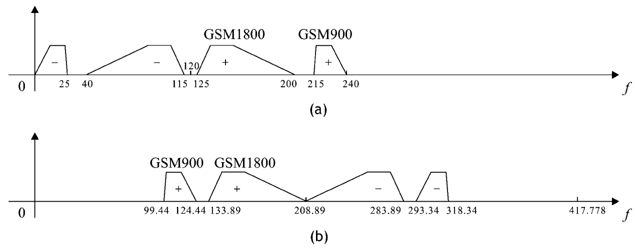

Fig. 3. An example of the replica in $[0, f_s)$ for (a) $f_{s,\min} = 240$ MHz without an ordering constraint, and (b) $f_{s,\min} = 417.778$ MHz with an ordering constraint.

[8] is more efficient. (The method in [8] is not compared in Table I as it does not compute minimum sampling frequency without ordering restriction.) Comparing Tables I and II, we can see that the minimum sampling frequency without a constraint can be much smaller than that with a constraint. Fig. 3 shows the replica in $[0, f_s)$ with a constraint ($f_{s,\min} = 417.78$ MHz) and without a constraint ($f_{s,\min} = 240$ MHz) when the bandpass signals are GSM 900 and GSM1800 as in the first case of Table II.

VI. Conclusion

We have proposed a new algorithm for finding the minimum sampling frequency for multiband signals. We have derived a new set of conditions for alias-free sampling. These conditions lead to an iterative algorithm for finding the minimum sampling frequency. There is no need to consider ordering of the signal bands in the folded spectrum in the implementation of algorithm. The method can be generalized to find alias-free sampling frequency intervals and to find the minimum sampling frequency when the ordering of replicas is constrained.

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Cooperative Interference Management With MISO Beamforming

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***Abstract*—In this correspondence, we study the downlink transmission in a multi-cell system, where multiple base stations (BSs) each with multiple antennas cooperatively design their respective transmit beamforming vectors to optimize the overall system performance. For simplicity, it is assumed that all mobile stations (MSs) are equipped with a single antenna each, and there is one active MS in each cell at one time. Accordingly, the system of interests can be modeled by a multiple-input single-output (MISO) Gaussian interference channel (IC), termed as MISO-IC, with interference treated as noise. We propose a new method to characterize different rate-tuples for active MSs on the Pareto boundary of the achievable rate region for the MISO-IC, by exploring the relationship between the MISO-IC and the cognitive radio (CR) MISO channel. We show that each Pareto-boundary rate-tuple of the MISO-IC can be achieved in a decentralized manner when each of the BSs attains its own channel capacity subject to a certain set of interference-power constraints (also known as interference-temperature constraints in the CR system) at the other MS receivers. Furthermore, we show that this result leads to a new decentralized algorithm for implementing the multi-cell cooperative downlink beamforming.**

***Index Terms*—Beamforming, cooperative multi-cell system, interference channel, multi-antenna, Pareto optimal, rate region.**

I. Introduction

Conventional wireless mobile networks are designed with a cellular architecture, where base stations (BSs) from different cells control communications for their associated mobile stations (MSs) independently. The resulting inter-cell interference is treated as

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additive noise and minimized by applying a predesigned frequency reuse pattern such that the same frequency band is reused only by non-adjacent cells. Due to the rapidly growing demand for high-rate wireless multimedia applications, conventional cellular networks have been pushed towards their throughput limits. Consequently, many beyond-3G wireless technologies such as WiMAX and 3GPP UMTS Long Term Evolution (LTE) have relaxed the constraint on the frequency reuse such that the total frequency band becomes available for reuse by all cells. However, this factor-one frequency reuse pattern renders the overall network performance limited by the inter-cell interference; consequently, more sophisticated interference management techniques with multi-cell cooperation become crucial. Among others, one effective method to cope with the inter-cell interference in the cellular network is via joint signal processing across different BSs. In this correspondence, we study a particular type of multi-BS cooperation for the downlink transmission, where we are interested in evaluating the benefit in terms of network throughput by cooperatively optimizing the transmit beamforming vectors for different BSs each with multiple antennas. Notice that the problem setup of our work is different from that for a fully cooperative multi-cell system considered in, e.g., [1]–[6], where a central processing unit is assumed with the global knowledge of all the required downlink channels and user messages to jointly design the transmitted signals for all BSs. In contrast, our work focuses on the decentralized implementation of the multi-cell cooperative downlink beamforming assuming only the local message and neighboring-channel knowledge at each BS, which is more practical than implementing the full baseband-level coordination. It is worth noting that decentralized multi-cell cooperative downlink beamforming has been studied in [7] to minimize the total power consumption of all BSs to meet with MSs' individual signal-to-interference-plus-noise ratio (SINR) targets, based on the uplink-downlink beamforming duality. In this work, we provide a different design approach for rate-optimal strategies in decentralized multi-cell cooperative beamforming.

For the purpose of exposition, in this work we consider a simplified scenario, where each MS is equipped with a single antenna, and at any given time there is only one active MS in each cell (over a particular frequency band). Accordingly, we can model the multi-cell cooperative downlink transmission system as a multiple-input single-output (MISO) Gaussian interference channel (IC), termed as MISO-IC. From an information-theoretic viewpoint, the capacity region of the Gaussian IC, which constitutes all the simultaneously achievable rates for all users, is still unknown in general [8], while significant progresses have recently been made on approaching this limit [9]. Capacity-approaching techniques for the Gaussian IC in general require certain signal-level encoding/decoding cooperations among the users, while a more pragmatic approach that leads to suboptimal achievable rates of the users is to allow only single-user encoding and decoding by treating the interference from other users as additive Gaussian noise. In this work, we adopt the latter approach to study the design of cooperative transmit beamforming for the MISO-IC. Particularly, we focus on the design criterion to achieve different rate-tuples for the users on the Pareto boundary of the achievable rate region for the MISO-IC. Due to the coupled signal structure, the achievable rate region for the MISO-IC with interference treated as noise is in general a non-convex set,[1] which renders the joint optimization of beamforming vectors to achieve different Pareto-boundary rate-tuples a challenging task. Note that this problem has been studied in [10], where for the special two-user case, it was shown that the optimal transmit beamforming vector to achieve a Pareto-boundary rate-pair for the MISO-IC can be expressed as a linear combination of the zero-forcing (ZF) and maximum-ratio transmission (MRT) beamformers. The rate maximization for the IC with interference treated as noise has also been studied in the literature via various "pricing" algorithms (see, e.g., [11] and references therein), while in general the price-based approach does not achieve the Pareto-optimal rates for the MISO-IC. In [12], the maximum sum-rate for the Gaussian IC is characterized in terms of degrees of freedom (DoF) over the interference-limited regime.

In this correspondence, we develop a new *parametrical* characterization of the Pareto boundary for the MISO-IC in terms of the interference-power levels at all receivers caused by different transmitters, also known as the *interference temperature* (IT) levels in the newly emerging "cognitive radio (CR)" type of applications [13]. We show that each Pareto-boundary rate-tuple can be achieved in a decentralized manner when each of the users maximizes its own MISO channel capacity subject to a certain set of IT constraints at the other users' receivers, which is identical to the CR MISO channel transmit optimization problem studied in [14] and thus shares the same solution structure. We derive new closed-form solutions for the optimal transmit covariance matrices of all users to achieve an arbitrary rate-tuple on the Pareto boundary of the MISO-IC rate region, from which we see that the optimal transmit covariance matrices should all be *rank-one* (i.e., beamforming is optimal).[2] Furthermore, we derive the conditions that are necessary for any particular set of mutual IT constraints across all users to guarantee a Pareto-optimal rate-tuple for the MISO-IC. Based on these conditions, we propose a new *decentralized* algorithm for implementing the multi-cell cooperative downlink beamforming. For this algorithm, all different pairs of BSs independently search for their mutually desirable IT constraints (with those for the MSs associated with the other BSs fixed), under which their respective beamforming vectors are optimized to maximize the individual transmit rates. This algorithm improves the rates for the BSs in a pairwise manner until the transmit rates for all BSs converge with their mutual IT levels.

Notation: $\boldsymbol{I}$ and $\boldsymbol{0}$ denote the identity matrix and the all-zero matrix, respectively, with appropriate dimensions. For a square matrix $\boldsymbol{S}$, $\mathrm{Tr}(\boldsymbol{S})$, $|\boldsymbol{S}|$, $\boldsymbol{S}^{-1}$, and $\boldsymbol{S}^{1/2}$ denote the trace, determinant, inverse, and square-root of $\boldsymbol{S}$, respectively; and $\boldsymbol{S} \succeq 0$ means that $\boldsymbol{S}$ is positive semi-definite [16]. $\mathrm{Diag}(\boldsymbol{a})$ denotes a diagonal matrix with the diagonal elements given by $\boldsymbol{a}$. For a matrix $\boldsymbol{M}$ of arbitrary size, $\boldsymbol{M}^H$, $\boldsymbol{M}^T$, and $\mathrm{Rank}(\boldsymbol{M})$ denote the Hermitian transpose, transpose, and rank of $\boldsymbol{M}$, respectively. $\mathbb{E}[\cdot]$ denotes the statistical expectation. The distribution of a circularly symmetric complex Gaussian (CSCG) random vector with the mean vector $\boldsymbol{x}$ and the covariance matrix $\boldsymbol{\Sigma}$ is denoted by $\mathcal{CN}(\boldsymbol{x}, \boldsymbol{\Sigma})$; and $\sim$ stands for "distributed as". $\mathbb{C}^{m\times n}$ denotes the space of $m \times n$ complex matrices. $\|\boldsymbol{x}\|$ denotes the Euclidean norm of a complex vector (scalar) $\boldsymbol{x}$. The $\log(\cdot)$ function is with base 2 by default.

II. System Model

We consider the downlink transmission in a cellular network consisting of K cells, each having a multi-antenna BS to transmit independent messages to one active single-antenna MS. It is assumed that all BSs share the same narrowband spectrum for downlink transmission. Accordingly, the system under consideration can be modeled by a K-user MISO-IC. It is assumed that the BS in the kth cell, $k = 1, \ldots, K$, is equipped with M_k transmitting antennas, $M_k \geq 1$. The discrete-time baseband signal received by the active MS in the kth cell is then given by

$$y_k = \boldsymbol{h}_{kk}^H \boldsymbol{x}_k + \sum_{j\neq k} \boldsymbol{h}_{jk}^H \boldsymbol{x}_j + z_k \tag{1}$$

where $\boldsymbol{x}_k \in \mathbb{C}^{M_k\times 1}$ denotes the transmitted signal from the kth BS; $\boldsymbol{h}_{kk}^H \in \mathbb{C}^{1\times M_k}$ denotes the direct-link channel for the kth MS, while $\boldsymbol{h}_{jk}^H \in \mathbb{C}^{1\times M_j}$ denotes the cross-link channel from the jth BS to the kth MS, $j \neq k$; and z_k denotes the receiver noise. It is assumed that $z_k \sim \mathcal{CN}(0, \sigma_k^2), \forall k$, and all z_k's are independent.

[1]It is noted that the non-convex rate region is obtained without time-sharing (convex-hull operation) between different achievable rate-tuples. With time-sharing, the achievable rate region will become a convex set.

[2]We thank the anonymous reviewer who brought our attention to [15], in which the authors also showed the optimality of beamforming to achieve the Pareto-boundary rates for the Gaussian MISO-IC with interference treated as noise, via a different proof technique.

We assume independent encoding across different BSs and thus $\boldsymbol{x}_k$'s are independent over k. It is further assumed that a Gaussian codebook is used at each BS and $\boldsymbol{x}_k \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{S}_k), \quad k = 1, \ldots, K$, where $\boldsymbol{S}_k = \mathbb{E}[\boldsymbol{x}_k \boldsymbol{x}_k^H]$ denotes the transmit covariance matrix for the kth BS, with $\boldsymbol{S}_k \in \mathbb{C}^{M_k \times M_k}$ and $\boldsymbol{S}_k \succeq 0$. Notice that the CSCG distribution has been assumed for all the transmitted signals.[3] Furthermore, the interferences at all the receivers caused by different transmitters are treated as Gaussian noises. Thus, for a given set of transmit covariance matrices of all BSs, $\boldsymbol{S}_1, \ldots, \boldsymbol{S}_K$, the achievable rate of the kth MS is expressed as

$$R_k(\boldsymbol{S}_1, \ldots, \boldsymbol{S}_K) = \log\left(1 + \frac{\boldsymbol{h}_{kk}^H \boldsymbol{S}_k \boldsymbol{h}_{kk}}{\sum_{j \neq k} \boldsymbol{h}_{jk}^H \boldsymbol{S}_j \boldsymbol{h}_{jk} + \sigma_k^2}\right). \quad (2)$$

Next, we define the achievable rate region for the MISO-IC to be the set of rate-tuples for all MSs that can be simultaneously achievable under a given set of transmit-power constraints for the BSs, denoted by $P_1, \ldots, P_K$:

$$\mathcal{R} \triangleq \bigcup_{\{\boldsymbol{S}_k\}: \mathrm{Tr}(\boldsymbol{S}_k) \leq P_k, k=1,\ldots,K} \{(r_1, \ldots, r_K) : 0 \leq r_k \leq R_k(\boldsymbol{S}_1, \ldots, \boldsymbol{S}_K), \quad k = 1, \ldots, K\}. \quad (3)$$

The upper-right boundary of this region is called the *Pareto boundary*, since it consists of rate-tuples at which it is impossible to improve a particular user's rate, without simultaneously decreasing the rate of at least one of the other users. More precisely, the Pareto optimality of a rate-tuple is defined as follows [10].

Definition 2.1: A rate-tuple $(r_1, \ldots, r_K)$ is *Pareto optimal* if there is no other rate-tuple $(r_1', \ldots, r_K')$ with $(r_1', \ldots, r_K') \geq (r_1, \ldots, r_K)$ and $(r_1', \ldots, r_K') \neq (r_1, \ldots, r_K)$ (the inequality is component-wise).

In this work, we consider the scenario where multiple BSs in the cellular network cooperatively design their transmit covariance matrices in order to minimize the effect of the inter-cell interference on the overall network throughput. In particular, we are interested in the design criterion to achieve different Pareto-optimal rate-tuples for the corresponding MISO-IC defined as above.

It is worth noting that in prior works on characterizing the Pareto boundary for the MISO-IC with interference treated as noise (see, e.g., [10] and references therein), it has been assumed (without proof) that the optimal transmit strategy for users to achieve any rate-tuple on the Pareto boundary is *beamforming*, i.e., $\boldsymbol{S}_k$ is a rank-one matrix for all k's. Under this assumption, we can express $\boldsymbol{S}_k$ as $\boldsymbol{S}_k = \boldsymbol{w}_k \boldsymbol{w}_k^H, \quad k = 1, \ldots, K$, where $\boldsymbol{w}_k \in \mathbb{C}^{M_k \times 1}$ denotes the beamforming vector for the kth user. Similarly as in the general case with $\mathrm{Rank}(\boldsymbol{S}_k) \geq 1$, the achievable rates and rate region of the MISO-IC with transmit beamforming (BF) can be defined in terms of $\boldsymbol{w}_k$'s. However, it is not evident whether the BF case bears the same Pareto boundary as the general case with $\mathrm{Rank}(\boldsymbol{S}_k) \geq 1$ for the MISO-IC. In this work, we will show that this is indeed the case (see Section III). Accordingly, we can choose to use the rate and rate-region expressions in terms of either $\boldsymbol{S}_k$'s or $\boldsymbol{w}_k$'s to characterize the Pareto boundary of the MISO-IC, for the rest of this correspondence.

In the following, we review some existing approaches to characterize the Pareto boundary for the MISO-IC with interference treated as noise. For the purpose of illustration, in Fig. 1, we show the achievable rate region for a two-user MISO Gaussian IC with interference treated as noise (prior to any time-sharing of achievable rate-pairs), which is observed to be non-convex. A commonly adopted method to obtain the Pareto boundary for the MISO-IC is via solving a sequence of weighted sum-rate maximization (WSRMax) problems, each for a given set of user rate weights, $\mu_k \geq 0, \quad \forall k$, and given by

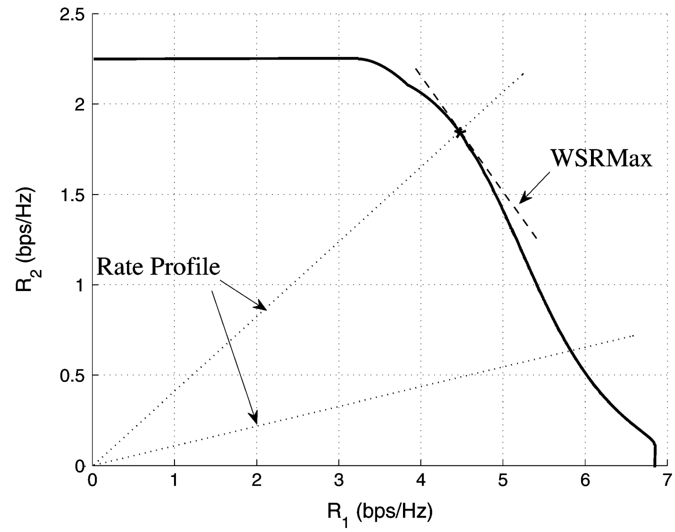

Fig. 1. Achievable rate region and Pareto boundary for a two-user MISO Gaussian IC with interference treated as noise.

$$\begin{aligned} \underset{\{\boldsymbol{w}_k\}}{\texttt{Max.}} \quad & \sum_{k=1}^{K} \mu_k \log(1 + \gamma_k(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)) \\ \texttt{s.t.} \quad & \|\boldsymbol{w}_k\|^2 \leq P_k, \quad k = 1, \ldots, K \end{aligned} \quad (4)$$

where $\gamma_k(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)$ is the receiver SINR for the kth user defined as

$$\gamma_k(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K) = \frac{\|\boldsymbol{h}_{kk}^H \boldsymbol{w}_k\|^2}{\sum_{j \neq k} \|\boldsymbol{h}_{jk}^H \boldsymbol{w}_j\|^2 + \sigma_k^2}, \quad k = 1, \ldots, K. \quad (5)$$

This problem can be shown non-convex, and thus cannot be solved efficiently. Moreover, the WSRMax method cannot guarantee the finding of all Pareto-boundary points for the MISO-IC (cf. Fig. 1).

An alternative method to characterize the *complete* Pareto boundary for the MISO-IC is based on the concept of *rate profile* [18]. Specifically, any rate-tuple on the Pareto boundary of the rate region can be obtained via solving the following optimization problem with a particular rate-profile vector, $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_K)$:

$$\begin{aligned} \underset{R_{\mathrm{sum}}, \{\boldsymbol{w}_k\}}{\texttt{Max.}} \quad & R_{\mathrm{sum}} \\ \texttt{s.t.} \quad & \log(1 + \gamma_k(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)) \geq \alpha_k R_{\mathrm{sum}}, \\ & \qquad k = 1, \ldots, K \\ & \|\boldsymbol{w}_k\|^2 \leq P_k, \quad k = 1, \ldots, K \end{aligned} \quad (6)$$

with α_k denoting the target ratio between the kth user's achievable rate and the users' sum-rate, R_{sum}. Without loss of generality, we assume that $\alpha_k \geq 0, \forall k$, and $\sum_{k=1}^{K} \alpha_k = 1$. For a given $\boldsymbol{\alpha}$, let the optimal solution of Problem (6) be denoted by $R_{\mathrm{sum}}^{\star}$. Then, it follows that $R_{\mathrm{sum}}^{\star} \cdot \boldsymbol{\alpha}$ must be the corresponding Pareto-optimal rate-tuple, which can be geometrically viewed as (cf. Fig. 1) the intersection between a ray in the direction of $\boldsymbol{\alpha}$ and the Pareto boundary of the rate region. Thereby, with

[3]It is worth noting that in [17] the authors point out that the CSCG distribution for the transmitted signals is in general non-optimal for the Gaussian IC with interference treated as noise, since it can be shown that the complex Gaussian but not circularly symmetric distribution can achieve larger rates than the symmetric distribution for some particular channel realizations.

different $\boldsymbol{\alpha}$'s, solving Problem (6) yields the complete Pareto boundary for the rate region, which does not need to be convex.

Next, we show that Problem (6) is solvable via solving a sequence of feasibility problems each for a fixed r_{sum} and given by

$$\begin{aligned} \texttt{Find} \quad & \{\boldsymbol{w}_k\} \\ \texttt{s.t.} \quad & \log(1+\gamma_k(\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K)) \geq \alpha_k r_{\text{sum}}, \\ & \qquad\qquad k=1,\ldots,K \\ & \|\boldsymbol{w}_k\|^2 \leq P_k, \quad k=1,\ldots,K. \end{aligned} \tag{7}$$

If the above problem is feasible for a given sum-rate target,r_{sum}, it follows that $R_{\text{sum}}^\star \geq r_{\text{sum}}$; otherwise, $R_{\text{sum}}^\star < r_{\text{sum}}$. Thus, by solving Problem (7) with different r_{sum}'s and applying the simple bisection method [16] over r_{sum}, $R_{\text{sum}}^\star$ can be obtained for Problem (6). Let $\bar{\gamma}_k = 2^{\alpha_k r_{\text{sum}}} - 1, \quad k=1,\ldots K$. Then, for Problem (7), we can replace the rate constraints by the equivalent SINR constraints:

$$\gamma_k(\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K) \geq \bar{\gamma}_k, \quad k=1,\ldots,K. \tag{8}$$

Similarly as shown in [19], the resultant feasibility problem can be transformed into a second-order cone programming (SOCP) problem, which is convex and can be solved efficiently [20].

III. Characterizing Pareto Boundary for MISO-IC via Interference Temperature Control

In this section, instead of investigating centralized approaches, we present a new method to characterize the Pareto boundary for the MISO-IC in a distributed fashion, by exploring its relationship with the CR MISO channel [14]. We start with the general-rank transmit covariance matrices $\boldsymbol{S}_k$'s for the MISO-IC. First, we introduce a set of auxiliary variables, $\Gamma_{kj}, \quad k=1,\ldots,K, \quad j=1,\ldots,K, j\neq k$, where Γ_{kj} is called the interference-power or interference-temperature (IT) constraint from the kth BS to jth MS, $j \neq k$, $\Gamma_{kj} \geq 0$. For notational convenience, let $\boldsymbol{\Gamma}$ be the vector consisting of all $K(K-1)$ different Γ_{kj}'s, and $\boldsymbol{\Gamma}_k$ be the vector consisting of all $2(K-1)$ different Γ_{kj}'s and Γ_{jk}'s, $j=1,\ldots,K, j\neq k$, for any given $k \in \{1,\ldots,K\}$.

Next, we consider a set of parallel transmit covariance optimization problems, each for one of the K BSs in the MISO-IC expressed as

$$\begin{aligned} \underset{\boldsymbol{S}_k}{\texttt{Max.}} \quad & \log\left(1+\frac{\boldsymbol{h}_{kk}^H\boldsymbol{S}_k\boldsymbol{h}_{kk}}{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}\right) \\ \texttt{s.t.} \quad & \boldsymbol{h}_{kj}^H\boldsymbol{S}_k\boldsymbol{h}_{kj} \leq \Gamma_{kj}, \forall j\neq k \\ & \text{Tr}(\boldsymbol{S}_k) \leq P_k, \boldsymbol{S}_k \succeq 0 \end{aligned} \tag{9}$$

where $k \in \{1,\ldots,K\}$. Note that in the above problem for a given k, $\boldsymbol{\Gamma}_k$ is fixed. For notational convenience, we denote the optimal value of this problem as $C_k(\boldsymbol{\Gamma}_k)$. If in the objective function of (9) we set $\Gamma_{jk} = \boldsymbol{h}_{jk}^H\boldsymbol{S}_j\boldsymbol{h}_{jk}, \quad \forall j\neq k$, $C_k(\boldsymbol{\Gamma}_k)$ becomes equal to the maximum achievable rate of an equivalent MISO CR channel [14], where the kth BS is the so-called "secondary" user transmitter, and all the other $K-1$ BSs, indexed by $j=1,\ldots,K, j\neq k$, are the "primary" user transmitters, each of which has a transmit covariance matrix, $\boldsymbol{S}_j$, and its intended "primary" user receiver is protected by the secondary user via the IT constraint: $\boldsymbol{h}_{kj}^H\boldsymbol{S}_k\boldsymbol{h}_{kj} \leq \Gamma_{kj}$. In [14], it was proved that the solution for Problem (9) is rank-one, i.e., beamforming is optimal, and in the special case of $K=2$ (i.e., one single primary user), a closed-form solution for the optimal beamforming vector was derived. In the following proposition, we provide a new closed-form solution for Problem (9) with arbitrary values of K, from which we can easily infer that beamforming is indeed optimal.

Proposition 3.1: The optimal solution of Problem (9) is *rank-one*, i.e., $\boldsymbol{S}_k = \boldsymbol{w}_k\boldsymbol{w}_k^H$, and

$$\boldsymbol{w}_k = \left(\sum_{j\neq k}\lambda_{kj}\boldsymbol{h}_{kj}\boldsymbol{h}_{kj}^H + \lambda_{kk}\boldsymbol{I}\right)^{-1}\boldsymbol{h}_{kk}\sqrt{p_k} \tag{10}$$

where $\lambda_{kj}, j\neq k$, and λ_{kk}, are non-negative constants (dual variables) corresponding to the kth BS's IT constraint for the jth MS and its own transmit-power constraint, respectively, which are obtained as the optimal solutions for the dual variables in the dual problem of Problem (9); and p_k is given by

$$p_k = \left(\frac{1}{\ln 2} - \frac{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}{\|\boldsymbol{A}_k\boldsymbol{h}_{kk}\|^2}\right)^+ \frac{1}{\|\boldsymbol{A}_k\boldsymbol{h}_{kk}\|^2} \tag{11}$$

where $\boldsymbol{A}_k \triangleq (\sum_{j\neq k}\lambda_{kj}\boldsymbol{h}_{kj}\boldsymbol{h}_{kj}^H + \lambda_{kk}\boldsymbol{I})^{-1/2}$ and $(x)^+ \triangleq \max(0,x)$.

Proof: Please see Appendix I. ∎

Now, we are ready to present a *parametrical* characterization of the Pareto boundary for the MISO-IC in terms of $\boldsymbol{\Gamma}$ as follows.

Proposition 3.2: For any rate-tuple $(R_1,\ldots,R_K)$ on the Pareto boundary of the MISO-IC rate region defined in (3), which is achievable with a set of transmit covariance matrices, $\boldsymbol{S}_1,\ldots,\boldsymbol{S}_K$, there is a corresponding interference-power constraint vector, $\boldsymbol{\Gamma} \geq 0$, with $\Gamma_{kj} = \boldsymbol{h}_{kj}^H\boldsymbol{S}_k\boldsymbol{h}_{kj}, \quad \forall j\neq k, j\in\{1,\ldots,K\}$, and $k\in\{1,\ldots,K\}$, such that $R_k = C_k(\boldsymbol{\Gamma}_k), \quad \forall k$, and $\boldsymbol{S}_k$ is the optimal solution of Problem (9) for the given k.

Proof: Please see Appendix II. ∎

From Proposition 3.2, it follows that the Pareto boundary for the MISO-IC is parameterized in terms of a lower-dimensional manifold over the non-negative real vector $\boldsymbol{\Gamma}$, in comparison with that over the complex transmit covariance matrices, $\boldsymbol{S}_k$'s, or with that over the complex beamforming vectors, $\boldsymbol{w}_k$'s. Furthermore, by combining Propositions 3.1 and 3.2, it follows that *beamforming* is indeed optimal to achieve any rate-tuple on the MISO-IC Pareto boundary.

Remark 1: It is worth noting that the dimensionality reduction approach proposed in this work for characterizing the Pareto boundary of the MISO-IC is in spirit similar to that proposed in [10], where it has been shown that the transmit beamforming vectors to achieve any Pareto-boundary rate-tuple of the K-user MISO-IC with interference treated as noise can be expressed in the following forms:

$$\boldsymbol{w}_k = \sum_{j=1}^{K}\xi_{kj}\boldsymbol{h}_{kj}, \quad k=1,\ldots,K \tag{12}$$

where ξ_{kj}'s are complex coefficients. Note that under the assumption of independent $\boldsymbol{h}_{kj}$'s, the above beamforming structure is non-trivial only when $M_k > K$. For this case, from Remark 2 in Appendix I, it is known that for the optimal beamforming structure given in (10), we have $\lambda_{kk} > 0$. With this and by applying the matrix inversion lemma [21], it can be shown (the detailed proof is omitted here for brevity) that the optimal beamforming structure given by (10) is indeed in accordance with that given by (12). The main difference for these two methods to characterize the Pareto boundary for the MISO-IC lies in their adopted parameters: The method in our work uses $K(K-1)$ real Γ_{kj}'s, while that in [10] uses $K(K-1)$ complex ξ_{kj}'s. Note that Γ_{kj} corresponds to the IT constraint from the kth user transmitter to the jth user receiver, whereas there is no practical meaning associated with ξ_{kj}. Consequently, as will be shown next, the proposed method in our work leads to a practical algorithm to implement the multi-cell cooperative downlink beamforming, via iteratively searching for mutually desirable IT constraints between different pairs of BSs.

IV. Decentralized Algorithm For Multi-Cell Cooperative Beamforming

In this section, we develop a new *decentralized* algorithm that practically implements the multi-cell cooperative downlink beamforming based on the results derived in the previous section. It is assumed that each BS in the cellular network has the perfect knowledge of the channels from it to all MSs. Furthermore, it is assumed that all BSs operate according to the same protocol described as follows. Initially, a set of prescribed IT constraints in $\mathbf{\Gamma}$ are set over the whole network, and the kth BS is informed of its corresponding $\mathbf{\Gamma}_k, k = 1, \ldots, K$. Accordingly, each BS sets its own transmit beamforming vector via solving Problem (9) and sets its transmit rate equal to the optimal objective value of Problem (9), which is achievable for its MS since the actual IT levels from the other BSs must be below their prescribed constraints. Then, by assuming that there is an error-free link between each pair of BSs, all different pairs of BSs start to communicate with each other for updating their mutual IT constraints (the details are given later in this section), under which each pair of BSs reset their respective beamforming vectors via solving Problem (9) such that the achievable rates for their MSs both get improved. Notice that each pair of updating BSs keeps the IT constraints for the MSs associated with the other BSs excluding this pair fixed; and as a result, the transmit rates for all the other MSs are not affected. Therefore, the above algorithm can be implemented in a pairwise decentralized manner across the BSs, while it converges when there are no incentives for all different pairs of BSs to further update their mutual IT constraints.

Next, we focus on the key issue on how to update the mutual IT constraints for each particular pair of BSs to guarantee the rate increase for both of their MSs. To resolve this problem, in the following proposition, we first provide the necessary conditions for any given $\mathbf{\Gamma} \geq 0$ (component-wise) to correspond to a Pareto-optimal rate-tuple for the MISO-IC, which will lead to a simple rule for updating the mutual IT constraints between different pairs of BSs. Note that from Proposition 3.2, it follows that for any Pareto-optimal rate-tuple of the MISO-IC, there must exist a $\mathbf{\Gamma}$ such that the optimal solutions of the problems given in (9) for all k's are the same as those for the general formulation of MISO-IC to achieve this rate-tuple. However, for any given $\mathbf{\Gamma} \geq 0$, it remains unknown whether this value of $\mathbf{\Gamma}$ will correspond to a Pareto-optimal rate-tuple.

Proposition 4.1: For an arbitrarily chosen $\mathbf{\Gamma} \geq 0$, if the optimal rate values of the problems in (9) for all k's, $C_k(\mathbf{\Gamma}_k)$'s, are Pareto-optimal on the boundary of the MISO-IC rate region defined in (3), then for any pair of $(i, j), \quad i \in \{1, \ldots, K\}, j \in \{1, \ldots, K\}$, and $i \neq j$, it must hold that $|\boldsymbol{D}_{ij}| = 0$, where $\boldsymbol{D}_{ij}$ is defined as

$$\boldsymbol{D}_{ij} = \begin{bmatrix} \frac{\partial C_i(\mathbf{\Gamma}_i)}{\partial \Gamma_{ij}} & \frac{\partial C_i(\mathbf{\Gamma}_i)}{\partial \Gamma_{ji}} \\ \frac{\partial C_j(\mathbf{\Gamma}_j)}{\partial \Gamma_{ij}} & \frac{\partial C_j(\mathbf{\Gamma}_j)}{\partial \Gamma_{ji}} \end{bmatrix}. \tag{13}$$

Proof: Please see Appendix III. ■

Note that $\boldsymbol{D}_{ij}$'s for all different pairs of (i, j) can be obtained from the (primal and dual) solutions of the problems given in (9) for all k's with the given $\mathbf{\Gamma}$ (for the details, please refer to Appendix I). More specifically, we have

$$\frac{\partial C_i(\mathbf{\Gamma}_i)}{\partial \Gamma_{ij}} = \lambda_{ij} \tag{14}$$

where λ_{ij} is the solution for the dual problem of Problem (9) with $k = i$, which corresponds to the jth IT constraint, and from the objective function of Problem (9),

$$\frac{\partial C_i(\mathbf{\Gamma}_i)}{\partial \Gamma_{ji}} = \frac{-\boldsymbol{h}_{ii}^H \boldsymbol{S}_i^\star \boldsymbol{h}_{ii}}{\ln 2 \left(\sum_{l \neq i} \Gamma_{li} + \sigma_i^2\right)\left(\sum_{l \neq i} \Gamma_{li} + \sigma_i^2 + \boldsymbol{h}_{ii}^H \boldsymbol{S}_i^\star \boldsymbol{h}_{ii}\right)} \tag{15}$$

where $\boldsymbol{S}_i^\star$ is the optimal solution of Problem (9) with $k = i$. Similarly, $(\partial C_j(\mathbf{\Gamma}_j))/(\partial \Gamma_{ij})$ and $(\partial C_j(\mathbf{\Gamma}_j))/(\partial \Gamma_{ji})$ can be obtained from solving Problem (9) via the Lagrange duality method with $k = j$.

From Proposition 4.1, the following observations can be easily obtained (the proofs are omitted for brevity):

- for any particular $\mathbf{\Gamma}$ that corresponds to a Pareto-optimal rate-tuple, it must hold that $\Gamma_{ij} \leq \bar{\Gamma}_{ij}, \quad \forall i, j, i \neq j$, where $\bar{\Gamma}_{ij} = (\|\boldsymbol{h}_{ij}^H \boldsymbol{h}_{ii}\|^2 P_i)/(\|\boldsymbol{h}_{ii}\|^2)$ corresponds to the case of using maximum transmit power with MRT beamforming for the ith BS;
- for any particular $\mathbf{\Gamma}$ that corresponds to a Pareto-optimal rate-tuple, it must hold that $\boldsymbol{h}_{ij}^H \boldsymbol{S}_i^\star \boldsymbol{h}_{ij} = \Gamma_{ij}, \quad \forall i, j, i \neq j$, i.e., the IT constraints across all BSs must be tight.

From the above observations, we see that if we are only interested in the values of $\mathbf{\Gamma}$ that correspond to Pareto-optimal rate-tuples for the MISO-IC, it is sufficient for us to focus on the subset of $\mathbf{\Gamma}$ within the set $\mathbf{\Gamma} \geq 0$, in which $\Gamma_{ij} \leq \bar{\Gamma}_{ij}$ and $\Gamma_{ij} = \boldsymbol{h}_{ij}^H \boldsymbol{S}_i^\star \boldsymbol{h}_{ij}, \quad \forall i, j, i \neq j$.

Based on Proposition 4.1, we can develop a simple rule for different pairs of BSs in the cooperative multi-cell system to update their mutual IT constraints for improving both of their transmit rates, while keeping those of the other BSs unchanged. From the Proof of Proposition 4.1 given in Appendix III, it follows that the method for any updating BS pair (i, j) to fulfill the above requirements is via changing Γ_{ij} and Γ_{ji} according to (38). Note that in general, the choice for $\boldsymbol{d}_{ij}$ in (38) to make $\boldsymbol{D}_{ij}\boldsymbol{d}_{ij} > 0$ is not unique. For notational conciseness, let $\boldsymbol{D}_{ij} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; it can then be shown that one particular choice for $\boldsymbol{d}_{ij}$ is

$$\boldsymbol{d}_{ij} = \text{sign}(ad - bc) \cdot [\alpha_{ij} d - b, a - \alpha_{ij} c]^T \tag{16}$$

where $\text{sign}(x) = 1$ if $x \geq 0$ and $= -1$ otherwise; $\alpha_{ij} \geq 0$ is a constant that controls the ratio between the rate increments for the ith and jth BSs. It can be easily verified that when $\alpha_{ij} > 1$, a larger rate increment is resulted for the ith BS than that for the jth BS, and *vice versa* when $\alpha_{ij} < 1$ (provided that the step-size δ_{ij} in (38) is sufficiently small).

More specifically, the procedure for any BS pair $(i, j), \quad i \neq j, i \in \{1, \ldots, K\}$, and $j \in \{1, \ldots, K\}$, to update their mutual IT constraints is given as follows. First, the ith BS computes the elements a and b in $\boldsymbol{D}_{ij}$ according to (14) and (15), respectively, with the present value of $\mathbf{\Gamma}_i$. Similarly, the jth BS computes c and d with the present value of $\mathbf{\Gamma}_j$. Next, the ith BS sends a and b to the jth BS, while the jth BS sends c and d to the ith BS. Then, assuming that α_{ij} and δ_{ij} are preassigned values known to these two BSs, they can both compute $\boldsymbol{d}_{ij}$ according to (16) and update Γ_{ij} and Γ_{ji} according to (38) in Appendix III. Last, with the updated values Γ'_{ij} and Γ'_{ji}, these two BSs reset their respective beamforming vectors and transmit rates via solving (9) independently. Note that the above operation requires only local information (scalar) exchanges between different pairs of BSs, and thus can be implemented at a very low cost in a cellular system. One version of the decentralized algorithm for cooperative downlink beamforming in a multi-cell system is described in Table I. Since in each iteration of the algorithm the achievable rates for the pair of updating BSs both improve and those for all other BSs are unaffected (non-decreasing), and the maximum achievable rates for all BSs are bounded by finite Pareto-optimal values, the convergence of this algorithm is ensured.

Example 4.1: In Fig. 2 (with the same two-user MISO-IC as for Fig. 1), we show the Pareto boundary for an example MISO-IC with $K = 2$, $M_1 = M_2 = 3$, $P_1 = 5, P_2 = 1$, and $\sigma_1^2 = \sigma_2^2 = 1$, which is obtained by the proposed method in this correspondence, i.e., solving the problems given in (9) for $k = 1, 2$, and a given pair of values Γ_{12} and Γ_{21} with $0 \leq \Gamma_{12} \leq \bar{\Gamma}_{12}$ and $0 \leq \Gamma_{21} \leq \bar{\Gamma}_{21}$, and

TABLE I
ALGORITHM FOR COOPERATIVE DOWNLINK BEAMFORMING

Initialize $\boldsymbol{\Gamma} \geq 0$ in the network
BS k sets $\boldsymbol{w}_k$ via solving (9) with the given $\boldsymbol{\Gamma}_k$, $k = 1, \ldots, K$
Repeat
  For $i = 1, \ldots, K$, $j = 1, \ldots, K, j \neq i$,
    BS i computes a and b in $\boldsymbol{D}_{ij}$ (cf. (14), (15)) with the given $\boldsymbol{\Gamma}_i$
    Similarly, BS j computes d and c in $\boldsymbol{D}_{ij}$ with the given $\boldsymbol{\Gamma}_j$
    BS i sends a and b to BS j
    BS j sends c and d to BS i
    BS i/j computes $\boldsymbol{d}_{ij}$ (cf. (16)), then updates Γ_{ij} and Γ_{ji} (cf. (38))
    BS i/j resets $\boldsymbol{w}_i/\boldsymbol{w}_j$ via solving (9) with the updated $\boldsymbol{\Gamma}_i/\boldsymbol{\Gamma}_j$
  End For
Until $|\boldsymbol{D}_{ij}| = 0, \forall i \neq j$.

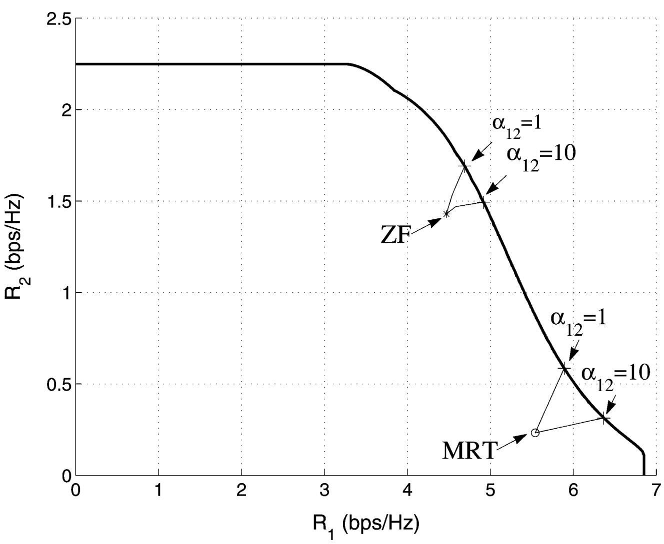

Fig. 2. Achievable rates for the proposed algorithm in a two-user MISO Gaussian IC with interference treated as noise.

then taking a closure operation over the resultant rate-pairs with all different values of Γ_{12} and Γ_{21} within their respective ranges. We demonstrate the effectiveness of the proposed decentralized algorithm for implementing the multi-cell cooperative downlink beamforming with two initial rate-pairs, indicated by "ZF" and "MRT" in Fig. 2, which are obtained when both BSs adopt the ZF and the MRT beamforming vectors, respectively, with their maximum transmit powers. It is observed that the achievable rates for both MSs increase with iterations and finally converge to a Pareto-optimal rate-pair.[4] Comparing the two cases with $\alpha_{12} = 1$ and $\alpha_{12} = 10$, it is observed that a larger value of α_{12} results in larger rate values for the first MS in the converged rate-pairs, which is in accordance with our previous discussion.

V. Concluding Remarks

In this correspondence, based on the concept of interference temperature (IT) and under a cellular downlink setup, we have developed a new method to characterize the complete Pareto boundary of the achievable rate region for the K-user Gaussian MISO-IC with interference treated as noise. It is shown that the proposed method also leads to a new decentralized algorithm for implementing the downlink beamforming in a cooperative multi-cell system to achieve maximal rates with a prescribed fairness guarantee.

There are a number of directions along which the developed results in this work can be further investigated. First, it would be interesting to extend the multi-cell cooperative beamforming design based on the principle of IT to the scenario where each BS supports simultaneous transmissions to *multiple* active MSs each with a single antenna or multiple antennas. Second, it remains yet to be proved whether the necessary conditions derived in this work for any particular set of IT constraints across the BSs to guarantee a Pareto-optimal rate-tuple for the MISO-IC are also *sufficient*, even for the special two-user case. This proof is essential for the proposed downlink beamforming algorithm to achieve the global convergence (Pareto-optimal rates). Last but not least, it is pertinent to analyze the proposed decentralized algorithm that iteratively updates the mutual IT constraints between different pairs of BSs from a *game-theoretical* viewpoint.

[4]We have verified with a large number of random channels and a variety of system parameters that the proposed algorithm always converges to Pareto-optimal rate-pairs for the two-user MISO-IC with randomly selected initial rate-pairs. However, we could not prove this result in general by, e.g., showing that the conditions given in Proposition 4.1 are not only necessary (as proved in this work) but also *sufficient* for any given $\boldsymbol{\Gamma}$ to achieve a Pareto-optimal rate-tuple for the MISO-IC.

Appendix I
Proof of Proposition 3.1

It can be verified that Problem (9) is convex, and thus it can be solved by the standard Lagrange duality method [16]. Let λ_{kj}, $j \neq k$, and λ_{kk} be the non-negative dual variables for Problem (9) associated with the kth BS's IT constraint for the jth MS and its own transmit-power constraint, respectively. The Lagrangian function for this problem can be written as

$$L(\boldsymbol{S}_k, \boldsymbol{\lambda}_k) = \log\left(1 + \frac{\boldsymbol{h}_{kk}^H \boldsymbol{S}_k \boldsymbol{h}_{kk}}{\sum_{j\neq k} \Gamma_{jk} + \sigma_k^2}\right) - \sum_{j\neq k} \lambda_{kj}(\boldsymbol{h}_{kj}^H \boldsymbol{S}_k \boldsymbol{h}_{kj} - \Gamma_{kj}) - \lambda_{kk}(\mathrm{Tr}(\boldsymbol{S}_k) - P_k) \quad (17)$$

where $\boldsymbol{\lambda}_k = [\lambda_{k1}, \ldots, \lambda_{kK}]$. The dual function of Problem (9) is given by

$$g(\boldsymbol{\lambda}_k) = \max_{\boldsymbol{S}_k \succeq 0} L(\boldsymbol{S}_k, \boldsymbol{\lambda}_k). \quad (18)$$

Accordingly, the dual problem is defined as

$$\min_{\boldsymbol{\lambda}_k \geq 0} g(\boldsymbol{\lambda}_k) \quad (19)$$

where $\boldsymbol{\lambda}_k \geq 0$ means component-wise non-negative. Since Problem (9) is convex with strictly feasible points [16], the duality gap between its optimal value and that of the dual problem is zero; thus, Problem (9) can be equivalently solved via solving its dual problem. In order to solve the dual problem, we need to obtain the dual function $g(\boldsymbol{\lambda}_k)$ for any given $\boldsymbol{\lambda}_k \geq 0$. This can be done by solving the maximization problem given in (18), which, according to (17), can be explicitly written as (by discarding irrelevant constant terms)

$$\begin{aligned} \mathop{\mathtt{Max.}}_{\boldsymbol{S}_k} \quad & \log\left(1 + \frac{\boldsymbol{h}_{kk}^H \boldsymbol{S}_k \boldsymbol{h}_{kk}}{\sum_{j\neq k} \Gamma_{jk} + \sigma_k^2}\right) - \mathrm{Tr}(\boldsymbol{B}_k(\boldsymbol{\lambda}_k)\boldsymbol{S}_k) \\ \mathtt{s.t.} \quad & \boldsymbol{S}_k \succeq 0 \end{aligned} \quad (20)$$

where $\boldsymbol{B}_k(\boldsymbol{\lambda}_k) \triangleq \sum_{j\neq k} \lambda_{kj} \boldsymbol{h}_{kj} \boldsymbol{h}_{kj}^H + \lambda_{kk}\boldsymbol{I}$ and $\boldsymbol{B}_k(\boldsymbol{\lambda}_k) \succeq 0$ of dimension $M_k \times M_k$. In order for Problem (20) to have a bounded objective value, it is shown as follows that $\boldsymbol{B}_k(\boldsymbol{\lambda}_k)$ should be a full-rank matrix. Suppose that $\boldsymbol{B}_k(\boldsymbol{\lambda}_k)$ is rank-deficient, such that we could define $\boldsymbol{S}_k = q_k \boldsymbol{v}_k \boldsymbol{v}_k^H$, where $q_k \geq 0$ and $\boldsymbol{v}_k \in \mathbb{C}^{M_k \times 1}$ satisfying

$\|\boldsymbol{v}_k\| = 1$ and $\boldsymbol{B}_k(\boldsymbol{\lambda}_k)\boldsymbol{v}_k = \boldsymbol{0}$. Thereby, the objective function of Problem (20) reduces to

$$\log\left(1+\frac{q_k\|\boldsymbol{h}_{kk}^H\boldsymbol{v}_k\|^2}{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}\right). \tag{21}$$

Due to the independence of $\boldsymbol{h}_{kk}$ and $\boldsymbol{h}_{kj}$'s, and thus the independence of $\boldsymbol{h}_{kk}$ and $\boldsymbol{v}_k$, it follows that $\|\boldsymbol{h}_{kk}^H\boldsymbol{v}_k\| > 0$ with probability one such that (21) goes to infinity by letting $q_k \to \infty$. Since the optimal value of Problem (9) must be bounded, without loss of generality, we only need to consider the subset of $\boldsymbol{\lambda}_k$ in the set $\boldsymbol{\lambda}_k \geq 0$ to make $\boldsymbol{B}_k(\boldsymbol{\lambda}_k)$ full-rank.

Remark 2: Note that from the definition of $\boldsymbol{B}_k(\boldsymbol{\lambda}_k)$ and the Karush–Kuhn–Tucker (KKT) optimality conditions [16] of Problem (9), it follows that $\boldsymbol{B}_k(\boldsymbol{\lambda}_k)$ is full-rank only when either of the following two cases occurs:

- $\lambda_{kk} > 0$: in this case, the transmit power constraint for the kth BS is tight for Problem (9);
- $\lambda_{kk} = 0$, but there are at least M_k λ_{kj}'s, $j \neq k$, having $\lambda_{kj} > 0$: in this case, regardless of whether the transmit power constraint for the kth BS is tight, there are at least M_k out of the $K-1$ IT constraints of the kth BS are tight in Problem (9). Note that this case can be true only when $M_k \leq K-1$.

From the above discussions, it is known that $(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1}$ exists. Thus, we can introduce a new variable $\bar{\boldsymbol{S}}_k$ for Problem (20) as

$$\bar{\boldsymbol{S}}_k = (\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{1/2}\boldsymbol{S}_k(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{1/2} \tag{22}$$

and substituting it into (20) yields

$$\begin{aligned}\underset{\bar{\boldsymbol{S}}_k}{\mathtt{Max.}}\quad & \log\left(1+\frac{\boldsymbol{h}_{kk}^H(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\bar{\boldsymbol{S}}_k(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\boldsymbol{h}_{kk}}{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}\right)\\ & -\mathrm{Tr}(\bar{\boldsymbol{S}}_k)\\ \mathtt{s.t.}\quad & \bar{\boldsymbol{S}}_k \succeq 0.\end{aligned} \tag{23}$$

Without loss of generality, we can express $\bar{\boldsymbol{S}}_k$ into its eigenvalue decomposition (EVD) as $\bar{\boldsymbol{S}}_k = \boldsymbol{U}_k\boldsymbol{\Theta}_k U_k^H$, where $\boldsymbol{U}_k = [\boldsymbol{u}_{k1},\ldots,\boldsymbol{u}_{kM_k}] \in \mathbb{C}^{M_k\times M_k}$ is unitary and $\boldsymbol{\Theta}_k = \mathrm{Diag}([\theta_{k1},\ldots,\theta_{kM_k}]) \succeq 0$. Substituting the ED of $\bar{\boldsymbol{S}}_k$ into (23) yields

$$\begin{aligned}\underset{\boldsymbol{U}_k,\boldsymbol{\Theta}_k}{\mathrm{Max.}}\quad & \log\left(1+\frac{\sum_{i=1}^{M_k}\theta_{ki}\|\boldsymbol{h}_{kk}^H(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\boldsymbol{u}_{ki}\|^2}{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}\right)\\ & -\sum_{i=1}^{M_k}\theta_{ki}\\ \mathrm{s.t.}\quad & \|\boldsymbol{u}_{ki}\| = 1, \forall i, \boldsymbol{u}_{ki}^H\boldsymbol{u}_{kl} = 0, \forall l \neq i\\ & \theta_{ki} \geq 0, \forall i.\end{aligned} \tag{24}$$

For any given $\boldsymbol{U}_k$, it can be verified that the optimal solution of $\boldsymbol{\Theta}_k$ for Problem (24) is given by

$$\theta_{ki} = \begin{cases}\left(\frac{1}{\ln 2}-\frac{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}{\|\boldsymbol{h}_{kk}^H(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\boldsymbol{u}_{ki}\|^2}\right)^+ & \text{if } i = i^\star\\ 0 & \text{otherwise.}\end{cases} \tag{25}$$

where $i^\star = \arg\max_{l\in\{1,\ldots,M_k\}}\|\boldsymbol{h}_{kk}^H(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\boldsymbol{u}_{kl}\|$. Thus, it follows that for the optimal solution of Problem (23), $\mathrm{Rank}(\bar{\boldsymbol{S}}_k) \leq 1$. Furthermore, let i' denote the index of i for which $\theta_{ki'} \geq 0$. The objective function of Problem (24) reduces to

$$\log\left(1+\frac{\theta_{ki'}\|\boldsymbol{h}_{kk}^H(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\boldsymbol{u}_{ki'}\|^2}{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}\right)-\theta_{ki'}. \tag{26}$$

Clearly, the above function is maximized with any $\theta_{ki'} > 0$ when

$$\boldsymbol{u}_{ki'} = \frac{(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\boldsymbol{h}_{kk}}{\|(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\boldsymbol{h}_{kk}\|}. \tag{27}$$

From (25) and (27), it follows that the optimal solution for Problem (23) is

$$\begin{aligned}\bar{\boldsymbol{S}}_k = & \frac{\left(\frac{1}{\ln 2}-\frac{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}{\|\boldsymbol{h}_{kk}^H(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\|^2}\right)^+}{\|(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\boldsymbol{h}_{kk}\|^2}\\ & \times(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}\boldsymbol{h}_{kk}\boldsymbol{h}_{kk}^H(\boldsymbol{B}_k(\boldsymbol{\lambda}_k))^{-1/2}.\end{aligned} \tag{28}$$

Combining the above solution and (22), it can be shown that the optimal solution $\boldsymbol{S}_k$ for Problem (9) is as given by Proposition 3.1.

With the obtained dual function $g(\boldsymbol{\lambda}_k)$ for any given $\boldsymbol{\lambda}_k$, the dual problem (19) can be solved by searching over $\boldsymbol{\lambda}_k \geq 0$ to minimize $g(\boldsymbol{\lambda}_k)$. This can be done via, e.g., the ellipsoid method [22], by utilizing the subgradient of $g(\boldsymbol{\lambda}_k)$ that is obtained from (17) as $\Gamma_{kj} - \boldsymbol{h}_{kj}^H\boldsymbol{S}_k^\star\boldsymbol{h}_{kj}$ for $\lambda_{kj}, j \neq k$ and $P_k - \mathrm{Tr}(\boldsymbol{S}_k^\star)$ for λ_{kk}, where $\boldsymbol{S}_k^\star$ is the optimal solution for Problem (20) with the given $\boldsymbol{\lambda}_k$. When $\boldsymbol{\lambda}_k$ converges to the optimal solution for the dual problem, the corresponding $\boldsymbol{S}_k^\star$ becomes the optimal solution for Problem (9). Proposition 3.1 thus follows.

APPENDIX II
PROOF OF PROPOSITION 3.2

Since the given set of $\boldsymbol{S}_1,\ldots,\boldsymbol{S}_K$ achieves the Pareto-optimal rate-tuple $(R_1,\ldots,R_K)$ for the MISO-IC, from (2) and (3) it follows that for any $k \in \{1,\ldots,K\}$

$$R_k = \log\left(1+\frac{\boldsymbol{h}_{kk}^H\boldsymbol{S}_k\boldsymbol{h}_{kk}}{\sum_{j\neq k}\boldsymbol{h}_{jk}^H\boldsymbol{S}_j\boldsymbol{h}_{jk}+\sigma_k^2}\right). \tag{29}$$

Since $\Gamma_{jk} = \boldsymbol{h}_{jk}^H\boldsymbol{S}_j\boldsymbol{h}_{jk}, \quad \forall j \neq k$, (29) can be rewritten as

$$R_k = \log\left(1+\frac{\boldsymbol{h}_{kk}^H\boldsymbol{S}_k\boldsymbol{h}_{kk}}{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}\right). \tag{30}$$

Note that (30) is the same as the objective function of Problem (9) for the given k. Furthermore, from (3) it follows that $\mathrm{Tr}(\boldsymbol{S}_k) \leq P_k$. Using this and the fact that $\Gamma_{kj} = \boldsymbol{h}_{kj}^H\boldsymbol{S}_k\boldsymbol{h}_{kj}, \quad \forall j \neq k$, it follows that $\boldsymbol{S}_k$ satisfies the constraints given in Problem (9) for the given k. Therefore, $\boldsymbol{S}_k$ must be a feasible solution for Problem (9) with the given k and $\boldsymbol{\Gamma}_k$.

Next, we need to prove that $\boldsymbol{S}_k$ is indeed the optimal solution of Problem (9) for any given k, and thus the corresponding achievable rate R_k is equal to the optimal value of Problem (9), which is $C_k(\boldsymbol{\Gamma}_k)$. We prove this result by contradiction. Suppose that the optimal solution for Problem (9), denoted by $\boldsymbol{S}_k^\star$, is not equal to $\boldsymbol{S}_k$ for a given k. Thus, we have

$$R_k < \log\left(1+\frac{\boldsymbol{h}_{kk}^H\boldsymbol{S}_k^\star\boldsymbol{h}_{kk}}{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}\right) \tag{31}$$

$$= \log\left(1+\frac{\boldsymbol{h}_{kk}^H\boldsymbol{S}_k^\star\boldsymbol{h}_{kk}}{\sum_{j\neq k}\boldsymbol{h}_{jk}^H\boldsymbol{S}_j\boldsymbol{h}_{jk}+\sigma_k^2}\right) \triangleq r_k. \tag{32}$$

Furthermore, since $\boldsymbol{h}_{kj}^H\boldsymbol{S}_k^\star\boldsymbol{h}_{kj} \leq \Gamma_{kj}, \quad \forall j \neq k$, we have for any $j \neq k$,

$$R_j = \log\left(1+\frac{\boldsymbol{h}_{jj}^H\boldsymbol{S}_j\boldsymbol{h}_{jj}}{\sum_{i\neq j}\boldsymbol{h}_{ij}^H\boldsymbol{S}_i\boldsymbol{h}_{ij}+\sigma_j^2}\right) \tag{33}$$

$$= \log\left(1+\frac{\boldsymbol{h}_{jj}^H\boldsymbol{S}_j\boldsymbol{h}_{jj}}{\sum_{i\neq j}\Gamma_{ij}+\sigma_j^2}\right) \tag{34}$$

$$\leq \log\left(1+\frac{\boldsymbol{h}_{jj}^H\boldsymbol{S}_j\boldsymbol{h}_{jj}}{\sum_{i\neq j,k}\Gamma_{ij}+\boldsymbol{h}_{kj}^H\boldsymbol{S}_k^\star\boldsymbol{h}_{kj}+\sigma_j^2}\right) \quad (35)$$

$$=\log\left(1+\frac{\boldsymbol{h}_{jj}^H\boldsymbol{S}_j\boldsymbol{h}_{jj}}{\sum_{i\neq j,k}\boldsymbol{h}_{ij}^H\boldsymbol{S}_i\boldsymbol{h}_{ij}+\boldsymbol{h}_{kj}^H\boldsymbol{S}_k^\star\boldsymbol{h}_{kj}+\sigma_j^2}\right)\triangleq r_j. \quad (36)$$

Thus, for another set of transmit covariance matrices given by $\boldsymbol{S}_1,\ldots,\boldsymbol{S}_{k-1},\boldsymbol{S}_k^\star,\boldsymbol{S}_{k+1},\ldots,\boldsymbol{S}_K$, the corresponding achievable rate-tuple for the MISO-IC, $(r_1,\ldots,r_K)$, satisfies that $r_k > R_k$ and $r_j \geq R_j, \quad \forall j\neq k$, which contradicts the fact that $(R_1,\ldots,R_K)$ is a Pareto-optimal rate-tuple for the MISO-IC. Hence, the presumption that $\boldsymbol{S}_k\neq\boldsymbol{S}_k^\star$ for any given k cannot be true. Thus, we have $\boldsymbol{S}_k=\boldsymbol{S}_k^\star$ and $R_k=C_k(\boldsymbol{\Gamma}_k), \forall k$. Proposition 3.2 thus follows.

Appendix III
Proof of Proposition 4.1

As given in Proposition 4.1, with $\boldsymbol{\Gamma}$, the corresponding optimal values of the problems in (9) for all k's, $C_k(\boldsymbol{\Gamma}_k)$'s, correspond to a Pareto-optimal rate-tuple for the MISO-IC, denoted by $(R_1,\ldots,R_K)$. Let $\boldsymbol{S}_1,\ldots,\boldsymbol{S}_K$ denote the set of optimal solutions for the problems in (9). We thus have

$$C_k(\boldsymbol{\Gamma}_k)=R_k=\log\left(1+\frac{\boldsymbol{h}_{kk}^H\boldsymbol{S}_k\boldsymbol{h}_{kk}}{\sum_{j\neq k}\Gamma_{jk}+\sigma_k^2}\right),\quad k=1,\ldots,K. \quad (37)$$

Next, we prove Proposition 4.1 by contradiction. Suppose that there exists a pair of (i,j) with $|\boldsymbol{D}_{ij}|\neq 0$, where $\boldsymbol{D}_{ij}$ is defined in (13). Define a new $\boldsymbol{\Gamma}'$ over $\boldsymbol{\Gamma}$, where all the elements in $\boldsymbol{\Gamma}$ remain unchanged except $[\Gamma_{ij},\Gamma_{ji}]^T$ being replaced by

$$[\Gamma'_{ij},\Gamma'_{ji}]^T=[\Gamma_{ij},\Gamma_{ji}]^T+\delta_{ij}\cdot\boldsymbol{d}_{ij} \quad (38)$$

where $\delta_{ij}>0$ is a small step-size, and $\boldsymbol{d}_{ij}$ is any vector that satisfies $\boldsymbol{D}_{ij}\boldsymbol{d}_{ij}>0$ (component-wise), with one possible value for such $\boldsymbol{d}_{ij}$ is given by (16) in the main text. With $\boldsymbol{\Gamma}'$, the optimal solutions for the problems in (9) remain unchanged $\forall k\neq i,j$, while for those with $k=i$ and $k=j$, the optimal solutions are changed to be $\boldsymbol{S}_i^\star$ and $\boldsymbol{S}_j^\star$, respectively. Accordingly, the new achievable rates in the MISO-IC for any $k\neq i,j$ are given by

$$r_k=\log\left(1+\frac{\boldsymbol{h}_{kk}^H\boldsymbol{S}_k\boldsymbol{h}_{kk}}{\sum_{l\neq k,i,j}\boldsymbol{h}_{lk}^H\boldsymbol{S}_l\boldsymbol{h}_{lk}+\boldsymbol{h}_{ik}^H\boldsymbol{S}_i^\star\boldsymbol{h}_{ik}+\boldsymbol{h}_{jk}^H\boldsymbol{S}_j^\star\boldsymbol{h}_{jk}+\sigma_k^2}\right) \quad (39)$$

$$=\log\left(1+\frac{\boldsymbol{h}_{kk}^H\boldsymbol{S}_k\boldsymbol{h}_{kk}}{\sum_{l\neq k,i,j}\Gamma_{lk}+\boldsymbol{h}_{ik}^H\boldsymbol{S}_i^\star\boldsymbol{h}_{ik}+\boldsymbol{h}_{jk}^H\boldsymbol{S}_j^\star\boldsymbol{h}_{jk}+\sigma_k^2}\right) \quad (40)$$

$$\geq R_k \quad (41)$$

where (41) is due to (37) and the facts that $\boldsymbol{h}_{ik}^H\boldsymbol{S}_i^\star\boldsymbol{h}_{ik}\leq\Gamma_{ik}$ and $\boldsymbol{h}_{jk}^H\boldsymbol{S}_j^\star\boldsymbol{h}_{jk}\leq\Gamma_{jk}$. Also, it can be shown that

$$r_i=\log\left(1+\frac{\boldsymbol{h}_{ii}^H\boldsymbol{S}_i^\star\boldsymbol{h}_{ii}}{\sum_{l\neq i,j}\boldsymbol{h}_{li}^H\boldsymbol{S}_l\boldsymbol{h}_{li}+\boldsymbol{h}_{ji}^H\boldsymbol{S}_j^\star\boldsymbol{h}_{ji}+\sigma_i^2}\right) \quad (42)$$

$$=\log\left(1+\frac{\boldsymbol{h}_{ii}^H\boldsymbol{S}_i^\star\boldsymbol{h}_{ii}}{\sum_{l\neq i,j}\Gamma_{li}+\boldsymbol{h}_{ji}^H\boldsymbol{S}_j^\star\boldsymbol{h}_{ji}+\sigma_i^2}\right) \quad (43)$$

$$\geq C_i(\boldsymbol{\Gamma}'_i) \quad (44)$$

where (44) is due to the facts that $\boldsymbol{h}_{ji}^H\boldsymbol{S}_j^\star\boldsymbol{h}_{ji}\leq\Gamma'_{ji}$ and $\boldsymbol{S}_i^\star$ achieves the optimal value of Problem (9) with $k=i$ and the given $\boldsymbol{\Gamma}'_i$, denoted by $C_i(\boldsymbol{\Gamma}'_i)$. Similarly, it can be shown that $r_j\geq C_j(\boldsymbol{\Gamma}'_j)$. Thus, from (38) and $\boldsymbol{D}_{ij}\boldsymbol{d}_{ij}>0$, it follows that with sufficiently small δ_{ij}

$$\begin{bmatrix} r_i\\ r_j\end{bmatrix}\geq\begin{bmatrix} C_i(\boldsymbol{\Gamma}'_i)\\ C_j(\boldsymbol{\Gamma}'_j)\end{bmatrix} \quad (45)$$

$$\cong\begin{bmatrix} C_i(\boldsymbol{\Gamma}_i)\\ C_j(\boldsymbol{\Gamma}_j)\end{bmatrix}+\delta_{ij}\boldsymbol{D}_{ij}\boldsymbol{d}_{ij} \quad (46)$$

$$>\begin{bmatrix} R_i\\ R_j\end{bmatrix}. \quad (47)$$

Therefore, we have found a new set of achievable rate-tuple for the MISO-IC with $\boldsymbol{\Gamma}'$, $(r_1,\ldots,r_K)$, which has $r_i > R_i$, $r_j > R_j$, and $r_k\geq R_k, \quad \forall k\neq i,j$. Clearly, this contradicts the fact that $(R_1,\ldots,R_K)$ is Pareto-optimal for the MISO-IC. Thus, the presumption that there exists a pair of (i,j) with $|\boldsymbol{D}_{ij}|\neq 0$ cannot be true. Proposition 4.1 thus follows.

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Optimality of Beamforming for MIMO Multiple Access Channels Via Virtual Representation

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***Abstract*—In this correspondence, we consider the optimality of beamforming for achieving the ergodic capacity of multiple-input multiple-output (MIMO) multiple access channel (MAC) via virtual representation (VR) model. We assume that the receiver knows the channel state information (CSI) perfectly but that the transmitter knows only partial CSI, i.e., the channel statistics. For the single-user case, we prove that the capacity-achieving beamforming angle (c.b.a.) is unique, and there exists a signal-to-noise ratio (SNR) threshold below which beamforming is optimal and above which beamforming is strictly suboptimal. For the multi-user case, we show that the c.b.a is not unique and we obtain explicit conditions that determine the beamforming angles for a special class of correlated MAC-VR models. Under mild conditions, we show that a large class of power allocation schemes can achieve the sum-capacity within a constant as the number of users in the system becomes large. The beamforming scheme, in particular, is shown to be asymptotically capacity-achieving only for certain MAC-VR models.**

***Index Terms*—Beamforming, multiple access, multiple-input multiple-output, sum-capacity, power allocation, virtual representation.**

I. Introduction

The multiple-input multiple-output (MIMO) techniques provide powerful means to improve reliability and capacity of wireless channels. Significant amount of work has been done to study optimal input distributions and the channel capacity of single-user and multi-user MIMO channels (see, e.g., [1]–[7]). Several models have been adopted to capture the spatial correlation between the channel gains corresponding to different transmit-receive antenna pairs. These models include the i.i.d. model [1], the Kronecker model [2], [8]–[10], the virtual representation (VR) model [4], [11], and the unitary-independent-unitary (UIU) model [5]. The i.i.d. model assumes that the channel gains are independent and identically distributed (i.i.d.), and the Kronecker model assumes that the correlation between the channel gains can be written in terms of the product of the transmit correlation and the receive correlation. These two models apply only to wireless environments with rich or locally rich scattering at either the transmitter or the receiver. The VR and UIU models are more general, and both transform the MIMO channel to a domain such that the channel gains can be justified to be approximately independent.

In this correspondence, we adopt the VR model [11], which represents the MIMO channel in a virtual *angular* domain with each channel gain corresponding to one virtual transmit and receive angle pair. The channel gains in the angular domain can be justified to be approximately independent of each other, although not necessarily identically distributed, because they include different signal paths (corresponding to different transmit and receive angle pairs) with independent random phases.

The single-user MIMO channel based on VR was studied in [4]. In this correspondence, we generalize this study to the MIMO multiple access channel (MAC) based on VR, denoted by MAC-VR. We first characterize the optimal input distribution that achieves the sum-capacity. Then we study the optimality of beamforming, which is a simple scalar coding strategy desirable in practice. We first strengthen the conditions for the optimality of beamforming for the single-user VR model in [4] by proving that there exists a signal-to-noise ratio (SNR) threshold below which beamforming is optimal and above which beamforming is strictly suboptimal. This result was illustrated in [4] only numerically. For the multi-user case, we present an example to show that the capacity-achieving beamforming angle (c.b.a) of a given user may vary with SNR and beamforming angles of other users. This is in contrast to the single-user case in which the c.b.a. is independent of SNR. We also derive explicit conditions to determine possible c.b.a. for certain MAC-VR channels. For systems with K users, we show that as K goes to infinity, the sum-rates achieved by a large class of power allocation schemes are within a constant of the sum-capacity, and they grow in the order of $n_r \log K$, where n_r is the number of receive antennas. Furthermore, we obtain conditions under which beamforming is asymptotically capacity-achieving.

Our study for the single-user case generalizes that in [2], [6] for the Kronecker model, and is different from [12] for the double-scattering model [13]. Our study for the MAC-VR also differs from [7] which assumes perfect channel state information at the transmitter, and from [14], which assumes finite feedback. We also note that the results we derive for the MAC-VR are applicable to the MIMO-MAC Kronecker (MAC-Kr) model in [9]. However, certain results valid for the MAC-Kr may not hold for the MAC-VR as demonstrated in later sections.

II. Channel Model and Virtual Representation

We consider the K-user MIMO MAC, in which K users transmit to one base station (BS) with each user equipped with n_t antennas and the BS equipped with n_r antennas. The channel between each user k and the BS is assumed to be a frequency-flat, MIMO fading channel. The received signal at the BS is an n_r-dimensional vector $Y \in \mathcal{C}^{n_r}$ and is given by

$$Y = \sum_{k=1}^{K} \sqrt{\frac{p^k}{n_t}} H^k X^k + W \tag{1}$$

where $X^k \in \mathcal{C}^{n_t}$ is the input vector of user k that satisfies the power constraint $E[X^{k\dagger} X^k] \leq n_t$, $(\cdot)^\dagger$ denotes the Hermitian operator, p^k represents the effective SNR of user k at each receive antenna, $W \in \mathcal{C}^{n_r}$ is a proper complex Gaussian noise vector that consists of i.i.d. entries with zero-mean and unit-variance, and $H^k \in \mathcal{C}^{n_r \times n_t}$

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