number of points evenly spaced in the state space to deal with a single-leg air cargo RM problem considering the stochastic volume and weight of shipments. The values of the non-grid points are estimated by the linear approximation based on the plane defined by the neighboring grid points. Basically, their technique is only applicable to the case of two dimensions. For the present study, the limited points in the state space are randomly sampled and used in a non-linear regression for curve fitting, which can deal with the case with a dimension higher than two.

3.2. Control policy based on a parameterized function

The expected revenue function $V_t(x)$ in (1) has some special features due to the nature of RM problems. First, it is monotonously increasing with respect to the number of leg seats x , as more seats always generate more revenue. Second, the marginal benefit (revenue contribution) of a seat on a leg is diminishing, and the expected revenue function should reach a fixed value if the available seats on the legs are enough to accommodate all possible booking requests. These features provide the basis to choose the parameterized function $g_t(x)$ with an exponential form defined in (3) to approximate the expected revenue function $V_t(x)$ in (1). This exponential function is chosen because of its simplicity. In addition, fitting exponential functions by non-linear regression is a well-studied issue (e.g., Seber and Wild, 1989).

$$g_t(\mathbf{x}) = A_t \left(1 - \sum_{i=1}^I b_i^i e^{-a_i^i x_i} \right) \quad (3)$$

$$A_t = \sum_{j=1}^J \left(F_j \sum_{\tau=1}^t P_\tau^j \right) \quad (4)$$

- $g_t(x)$: parameterized function to approximate the expected revenue given the available seats on the legs x at period t

This function is horizontally asymptotic to parameter A_t , which is computed by (4) to represent the maximum possible revenue, given the possible booking requests from period t to the end of the booking process, by assuming there are an infinite number of leg seats. The other two leg-specific parameters (a_i^i and b_i^i) are used to model the rising characteristic of the approximate function with respect to the number of seats x_i on the leg. Their values can be determined by the DP-based procedure specified by (5) and (6).

The procedure to estimate the parameters of the approximate function has a structure similar to the original DP formulation of the network RM problem, which suffers from an enormous amount of points in the state space. However, for the purpose of curve fitting, not many data points are actually required. For example, the well-adopted regression analysis does not rely on a massive amount of data. Thus, the concept of sampling is introduced, and only a limited number of points in the state space are evaluated to determine the parameterized function. The related computation can be represented by (5) and (6), and the whole procedure is illustrated in Fig. 2.

$$W_t(\mathbf{x}_t^h) = P_t^0 g_{t-1}(\mathbf{x}_t^h) + \sum_{j=1}^J P_t^j R_t(F_j, g_{t-1}(\mathbf{x}_t^h)) \quad (5)$$

$$R_t(F_j, g_{t-1}(\mathbf{x}_t^h)) = \begin{cases} F_j + g_{t-1}(\mathbf{x}_t^h - \mathbf{S}^j) & \text{if } F_j \geq g_{t-1}(\mathbf{x}_t^h) - g_{t-1}(\mathbf{x}_t^h - \mathbf{S}^j) \\ g_{t-1}(\mathbf{x}_t^h) & \text{otherwise} \end{cases} \quad (6)$$

for $h = 1, \dots, H$

- \mathbf{x}_t^h : one sampling point in the state space at period t , and H is the number of total sampling points.
- $W_t(\mathbf{x}_t^h)$: expected revenue for the sampling point \mathbf{x}_t^h under the control rule of $g_{t-1}(x)$.
- $R_t(F_j, g_{t-1}(x))$: revenue function at period t evaluated on the basis of $g_{t-1}(x)$ and under the control rule of $g_{t-1}(x)$.

Given the boundary condition $g_0(x) = 0$, for each time period t , a limited number (H) of data points in the state space are randomly sampled. For each sampling point, the expected revenue is evaluated based on the parameterized function of the previous stage $g_{t-1}(x)$ as shown in (5) and (6). Once all of the sampling points are evaluated, the parameterized function for the current stage $g_t(x)$ can be established by any curve fitting technique to determine the values of the parameters of all legs (a_i^i and b_i^i , $\forall i$). For this study, the *nlinfit* (non-linear fit) function of a software package was used (MATLAB, 2009).

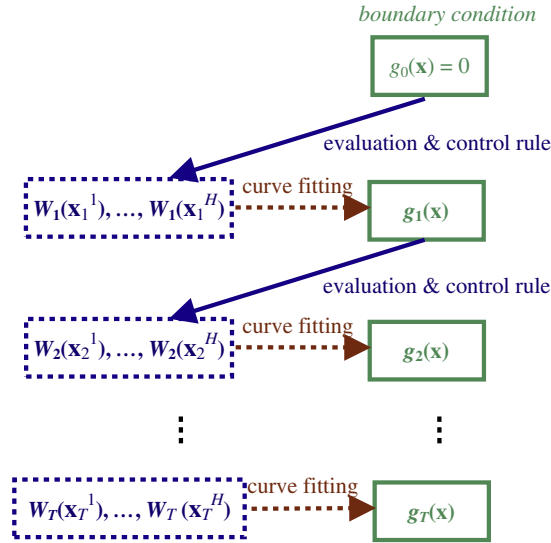


Fig. 2. Generation of the parameterized functions by sampling.

Once the recursive process of (5) and (6) is completed from $t = 1$ to T , the parameterized function for each time period $g_t(x)$ is available. The control rule for all possible situations (the whole state space) can then be determined by (7), which is similar to the optimal control specified by (2). As the computation of (3) and (7) is nominal for today’s personal computers, the accept/deny decision for a booking request during the booking process can be made right away based on the stored parameters. The decision process is similar to that of the bid-price control, but the computational effort as well as the data storage requirement is slightly higher. However, no update is required, unless it is found that the initial demand assumption has changed.

$$F_j \geq g_{t-1}(\mathbf{x}) - g_{t-1}(\mathbf{x} - \mathbf{S}^j) \tag{7}$$

It is worth noting that the control decision in (2) and (7) is based on the relative difference due to state change (i.e., the opportunity cost), and not on the absolute value. Thus, even though there is an approximation error for $g_t(x)$, a good control decision can still be generated as long as the approximated opportunity cost is not far from the true value.

3.3. DP model for determining the expected revenue of a policy

A numerical experiment, presented in the next section, was conducted to verify the effectiveness of the developed algorithm. The size of the test problem is small, so the optimal control policy as well as the maximum expected revenue can be determined by (1) and (2). At the same time, based on (5) and (6), the parameterized functions for all time periods, i.e., $g_t(x) \forall t$, can be established. Since the problem size is small, the expected revenue associated with this policy can be determined by the DP model of (8) and (9). The whole procedure is illustrated in Fig. 3.

$$Z_t(\mathbf{x}) = P_t^0 Z_{t-1}(\mathbf{x}) + \sum_{j=1}^J P_t^j Y_t(F_j, g_{t-1}(\mathbf{x})) \tag{8}$$

$$Y_t(F_j, g_{t-1}(\mathbf{x})) = \begin{cases} F_j + Z_{t-1}(\mathbf{x} - \mathbf{S}^j) & \text{if } F_j \geq g_{t-1}(\mathbf{x}) - g_{t-1}(\mathbf{x} - \mathbf{S}^j) \\ Z_{t-1}(\mathbf{x}) & \text{otherwise} \end{cases} \tag{9}$$

- $Z_t(x)$: expected revenue under the control rule of $g_{t-1}(x)$ given the available seats on the legs x at period t .
- $Y_t(F_j, g_{t-1}(x))$: revenue function at period t evaluated on the basis of $Z_{t-1}(x)$ and under the control rule of $g_{t-1}(x)$.

As a benchmarking policy, the first-come-first-served (FCFS) policy (effectively with no RM control) was tested to evaluate the benefit of the RM policies. The expected revenue under the FCFS control can be determined by a procedure similar to that in (8) and (9). However, the decision rule in (9) is changed to a booking request being always accepted as long as the available seats on the legs can accommodate it. (Alternatively, the FCFS control is identical to the case that $g_t(x)$ are set as zero for all time periods in Fig. 3.)

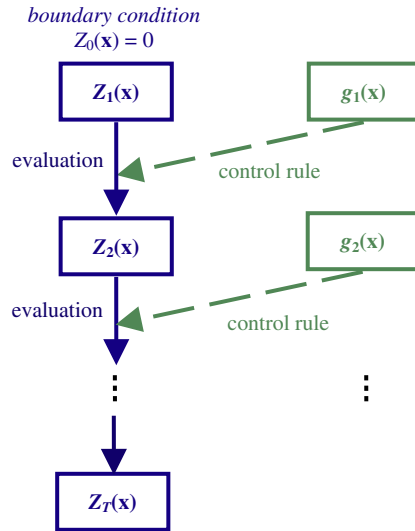


Fig. 3. Determination of the expected revenue of a policy.

4. Numerical experiment

One small two-leg network from Bertsimas and Popescu (2003), as shown in Fig. 4, was used to design the test problems, for which the optimal control policy can be derived by a usual personal computer within seconds. The three nodes in the network are *o*, *h*, *d*, and the two legs are denoted by *oh* and *hd* with the capacity of C_{oh} and C_{hd} respectively. Three OD pairs combined with two fare classes result in six ODFs. The related information of the test problem is summarized as follows.

- Capacity $C = (C_{oh}, C_{hd}) = (100, 100)$.
- Fares $(F_{oh}^{high}, F_{oh}^{low}, F_{hd}^{high}, F_{hd}^{low}, F_{ohd}^{high}, F_{ohd}^{low}) = (30, 25, 25, 20, 50, 35)$.
- Number of time periods, $T = 400$.
- Request probabilities $(P_{oh}^{high}, P_{oh}^{low}, P_{hd}^{high}, P_{hd}^{low}, P_{ohd}^{high}, P_{ohd}^{low})$ for all time periods were set as (0.2, 0.2, 0.15, 0.15, 0.05, 0.05).

The results of the experiment are summarized in Table 1. The maximum expected revenue can be determined by solving the DP model in (1) for the optimal control, and its value, that is, $V_{400}(100,100)$, is 5267.7. The expected revenue under the FCFS control is 4850.3, suggesting that the benefit of applying the optimal RM is about 8%. As for the expected revenue for the approximate control policy based on the parameterized function, a series of experiments were designed to examine the

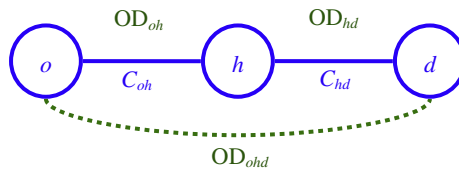


Fig. 4. Two-leg network in the numerical experiment.

Table 1
Performance for the policies with different numbers of sampling points.

Policy	Expected revenue	Solution quality (%)	Standard deviation
Optimal	5267.70	–	
FCFS	4850.30	–7.92	
Approximate ($H = 1250$)	5196.15	–1.36	1.670
Approximate ($H = 500$)	5195.83	–1.36	1.805
Approximate ($H = 300$)	5196.23	–1.36	4.640
Approximate ($H = 200$)	5193.83	–1.40	6.345
Approximate ($H = 80$)	5190.09	–1.47	17.964

effect of the number of sampling points. Given the total $100^2 = 10,000$ data points in the state space, H (i.e., the number of sampling points) was chosen to be 1250, 500, 300, 200 and 80, respectively. Due to the randomness of the sampling process, the estimated parameters are different from one run to another. For each sampling size, the procedure based on (5) and (6) for parameter estimation and policy establishment was performed 20 times, and the expected revenue of the DP model of (8) and (9) for each run was computed. The mean value relative to the maximum value achieved by the optimal control and the standard deviation were used to present the performance for a chosen sampling size.

In general, the quality of the solution is good, as the gaps between the optimal control and the control policies based on parameterized functions are all less than 1.5%. As expected, the solution quality increases if the number of sampling points is increased. However, the improvement is not very significant when compared with the increased computational effort required. This result is consistent with the experience with the regression analysis, which usually does not need an extremely large number of observations to achieve a good fit. Nonetheless, a policy based on a larger number of sampling points tends to result in a more stable performance, as the standard deviation is smaller.

How the parameterized function reacts to the change of demand and resource from period to period is critical in approximating the opportunity cost and then the control policy. The values of the parameters (in log scale) in one run of the case with 200 sampling points are shown in Fig. 5. In general, the values of the exponential parameters (a_i) decrease from the departure to the beginning of booking process ($t = 0$ to T). The larger values for the periods close to the departure indicate that, if there are fewer periods and fewer booking requests to sell a seat, the exponential function is easier to saturate as the number of seats increases. As the demand level and the fare are higher for the ODFs related to the first leg, its exponential parameter and weighting parameter (a_1 and b_1 respectively) are larger than those of the second leg for the same period.

In order to understand the performance of the approximate control policy under various demand–supply levels and demand patterns, the control policy based on the parameterized function was applied to two series of test problems. In particular, the number of sampling points was chosen as 200. The request probabilities were first scaled by a factor (n) of 0.9–1.1, and the results are summarized in Table 2. In general, the solution quality under various demand–supply levels is consistent with that in the base case, although the performance of the proposed algorithm is slightly degraded when demand level is low.

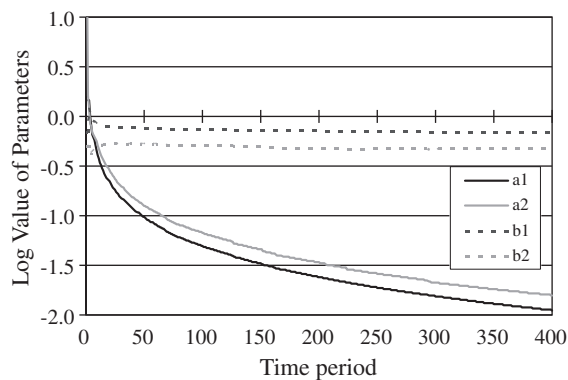


Fig. 5. Estimations of function parameters through time periods.

Table 2
Solution quality and demand–supply level.

Scaling factor, n	Policy	Expected revenue	Solution quality (%)
1.10	Optimal	5317.20	–
	FCFS	4850.30	–8.78
	Approximate	5256.61	–1.14
1.05	Optimal	5295.20	–
	FCFS	4850.30	–8.40
	Approximate	5224.04	–1.34
1.00	Optimal	5267.70	–
	FCFS	4850.30	–7.92
	Approximate	5193.71	–1.40
0.95	Optimal	5234.50	–
	FCFS	4850.30	–7.34
	Approximate	5153.26	–1.55
0.90	Optimal	5197.00	–
	FCFS	4850.20	–6.67
	Approximate	5108.46	–1.70

Table 3
Solution quality and demand arrival pattern.

Low-before-high factor, m (%)	Policy	Expected revenue	Solution quality (%)
20	Optimal	5259.20	–
	FCFS	4755.10	–9.59
	Approximate	5161.10	–1.87
10	Optimal	5263.90	–
	FCFS	4802.70	–8.76
	Approximate	5180.71	–1.58
0	Optimal	5267.70	–
	FCFS	4850.30	–7.92
	Approximate	5193.71	–1.40
–10	Optimal	5271.00	–
	FCFS	4897.90	–7.08
	Approximate	5204.58	–1.26
–20	Optimal	5273.80	–
	FCFS	4945.50	–6.23
	Approximate	5207.07	–1.27

To change the demand arrival pattern, the request probabilities of the three high-fare ODFs were adjusted by the low-before-high factor (m) according to the linearly relation in (10). The cases of $m > 0$ are for the usual situation that high-fare booking requests, possibly made by business travelers, tend to arrive late. On the other hand, the request probabilities of the three low-fare ODFs were adjusted in a reverse fashion so as to make the overall expected number of booking requests remain the same as that in the base case. Based on the results summarized in Table 3, the solution quality for various arrival patterns is consistent with that in the base case. The result in the numerical experiment suggests that the new approach with the approximation by parameterized functions and by using the sampling method for curve fitting can be a promising alternative for the network RM problem for airlines, as the balanced solution quality and computational load should be acceptable for the industry.

$$P_t^j = P_t^j \left(1 + m \left(1 - \frac{t}{T/2} \right) \right) \quad (10)$$

5. Conclusions

Due to the current hub-and-spoke operation of the airline industry, research in the network RM problem for airlines is very important. The mainstream approaches, bid price and virtual nesting, have some limitations such as the inaccuracy due to their suboptimal nature and the interruption of the operation as a result of the required updates. This study developed a solution algorithm that generates a seat control policy by approximating the expected revenue function in a DP model. In order to deal with the issue of dimensionality in the context of a network, this study used a suitable parameterized function and a sampling approach to achieve the approximation.

Since the numerical experiment is based on a small two-leg network, it was possible to use a DP model to determine the expected revenue of the optimal control policy and the control policy developed by this study. The quality of the solution is consistent and quite close to that of the optimal control. It is worth noting that only a few sampling points are needed to achieve a good solution quality. This new approach can be used as an alternative to the popular mainstream approaches for the network RM problem for the airline industry, and should be an inspiring concept for other types of multi-resource RM problems.

In terms of future research directions, there are some issues that require further attention. First, at this moment, the parameterized function used in this study is an exponential-type function, which is simple and appears to work well. Nonetheless, it is possible that some other types of functions satisfying the characteristics of the expected revenue function in the DP model can achieve better control. In addition, there is a chance that a customized non-linear regression procedure, replacing the built-in function in the software package, can raise the computational efficiency or solution quality. Finally, in order to fully verify the strength and the limitation of the proposed algorithm, a simulation experiment based on large-scale problems with real operational data is being considered.

Acknowledgements

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