Nonlinear Dyn (2010) 62: 105–117
DOI 10.1007/s11071-010-9702-x

ORIGINAL PAPER

Yang and Yin parameters in the Lorenz system

Zheng-Ming Ge · Shih-Yu Li

Received: 19 January 2010 / Accepted: 12 March 2010 / Published online: 3 April 2010

Abstract The history of the Lorenz system is firstly discussed in this paper. In Chinese philosophy, *Yin* is the negative, historical, or feminine principle in nature, while *Yang* is the positive, contemporary, or masculine principle in nature. *Yin* and *Yan* are two fundamental opposites in Chinese philosophy (therefore, in this paper, these words "Yin parameter," "Yang parameter," "historical system," and "contemporary system" are used to represent the "positive parameter," "negative parameter," "time reversed ($-t$) system," and "time forward (t) system," respectively). Simulation results show that chaos of historical Lorenz system can be generated when using "*Yin*" parameters. To our best knowledge, most characters of contemporary Lorenz system are studied in detail, but there are no articles in making a thorough inquiry about the history of Lorenz system. As a result, the chaos of historical Lorenz system with "*Yin* parameters" is introduced in this paper and various kinds of phenomena in the historical Lorenz system are investigated by Lyapunov exponents, phase portraits, and bifurcation diagrams.

Keywords Time reversed Lorenz system · Yin parameters · Lyapunov exponent · Chaos

Z.-M. Ge (✉) · S.-Y. Li
Department of Mechanical Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan
e-mail: zmg@cc.nctu.edu.tw

1 Introduction

Nonlinear dynamics, commonly called the chaos theory, changes the scientific way of looking at the dynamics of natural and social systems, which has been intensively studied over the past several decades. The phenomenon of chaos has attracted widespread attention amongst mathematicians, physicists, and engineers. Chaos has also been extensively studied in many fields, such as chemical reactions, power converters, biological systems, information processing, secure communications, etc. [1–9]. While many researchers analyze complicated, physically motivated configurations, there is also a need to investigate simple equations which may capture the essence of chaos in a less involved setting, thereby aiding the understanding of essential characteristics. The original investigation of an extraordinary three-dimensional nonlinear system by the mathematical meteorologist Lorenz [10] who discovered chaos in a simple system of three autonomous ordinary differential equations in order to describe the simplified Rayleigh–Benard problem in 1963 (which is called the contemporary Lorenz system in this paper) is the most popular system for studying.

There are a lot of articles in studying the contemporary Lorenz system [11–15]. Although the contemporary Lorenz system has been analyzed in detail, there are no articles in looking into the history of the Lorenz system. In this paper, we find out that there are rich dynamics in this historical Lorenz system.

In Chinese philosophy, *Yin* is the negative, historical, or feminine principle in nature, while *Yang* is the positive, contemporary, or masculine principle in nature. *Yin* and *Yan* are two fundamental opposites in Chinese philosophy. In this paper, the historical Lorenz system is introduced and the chaotic behavior with *Yin* parameters is investigated by phase portrait, Lyapunov exponents, and bifurcation diagrams in the following simulation results. We use positive, i.e., *Yang*, parameters for the contemporary Lorenz system, and negative, i.e., *Yin*, parameters for the historical Lorenz system.

The layout of the rest of the paper is as follows. In Sect. 2, the contemporary Lorenz system is reviewed. In Sect. 3, the historical Lorenz system is introduced. In Sect. 4, three simulation cases of historical and contemporary Lorenz systems are given for comparing and observation. In Sect. 5, the family of Yin Lorenz system is further studied. In Sect. 6, conclusions are given.

2 Contemporary Lorenz system

Before introducing the historical Lorenz equation, the contemporary Lorenz system [1] can be recalled firstly in (1) as follows:

$$\begin{cases} \frac{dx_1(t)}{dt} = a(x_2(t) - x_1(t)), \\ \frac{dx_2(t)}{dt} = cx_1(t) - x_1(t)x_3(t) - x_2(t), \\ \frac{dx_3(t)}{dt} = x_1(t)x_2(t) - bx_3(t) \end{cases} \tag{1}$$

when initial condition $(x_{10}, x_{20}, x_{30}) = (-0.1, 0.2, 0.3)$ and parameters $a = 10$, $b = 8/3$ and $c = 28$, chaos of the contemporary Lorenz system appears. The chaotic behavior of (1) is shown in Fig. 1.

3 Historical Lorenz system

Historical Lorenz equations are:

$$\begin{cases} \frac{dx_1(-t)}{d(-t)} = a(x_2(-t) - x_1(-t)), \\ \frac{dx_2(-t)}{d(-t)} = cx_1(-t) - x_1(-t)x_3(-t) - x_2(-t), \\ \frac{dx_3(-t)}{d(-t)} = x_1(-t)x_2(-t) - bx_3(-t) \end{cases} \tag{2}$$

It is clear that in the left-hand sides of (2), the derivatives are taken with the back-time. It means (2) aims to find out the historical behavior of the Lorenz system and to comprehend the relation between history and present. The simulation results are arranged in Table 1.

Table 1 Dynamic behaviors of historical Lorenz system for different signs of parameters

a	*b*	*c*	States
−	+	+	Approach to infinite
+	−	+	Approach to infinite
+	+	−	Periodic
−	−	+	Approach to infinite
−	+	−	Approach to infinite
−	−	−	Chaos and periodic

Table 2 Range of parameter *c* of contemporary Lorenz system

20.0–24.1	Converge to a fixed point
24.1–70.0	Chaos

Table 1 shows the dynamic behaviors of historical Lorenz system for different signs of parameters. An awe-inspiring phenomenon is discovered. When initial condition $(x_{10}, x_{20}, x_{30}) = (-0.1, 0.2, 0.3)$ and parameters $a = -10$, $b = -8/3$ and $c = -28$, chaos of the historical Lorenz system appears. Therefore, we call these parameters *Yin* parameters. In Chinese philosophy, *Yin* is the negative, past, or feminine principle in nature, while *Yang* is the positive, present, or masculine principle in nature. *Yin* and *Yang* are two fundamental opposites in Chinese philosophy. Consequently, the positive values of parameters, $a = 10$, $b = 8/3$, and $c = 28$, in the contemporary Lorenz system can be called *Yang* parameters. The chaotic behavior of (2) is shown in Fig. 2.

4 Simulation results

In order to study the difference and similarity between contemporary and historical Lorenz system, the bifurcation diagram and Lyapunov exponents are used. The simulation results are divided into three parts:

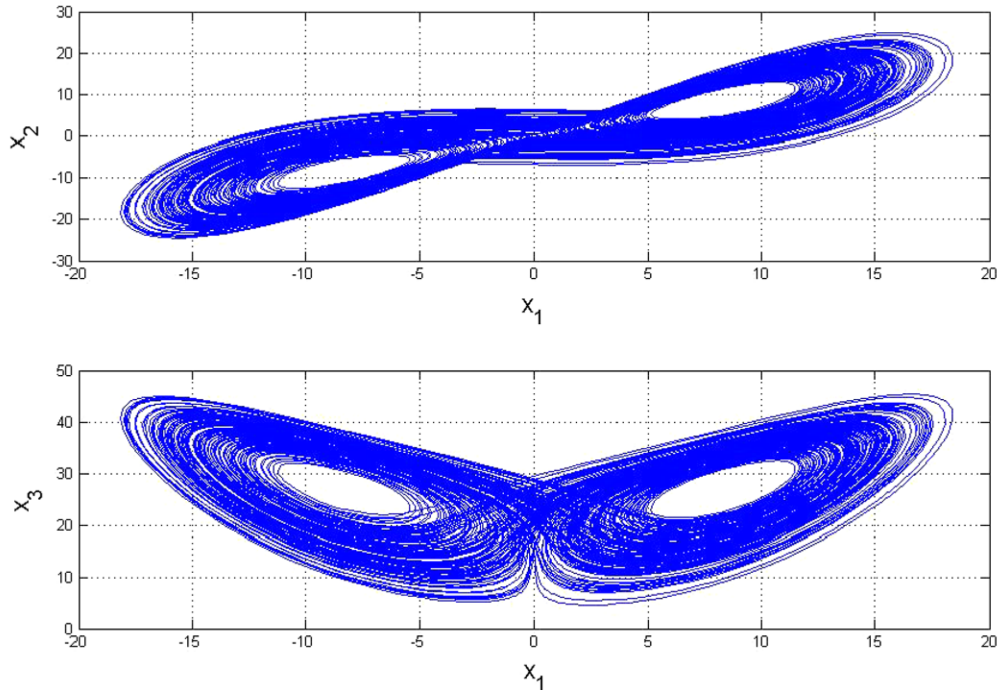

Fig. 1 Projections of phase portrait of chaotic contemporary Lorenz system with $a = 10$, $b = 8/3$ and $c = 28$

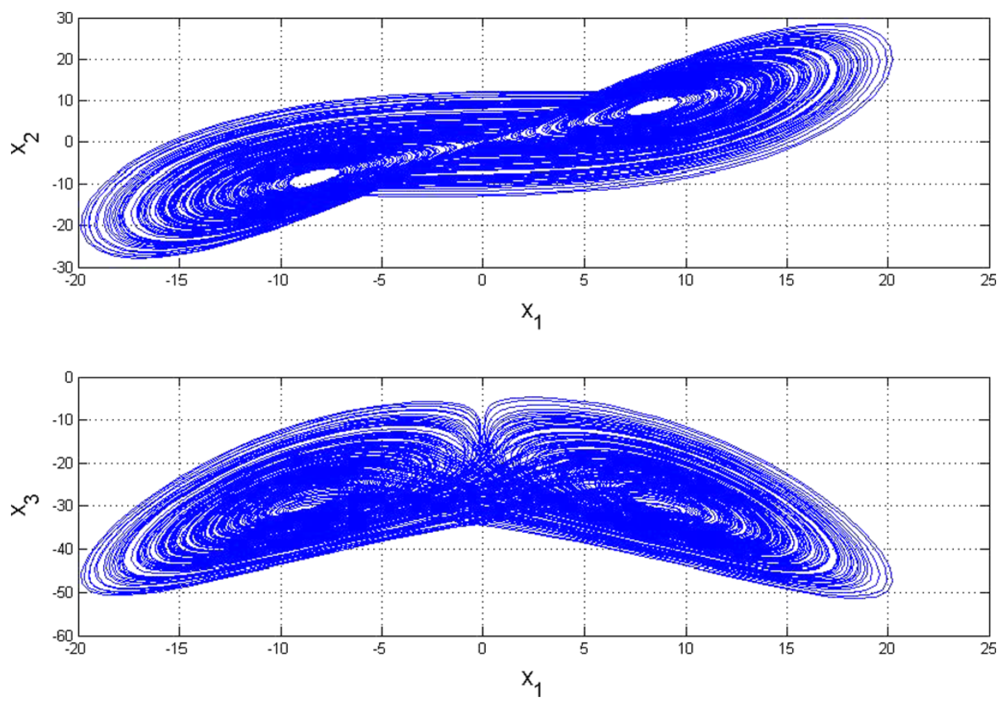

Fig. 2 Projections of phase portrait of chaotic historical Lorenz system with Yin parameters $a = -10$, $b = -8/3$, and $c = -28$

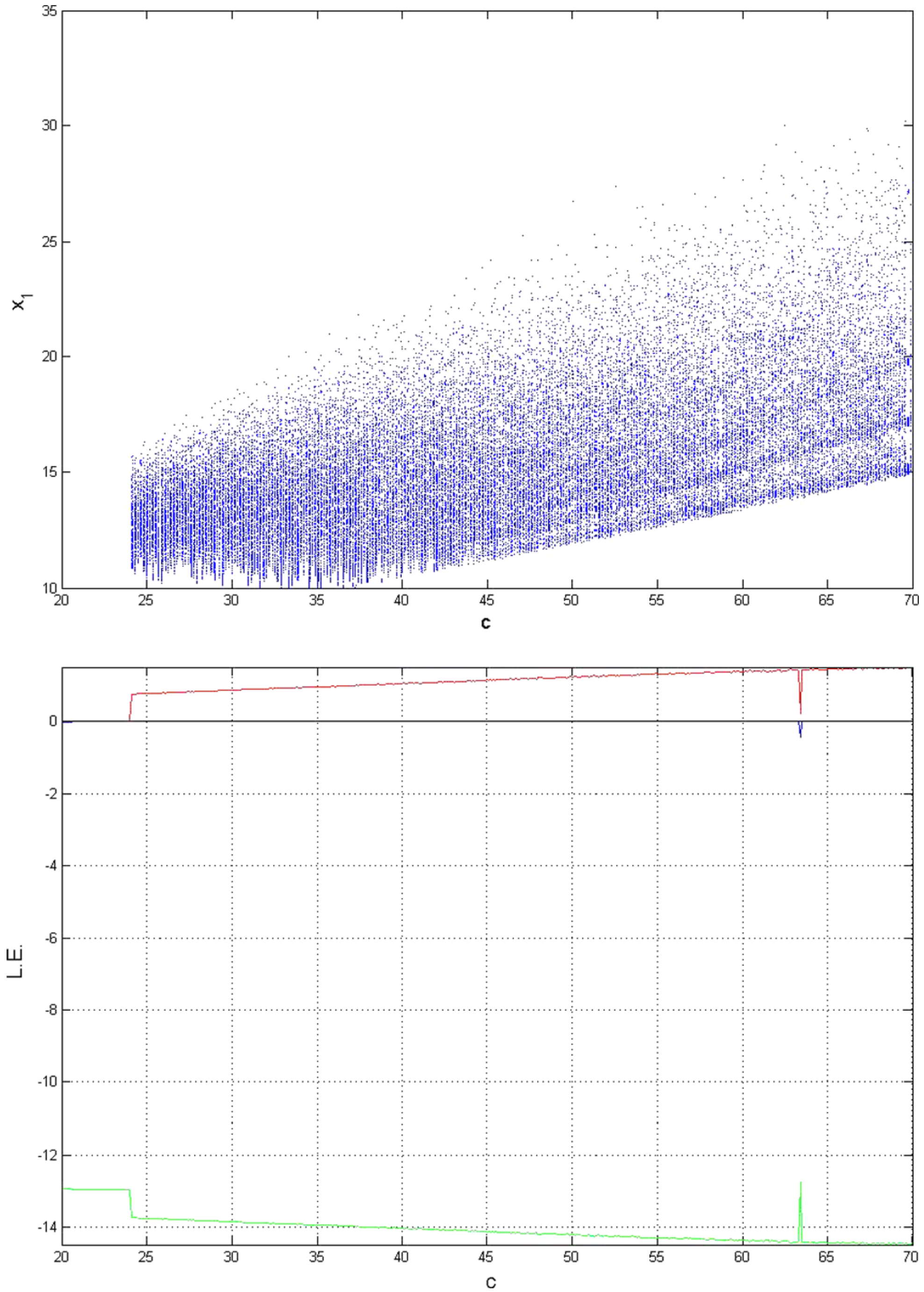

Fig. 3 Bifurcation diagram and Lyapunov exponents of chaotic contemporary Lorenz system with $b = 8/3$ and $a = 10$

Part1: parameter c is varied and a, b are fixed, the simulation results are shown in Figs. 3 and 4, and Tables 2 and 3.

Tables 2 and 3 show the different dynamics between contemporary and historical Lorenz systems with different ranges of parameter c. In Table 2, the

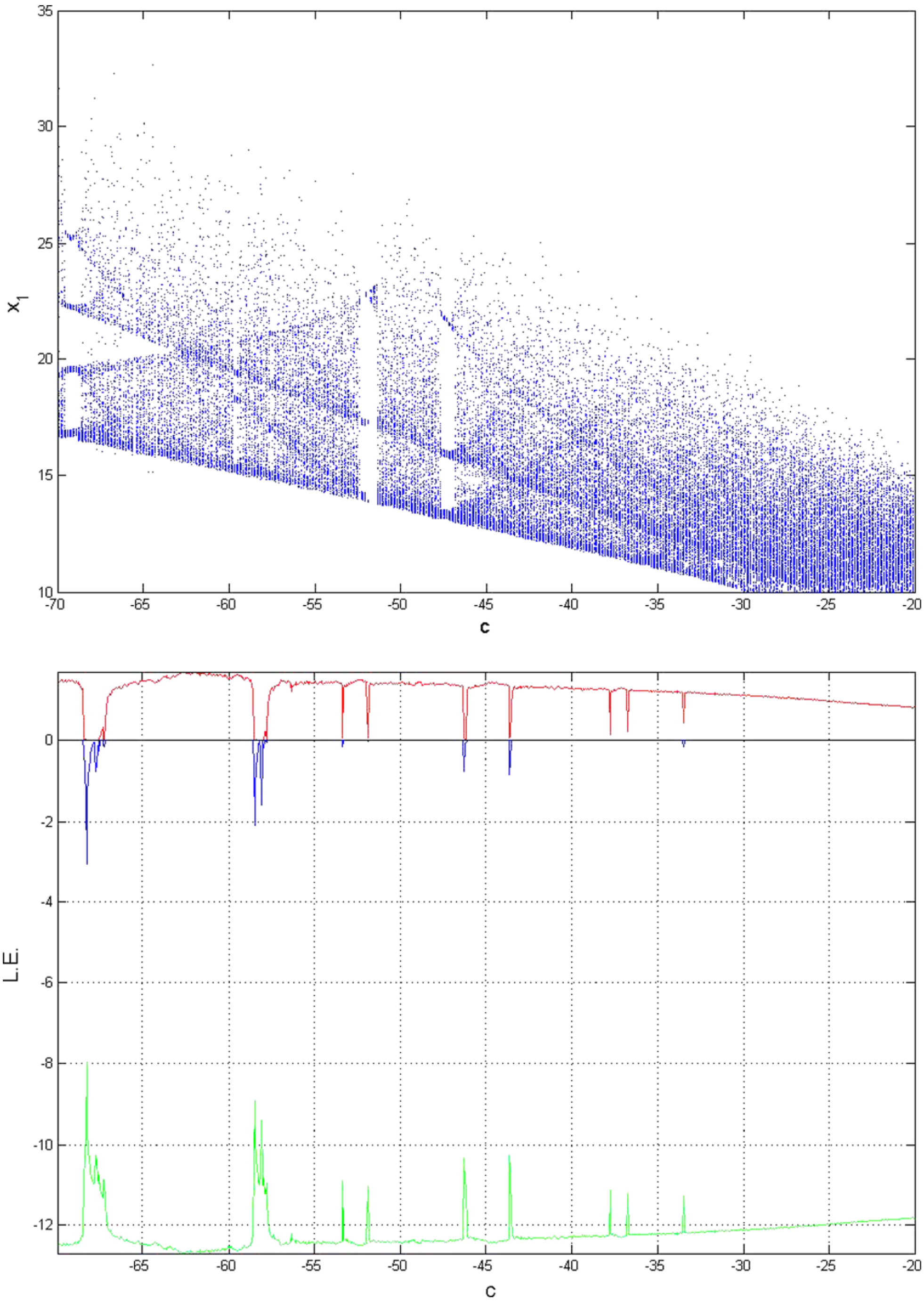

Fig. 4 Bifurcation diagram and Lyapunov exponents of chaotic historical Lorenz system with $b = -8/3$ and $a = -10$

behaviors of contemporary Lorenz system are varied with parameter c, become either chaos or converging to a fixed point. When $20.0 \le c \le 24.1$, contemporary Lorenz system is going to converge to a fixed point. When $24.1 \le c \le 70$, chaos appears. Table 3 shows that when parameter c are -20.0–(-46.8), -47.7–(-51.3), -52.4–(-59.5), -59.8–(-68.3) and -69.64–(-70), the chaotic behavior is

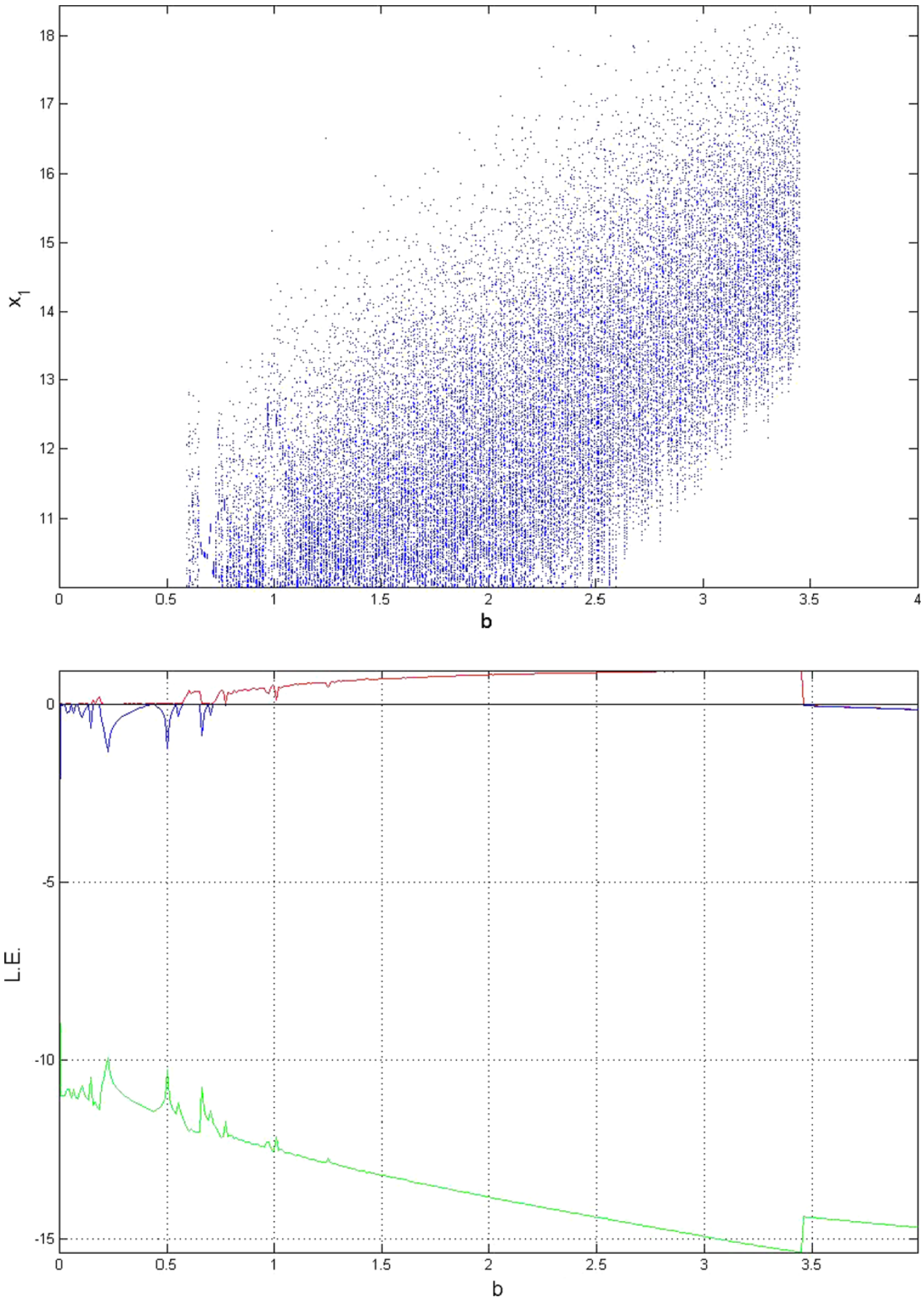

Fig. 5 Bifurcation diagram and Lyapunov exponents of chaotic contemporary Lorenz system with $c = 28$ and $a = 10$

shown in historical Lorenz system. When parameter c are -46.8–(-47.7), -51.3–(-52.4), -59.5–(-59.8) and -68.3–(-69.6), the behaviors of historical Lorenz system are periodic trajectories. Comparing Tables 2 and 3, it can be found out that there are only two cases, chaos and fixed point, in the contemporary Lorenz system for parameter c in range 20 to 70, but there exists chaotic behavior and periodic trajectory in historical Lorenz system with parameter c in range 20 to 70.

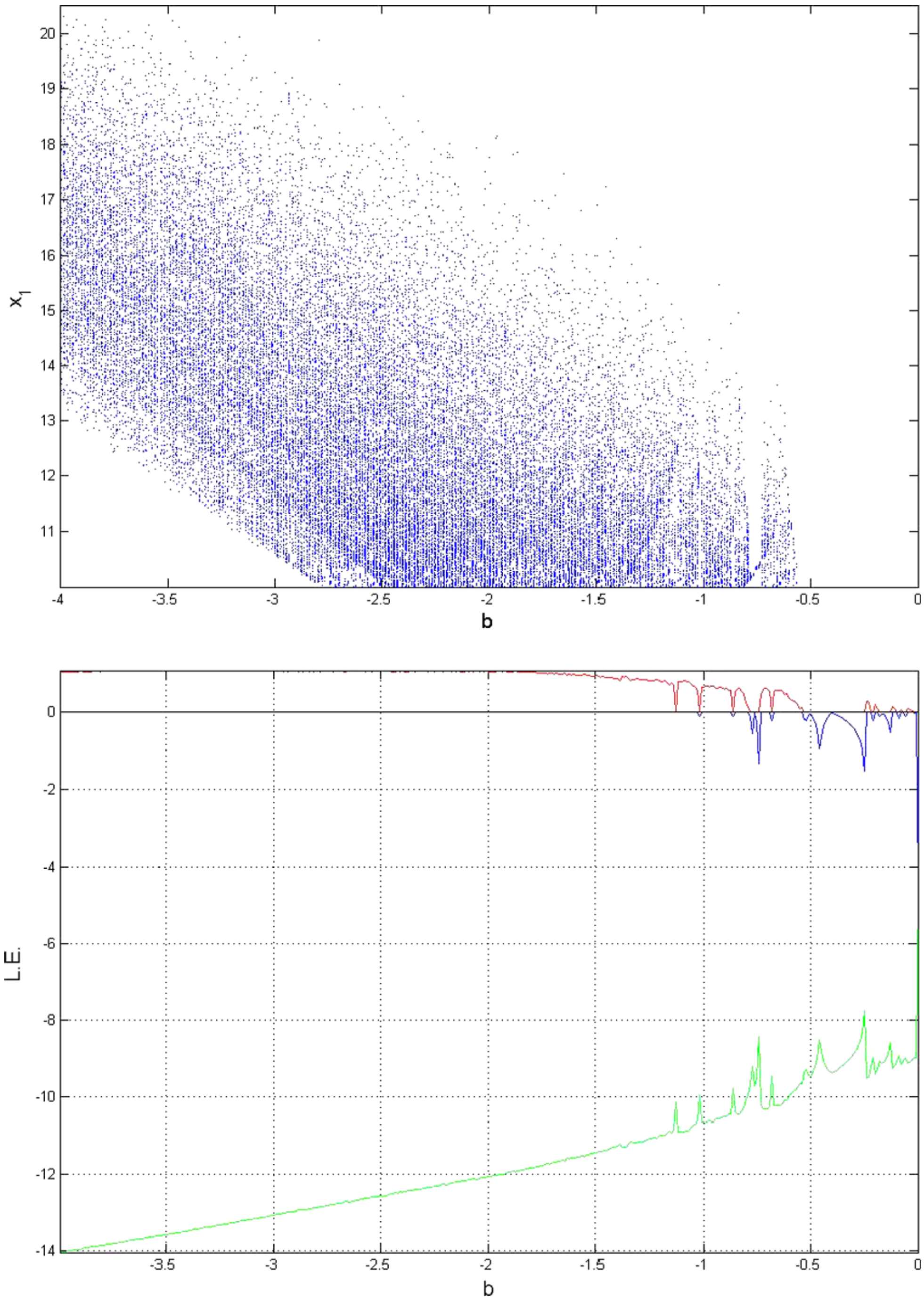

Fig. 6 Bifurcation diagram and Lyapunov exponents of chaotic historical Lorenz system with $c = -28$ and $a = -10$

Part2: parameter b is varied and a, c are fixed, the simulation results are shown in Figs. 5 and 6, Tables 4 and 5.

Tables 4 and 5 show that the behaviors of contemporary and the historical Lorenz system are similar but not the same.

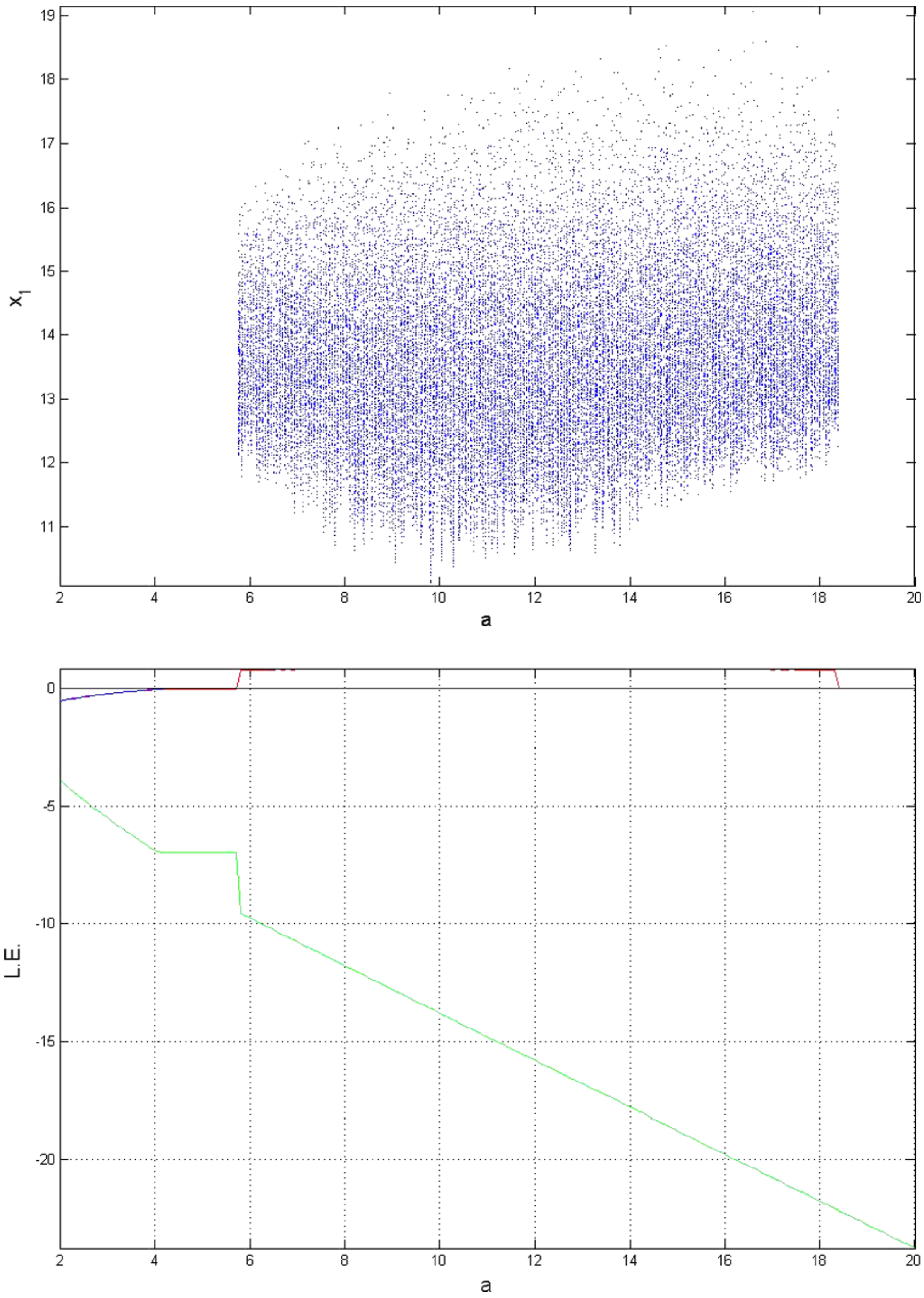

Fig. 7 Bifurcation diagram and Lyapunov exponents of chaotic contemporary Lorenz system with $b = 8/3$ and $c = 28$

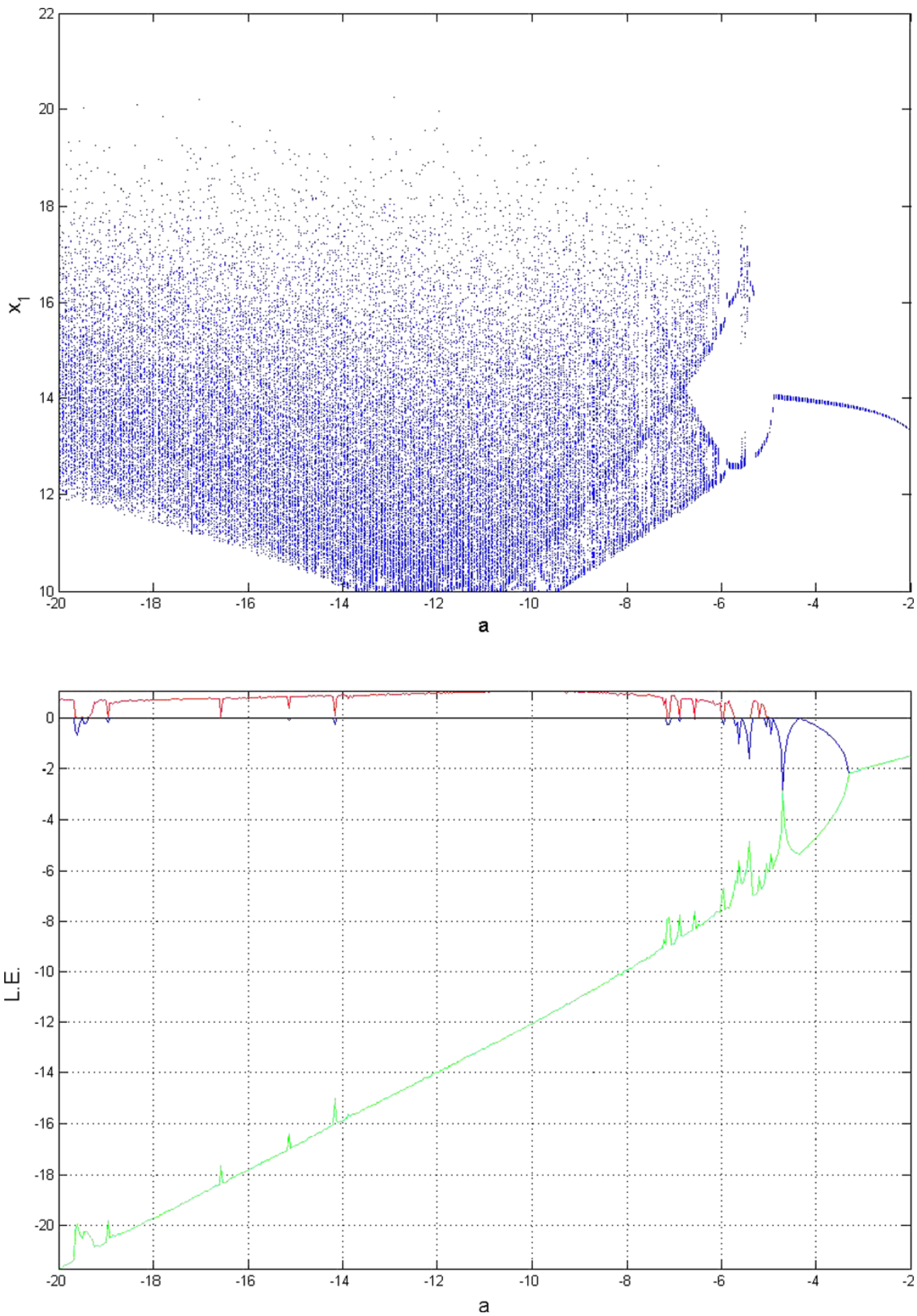

Fig. 8 Bifurcation diagram and Lyapunov exponents of chaotic historical Lorenz system with $b = -8/3$ and $c = -28$

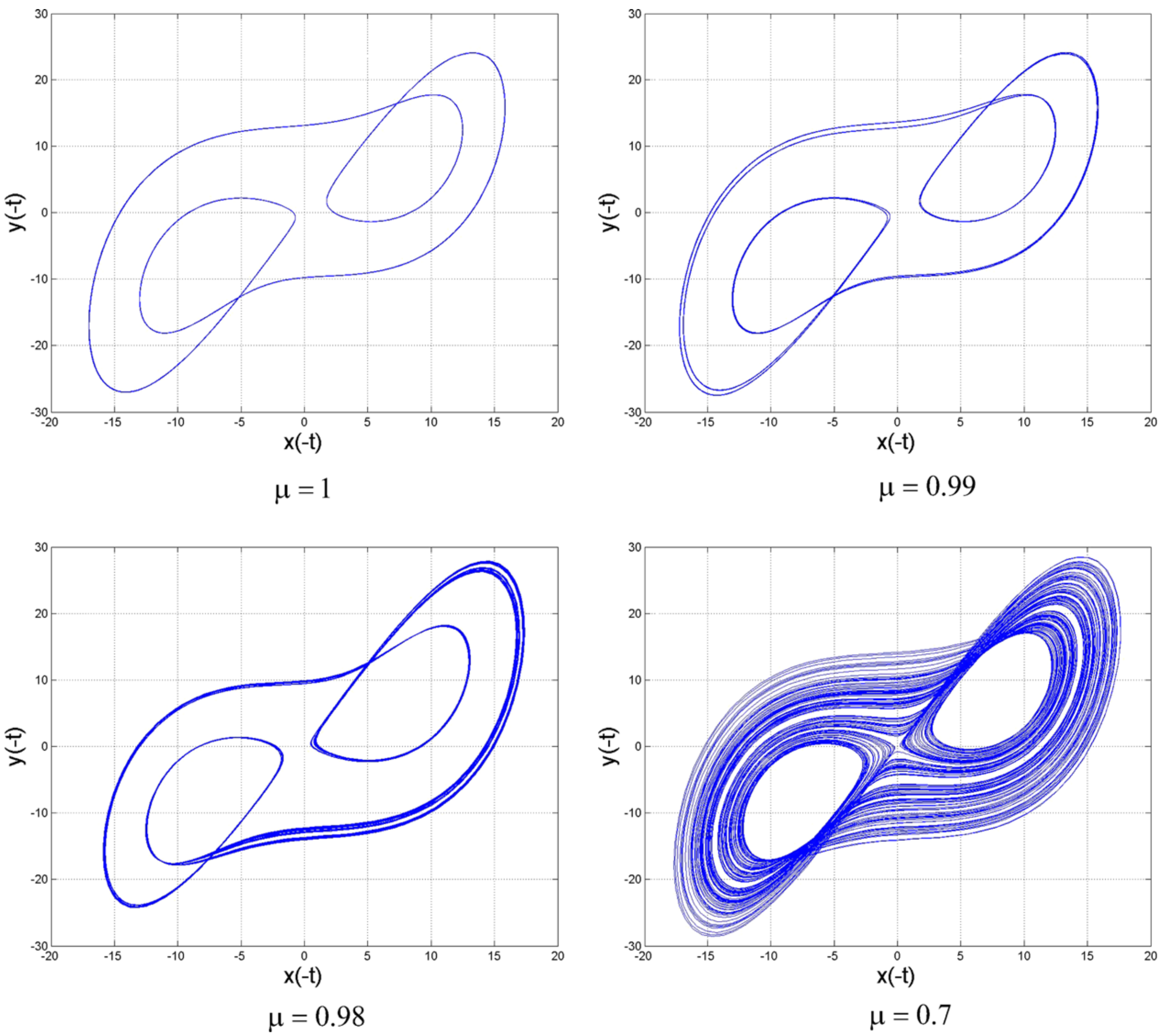

Fig. 9 Projections of phase portraits of family of Yin Lorenz system with $a=-6$, $b=-8/3$ and $c=-28$

Part3: parameter a is varied and b, c are fixed, the simulation results are shown in Figs. 7 and 8, Tables 6 and 7.

In Tables 6 and 7, the behaviors of contemporary and historical Lorenz system are very different. In Table 6, chaotic behavior is only existed in contemporary Lorenz system in range of $5.760 \le a \le 18.368$. In Table 7, chaos and periodic trajectory appear alternatively in historical Lorenz system for different a.

5 Family of *Yin* Lorenz system

In this section, furthermore, one-parameter family of system (3) is presented as well and can be described as follows:

$$\begin{cases} \frac{dx(-t)}{d(-t)} = a(y(-t)-x(-t)), \\ \frac{dy(-t)}{d(-t)} = cx(-t)-x(-t)z(-t)-\mu y(-t), \\ \frac{dz(-t)}{d(-t)} = x(-t)y(-t)-bz(-t) \end{cases} \tag{3}$$

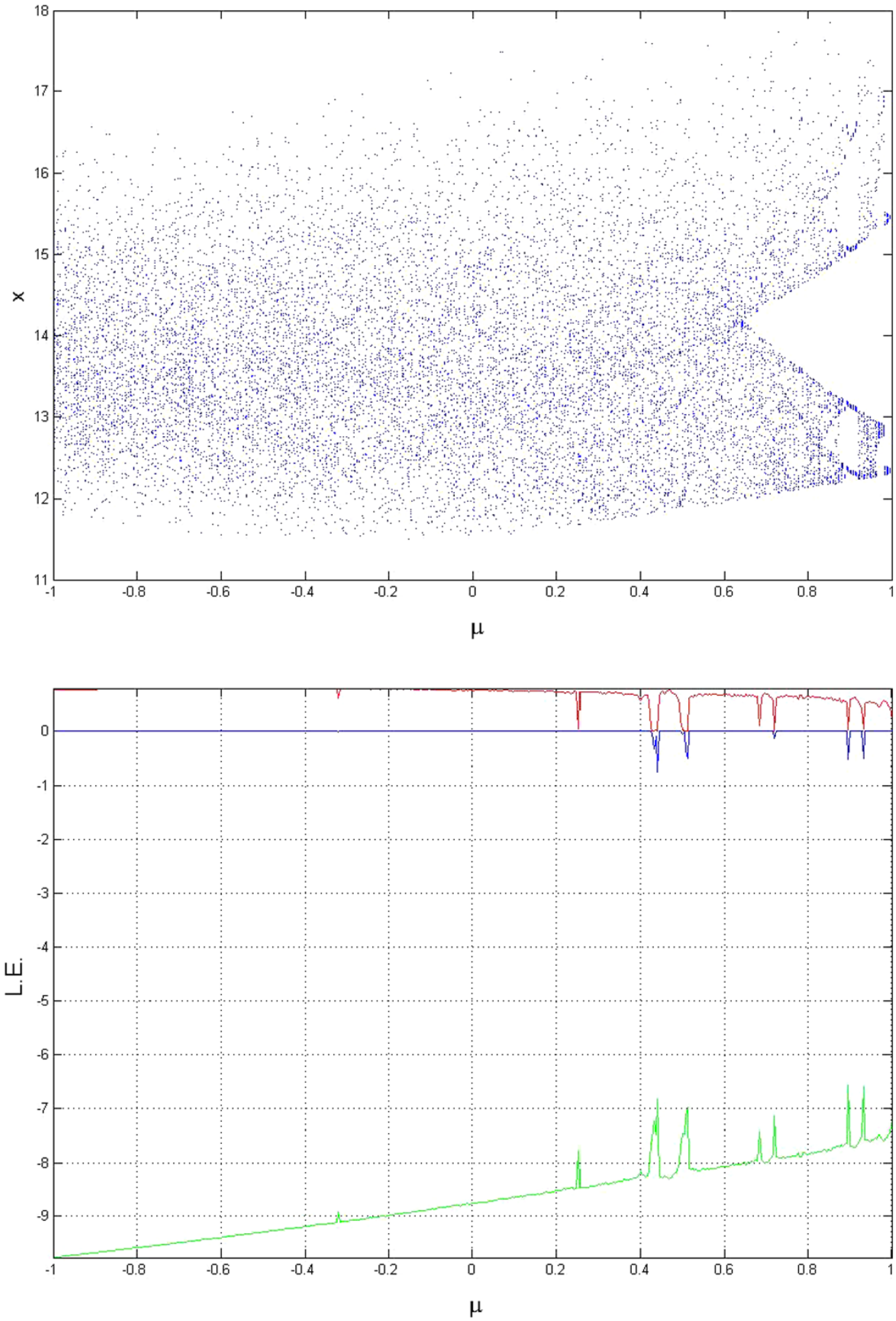

Fig. 10 Bifurcation diagram and Lyapunov exponents of family of Yin Lorenz system with $a = -6$, $b = -8/3$, and $c = -28$

Table 3 Range of parameter c of historical Lorenz system

−20.0–(−46.8)	Chaos
−46.8–(−47.7)	Periodic trajectory
−47.7–(−51.3)	Chaos
−51.3–(−52.4)	Periodic trajectory
−52.4–(−59.5)	Chaos
−59.5–(−59.8)	Periodic trajectory
−59.8–(−68.3)	Chaos
−68.3–(−69.6)	Periodic trajectory
−69.6–(−70)	Chaos

Table 4 Range of parameter b of contemporary Lorenz system

0–0.592	Converge to a fixed point
0.592–0.648	Chaos
0.648–0.720	Periodic trajectory
0.720–3.448	Chaos
3.448–4	Converge to a fixed point

Table 5 Range of parameter b of historical Lorenz system

0–(−0.568)	Converge to a fixed point
−0.568–(−0.728)	Chaos
−0.728–(−0.792)	Periodic trajectory
−0.792–(−4.000)	Chaos

Table 6 Range of parameter a of contemporary Lorenz system

5.000–5.760	Converge to a fixed point
5.760–18.368	Chaos
18.368–20.000	Converge to a fixed point

where $\mu \in [-1, 1]$. We choose initial condition $(x_0, y_0, z_0) = (-0.1, 0.2, 0.3)$ and *Yin* parameters $a = -6$, $b = -8/3$ and $c = -28$, the projection of phase portraits, bifurcation diagrams, and Lyapunov exponents with $\mu \in [-1, 1]$ are shown in Figs. 9 and 10. In observation of Figs. 9 and 10, it is clear that there are periodic and chaotic motions in such a family system when μ is varying.

6 Conclusions

In this paper, the *Yin* Lorenz system with "*Yin* parameters" and its one-parameter family are firstly introduced. When the transformation from $(x(t), y(t), z(t), t)$ to $(x(-t), y(-t), z(-t), -t)$ is made, simulation results show that chaos of the *Yin* Lorenz system can be generated via using "*Yin*" parameters $(-a, -c, -b)$. Via numerical simulation, the *Yin* Lorenz system is compared with the *Yang* Lorenz system and we found out there are similarities and differences between them. The approximate symmetry of Lyapunov exponents is most prominent in Figs. 5 and 6. This paper explores the another half battle field for chaos study, and will prove to have epoch-making significance in the future.

Table 7 Range of parameter a of historical Lorenz system

−5.00–(−5.45)	Periodic trajectory (one attractor to two attractors)
−5.45–(−5.60)	Chaos
−5.60–(−6.05)	Periodic trajectory
−6.05–(−6.17)	Chaos
−6.17–(−6.35)	Periodic trajectory
−6.35–(−7.58)	Chaos
−7.58–(−7.76)	Periodic trajectory
−7.76–(−20)	Chaos

Acknowledgements This research was supported by the National Science Council, Republic of China, under Grant Number NSC 96-2221-E-009-144-MY3.

References

1. Lacitignola, D., Petrosillo, I., Zurlini, G.: Time-dependent regimes of a tourism-based social–ecological system: period-doubling route to chaos. Ecol. Complex. **7**, 44–54 (2010)
2. Elnashaie, S.S.E.H., Grace, J.R.: Complexity, bifurcation and chaos in natural and man-made lumped and distributed systems. Chem. Eng. Sci. **62**, 3295–3325 (2007)
3. Jovic, B., Unsworth, C.P., Sandhu, G.S., Berber, S.M.: A robust sequence synchronization unit for multi-user DS-CDMA chaos-based communication systems. Signal Process. **87**, 1692–1708 (2007)
4. Ge, Z.M., Chen, C.C.: Phase synchronization of coupled chaotic multiple time scales systems. Chaos Solitons Fractals **20**, 639–647 (2004)
5. Ge, Z.M., Cheng, J.W.: Chaos synchronization and parameter identification of three time scales brushless DC motor system. Chaos Solitons Fractals **24**, 597–616 (2005)
6. Wang, Y., Wong, K.W., Liao, X., Chen, G.: A new chaos-based fast image encryption algorithm. Appl. Soft Comput. (in press)
7. Fallahi, K., Leung, H.: A chaos secure communication scheme based on multiplication modulation. Commun. Nonlinear Sci. Numer. Simul. **15**, 368–383 (2010)

8. Yu, W.: A new chaotic system with fractional order and its projective synchronization. Nonlinear Dyn. **48**, 165–174 (2007)
9. Chen, H.K., Sheu, L.J.: The transient ladder synchronization of chaotic systems. Phys. Lett. A **355**, 207–211 (2006)
10. Lorenz, E.N.: Deterministic non-periodic flows. J. Atoms. **20**, 130–141 (1963)
11. Cox, S.M.: The transition to chaos in an asymmetric perturbation of the Lorenz system. Phys. Lett. A **144**, 325–328 (1990)
12. Chen, C.-C., Tsai, C.-H., Fu, C.-C.: Rich dynamics in self-interacting Lorenz systems. Phys. Lett. A **194**, 265–271 (1994)
13. Liu, Y., Barbosa, L.C.: Periodic locking in coupled Lorenz systems. Phys. Lett. A **197**, 13–18 (1995)
14. Wang, L.: 3-scroll and 4-scroll chaotic attractors generated from a new 3-D quadratic autonomous system. Nonlinear Dyn. **56**, 453–462 (2009)
15. Cang, S., Qi, G., Chen, Z.: A four-wing hyper-chaotic attractor and transient chaos generated from a new 4-D quadratic autonomous system. Nonlinear Dyn. **59**, 515–527 (2010)