# Scheduling for Multiuser MIMO Broadcast Systems: Transmit or Receive Beamforming?

Li-Chun Wang, *Senior Member, IEEE*, and Chu-Jung Yeh, *Student Member, IEEE*

***Abstract*—In this paper, we present an approximation formula and the close-form expression for the sum rate of the transmit and the receive zero-forcing (ZF) multiple-input multiple-output (MIMO) broadcast systems with user selection, respectively. Instead of assuming a large number of users to obtain a scaling law as most current work, we derive the sum rate formulas of the ZF MIMO broadcast systems with a small number of scheduled users. By analysis and simulations, we find that when taking the variations of feedback channel into account, the receive ZF MIMO broadcast system is more robust to feedback errors and can deliver equal or even higher sum rate than the transmit ZF MIMO broadcast system. We discuss whether a feedback channel is suitable to send channel state information (CSI) for calculating transmit antenna beamforming weights, or suitable to send CSI for selecting users in the receive ZF MIMO broadcast system. Our results show that as the variation of feedback channel errors increases from 0.5 to 1.5, the receive ZF $3 \times 3$ MIMO broadcast system can provide 36% to 116% higher sum rate than the transmit ZF $3 \times 3$ MIMO broadcast system in the case of 20 users at signal to noise ratio (SNR) equal to 20 dB. Providing that more feedback bandwidth and an error-free feedback channel are available, the transmit ZF MIMO broadcast system can achieve higher sum rate than the receive ZF MIMO broadcast system.**

***Index Terms*—MIMO systems, zero-forcing beamforming, zero-forcing receiver, scheduling, MIMO broadcast channels.**

## I. Introduction

MULTIPLE-input multiple-output (MIMO) broadcast antenna techniques can transmit personalized data streams to multiple users concurrently in the point-to-multipoint scenario [1], [2]. Unlike a TV broadcast system, the MIMO broadcast system transmits *different* personalized data streams to a group of selected users. For $M_T$ transmit antennas and $K$ users, the capacity of the MIMO broadcast system is $\min\{M_T, K\}$ times higher than that of a time division multiple access (TDMA) scheduling system which selects one user at a time [3], [4]. When complete channel state information (CSI) is available at the transmitter, the dirty paper coding (DPC) scheme can maximize the sum rate of the MIMO broadcast system [5]–[8]. Although it is the optimal rate-achieving scheme, the DPC MIMO broadcast system faces the serious computation complexity issue, and requires huge amount of feedback information.

Thus, two types of suboptimal MIMO broadcast systems were proposed in the literature: (1) the orthogonal random beamforming [9]–[12] and (2) zero-forcing (ZF) based beamforming[1] [2].

- Firstly, recent research works regarding the orthogonal random beamforming for MIMO broadcast systems are briefly introduced as follows. In [9], it was proved that the orthogonal random beamforming strategy can asymptotically achieve the same throughput slope of DPC when the number of users increases. To solve the difficulty of calculating the random beamformer's weights for a large number of users, some low-complexity random beamformer approaches were proposed in [10]–[12].
- Secondly, we introduce the recent research results about the ZF-based beamforming for MIMO broadcast systems. In [2], a QR-based ZF-DPC MIMO broadcast system was proposed to maximize the sum rate of the MIMO broadcast system. Furthermore, the channel-inverse-based ZF beamforming was also proposed in [2], which is easier in calculating the beamforming weights than the ZF-DPC scheme. However, the effects of user selection and user ordering were not considered in [2], and the number of users is assumed to be smaller than that of transmit antennas. Thus, many researches aimed to relax this assumption and examine a more general MIMO broadcast system when the number of users is larger than the number of transmit antennas. The authors of [13] proposed a greedy user-selection procedure for the ZF-DPC MIMO broadcast systems. In [14], it was shown that the slope of throughput against SNR in dB for the greedy ZF-DPC MIMO broadcast system is the same as that for the capacity-achieving DPC strategy. In [15], it was proved that the channel-inverse-based transmit ZF beamforming combined with multiuser scheduling can asymptotically approach the capacity of the DPC-type MIMO broadcast system when the number of users approaches infinity. To overcome the prohibitively high complexity of exhaustively searching users, [14]–[17] proposed low-complexity and effective user selection approaches for the MIMO broadcast systems.

Generally speaking, the objective of orthogonal random beamforming is to select a group of users to maximize their signal

Manuscript received September 11, 2008; revised February 17, 2009, October 5, 2009, and March 13, 2010; accepted July 1, 2010. The associate editor coordinating the review of this paper and approving it for publication was J. Lu.

This work is sponsored by the National Science Council of Taiwan under grants NSC 97-2221-E-009-099-MY3, by the Ministry of Education of Taiwan under the MoE ATU Program, by the Academia-Research Cooperation Program of MoEA, and by the MediaTek Research Center at National Chiao Tung University, Taiwan.

L.-C. Wang is with the Department of Electrical Engineering, and the Institute of Communications Engineering, National Chiao Tung University, Taiwan (e-mail: lichun@cc.nctu.edu.tw).

C.-J. Yeh is with the Institute of Communications Engineering, National Chiao Tung University, Taiwan.

Digital Object Identifier 10.1109/TWC.2010.072110.081226

[1]Here we use beamforming to represent the situation when antenna elements are multiplied by certain weights. It is not implied that a physical beam pattern is formed.

to interference plus noise ratio (SINR) according to partial CSI, whereas ZF-based beamforming is to nullify the mutual interference among users according to complete CSI.

In this paper, we define the **transmit ZF scheduler** as the channel-inverse-based transmit ZF beamforming combined with multiuser scheduling, where the beamforming weights are multiplied at the transmit antennas and scheduling is an opportunistic transmission technique to exploit multiuser diversity [18], [19]. Furthermore, we investigate another type of ZF MIMO broadcast systems – the **receive ZF scheduler**, where the ZF algorithm is implemented at the multiple receive antennas of the user terminal to cancel the inter-stream interference. In the IEEE 802.11n wireless local area networks, 3GPP LTE (Long Term Evolution) and WiMAX (Worldwide Interoperability for Microwave Access) broadband cellular radio systems, a user terminal is equipped with multiple antennas. Thus, [20]–[22] exploited the advantage of multiple receive antennas and showed that receive ZF scheduler can be also used in MIMO broadcast systems.

The objective of this paper is to quantitatively compare the sum rate and the feedback requirements for the transmit and the receive ZF MIMO broadcast systems with user selection, where both base stations and user terminals are equipped with multiple antennas. Our goal is not to claim that one scheme outperforms the other, but try to suggest a feasible MIMO broadcast system subject to the constraint of feedback bandwidth. It is too costly to use frequency spectrum not for transmitting user's data, but only for sending CSI. Thus, we put an emphasis on the right usage of feedback information and its robustness to channel variations. Specifically, we discuss whether feedback CSI is suitable for selecting users or for calculating antenna beamforming weights. The contributions of this paper can be summarized as follows:

- Subject to feedback channel variations and the amount of feedback information, we quantitatively compare the sum rate of the receive ZF scheduler and the three considered transmit MIMO broadcast systems (including QR-based ZF-DPC, channel-inverse-based ZF beamforming, and the block diagonalization approach [23]). We find that utilizing feedback CSI for **user selection** in receive ZF MIMO broadcast systems is more robust to feedback channel variations compared with utilizing feedback CSI for calculating **antenna beamforming weights** in transmit MIMO broadcast systems. As a result, the receive ZF MIMO broadcast system can deliver the same or even higher sum rate than the transmit MIMO broadcast systems, especially in the presence of channel variations. If an error-free feedback channel is available, the transmit ZF MIMO broadcast system can achieve higher sum rate than the receive ZF MIMO broadcast system.
- Different from the sum rate scalability laws for a large number of users in [3] [4] [15] [21], the derived analytical expression for the sum rate of the transmit ZF scheduler is applicable for a small number of scheduled users. Furthermore, the newly derived sum rate analysis formula of the receive ZF scheduler based on water-filling power allocation is also more general in comparison with the receive ZF scheduler based on equal power allocation [22].

The rest of this paper is organized as follows. Section II introduces the related work about MIMO broadcast systems. In Section III, we describe the considered system model and briefly review the basic principles of the transmit and receive ZF beamforming. In Section IV, we evaluate the sum rate of the transmit ZF scheduler when each user is equipped with only one antenna. For user terminals having multiple antennas, a modified antenna selection algorithm is provided. In Section V we derive the analytical closed-form expression for the sum rate of the receive ZF scheduler. Section VI provides numerical results. In Section VII the performance issues of feedback requirements, feedback errors, and scheduling complexity are discussed. Finally, we offer our concluding remarks in Section VIII.

## II. Related Work

In the literature, the sum rate analysis of the transmit MIMO broadcast systems for a large number of users has been studied extensively. It has been shown that the MIMO broadcast system using ZF beamforming [15] (or called the transmit ZF scheduler in this paper) as well as the MIMO broadcast system using orthogonal random beamforming [9] can achieve the same asymptotic sum rate as that of DPC. For $M_T$ transmit antennas and $K$ users equipped with $M_R$ receive antennas, using transmit beamforming and DPC can achieve the same $M_T \log\log(KM_R)$ scaling law for MIMO broadcast systems [3], [4]. Furthermore, [17] proposed a vector feedback mechanism using singular value decomposition (SVD) and analyzed the sum rate of transmit ZF MIMO broadcast systems based on SVD vector feedback. [24] provided asymptotical sum rate analysis for transmit MIMO broadcast systems using the feedback-based scheduling architecture according to the quantized CSI, where a strong form of throughput optimality of the proposed MIMO broadcast system was demonstrated. Much of the above analysis has focused on the asymptotical sum rate of MIMO broadcast systems with user selection by the extreme value theorem, while [25] showed the asymptotic sum rate for the finite large number of users. To our knowledge, however, the sum rate analysis for the transmit MIMO broadcast system for a small number of users has rarely seen.

Recently, the sum rate analysis of transmit MIMO broadcast system with respect to different feedback assumptions has become a hot research subject. [26] investigated the effect of imperfect received CSI on a ZF-based MIMO broadcast system with random user selection, and derive the capacity bound with analog or digital feedback information. Under the assumption of random user selection and equal power allocation the effects of delayed and quantized CSI on the sum rate of ZF-based MIMO broadcast systems were further analyzed in [27]. For the ZF-based receive beamforming implemented at the base station, [28] derived the asymptotic rate scaling with the uplink limited feedback system.

Now let us discuss the receive beamforming MIMO broadcast systems. [21] demonstrated that the sum rate of the receive ZF scheduler scales with the same slope as the optimal DPC scheme when the number of users approaches infinity. For a finite number of users, a close-form sum rate expression was derived for the receive ZF scheduler based on equal power allocation [22]. The receive ZF scheduler only requires *vector* feedback for user selection [21], [29], whereas the transmit ZF scheduler requires channel matrix feedback or SVD-based vector feedback [17].

Joint transmitter precoding and receiver processing is another method to broadcast personalized data in a point-to-multipoint scenario when multiple receive antennas are available. The block-diagonalization (BD) joint transmitter and receiver beamforming approach consists of a precoding matrix for cancelling the inter-user interference and an equalizer matrix for cancelling the inter-antenna interference [30]–[34]. One key challenge for the BD-based precoder is its high complexity. Both the precoding matrix and the equalizer matrix are determined at the base station. Extra overhead is required to send the information of the equalizer matrix to serving users. The BD-based precoder also needs complete channel matrix information as the transmit ZF beamforming. In [31] an opportunistic user selection algorithm was proposed to select a group of users with the highest sum rate. Based on [31], two low-complexity user/antenna selection algorithms were proposed to select the number of data streams for each user adaptively and thus realize multi-mode transmissions [32]. Based on the search tree concept, [33] suggested an efficient search and scheduling algorithm. To reduce search space, [34] proposed a simplified user selection and receive antenna selection algorithm according to a spatial correlation threshold.

## III. Background

We consider a multiuser MIMO broadcast system with an $M_T$- transmit-antenna base station and $K$ users each of which has $M_R$ receive antennas as shown in Fig. 1. The base station is designed to transmit different data streams up to $M_T$ users simultaneously. For handling the inter-user interference in the multiuser MIMO broadcast systems, beamforming can be implemented at either the transmitter side or the receiver side.

### A. Transmit ZF Beamforming of MIMO Systems

The transmit ZF beamforming multiples the beamforming weights at the transmitter to decouple the MIMO channel matrix $\mathbf{H}$ into parallel subchannels. Based on the ZF principle, the beamforming matrix is written as $\mathbf{W} = \mathbf{H}(\mathcal{S})^T(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^T)^{-1}$, where $\mathcal{S}$ represents the subset of the total receive antennas $\mathcal{U}$, and $(\cdot)^T$ represents the conjugate transpose operation. Note that $|\mathcal{U}| = KM_R$ and $|\mathcal{S}| = M_T$. Then, the received signal vector becomes

$$\begin{aligned}\mathbf{y} &= \mathbf{H}(\mathcal{S})\mathbf{W}\mathbf{x} + \mathbf{n} \\ &= \mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^T(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^T)^{-1}\mathbf{x} + \mathbf{n} = \mathbf{x} + \mathbf{n}\ ,\end{aligned} \quad (1)$$

where $\mathbf{x}$ is the transmit vector, $\mathbf{n} \in \mathbb{C}^{M_T \times 1}$ is the circularly complex additive white Gaussian noise vector with covariance matrix $\mathrm{E}[\mathbf{n}\mathbf{n}^T] = \sigma^2\mathbf{I}$. According to [2], the sum rate of the transmit ZF-based MIMO broadcast system is given by

$$R_{\text{ZFB}}(\mathcal{S}) = \sum_{i\in\mathcal{S}} [\log(\mu b_i)]_+ \ , \quad (2)$$

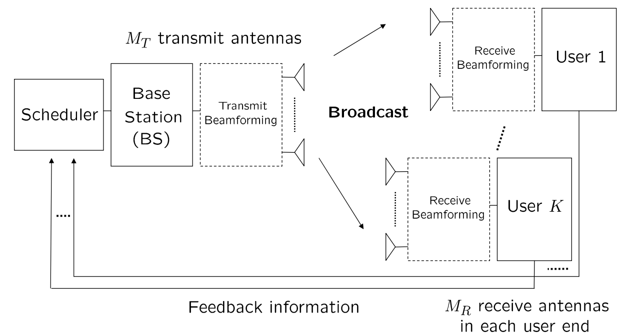



Fig. 1. System model of multiuser MIMO broadcast systems with $M_T$ transmit antennas and $M_R$ receive antennas per user.

where $\mu$ is the water-level satisfying the criterion $\sum_{i\in\mathcal{S}}\left[\mu - \frac{1}{b_i}\right]_+ = P_T$, $[z]_+$ represents $\max\{z, 0\}$, and the effective channel gain $b_i$ of the $i$-th subchannel is

$$b_i = \frac{1}{\left[\left(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^T\right)^{-1}\right]_{ii}} \ . \quad (3)$$

Here $[\mathbf{A}]_{ij}$ denotes the $(i, j)$ entry of the matrix $\mathbf{A}$. Note that the transmit ZF beamforming weighting matrix $\mathbf{W} = \mathbf{H}(\mathcal{S})^T(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^T)^{-1}$ leads to higher transmission power equivalent to the noise enhancement effect of the receive ZF beamforming.

### B. Receive ZF Beamforming of MIMO Systems

With ZF algorithm implemented at the receiver end, data streams can be independently decoded to recover the spatially multiplexed signals. For a certain user terminal $k$, by multiplying received signal $\mathbf{y}_k$ with the pseudo-inverse $\mathbf{H}_k^\dagger = (\mathbf{H}_k^T\mathbf{H})_k^{-1}\mathbf{H}_k^T$, the decoded received signal $\hat{\mathbf{y}}_k$ becomes

$$\hat{\mathbf{y}}_k = \mathbf{H}_k^\dagger\mathbf{y}_k = \mathbf{x} + \mathbf{H}_k^\dagger\mathbf{n}_k \ . \quad (4)$$

The noise covariance matrix after the ZF receiver becomes $\sigma^2[(\mathbf{H}_k^T\mathbf{H}_k)^{-1}]^T$, which may have nonzero off-diagonal elements and results in correlated noise across different data streams. To lower complexity, the noise correlation is usually ignored and each stream is decoded independently [20], [35]. With the noise power per subchannel $\sigma^2[(\mathbf{H}_k^T\mathbf{H}_k)^{-1}]_{ii}$, the equal power principle results in the output (SNR) at the $i$-th subchannel for user terminal $k$ as

$$\gamma_i^k = \frac{P_T}{M_T\sigma^2\left[\left(\mathbf{H}_k^T\mathbf{H}_k\right)^{-1}\right]_{ii}} \ , \quad i = 1, \cdots, M_T \quad (5)$$

where $d_i = 1/\left[\left(\mathbf{H}_k^T\mathbf{H}_k\right)^{-1}\right]_{ii}$ is defined as the effective channel gain of the $i$-th subchannel [21]. Because $\{d_i\}_{i=1}^{M_T}$ are characterized as Chi-square distributed random variables with $2(M_R - M_T + 1)$ degrees of freedom [22], [35], [36], the probability distribution function (PDF) of $d_i$ is

$$f_{d_i}(d) = \frac{d^{M_R - M_T}e^{-d}}{(M_R - M_T)!} \ , \quad \text{for } i = 1, \ldots, M_T \ . \quad (6)$$

### C. Sum Rate with Long-Term Power Constraint

Let $\mu_0$ be the solution of the water-filling equation for the long-term power constraint

$$E\left[\sum_{i=1}^{M_T}[\mu - \frac{1}{z_i}]_+\right] = \sum_{i=1}^{M_T} E\left[\mu - \frac{1}{z_i}\right]_+ = P_T \ . \quad (7)$$

Subject to this long-term power constraint with water-level solution $\mu_0$, the average sum rate with the water-filling power allocation is

$$\begin{aligned} R_{\text{sum rate}} &= E\left[\sum_{i=1}^{M_T}[\log(\mu_0 z_i)]_+\right] \\ &= \sum_{i=1}^{M_T} E\left[\log(\mu_0 z_i)\right]_+ \\ &= \sum_{i=1}^{M_T} \int_{1/\mu_0}^{\infty} \log(\mu_0 z) f_{z_i}(z) dz \ , \end{aligned} \quad (8)$$

where $f_{z_i}(z)$ represents the PDF of effective channel gain $z_i$ for the transmit ZF beamforming or receive ZF beamforming.

## IV. Scheduling for Transmit ZF Beamforming

Because the number of users ($K$) is usually larger than the number of transmit antennas ($M_T$), scheduling is necessary for the transmit MIMO broadcast systems to select $M_T$ receive antennas out of $KM_R$ antennas. Scheduling can exploit the multiuser diversity gain for the transmit ZF-based MIMO broadcast system. For large $K$, the sum rate of the transmit ZF-based MIMO broadcast system can asymptotically achieve the sum rate of DPC (denoted by $R_{\text{DPC}}$) in slope [15], that is,

$$E[R_{\text{ZFB}}] \sim M_T \log\left(1 + \frac{P_T}{M_T}\log\sum_{k=1}^{K} M_R^k\right) \sim E[R_{\text{DPC}}] \ , \quad (9)$$

where $M_R^k$ denotes the number of receive antennas of the $k$th user. Note that we consider the case where all users have the same number of receive antennas. A stronger convergence result was shown in [37]

$$\lim_{K\to\infty}(R_{\text{DPC}} - R_{\text{ZFB}}) = 0 \ . \quad (10)$$

The maximal sum rate scheduling algorithm selects the user set according to

$$R_{\text{ZFB}}^{\max} = \max_{\mathcal{S}\subset\mathcal{U}:\ |\mathcal{S}|=M_T} R_{\text{ZFB}}(\mathcal{S}) \ . \quad (11)$$

For the case $M_R = 1$, this combinatorial optimization problem is to select the best one from $\binom{K}{M_T}$ combinations. The closed-form expression for the sum rate of transmit ZF-based MIMO broadcast systems is not easy to obtain. One of the goals in this paper is to develop an estimation approach to analyze the sum rate of the downlink transmit ZF-based MIMO broadcast systems.

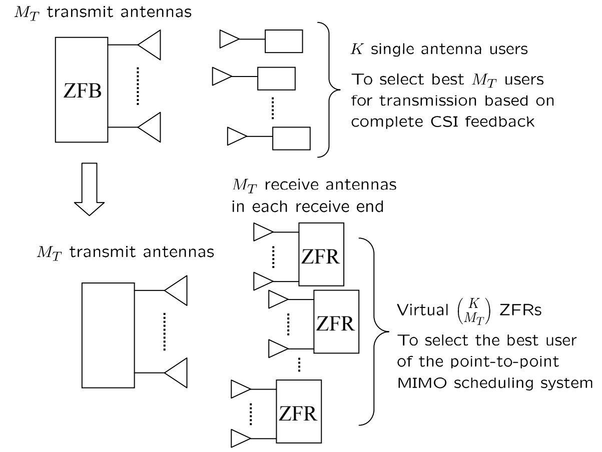



Fig. 2. The modified problem model for the transmit ZF beamforming with the exhaustive-search based scheduling.

### A. Sum Rate Estimation with Exhaustively Searched Users

First we reformulate the transmit ZF-based MIMO broadcast system into the point-to-point MIMO scheduling system. Specifically, a transmit ZF-based MIMO broadcast system with $M_T$ transmit antennas at the base station and $K$ users with single antenna is translated into a receive ZF MIMO system with $M_T$ transmit antennas at the base station and $M_T$ receive antennas, as shown in Fig. 2. In (11), exhaustive search in a transmit ZF-based MIMO broadcast system selects the best $M_T$ antennas from $K$ users. Thus, each of the $\binom{K}{M_T}$ combinations can be viewed as a virtual receive ZF MIMO user with $M_T$ antennas. As a result, the sum rate of the transmit ZF-based MIMO broadcast system with exhaustive user search can be approximated by the receive ZF MIMO system with one user at a time using TDMA scheduling. This approximation will be considered in two scenarios, both of which can lead to the closed-form sum rate performance and will be verified by simulations later.

- In the low SNR region, because of the property of the logarithm function $\log_2(1+x) \approx x\log_2 e$ for $x \approx 0$, the ideal policy for achieving the maximal sum rate for the point-to-point TDMA-based scheduling is to find a user having the maximal strongest subchannel and to allocate all the power only to the strongest subchannel. This principle coincides with the max-max scheduling scheme. That is, a user with the best effective channel gain will be selected most likely.
- In the high SNR region, the property of the logarithm function is $\log_2(1+x) \approx \log_2 x$ for $x \gg 1$. Therefore, improving all subchannels with suitable scheduling gains and corresponding power will yield the maximal sum rate. It is implied that no subchannel will be omitted in each scheduling run. From [38] and [39], the max-min scheduling scheme can uniformly provide the scheduling gains to all subchannels and deliver the maximal sum rate approximately. Thus, we use the max-min scheduling to approximate the sum rate of the transmit ZF-based MIMO broadcast system in the high SNR region.

In [22], the *order statistics* analysis techniques were applied to derive the closed-form expression for the sum-rate capacity of the point-to-point MIMO system with ZF receiver based on the max-max and max-min scheduling. From the above discussions, we present the approximate sum-rate estimations of the transmit ZF-based MIMO broadcast system as follows.

*1) Low SNR Region (Max-Max Approach):* The max-max scheduling algorithm selects the target user with the maximal strongest subchannel among virtual $\binom{K}{M_T}$ users at each time slot. Denote $\{b_i^k\}_{i=1}^{M_T}$ as the set of all subchannel effective channel gains for the $k$th virtual user ($k = 1, \ldots, Z = \binom{K}{M_T}$) and $b_{1:M_T}^k \leq \ldots \leq b_{M_T:M_T}^k$ as the ordered effective channel gains in ascending order of magnitude. With the information of $\{b_{M_T:M_T}^k\}_{k=1}^Z$ from all users, the transmitter chooses the target user according to

$$k^* = \arg\max_k b_{M_T:M_T}^k \quad . \tag{12}$$

After determining the target user $k^*$, we have $\tilde{b}_{i:M_T}^{\text{max}} = b_{i:M_T}^{k^*}$ for $i = 1, \ldots, M_T$ where the superscript *max* denotes the max-max approach. Based on the order statistics analysis proposed in [38], we can obtain the PDFs of $\{\tilde{b}_{i:M_T}^{\text{max}}\}_{i=1}^{M_T}$ as follows:

$$f_{\tilde{b}_{M_T:M_T}^{\text{max}}}(b_{M_T}) = ZM_T\, e^{-b_{M_T}} \left(1 - e^{-b_{M_T}}\right)^{ZM_T - 1} \quad , \tag{13}$$

and

$$\begin{aligned} &f_{\tilde{b}_{i:M_T}^{\text{max}}}(b_i) \\ &= \frac{ZM_T(M_T-1)!}{(i-1)!(M_T-i-1)!} \sum_{a_1=0}^{i-1} \binom{i-1}{a_1} \sum_{a_2=0}^{M_T(Z-1)} \binom{M_T(Z-1)}{a_2} \\ &\quad e^{-(a_1+a_2+1+M_T-i)b_i} \sum_{a_3=0}^{M_T-i-1} \binom{M_T-i-1}{a_3} \frac{(-1)^{a_1+a_2+a_3}}{a_2+a_3+1} \quad . \end{aligned} \tag{14}$$

From (13) and (14), we have all the PDFs of $\tilde{b}_{i:M_T}^{\text{max}}$ for $i = 1, \cdots, M_T$. Applying (13) and (14) to (8) and (7), we can obtain the sum rate and long-term power constraint equation, respectively.

*2) High SNR Region (Max-Min Approach):* Unlike the max-max scheme, the max-min scheduling selects the target user according to the maximal weakest subchannel among virtual $Z$ users. Based on the information of $\{b_{1:M_T}^k\}_{k=1}^Z$, the base station arranges the transmission during each time slot according to

$$k^* = \arg\max_k b_{1:M_T}^k \quad . \tag{15}$$

Once the target user $k^*$ is selected, we have $\tilde{b}_{i:M_T}^{\text{min}} = b_{i:M_T}^{k^*}$ for $i = 1, \ldots, M_T$ where the superscript *min* indicates the max-min scheduling. Similarly, we can get the PDFs of $\{\tilde{b}_{i:M_T}^{\text{min}}\}_{i=1}^{M_T}$ based on the analysis in [38] as follows:

$$f_{\tilde{b}_{1:M_T}^{\text{min}}}(b_1) = ZM_T\, e^{-b_1 M_T} \left(1 - e^{-b_1 M_T}\right)^{Z-1} \quad , \tag{16}$$

and

$$\begin{aligned} &f_{\tilde{b}_{i:M_T}^{\text{min}}}(b_i) \\ &= \frac{ZM_T \prod_{j=2}^{i}(M_T-j+1)}{(i-2)!} \sum_{a_1=0}^{Z-1} \sum_{a_2=2}^{i} (-1)^{i+a_1+a_2} \binom{Z-1}{a_1} \\ &\quad \binom{i-2}{a_2-2} \frac{\left[e^{-b_i(M_T-a_2+1)} - e^{-b_i(M_T+a_1 M_T)}\right]}{(M_T a_1 + a_2 - 1)} \quad . \end{aligned} \tag{17}$$

With all the PDFs of $\{\tilde{b}_{i:M_T}^{\text{min}}\}_{i=1}^{M_T}$, we can obtain the sum rate by substituting (16) and (17) into (8) and the long-term power constraint (7).

### B. Sum Rate Analysis with Randomly Searched Users

To compare with the transmit ZF scheduler with exhaustively searched users, the sum rate of the transmit ZF-based MIMO broadcast system with randomly searched user is also provided. The random user selection is also called the round robin (RR) scheduling policy, which selects users in turn and does not exploit the multiuser diversity gain. The average sum rate of the transmit ZF-based MIMO broadcast system with randomly selected user is given by

$$\begin{aligned} R_{\text{ZFB}} &= \sum_{i=1}^{M_T} E\left[\log(\mu_0 b_i)\right]_+ \\ &= M_T \int_{1/\mu_0}^{\infty} \log(\mu_0 z) e^{-z} dz = M_T \Gamma\left(0, \frac{1}{\mu_0}\right) \quad , \end{aligned} \tag{18}$$

where $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$ is the incomplete gamma function [40] and $\mu_0$ is the water-level solution satisfying the following long-term water-filling equation:

$$\begin{aligned} \sum_{i=1}^{M_T} E\left[\mu - \frac{1}{b_i}\right]_+ &= M_T \int_{1/\mu}^{\infty} \left(\mu - \frac{1}{z}\right) e^{-z} dz \\ &= M_T \left(\mu e^{-1/\mu} - \Gamma\left(0, \frac{1}{\mu}\right)\right) = P_T \quad . \end{aligned} \tag{19}$$

With $\mu_0$ the average sum rate in (18) becomes $M_T \mu_0 e^{-1/\mu_0} - P_T$.

### C. Scheduling for Transmit ZF Beamforming with Multiple Receive Antennas

In Section IV-A and Section IV-B, the sum rate formulas of the transmit ZF-based MIMO broadcast system with exhaustive and random user selection are derived for the case when a user has one single receive antenna. For $M_R > 1$, the analysis in Section IV-A and Section IV-B can also be extended by treating each antenna as a virtual user. That is, there are total $KM_R$ virtual users, each of which has one single receive antenna.

Although the sum rate analysis for the transmit ZF-based MIMO broadcast system with exhaustive user selection has been discussed in Section IV-A, the exhaustive search algorithm with large $K$ may not be practical because of the huge search space $\binom{KM_R}{M_T}$. Low-complexity suboptimal user selection algorithms were proposed for the transmit ZF-based MIMO broadcast system [14]–[16]. In our paper we extend the algorithm of [16] to the case with user terminals having multiple receive antennas. Figure 3 shows the sum rates of the transmit ZF-based MIMO broadcast system using various user selection algorithms, including the exhaustive search and the suboptimal algorithms proposed in [14], [16]. One can see that both suboptimal algorithms approach the same sum rate as the exhaustive user search.

The user scheduling algorithm extended from [16] is briefly explained as follows. First, the scheduler selects a user with

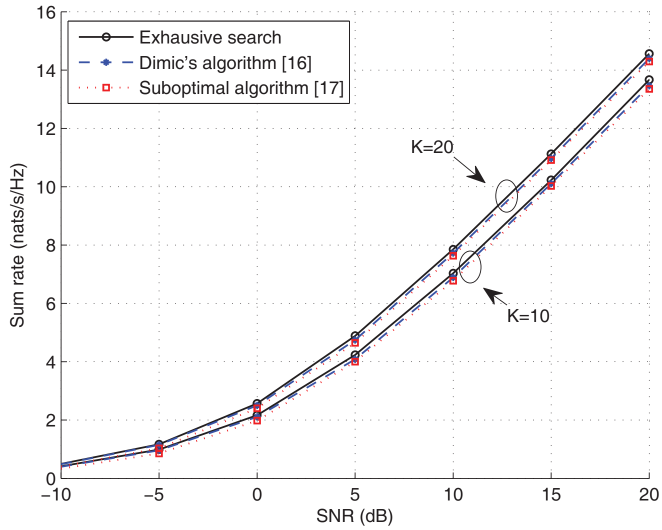



Fig. 3. Sum rate of the transmit ZF beamforming with the exhaustive-search based scheduling and the suboptimal scheduling algorithms when $K = 10, 20$ users, $M_T = 3$ and $M_R = 1$.

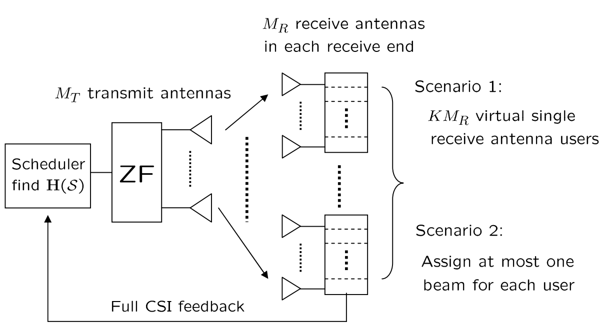



Fig. 4. Diagram of the transmit ZF beamforming with scheduling (or the transmit ZF scheduler) when multiple receive antennas are available.

the best channel quality. Next, the second user is selected so that the signal vectors of the two user are nearly orthogonal. Repeat this procedure until all the $M_T$ selected signal vectors are nearly orthogonal. We consider the following two scenarios for selecting receive antenna.

1) Scenario 1 (*users have multiple data streams*): In this case, we have $KM_R$ virtual users each of which is equipped with one single receive antenna as shown in Fig. 4.
2) Scenario 2 (*users have at most one data stream*): Referring to Fig. 4, this scenario is to prevent unfairness when some users have good channel quality in the short term.

The scheduling algorithm for the first scenario is described as:

- Step 0: Denote the individual channel vector as $\mathbf{h}_i$, where $i \in \mathcal{U} = \{1, 2, \ldots, KM_R\}$.
- Step 1: Initialize $\mathcal{U} = \{1, 2, \ldots, KM_R\}$, $\mathcal{S} = \phi$.
- Step 2: Find link $j$ such that

$$j = \arg\max_{i \in \mathcal{U}} ||\mathbf{h}_i|| \ , \quad \mathbf{H}(\mathcal{S}) = [\mathbf{h}_j] \ . \tag{20}$$

Set $\mathcal{S} = \{j\}$ and $\mathcal{U} = \mathcal{U} - \mathcal{S}$ .

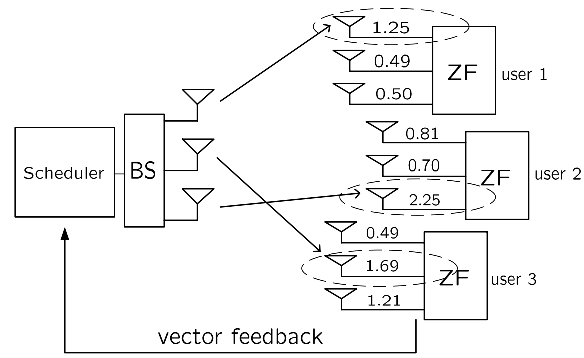



Fig. 5. An example of the receive ZF beamforming with scheduling (or the receive ZF scheduler) for $M_T = M_R = 3$ and $K = 3$.

- Step 3: Find $Null(\mathbf{H}(\mathcal{S})) = \mathbf{V}_2$ by the SVD of $\mathbf{H}(\mathcal{S}) = \mathbf{U\Sigma V}^T$, where $\mathbf{V}_2 = [\mathbf{v}_{r+1} \ \mathbf{v}_{r+2} \ \ldots \ \mathbf{v}_{M_T}]$ and $r = |\mathcal{S}|$.
- Step 4: Find link $j$ such that

$$j = \arg\max_{i \in \mathcal{U}} ||\mathbf{h}_i \cdot Null(\mathbf{H}(\mathcal{S}))|| \ , \quad \mathbf{H}(\mathcal{S}) = \begin{bmatrix} \mathbf{H}(\mathcal{S}) \\ \mathbf{h}_j \end{bmatrix} . \tag{21}$$

Set $\mathcal{S} = \mathcal{S} \bigcup \{j\}$ and $\mathcal{U} = \mathcal{U} - \{j\}$.
- Step 5: Iterate Steps 3 and 4 until $|\mathcal{S}| = M_T$.
- Power Loading Principle: Water-filling.

In scenario 2, the way of updating $\mathcal{U}$ is different. If any link is selected for a user, its other antennas will not be considered.

## V. Scheduling for Receive ZF Beamforming

### A. Scheduling Algorithm

Now we consider the receive ZF scheduler. Each user feedbacks the channel vector $\{\gamma_i^k\}_{i=1}^{M_T}$ to the transmitter for scheduling the target group of users, where $\gamma_i^k$ is the output SNR at the $i$th receive antenna of user $k$ defined in (5) under equal power allocation. Since the ZF receiver can change an $M_R \times M_T$ channel matrix into $M_T$ parallel channels, the scheduler at the transmitter assigns a transmit antenna to serve one of the selected target users. It is unnecessary to assign all the subchannels to a single user [20]–[22]. The scheduler transmits data packets to the target user $k^*$ via the $i$-th transmit antenna according to the criterion:

$$k^* = \arg\max_k \gamma_i^k \ . \tag{22}$$

Since there are $K$ spatially-independent choices for an arbitrary transmit antenna, such broadcast scheduling algorithm is called spatially-independent scheduling in [22] and termed the receive ZF scheduler in this paper. Figure 5 is an example of the receive ZF scheduler for $M_T = M_R = N = 3$ and $K = 3$. The output SNRs of users 1, 2, and 3 are [1.25, 0.49, 0.50], [0.81, 0.70, 2.25], and [0.49, 1.69, 1.21], respectively. The receive ZF scheduler allows each transmit antenna to select its target user according to (22) independently. Among the first antennas, user 1 has the highest SNR (i.e., 1.25). Similarly, the highest SNR of the second antennas among three users is 1.69 belonging to user 3. Also, the third antenna's SNR

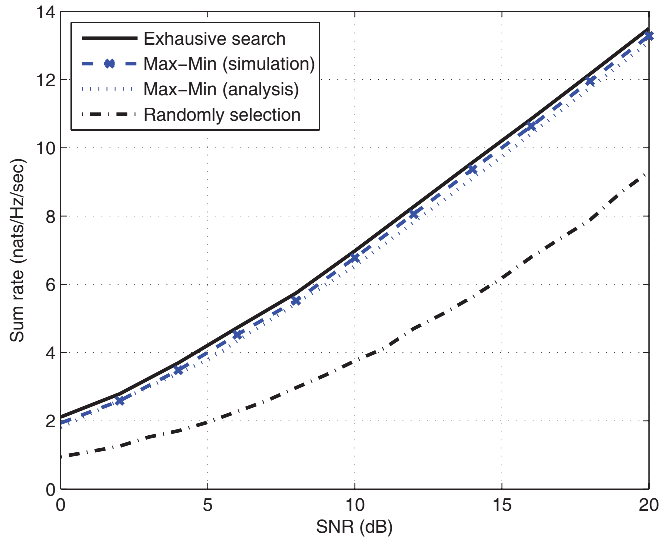



Fig. 6. Comparison of the sum rate of the transmit ZF beamforming with various multiuser scheduling policies for $K = 10$ and $M_T = 3$ in the high SNR region.

of user 2 is the largest (i.e., 2.25). Thus, the data streams of the first, the second, and the third antennas at the base station correspond to the first, the second, and the third antennas of users 1, 3, and 2, respectively.

### B. Sum Rate Analysis

Now the order statistics technique is applied to derive the closed-form expression for the sum-rate capacity of the receive ZF scheduler $R_{\text{ZFR}}$ for $M_T = M_R = N$ under the condition of equal power allocation. Denote $\tilde{\gamma}_i$ as the output SNR of the $i$-th selected subchannel, the PDF of $\tilde{\gamma}_i$ is

$$f_{\tilde{\gamma}_i}(\gamma_i) = \frac{KN}{\rho}\left(1 - e^{\frac{-N\gamma_i}{\rho}}\right)^{K-1} e^{\frac{-N\gamma_i}{\rho}}, \quad \text{for } i = 1, \ldots, N \ , \tag{23}$$

where $\rho = P_T/\sigma^2$. After independently scheduling across all transmit antennas in the space domain, $\{\tilde{\gamma}_i\}_{i=1}^N$ are i.i.d. with respect to $i$. The resulting sum rate is

$$R_{\text{ZFR}} = \frac{KN^2}{\rho}\sum_{i=0}^{K-1}\binom{K-1}{i}(-1)^i h\left(\frac{(i+1)N}{\rho}\right) \ , \tag{24}$$

where

$$h(x) \triangleq \int_0^\infty e^{-xt}\log(1+t)dt = \frac{e^x E_1(x)}{x} \ , \tag{25}$$

and $E_1(x) = \int_1^\infty e^{-xt}t^{-1}dt$ is the exponential integer function of the first order [40].

Since $\{\tilde{\gamma}_i\}_{i=1}^N$ are known at the base station through the feedback channel vector, all effective channel gains $\{\tilde{d}_i\}_{i=1}^N$ (as defined in Section III-B) of selected subchannels are available in the base station. Thus, the sum rate of the receive ZF scheduler can be further improved by using water-filling power allocation. From (5) and (23), the PDF of $\tilde{d}_i$ is $f_{\tilde{d}_i}(d_i) = K\left(1 - e^{-d_i}\right)^{K-1} e^{-d_i}$ and the long-term power constraint in (7) becomes

$$\begin{aligned}
&\sum_{i=1}^N E\left[\mu - \frac{1}{\tilde{d}_i}\right]_+ \\
&= KN\int_{1/\mu}^\infty \left(\mu - \frac{1}{z}\right)\left(1 - e^{-z}\right)^{K-1} e^{-z}dz \\
&\overset{(a)}{=} KN\sum_{i=0}^{K-1}\binom{K-1}{i}(-1)^i\int_{1/\mu}^\infty\left(\mu - \frac{1}{z}\right)e^{-z(i+1)}dz \\
&= KN\sum_{i=0}^{K-1}\binom{K-1}{i}(-1)^i\left(\frac{\mu}{1+i}e^{-(1+i)/\mu} - E_1\left(\frac{1+i}{\mu}\right)\right) \\
&= P_T \ ,
\end{aligned} \tag{26}$$

where (a) comes from the binomial expansion $(1-x)^n = \sum_{i=0}^n \binom{n}{i}(-1)^i x^i$. The resulting average sum rate of the receive ZF scheduler with water-filling power allocation is given by

$$\begin{aligned}
R_{\text{ZFR}}^{\text{water}} &= \sum_{i=1}^N E\left[\log(\tilde{\mu}_0\tilde{d}_i)\right]_+ \\
&= KN\int_{1/\tilde{\mu}_0}^\infty \log(\tilde{\mu}_0 z)\left(1 - e^{-z}\right)^{K-1} e^{-z}dz \\
&= KN\sum_{i=0}^{K-1}\binom{K-1}{i}(-1)^i\int_{1/\tilde{\mu}_0}^\infty \log(\tilde{\mu}_0 z)e^{-z(1+i)}dz \\
&= KN\sum_{i=0}^{K-1}\binom{K-1}{i}(-1)^i\frac{\Gamma\left(0, 1 + i/\tilde{\mu}_0\right)}{1+i} \ ,
\end{aligned} \tag{27}$$

where $\tilde{\mu}_0$ is the water-level solution which satisfies (26).

## VI. Numerical Results

### A. Sum Rate Approximation for Transmit ZF Scheduler with Exhaustive Searched Users

We first show the sum-rate capacity of the transmit ZF beamforming with random user selection and exhaustive search scheduling for various received SNRs when each user has only single antenna. For SNRs $= 0 \sim 20$ dB and $K = 10$ users, Fig. 6 compares the sum rate of the transmit ZF scheduler based on the exhaustive user selection with that based on the max-min analytical approach. As shown, the sum rate of the max-min analytical approach matches that of the exhaustive user selection quite well. At SNR = 5 dB, the sum rates of the exhaustive search and random user selection are 4 nats/s/Hz and 2 nats/s/Hz, respectively.

Figure 7 shows the sum rate of the transmit ZF scheduler in the low SNRs region from $-20$ to 0 dB and $K = 10$ users based on the exhaustive search and the max-max analytical approaches. As shown in the figure, the sum rate of the max-max can match that of the exhaustive search in the low SNR region. For comparison, the sum rate with random user selection is also shown in the figure. At SNR $= -5$ dB, the exhaustive user search can provide the sum rate of 1 nats/s/Hz, while the random user selection can provide only 0.4 nats/s/Hz. Basically, Figs. 6 and 7 show that even for $K = 10$ the impact of multiuser diversity is quite significant.

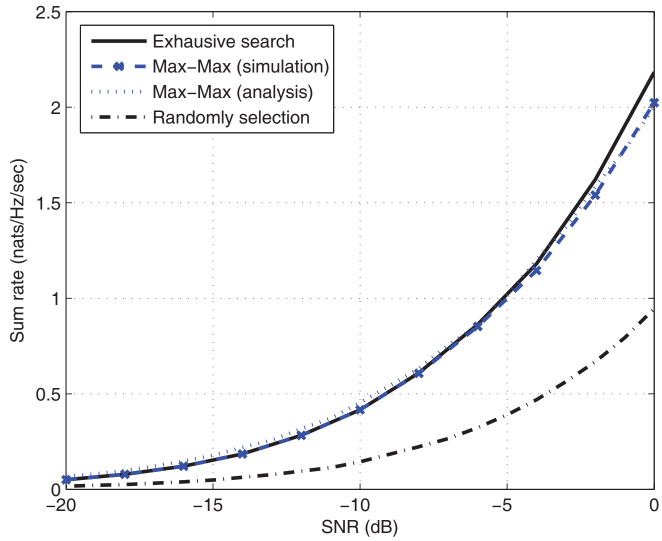



Fig. 7. Comparison of the sum rate of the transmit ZF beamforming with various multiuser scheduling policies for $K = 10$ and $M_T = 3$ in the low SNR region.

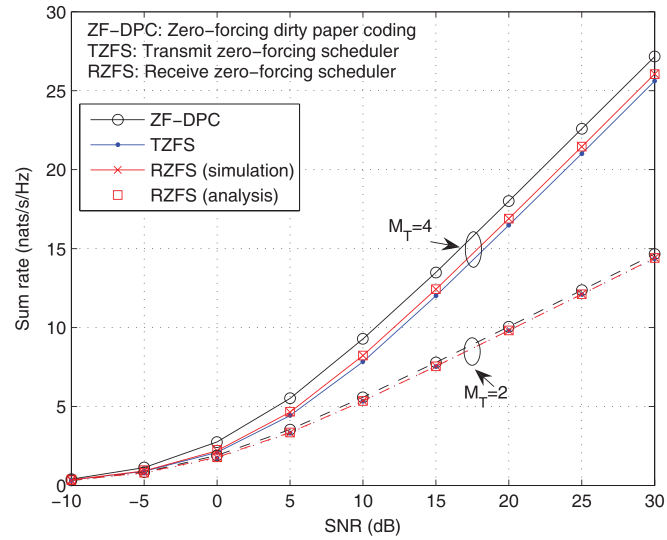



Fig. 8. Comparison of the sum rate for the receive ZF scheduler ($M_T = M_R = 4$ and $M_T = M_R = 2$) and the transmit ZF and ZF-DPC schedulers ($M_T = 4$, $M_R = 1$ and $M_T = 2$, $M_R = 1$) with $K = 10$.

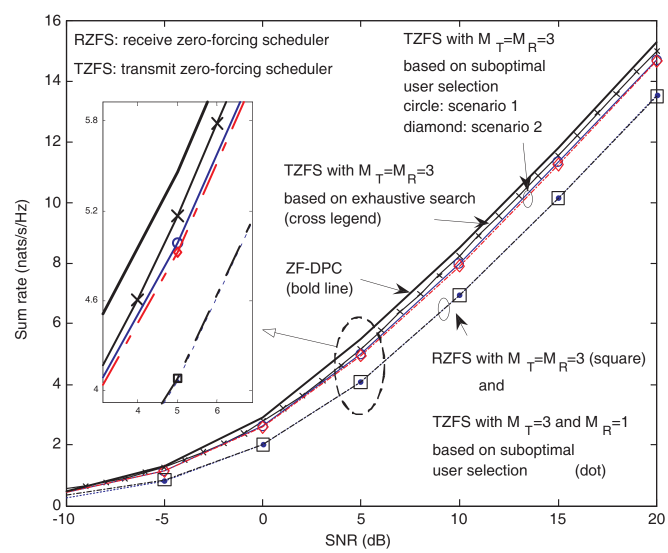



Fig. 9. Effects of the number of receive antennas on the sum rate of the transmit ZF scheduler for $K = 10$ and $M_T = 3$.

### *B. Sum Rate Comparison of Transmit and Receive ZF Schedulers*

Next, we compare the sum-rate capacity of the receive ZF scheduler with that of the transmit ZF scheduler for various SNRs. We consider two conditions: (1) under the similar available feedback but different antenna architectures; and (2) under the same antenna architecture but different feedback requirements. For comparison, we also provide the sum rate performance of ZF-DPC transmit beamforming [14] combined with the greedy multiuser scheduling algorithm [13].

In Fig. 8 the sum rate performance of the receive ZF scheduler is compared with both the transmit beamforming approaches (ZF-DPC and ZF) for different values of $M_T$ and $M_R$ as $K = 10$. We mainly focus on the transmit beamformings with $M_R = 1$ which has similar feedback overheads ($M_T$ complex values) compared to the receive ZF scheduler ($M_T$ scalar values). From the figure, we have the following observations:

- For the receive ZF scheduler, the provided sum rate analysis (27) (with the legends of square) matches the simulated results (with the legends of cross) quite well.
- With similar amount of feedback, transmit ZF-DPC has the highest sum rate and the transmit and receive ZF schedulers result in almost the same sum rate. However, as the number of transmit antennas decreases, the sum rate of transmit and receive beamformings are very close to each other.
- With similar available feedback, the transmit and receive ZF schedulers result in almost the same sum rate. The selecting degrees of freedom per link is $K$ for the receive ZF scheduler, and is $K-i+1$ for determining the $i$th link of the transmit ZF scheduler. Two schedulers have almost the same selecting degrees of freedom when $K \gg M_T$. With similar feedback overheads and selecting degrees of freedom, one can expect that two schedulers yield the similar sum rate.

Figure 9 compares the sum rate of the transmit ZF scheduler when user terminals are equipped with single and multiple receive antennas for $K = 10$ and $M_T = 3$. For multiple receive antennas $M_R = 3$, antenna selection scenarios 1 and 2 yield similar sum rates by the transmit ZF scheduler. For comparison, the sum rates of the transmit ZF-DPC, the transmit ZF beamforming based on the exhaustive user search and the receive ZF scheduler are plotted. From the figure, some observations can be made as follows:

- The suboptimal user selection methods can approach the performance of the complicated exhaustive search method for the transmit ZF scheduler.
- The transmit ZF scheduler with multiple receive antennas improves the sum rate by 20% over the transmit ZF scheduler with single antenna for SNR $= 0 \sim 5$ dB. The performance gain results from a higher multiuser

diversity gain because more receive antennas imply more virtual users. From (9), the extra channel gain induced by larger diversity is $\log M_R$.

## VII. Performance Issues

### A. Feedback Requirement

The amount of required feedback information is an important parameter for designing the transmit and receive multiuser MIMO broadcast systems. The receive ZF scheduler requires $M_T$ real scalar values, $\{\gamma_i^k\}_{i=1}^{M_T}$, in the feedback channel for each user $k \in \{1, \ldots, K\}$. For example user 1 in Fig. 5 feedbacks a channel state vector $[1.25, 0.49, 0.50]$. In general, complete channel matrix information is required for the transmit ZF scheduler to calculate antenna beamforming weights. For $M_R$ receive antennas, user $k$ needs to feedback $M_T M_R$ complex-valued entries of $\mathbf{H}_k$. Even if $M_R = 1$, the transmit ZF scheduler still needs to feedback $M_T$ complex-valued entries. Codebook-based feedback is another efficient channel quality feedback apparatus, where feedback information includes the channel direction information (CDI) and channel quality information (CQI) [24], [25], [41], [42]. The transmit ZF scheduler needs both CQI and CDI, but the receive ZF scheduler requires CQI only. Nevertheless, certain indices are required to informing each antenna of its corresponding CQI in the receive ZF scheduler. Providing that each user is equipped with multiple antennas, the transmit ZF scheduler also have the same overhead for sending the indices of each antenna's corresponding CQI as the receive ZF scheduler.

### B. Effects of Feedback CSI Variations

Now we evaluate the impacts of feedback CSI variations on the performance of the transmit and receive beamforming in MIMO broadcast systems. Feedback CSI variations are introduced to characterize the difference of the exact CSI and the measured CSI at a feedback channel. Feedback CSI variations may result from the noisy estimated information, the outdated information due to feedback delay, and quantization errors, etc. In this paper, we quantify feedback CSI variations by the coefficient of variations (CV), which is defined as the ratio of the standard deviation $\sigma_X$ of a random variable $X$ to its mean $E[X]$, i.e., CV $= \sigma_X/E[X]$. The case with CV $= 0$ is equivalent to perfect feedback CSI. The larger the CV value, the more the perturbation on the feedback information. In this paper, we model the combined effects of feedback CSI variations as a normal random variable $\Delta \sim \mathcal{N}(0, \sigma^2)$. For the perfect CSI $(x)$, the perturbed CSI $(x')$ received at the base station is written as $x' = x + \Delta$, in which CV $= \sigma/x$.

Figure 10 compares the sum rate performance of three transmit beamforming techniques (denoted by ZF-DPC, BD, TZFS) and the ZF-based receive beamforming (denoted by RZFS), in which for CV $= 0.5$, $M_T = M_R = 3$ and $K = 20$. The three considered transmit beamforming techniques ZF-DPC, the BD-based precoder, and the transmit ZF scheduler respectively adopt the greedy scheduling algorithm [13], the norm-based user/antenna selection algorithm [32], and the scheduling algorithm in Section IV-C. In [43], we only considered the ZF-based receive and transmit beamforming multiuser MIMO broadcast systems. In this figure, we have the following observations:

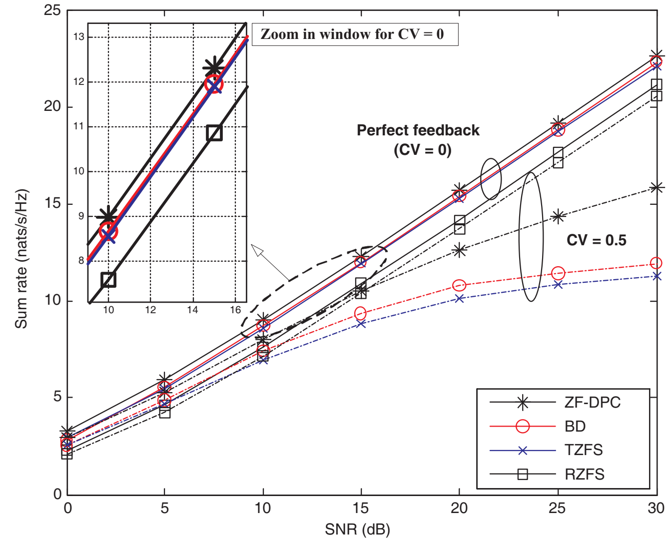



Fig. 10. Sensitivity to feedback channel variations on the sum rate of the transmit-type MIMO broadcast schedulers and receive ZF scheduler for $K = 20$, $M_T = 3$, and low variation (CV = 0.5).

- For CV $= 0$, the ZF-DPC, BD, and ZF based transmit beamformings result in a similar sum rate, and their sum rates are about 10 % (i.e., $1 \sim 1.5$ nats/s/Hz) higher than that of the receive ZF scheduler. For SNR = 20 dB, the sum rates for the ZF-DPC, BD, TZFS and RZFS are 15.73, 15.43, 15.28, and 14.21 nats/s/Hz, respectively. Among the three considered transmit beamforming techniques, ZF-DPC has the highest sum rate, and the BD-based transmit beamforming is only slightly better than the transmit ZF scheduler. The superiority of the BD-based transmit beamforming over the transmit ZF scheduler results from the joint processing of transmit precoding and receive equalizer.
- As the feedback CSI variation increases, the performance advantages of the transmit beamforming over the receive beamforming disappear. When CV increases from 0 to 0.5 at SNR $= 20$ dB, the sum rates of ZF-DPC, BD, transmit ZF scheduler decrease 19.7% to 12.63 nats/s/Hz, 30.4% to 10.73 nats/s/Hz, and 33.8% to 10.11 nats/s/Hz, respectively. By contrast, the sum rate of the receive ZF scheduler is only reduced 3.5% to 13.71 nats/s/Hz.

Figure 11 shows that the increase of the feedback CSI variation up to CV = 1.5 will degrade the sum rates of the three considered transmit beamforming techniques seriously, i.e. 38.8%, 56.9%, 61.1% for ZF-DPC, BD, and TZFS, respectively. However, the sum rate of the receive ZF beamforming only decreases 7%. From Figs. 10 and 11, one can observe that transmit beamforming is more sensitive to feedback CSI variations than the receive ZF beamforming.

### C. Usage of Feedback Channel State Information

Noteworthily, the feedback information in transmit beamforming is applied to *calculate the beamforming weight* for **interference cancellation**, whereas the feedback information

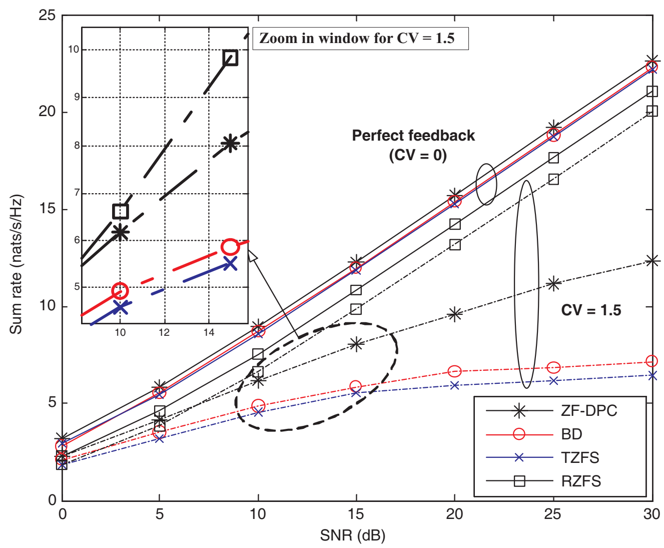



Fig. 11. Sensitivity to feedback channel variations on the sum rate of the transmit-type MIMO broadcast schedulers and receive ZF scheduler for $K = 20$, $M_T = 3$, and high variation (CV = 1.5).

in receive beamforming is to *select the appropriate users*. In this part, we further discuss why different usage of feedback CSI affects the sensitivity of MIMO broadcast systems with respect to feedback channel variations. Consider the ZF-DPC and ZF-based transmit beamforming techniques. The $i$-th received signal in (1) can be written as

$$y_i = \mathbf{h}(\mathcal{S})_i \mathbf{w}_i x_i + \underbrace{\mathbf{h}(\mathcal{S})_i \sum_{j=1, j\neq i}^{M_T} \mathbf{w}_j x_j}_{\text{Interference}} + n_i \ , \quad (28)$$

where $\mathbf{h}(\mathcal{S})_i$ is the channel vector of the $i$-th receiver and $\mathbf{w}_i$ is the $i$-th column of the beamforming matrix $\mathbf{W}$, respectively. To cancel the inter-stream interference, i.e., making $\mathbf{h}(\mathcal{S})_i \mathbf{w}_j = 0$ $(\forall j \neq i)$ in (28), the complete channel state matrix is required. More importantly, it is implied that large feedback channel variations make it difficult to cancel inter-stream interference and thus significantly affect the performance of the ZF-DPC and ZF-based transmit beamforming techniques.

For the BD-based transmit beamforming, the received signal of user $k$ with the BD precoder is represented as

$$\mathbf{y}_k = \mathbf{R}_k^T \mathbf{H}_k \mathbf{T}_k \mathbf{x}_k + \underbrace{\mathbf{R}_k^T \mathbf{H}_k \sum_{j=1, j\neq k}^{K} \mathbf{T}_j \mathbf{x}_j}_{\text{Interference}} + \mathbf{R}_k^T \mathbf{n}_k \ , \quad (29)$$

where $\mathbf{T}_k \in \mathcal{C}^{M_T \times L_k}$ and $\mathbf{R}_k \in \mathcal{C}^{M_R \times L_k}$ are the precoding matrix and the equalizer matrix, respectively, and $\mathbf{x}_k$ is the $L_k \times 1$ transmit vector for user $k$. The principle of the BD-based transmit beamforming is to design $\mathbf{T}_k \in \mathbf{U}^{M_T \times L_k}$ and $\mathbf{R}_k \in \mathbf{U}^{M_R \times L_k}$ so that

$$\mathbf{R}_k^T \mathbf{H}_k \mathbf{T}_j = 0 \qquad \forall 1 \leq k \neq j \leq K \ , \quad (30)$$

where $\mathbf{U}$ represents an unitary matrix. With perfect feedback information, the interference term in (29) can be cancelled as long as the conditions (30) are satisfied. Similar to the ZF-DPC and the ZF-based transmit beamforming techniques, the sum rate of the BD-based transmit beamforming is quite sensitive to the feedback channel errors due to the requirement of an accurate channel state matrix for inter-stream interference cancelation.

Now we consider the ZF-based receive beamforming. Assume that the received feedback information from $K$ users at the $i$-th transmit antenna be $\{\hat{\gamma}_i^k\}_{k=1}^K$ in which $\hat{\gamma}_i^k = \gamma_i^k + \varepsilon_i^k$, where $\gamma_i^k$ is the exact SNR value and $\varepsilon_i^k$ is the distortion term. The scheduler at the base station selects the target user for the $i$-th transmit antenna according to

$$\hat{k}^* = \arg\max_k \hat{\gamma}_i^k \ . \quad (31)$$

Compared to the correct decision in (22), the effect of feedback CSI variations may lead to $\hat{k}^* \neq k^*$, i.e., the best user may not be chosen. Unlike the transmit beamforming is affected by inter-stream interference, the ZF-based receive beamforming will not suffer from inter-stream interference. Although the best user is not chosen due to feedback CSI errors, the selected user (e.g., the second best user) can still receive reasonably good sum rate. Therefore, the sum rate scaling of the receive ZF scheduler can be maintained much better than the transmit beamforming techniques for MIMO broadcast system under the impact of feedback CSI variations.

### D. Complexity

Here the complexity of the link selection procedures and the search space for the transmit ZF scheduler is compared with those of the receive ZF scheduler. For the suboptimal selection algorithm of the transmit ZF beamforming, one 2-norm vector calculation $||\mathbf{h}_i||$ per $i \in \mathcal{U}$ in Step 2 is required. Next, we search the link with the maximum norm in $\mathcal{U}$ of which the complexity is proportional to $|\mathcal{U}|$. This link selection procedure will be finished only in the first run and determine the first link. Note that $\mathcal{U}$ will be updated when finishing one link search and the following Steps 3 and 4 will be iterated $M_T - 1$ times to determine the remaining $M_T - 1$ links. In Step 3, the orthonormal basis $Null(\mathbf{H}(\mathcal{S}))$ is calculated, which requires SVD computation. In Step 4, we need to perform a vector-matrix multiplication for a $1 \times M_T$ vector and an $M_T \times (M_T - n)$ matrix and a 2-norm calculation per $i \in \mathcal{U}$ to search for the link with the maximum norm $||\mathbf{h}_i \cdot Null(\mathbf{H}(\mathcal{S}))||$, where $n$ represents the number of iterations for $n = 1, \ldots, M_T - 1$. Therefore, the computational complexity per $i \in \mathcal{U}$ in Step 4 is a constant $c_1$ which is related to one vector-matrix multiplication and 2-norm calculation. Further, the size of search space for scenario 1 is equal to $\sum_{n=0}^{M_T-1} \binom{M_R K - n}{1}$. That is, the first link is chosen from $\binom{M_R K}{1}$, the second from $\binom{M_R K - 1}{1}$, and so on. Similarly, the size of search space for scenario 2 is equal to $\sum_{n=0}^{M_T-1} \binom{M_R K - n M_R}{1}$. To sum up, the complexity of the link selection procedures for scenario 1 is approximately $c_1 \sum_{n=1}^{M_T-1} \binom{M_R K - n}{1} + c_2(M_T - 1)$, where $c_2$ is a proportional constant related to SVD procedures in the $n$-th iteration and the computation of searching the maximum norm is omitted.

The computing complexity for the receive ZF scheduler lies in selecting a certain link with a highest SNR from $K$

TABLE I
PERFORMANCE COMPARISON AND TRADEOFF BETWEEN THE TRANSMIT AND RECEIVE ZF SCHEDULERS.

| | Transmit ZF scheduler (TZFS) | Receive ZF scheduler (RZFS) |
|---|---|---|
| Sum Rate | Achieve the same scaling law with DPC [15].<br>Better sum rate performance than RZFS. | Achieve the same scaling law with DPC [21].<br>Slight poorer sum rate performance than TZFS. |
| Feedback Requirement | $M_R \times M_T$ complex-valued entries.<br>Use for interference cancellation.<br>(matrix feedback; need CQI and CDI feedback) | $M_T$ real-valued scalar values.<br>Use for user selection.<br>(vector feedback; CQI feedback) |
| Effect of Feedback Error on sum rate | Sensitive to feedback channel variation.<br>Degradation largely especially in high SNR region.<br>Interference domination. | Robust to feedback channel variation.<br>Degradation slightly compared to TZFS.<br>Maintain sum rate scaling. |
| Search Space | $\sum_{n=0}^{M_T-1}\binom{M_R K-n}{1}$<br>More computational complexity at base station. | $\sum_{n=0}^{M_T-1}\binom{K-n}{1}$<br>Extra ZF implementation at each user side. |
| Note | Suitable for huge feedback to support higher sum rate. | As a candidate for scarce feedback environment. |

users. Specifically, the size of search space for the receive ZF scheduler is $\sum_{n=1}^{M_T}\binom{K}{1}$, which is close to search size of the transmit ZF scheduler with $M_R = 1$ for large $K$. However, the receive ZF scheduler induces the extra cost of implementing ZF algorithm at the user terminals. To sum up, since $\sum_{n=1}^{M_T}\binom{K}{1} < \sum_{n=0}^{M_T-1}\binom{M_R K-n}{1}$ for scenario 1 and $\sum_{n=1}^{M_T}\binom{K}{1} < \sum_{n=0}^{M_T-1}\binom{M_R K-nM_R}{1}$ for scenario 2, one can conclude that the size of search space of the receive ZF scheduler is much smaller than that of the transmit ZF scheduler.

### *E. Fairness Issue*

To achieve the fairness among users, one should maintain the same probability of obtaining services at each time slot. Consider the transmit ZF scheduler in scenario 2 as an example. To assign $M_T$ data streams, there are $(M_R K)(M_R K - M_R)\cdots(M_R K - M_R(M_T-1))$ possible outcomes. When all $M_T$ transmit antennas are assigned to other $(K-1)$ users, user $k$ is not served. Therefore, the probability $p_k$ of user $k$ to obtain services at each time slot is

$$\begin{aligned} p_k &= 1 - \Pr\{\text{user } k \text{ is not selected}\} \\ &= 1 - \frac{(M_R K - M_R)(M_R K - 2M_R)\cdots(M_R K - M_R M_T)}{(M_R K)(M_R K - M_R)\cdots(M_R K - M_R(M_T-1))} \\ &= \frac{M_T}{K}\ . \end{aligned} \tag{32}$$

As for the receive ZF scheduler, total $K^{M_T}$ possible outcomes for the $M_T$ transmit antennas assigning to $K$ users. The probability of user $k$ to obtain services at each time slot is $\left[1-(1-\frac{1}{K})^{M_T}\right] \approx \frac{M_T}{K}$ for $K \gg M_T$. Therefore, both the transmit and receive ZF scheduler can achieve the same fairness performance in terms of the probability of users to obtain services during a scheduling time slot.

In summary, simultaneously "broadcast" data to multiple users can be implemented by the transmit beamforming and receive beamforming. While there are enough feedback bandwidth and the accuracy of the feedback channel state can be ensured, transmit beamforming is a good multi-user MIMO broadcast strategy and have better sum rate performance than the ZF-based receive beamforming. However, because imperfect feedback may cause significant inter-stream interference, the ZF-DPC, BD, and ZF-based transmit beamformings may not be feasible in the presence of large feedback CSI variations, which will cause the loss of opportunity to exploit the potential of having multiple receive antennas at the user terminal. By contrast, in the case that feedback channel bandwidth is constrained and feedback channel states have certain uncertainty, then the ZF-based receive beamforming can be a good candidate for multi-user MIMO broadcast strategy at the cost of ZF implementation at the user side.

## VIII. CONCLUSION

In this paper, we compared the sum rate performance and feedback requirements when implementing the ZF technique at the transmitter end and receiver end for multiuser MIMO broadcast systems. Our contributions can be summarized as follows:

1) Analyze the sum-rate performance of the transmit ZF MIMO broadcast system for the finite users case, instead of the scaling law of the sum rate of MIMO broadcast systems for very large number of users.
2) Derive the closed-form expressions for the sum rate of the receive ZF scheduler with water-filling power allocation among transmit antennas.
3) Provide the performance tradeoff analysis between the transmit and receive ZF schedulers. As shown in Table I, both the transmit and receiver ZF schedulers can have the same scaling law as DPC for an extremely large number of users. However, the transmit ZF scheduler needs perfect CSI and is sensitive to imperfect CSI feedback. The receive ZF scheduler can relax the feedback requirement and is robust to feedback channel variations.

Implementing more antennas at user handsets become more feasible thanks to the advanced antenna technologies. For designing MIMO broadcast systems, the main superiority of transmit ZF scheduler is the better sum-rate performance with simpler implementation at user terminals, but transmit ZF scheduler is more sensitive to feedback channel variations. By contrast, the receive ZF scheduler is more robust to feedback channel variations with the cost of implementing ZF algorithm at user terminals. Hence, when feedback bandwidth is limited, it is more reasonable to adopt the receive ZF scheduler. When feedback bandwidth is sufficient, the transmit

ZF scheduler can be used. Recently, some codebook-based multi-user MIMO systems are also widely discussed, which allow simultaneous multiuser transmission based on a limited feedback mechanism. Thus, one of interesting future research topics extended from this work is to compare performance tradeoff of various codebook-based multiuser MIMO broadcast systems subject to feedback channel variations.

## REFERENCES


[1] M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. 29, no. 3, pp. 439–441, May 1983.

[2] G. Caire and S. Shamai, "On the achievable throughput of a multi-antenna Gaussian broadcast channel," *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1691–1706, July 2003.

[3] N. Jindal and A. Goldsmith, "Dirty-paper coding versus TDMA for MIMO broadcast channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 5, pp. 1783–1794, May 2005.

[4] M. Sharif and B. Hassibi, "A comparison of time-sharing, DPC, and beamforming for MIMO broadcast channels with many users," *IEEE Trans. Commun.*, vol. 55, no. 1, pp. 11–15, Jan. 2007.

[5] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2658–2668, Oct. 2003.

[6] P. Viswanath and D. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality," *IEEE Trans. Inf. Theory*, vol. 49, no. 8, pp. 1912–1921, Aug. 2003.

[7] H. Weingarten, Y. Steinberg, and S. Shamai (Shitz), "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," *IEEE Trans. Inf. Theory*, vol. 52, no. 9, pp. 3936–3964, Sep. 2006.

[8] H. Viswanathan, S. Venkatesan, and H. Huang, "Downlink capacity evaluation of cellular networks with known-interference cancellation," *IEEE Trans. Inf. Theory*, vol. 21, no. 5, pp. 802–811, June 2003.

[9] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channels with partial side information," *IEEE Trans. Inf. Theory*, vol. 51, no. 2, pp. 506–522, Feb. 2005.

[10] M. Kountouris, D. Gesbert, and T. Sälzer, "Enhanced multiuser random beamforming dealing with the not so large number of users case," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1536–1545, Oct. 2008.

[11] M. Kountouris, R. de Francisco, D. Gesbert, D. T. Slock, and T. Sälzer, "Low complexity scheduling and beamforming for multiuser MIMO systems," in *Proc. IEEE Signal Processing Advances in Wireless Commun.*, pp. 1–5, July 2006.

[12] R. de Francisco, M. Kountouris, D. T. Slock, and D. Gesbert, "Orthogonal linear beamforming in MIMO broadcast channels," in *Proc. IEEE Wireless Commun. and Networking Conf.*, pp. 1210–1215, Mar. 2007.

[13] Z. Tu and R. S. Blum, "Multiuser diversity for a dirty paper approach," *IEEE Commun. Lett.*, vol. 7, no. 8, pp. 370–372, Aug. 2003.

[14] G. Dimić and N. D. Sidiropoulos, "On downlink beamforming with greedy user selection: performance analysis and a simple new algorithm," *IEEE Trans. Signal Process.*, vol. 53, no. 10, pp. 3857–3868, Oct. 2005.

[15] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 528–541, Mar. 2006.

[16] J. Kim, S. Park, J. H. Lee, J. Lee, and H. Jung, "A scheduling algorithm combined with zero-forcing beamforming for a multiuser MIMO wireless system," in *Proc. IEEE Vehicular Technology Conf.*, vol. 1, pp. 211–215, Sep. 2005.

[17] A. Bayesteh and A. K. Khandani, "On the user selection for MIMO broadcast channels," *IEEE Trans. Inf. Theory*, vol. 54, no. 3, pp. 1086–1107, Mar. 2008.

[18] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277–1294, June 2002.

[19] N. Sharma and L. H. Ozarow, "A study of opportunism for multiple-antenna systems," *IEEE Trans. Inf. Theory*, vol. 51, no. 5, pp. 1804–1814, May 2005.

[20] R. W. Heath, Jr., M. Airy, and A. J. Paulraj, "Multiuser diversity for MIMO wireless systems with linear receivers," in *Proc. Asilomar Conf. Signals, Systems and Computers*, pp. 1194–1199, Nov. 2001.

[21] M. Airy, R. W. Heath, Jr., and S. Shakkottai, "Multi-user diversity for the multiple antenna broadcast channel with linear receivers: asymptotic analysis," in *Proc. Asilomar Conf. Signals, Systems, and Computers*, pp. 886–890, Nov. 2004.

[22] C. J. Chen and L. C. Wang, "Performance analysis of scheduling in multiuser MIMO system with zero-forcing receivers," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1435–1445, Sep. 2007.

[23] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multi-user MIMO channels," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 461–471, Feb. 2004.

[24] C. Swannack, G. W. Wornell, and E. Uysal-Biyikoglu, "MIMO broadcast scheduling with quantized channel state information," in *Proc. IEEE International Symposium on Information Theory*, pp. 1788–1792, July 2006.

[25] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-antenna downlink channels with limited feedback and user selection," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1478–1491, Sep. 2007.

[26] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran. "Multiuser MIMO achievable rates with downlink training and channel state feedback". [Online]. Available: http://arxiv.org/abs/0711.2642

[27] J. Zhang, R. W. Heath, Jr., M. Kountouris, and J. G. Andrews, "Mode switching for the multi-antenna broadcast channel based on delay and channel quantization," *EURASIP J. Advances Signal Process.*, vol. 2009, article ID 802548.

[28] K. Huang, R. W. Heath, Jr., and J. G. Andrews, "Uplink SDMA with limited feedback: throughput scaling," *EURASIP J. Advances Signal Process.*, vol. 2008, article ID 479357.

[29] L. C. Wang and C. J. Yeh, "Comparison of transmit and receive zero-forcing schedulers for MIMO broadcast channels," in *Proc. Wireless Personal Multimedia Commun.*, pp. 835–839, Sep. 2006.

[30] Z. Shen, R. Chen, J. G. Andrews, R. W. Heath, Jr., and B. L. Evans, "Sum capacity of multiuser MIMO broadcast channels with block diagonalization," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, pp. 2040–2045, June 2007.

[31] ——, "Low complexity user selection algorithms for multiuser MIMO systems with block diagonalization," *IEEE Trans. Signal Process.*, vol. 54, no. 9, pp. 3658–3663, Sep. 2006.

[32] R. Chen, Z. Shen, J. G. Andrews, and R. W. Heath, Jr., "Multimode transmission for multiuser MIMO systems with block diagonalization," *IEEE Trans. Signal Process.*, vol. 56, no. 7, pp. 3294–3302, July 2008.

[33] M. Fuchs, G. D. Galdo, and M. Haardt, "Low complexity space-time-frequency scheduling for MIMO systems with SDMA," *IEEE Trans. Veh. Technol.*, vol. 56, no. 5, pp. 2775–2784, Sep. 2007.

[34] S. Sigdel and W. A. Krzymień, "Simplified fair scheduling and antenna selection algorithms for multiuser MIMO orthogonal space-division multiplexing downlink," *IEEE Trans. Veh. Technol.*, vol. 58, no. 3, pp. 1329–1344, Mar. 2009.

[35] A. Paulraj, R. Nabar, and D. Gore., *Introduction to Space-Time Wireless Communications*. Cambridge University Press, 2003.

[36] D. A. Gore, R. W. Heath, Jr., and A. J. Paulraj, "Transmit selection in spatial multiplexing systems," *IEEE Commun. Lett.*, vol. 6, no. 11, pp. 491–493, Nov. 2002.

[37] A. Vakili, A. Dana, M. Sharif, and B. Hassibi, "Differentiated rate scheduling for MIMO broadcast channels," in *Proc. Allerton Conf. on Commun., Control, and Computation*, Sep. 2005.

[38] C. J. Chen and L. C. Wang, "Enhancing coverage and capacity for multiuser MIMO systems by utilizing scheduling," *IEEE Trans. Wireless Commun.*, vol. 5, no. 5, pp. 1148–1157, May 2006.

[39] L. C. Wang and C. J. Yeh, "Comparison of scalar feedback mechanisms in MIMO scheduling systems," in *Proc. Wireless Personal Multimedia Commun.*, pp. 1172–1176, Sep. 2005.

[40] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th edition. New York: Dover, 1970.

[41] P. Ding, D. Love, and M. Zoltowski, "Multiple antenna broadcast channels with shape feedback and limited feedback," *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3417–3428, July 2007.

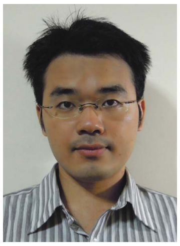
[42] K. Huang, J. G. Andrews, and R. W. Heath, Jr., "Performance of orthogonal beamforming with SDMA with limited feedback," *IEEE Trans. Veh. Technol.*, vol. 58, no. 1, pp. 152–164, Jan. 2009.

[43] L. C. Wang and C. J. Yeh, "Multi-user MIMO broadcast systems with imperfect feedbacks," in *Proc. International Wireless Commun. and Mobile Computing Conf.*, pp. 500–504, June 2009.

**Li-Chun Wang** received the B.S. degree from National Chiao Tung University , Taiwan, R.O.C. in 1986, the M.S. degree from National Taiwan University in 1988, and the Ms.Sci. and Ph.D. degrees from the Georgia Institute of Technology, Atlanta, in 1995, and 1996, respectively, all in electrical engineering.

From 1990 to 1992, he was with the Telecommunications Laboratories of the Ministry of Transportations and Communications in Taiwan (currently the Telecom Labs of Chunghwa Telecom Co.). In 1995, he was affiliated with Bell Northern Research of Northern Telecom, Inc., Richardson, TX. From 1996 to 2000, he was with AT&T Laboratories, where he was a Senior Technical Staff Member in the Wireless Communications Research Department. Since August 2000, he has joined the Department of Communications Engineering of National Chiao Tung University in Taiwan and became professor in 2005. His current research interests are in the areas of adaptive/cognitive wireless networks, radio network resource management, cross-layer optimization, and cooperative wireless communications networks.

Dr. Wang was a co-recipient (with Gordon L. Stüber and Chin-Tau Lea) of the 1997 IEEE Jack Neubauer Best Paper Award for his paper "Architecture Design, Frequency Planning, and Performance Analysis for a Microcell/Macrocell Overlaying System," IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, vol. 46, no. 4, pp. 836-848, 1997 (best systems paper published in 1997 by the IEEE Vehicular Technology Society). He has published over 150 journal and international conference papers and is holding three US patents. He served as an Associate Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS from 2001 to 2005, the Guest Editor of the Special Issue on "Mobile Computing and Networking" for IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS in 2005 and on "Radio Resource Management and Protocol Engineering in Future IEEE Broadband Networks" for *IEEE Wireless Communications Magazine* in 2006. He is holding eight US patents.



**Chu-Jung Yeh** received the B.S. degree in electrical engineering from National Dong Hwa University, Hualien, Taiwan, in 2004. He is currently working toward the Ph.D. degree with the Institute of Communications Engineering, National Chiao Tung University, Hsinchu, Taiwan. His current research interests include MIMO systems with scheduling, network MIMO systems, and resource management and performance analysis for cellular mobile networks.