



A new correlation and the review of two-phase flow pressure change across sudden expansion in small channels

Chi-Chuan Wang^c, Chih-Yung Tseng^{a,b}, Ing Youn Chen^{a,*}

^a Mechanical Engineering Department, National Yunlin University of Science and Technology, Yunlin 640, Taiwan

^b Energy & Environment Research Laboratories, Industrial Technology Research Institute, Hsinchu 310, Taiwan

^c Department of Mechanical Engineering, National Chiao Tung University, Hsinchu 300, Taiwan

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ABSTRACT

In this study, the authors first present a short overview on the single-phase flow across sudden expansion, followed by a thorough review of the relevant literature for two-phase flow across sudden expansion, and examination of the applicability of the existing correlations. Also, 282 data available from five publications are collected. It is found that none of the existing correlations can accurately predict the entire database. Most of the correlations highly over predict the data with a mini test section which has a Bond number being less than 0.1 in which the effect of surface tension dominates. Also, some of the correlations significantly under predict the data for very large test sections. Among the models/correlations being examined, the homogeneous model shows a poor predictive ability than the others, but it is handy for the engineering aspect. Hence by taking account the influences of Bond number, Weber number, Froude number, liquid Reynolds number, gas quality and area ratio into the original homogeneous model, a modified homogeneous model is proposed that considerably improves the predictive ability over existing correlations with a mean deviation of 23% and a standard deviation of 29% to all the data with wider ranges of operating parameters for application.

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1. Introduction

The flow of two-phase mixtures across sudden expansions and contractions is commonly seen among connection piping as well as relevant to many applications such as chemical reactors, power generation units, oil wells and petrochemical plants. It is well known that the gas–liquid interactions in sudden flow area changes such as pipeline connections and heat exchangers are a complex function of the flow rates of the two phases, their physical properties and pipe geometry. As the two-phase mixture flows through the sudden area changes, the flow might form a separation region at the sharp corner and results in an appreciable pressure loss due to irreversibility. Two-phase flow studies having constant cross-sectional pipes had been widely studied in the literatures, however, frictional performance arisen from singularities such as expansion and contraction are among the least studies of two-phase system [1]. On the other hand, the small and narrow channels are widely adopted in compact heat exchangers; or act as an integral part of CPU cold plate using the liquid cooling with or without phase change. Recently, the authors had investigated the two-phase flow pressure change across sudden contractions with small channels. A correlation was proposed that

considerably improves the predictive ability (with a mean deviation of 30%) over existing correlations with their 156 data and 357 available literature data [2].

Though, there are several correlations for the two-phase flow across sudden expansions available in the literature. Most of the correlations could only predict their own database, and extrapolating their correlations outside their database was uncertain. In this sense, one of the objectives of this study is to examine the applicability of existing correlations subject to sudden expansions. In the following, a short overview on the single-phase flow across sudden expansion is firstly given, followed by a thorough review of the relevant literatures for two-phase flow across sudden expansions. Based on the available data collected from the literatures, the applicability for each cited correlation/model is tested. It will be shown later that none of them is able to predict the entire database. Hence, a rationally based correlation incorporating with all the significant parameters is proposed to encapsulate the much larger database.

2. Review of literature

2.1. Single-phase pressure change across sudden expansion

For single-phase flow, this pressure change across the sudden expansion is mathematically given as a function of the enlargement loss coefficient (K_e) and the kinetic energy of the flow:

* Corresponding author. Address: 123 University Rd., Section 3, Douliu, Yunlin 640, Taiwan. Tel.: +886 5 5342601; fax: +886 5 5312062.

E-mail address: cheniy@yuntech.edu.tw (I.Y. Chen).

Nomenclature

Bo	Bond number defined Eq. (34)
C_h	Chisholm's factor defined Eq. (15)
d	internal diameter of circular tube (m)
g	gravity (m s^{-2})
f_e	total friction factor, defined in Eq. (23)
Fr	Froude number defined in Eq. (36)
G	total mass flux ($\text{kg m}^{-2} \text{s}^{-1}$)
K	loss coefficient
P	pressure, Pa
ΔP_e	pressure change across the sudden expansion (Pa)
ΔP_{ir}	irreversible pressure change, defined in Eq. (26) (Pa)
ΔP_{er}	reversible pressure change, defined in Eq. (27) (Pa)
P_o	average wall pressure (Pa)
P_w	local wall pressure (Pa)
r	correction factor given in Eq. (14)
Re	Reynolds number ($\rho u d / \mu$)
S	slip ratio
u	fluid mean velocity (m s^{-1})
u_G	actual gas velocity (m s^{-1})
u_L	actual liquid velocity (m s^{-1})
x	gas quality
X	Martinelli parameter
We	Weber number defined in Eq. (35)
Z, γ	coefficients defined in Eq. (31)

Greek symbols

Ω	correction factors given in Eq. (37)
α	mean void fraction
α_E	mean volumetric entrainment, defined in Eq. (18)
Φ	characteristic function defined in Eq. (14)
ρ	density (kg m^{-3})
ρ_{eff}	mean effect density, defined in Eq. (16) (kg m^{-3})
ρ'	fictitious mixture density, defined in Eq. (29) (kg m^{-3})
ρ''	fictitious mixture density, defined in Eq. (30) (kg m^{-3})
$\Delta\rho$	density difference between liquid and gas (kg m^{-3})
σ	surface tension (N m^{-1})
σ_A	flow cross-sectional area expansion ratio, $0 < \sigma_A < 1$
θ	function of the area ratio defined in Eq. (14)
μ	viscosity (N s m^{-2})

Subscripts

G	gas phase
h	two-phase homogeneous mixture
in	sudden expansion inlet
L	liquid phase
LO	all liquid flow
out	sudden expansion outlet

$$\Delta P_e = K_e \rho u^2 / 2 = K_e G^2 / (2\rho) \quad (1)$$

where u is the mean flow velocity, ρ is the fluid density, and mass flux, G is calculated based on the smaller cross-sectional area of the inlet tube, K_e is a function of Reynolds number and the passage cross-section area ratio ($\sigma_A < 1$), K_e decreases as σ_A is increased [3]. As σ_A approaches 0, K_e becomes 1. The enlargement pressure difference becomes a simple correlation by Delhaye [4] from a simplified momentum balance equation

$$\Delta P_e = \frac{G^2 \sigma_A (1 - \sigma_A)}{\rho} \quad (2)$$

Fig. 1 shows a typical change of static pressure along the axis for flow across the expansion. Due to the deceleration of the flow in the transitional region, the static pressure initially increases at the expansion area. After the pressure reaches the maximum, the pressure gradient merges with the downstream pressure gradient line. The pressure change at the sudden expansion is defined as the pressure difference for upstream and downstream fully developed pressure gradient lines extended to the expansion position, i.e., ΔP_{EXP} , as shown in Fig. 1.

2.2. Two-phase pressure change across sudden expansion

Romie [5], Richardson [6], Lottes [7], Mendler [8], and McGee [9] were among the first to investigate two-phase flow through sudden expansions. And this topic was continuously investigated by Chisholm and Sutherland [9], Delhaye [4], Wadle [11], Schmidt and Friedel [12], Attou et al. [13], Attou and Bolle [14], Abdelall et al. [15], and Chalfi et al. [1]. In the majority of the previous studies, the void fraction, area ratio and gas quality, as well as the densities of gas and liquid were used to estimate the pressure recovery across the sudden area expansion. The pressure change equations were obtained from the mass and momentum balances without considering the structure of the flow and the frictional effect on the pipe wall. Ahmed [16] had shown that once the flow is fully developed, the flow pattern, void fraction and pressure gradient can be characterized only by the pipe geometry and local flow conditions without memory of its formation. Recently, Ahmed et al. [17] considered the influence

of wall shear stress in the developing region immediately downstream of the expansion and the wall pressure on the downstream face of the expansion in the flow developing region on the pressure recovery. More recently, Ahmed et al. [18] included the length of the developing regime to the area ratio and the upstream liquid Reynolds number in their correlation.

By allowing a relative velocity between the momentum balances among phases, Romie [5] derived an expression for sudden enlargement

$$\Delta P_e = \frac{G^2 \sigma_A}{\rho_L} \left[(1-x)^2 \left(\frac{1}{(1-\alpha_{in})} - \frac{\sigma_A}{(1-\alpha_{out})} \right) + x^2 \frac{\rho_L}{\rho_G} \left(\frac{1}{\alpha_{in}} - \frac{\sigma_A}{\alpha_{out}} \right) \right] \quad (3)$$

where the subscripts, in denotes upstream of expansion, and out represents downstream of expansion.

If the void fraction remains unchanged, Eq. (3) is simplified to [4]:

$$\Delta P_e = \frac{G^2 \sigma_A (1 - \sigma_A)}{\rho_L} \left[\frac{(1-x)^2}{(1-\alpha)} + \frac{(\rho_L/\rho_G)x^2}{\alpha} \right] \quad (4)$$

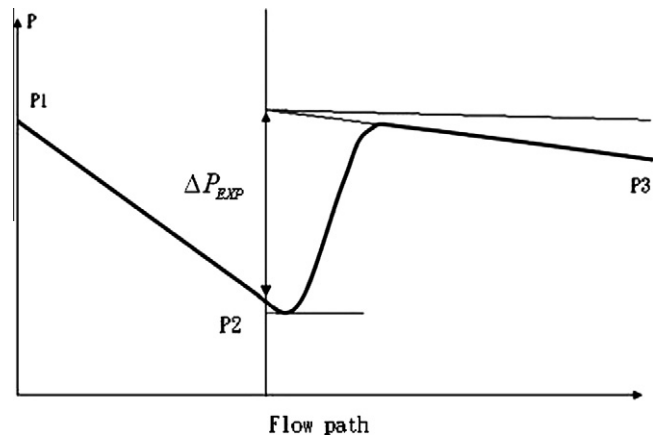


Fig. 1. Idealized pressure variations across sudden expansion.

McGee [9] measured pressure drops for steam–water mixtures flowing through sudden expansions with area ratios (σ_A) of 0.332, 0.546, and 0.607. His test conditions are summarized in Table 1. The predicted pressure change by Romie's equation (3) using the measured upstream and down stream void fractions is in fair agreement with the data. The predictive ability of his correlation against his 64 point data was in the order of $\pm 40\%$. A further simplification by neglecting the density change and replacing the void fractions, α_{in} and α_{out} by their average value, the predictive ability of Eq. (4) remains unchanged with a 39% standard deviation as shown in Table 2.

Considering a homogeneous flow from the momentum balance, Eq. (4) is reduced to [4]:

$$\Delta P_e = G^2 \sigma_A (1 - \sigma_A) \left[\frac{(1-x)}{\rho_L} + \frac{x}{\rho_G} \right] \quad (5)$$

However, the homogeneous model gave a very poor predictive ability against McGee's [9] database with a standard deviation exceeding 138% as shown in Table 1. Attou et al. [13] and Abdelall et al. [15] had pointed out that the homogeneous model tends to significantly overestimate the experimental results due to the assumption of no slip between the phases.

From the mechanical energy balance for the mixture with neglecting the friction dissipation, Delhaye [4] also derived the following expression:

$$\Delta P_e = \frac{G^2 (1 - \sigma_A^2)}{2} \left[\frac{(1-x)^3}{\rho_L^2 (1-\alpha)^2} + \frac{x^3}{\rho_G^2 \alpha^2} \right] \left[\frac{(1-x)}{\rho_L} + \frac{x}{\rho_G} \right]^{-1} \quad (6)$$

Eq. (6) allows a relative velocity between the phases. For homogeneous flow, the mechanical energy equation (6) is simplified to [4]:

$$\Delta P_e = \frac{G^2 (1 - \sigma_A^2)}{2} \left[\frac{(1-x)}{\rho_L} + \frac{x}{\rho_G} \right] \quad (7)$$

Richardson [6] simplified the energy balance model and assumed that the pressure recovery is proportional to the kinetic energies of the phases:

$$\Delta P_e = \frac{G^2 (1 - \sigma_A^2)}{2} \left[\frac{\sigma_A (1-x)^2}{\rho_L (1-\alpha)} \right] \quad (8)$$

Assuming the void fraction remains unchanged, Lottes [7] ignored the gas mass flow rate and assumed that all losses of dynamic pressure head can only take place in the liquid phase, yielding

$$\Delta P_e = \frac{G^2 \sigma_A (1 - \sigma_A)}{\rho_L (1 - \alpha)^2} \quad (9)$$

Mendler [8] also measured the pressure drops for steam–water mixtures flowing through sudden expansions with area ratios (σ_A) of 0.145, 0.264, and 0.693, respectively. His test conditions are listed in Table 1. Three correlations were proposed by using the least square method to test the database. Since the correlations were empirically obtained from his data, they were only recommended for use at $x < 0.15$, $\sigma_A < 0.5$, and a steam pressure < 41.2 bar. However, the predictions by homogeneous model and the simplified Romie's equation (4) by Delhaye [4] were fair against Mendler's data [8] with a standard deviation of 65% and 36%, respectively, as shown in Table 2.

Chisholm and Sutherland [10] also developed a heterogeneous model based on the momentum balance:

$$\Delta P_e = \frac{G^2 \sigma_A (1 - \sigma_A) (1-x)^2}{\rho_L} \left(1 + \frac{C_h}{X} + \frac{1}{X^2} \right) \quad (10)$$

where

$$X = \left(\frac{\rho_G}{\rho_L} \right)^{0.5} \frac{(1-x)}{x} \quad (11)$$

and

$$C_h = \left[1 - 0.5 \left(\frac{\rho_L - \rho_G}{\rho_L} \right)^{0.5} \right] \left[\left(\frac{\rho_L}{\rho_G} \right)^{0.5} + \left(\frac{\rho_G}{\rho_L} \right)^{0.5} \right] \quad (12)$$

The model of Chisholm and Sutherland [10] was compared with the air–water bubbly flow data ($\alpha < 0.35$) [13]. Their predictions remained reasonably good with the test data but with a slightly under estimation. The underestimation of the Chisholm model was also reported by Wadle [11].

Wadle [11] assumed the liquid is decelerated much less than the gas when it passes through the expansion due to its higher inertia. The pressure recovery at the sudden expansion is caused by the bulk deceleration effect and a formula was proposed to describe his pressure recovery data in an abrupt diffuser. The model includes an artificial constant K in connection with different working fluids ($K = 0.667$ for steam–water, $K = 0.83$ for air–water)

Table 1
Available data for two-phase pressure change across sudden expansions.

Researchers	Abdelall et al. [15]	Chalfi et al. [1]	Mendler [8]	McGee [9]	Schmidt and Friedel [12]
G (kg/m ² s)	3074–4212	506–664	692–5642	544–2480	1000–3000
x	0.002–0.013	0.0285–0.182	0.032–0.469	0.001–0.304	0.01–0.99
Working fluid	Air–water	Air–water	Steam–water	Steam–water	Air–water
d_{in} (mm)	0.84	0.84	9.55 12.9 17.63	8.64 11.68	17.2 19
σ_A	0.276	0.276	0.145 0.264 0.493	0.332 0.546 0.607	0.0568 0.0937
$Bo = \frac{\Delta \rho g d^2}{\sigma}$	0.095	0.095	Max–92.1 Min–19.8	Max–30.7 Min–13.5	Max–48.8 Min–40
$Fr = \frac{G^2}{\rho_m g d}$	Max – 1.67E+05 Min – 9.35E+03	Max – 9.19E+05 Min – 2.64E+04	Max – 3.76E+04 Min – 2.57E+01	Max – 9.57E+04 Min – 1.03E+01	Max – 1.72E+05 Min – 3.95E+01
$We = \frac{G^2 d}{\sigma \rho_m}$	Max – 1.33E+03 Min – 3.12E+02	Max – 5.94E+02 Min – 1.00E+02	Max – 4.55E+04 Min – 1.02E+03	Max – 1.55E+04 Min – 1.85E+02	Max – 8.36E+04 Min – 7.16E+02
$Re = \frac{G d}{\mu_L}$	Max – 3.53E+03 Min – 2.58E+03	Max – 5.71E+02 Min – 4.35E+02	Max – 4.95E+05 Min – 9.02E+04	Max – 1.37E+05 Min – 3.36E+04	Max – 6.30E+04 Min – 1.90E+04
ΔP_e (kPa)	Max – 4.62 Min – 0.726	Max – 3.71 Min – 0.801	Max – 15.6 Min – 0.575	Max – 14.4 Min – 0.069	Max – 36.0 Min – 0.2
Total points	14	24	90	64	90

Table 2

The standard deviations for the predictions of correlations to the data

Correlations	Data (%)					
	Abdellal et al. [15] 1 tube	Chalfi et al. [1] 1 tube	Mendler [8] 3 tubes	McGee [9] 3 tubes	Schmidt and Friedel [12] 2 tubes	Total 282 Points
Delhaye [4]	139	28	36	39	59	54
Homogenous	468	207	65	138	31	143
Richardson [6]	57	51	54	44	82	63
Chisholm and Sutherland [10]	250	77	37	56	49	74
Wadle [11]	178	46	52	66	123	92
Schmit and Friedel [12]	259	28	43	47	41	69
Attou and Bolle [14]	93	53	54	52	83	66
Abdelal et al. [15]	133	46	169	80	360	230

$$\Delta P_e = \frac{G^2 K (1 - \sigma_A^2)}{2} \left[\frac{(1-x)^2}{\rho_L} + \frac{x^2}{\rho_G} \right] \quad (13)$$

Attou et al. [13] mentioned that Wadle's model significantly overestimated their air–water bubbly flow data ($\alpha < 0.35$). Owen et al. [19] also noticed this phenomenon and claimed that, with $K = 0.22$ (very different from 0.667), Wadle's model [11] agreed quite well with their measurements. The difference of the K values was speculated by the difference of the expansion geometries and flow conditions having a lower mass flow rate. The parameter, K , is an empirical constant that summed up various influencing parameters. For further clarifying the individual effects, an adequate modeling is needed.

Attou and Bolle [14] simplified the jet line emerging from a sudden expansion as a straight line, and then the central flow is confined inside a conical diffuser. By applying the momentum balance within the boundary of the conical jet for an incompressible and adiabatic flow, they obtained a correlation applicable for two-phase flow pressure recovery from a sudden expansion:

$$\Delta P_e = G^2 \sigma_A (1 - \sigma_A) [\Phi \theta^r + (1 - \theta^r) / \rho_L] \quad (14)$$

where $\Phi = x^2 / (\alpha \rho_G) + (1-x)^2 / [(1-\alpha)\rho_L]$, $\theta = 3/[1 + \sigma_A 0.5 + \sigma_A]$ and r is a correction factor related to the physical properties of the mixture. For a gas quality of $x = 0$, Eq. (14) can be reduced to Eq. (2). The best fitting to the correction factor is $r = 1$ for steam–water mixture and $r = -1.4$ for air–water mixture. The predictions of Eq. (14) had been compared with air–water data and steam–water data. The mean quadratic errors were about 23.4%. The correlation is particularly good for small mass velocities, but it is inapplicable to high gas quality flows ($x > 0.2$).

Based on the momentum and mass transfer balances, Schmidt and Friedel [12] developed a new pressure drop model for sudden expansion which incorporated all of the relevant boundary conditions. In this model all the relevant physical parameters were also included in their sudden contraction paper [20]. Assuming constant properties, the equations for calculating the pressure change across the sudden expansion are simplified as the following

$$\Delta P_e = \frac{G^2 \left[\frac{\sigma_A - \sigma_A^2}{\rho_{eff}} - f_e \rho_{eff} \left(\frac{x}{\rho_G \alpha} - \frac{(1-x)}{\rho_L (1-\alpha)} \right)^2 (1 - \sigma_A^{0.5})^2 \right]}{1 - \Gamma_e (1 - \sigma_A)} \quad (15)$$

where

$$\frac{1}{\rho_{eff}} = \frac{x^2}{\rho_G \alpha} + \frac{(1-x)^2}{\rho_L (1-\alpha)} + \frac{\alpha_E \rho_L (1-\alpha)}{1 - \alpha_E} \left[\frac{x}{\rho_G \alpha} - \frac{1-x}{\rho_L (1-\alpha)} \right]^2 \quad (16)$$

$$\alpha = 1 - \frac{2(1-x)^2}{1 - 2x + \sqrt{1 + 4x(1-x) \left(\frac{\rho_L}{\rho_G} - 1 \right)}} \quad (17)$$

$$\alpha_E = \frac{1}{S} \left[1 - \frac{1-x}{1-x(1-0.05We^{0.27} Re^{0.05})} \right] \quad (18)$$

$$S = \frac{x}{1-x} \frac{(1-\alpha)}{\alpha} \frac{\rho_L}{\rho_G} \quad (19)$$

$$We = G^2 x^2 \frac{d}{\rho_G \sigma} \frac{(\rho_L - \rho_G)}{\rho_G} \quad (20)$$

$$Re = \frac{G(1-x)d}{\mu_L} \quad (21)$$

$$\Gamma_e = 1 - \sigma_A^{0.25} \quad (22)$$

$$f_e = 4.9 \times 10^{-3} x^2 (1-x)^2 \left(\frac{\mu_L}{\mu_G} \right)^{0.7} \quad (23)$$

The model predicted several experimental data sets from eight test sections with several conventional inlet (17.2–44.2 mm) and outlet (44.2–72.2 mm) diameters. The average of the logarithmic ratios of measured and predicted values was less than –3%, the scatter equaled to 61%. The water–air data for the test sections with area ratio of 0.0568 and 0.0937 conducted at 25 °C and 5 bar was shown in their paper [12] which are also listed in Table 1.

Recently, Abdelal et al. [15] investigated air–water pressure drops caused by abrupt flow area expansions in two small tubes. The tube diameters were 1.6 and 0.84 mm with a sudden area change ratio of 0.276. Their measured two-phase pressure difference indicated the occurrence of significant velocity slip. With the assumption of an ideal annular flow regime in accordance with minimum kinetic energy of the flowing mixture, the velocity slip ratio given by Zivi [21] is:

$$S = \frac{u_G}{u_L} = \frac{(1-\alpha)x\rho_L}{\alpha(1-x)\rho_G} = \left(\frac{\rho_L}{\rho_G} \right)^{1/3} \quad (24)$$

where u_G and u_L are the actual gas and liquid velocities of gas and liquid phases, respectively. In practice the slip ratio is actually varying along the flow path. The pressure drop across the sudden expansion (ΔP_e) is the combination of the reversible pressure change (ΔP_{eR}) and the irreversible pressure changes (ΔP_{eI}), i.e.,

$$\Delta P_e = \Delta P_{eR} + \Delta P_{eI} \quad (25)$$

For an incompressible and adiabatic flow,

$$\Delta P_{eR} = \frac{G^2}{2} \left(\frac{1}{\rho'^2} - \frac{\sigma_A}{\rho'^2} \right) \quad (26)$$

$$\Delta P_{eI} = 0.5 \frac{G^2}{\rho_L} \left[\frac{2\rho_L \sigma_A (\sigma_A - 1)}{\rho'} - \frac{\rho_h \rho_L (\sigma_A - 1)}{\rho'^2} \right] \quad (27)$$

where

$$\rho_h = \left[\frac{x}{\rho_G} + \frac{(1-x)}{\rho_L} \right]^{-1} \quad (28)$$

$$\rho' = \left[\frac{(1-x^2)}{\{\rho_L(1-\alpha)\}} + \frac{x^2}{(\rho_G\alpha)} \right]^{-1} \quad (29)$$

$$\rho'' = \left[\frac{(1-x)^3}{\rho_L^2(1-\alpha)^2} + \frac{x^3}{(\rho_G^2\alpha^2)} \right]^{-1/2} \quad (30)$$

An empirical relation between α and x , Eq. (24), must be used to relate the slip ratio for applying Eqs. (25)–(27). The predictions of Abdelall et al.'s slip flow model [15] are slightly higher than the experimental data, but the simplified homogeneous model, Eq. (5) significantly over predicts the data. The error margins for the predictions of the homogeneous model and Abdelall et al.'s slip flow model to their 14 data were not reported in their study.

Based on the same test facility of Abdelall et al. [15], Chalfi et al. [1] recently reported more data for single-phase and two-phase flow pressure drops caused by expansion and contraction using air and water. Their new data fell within an even lower all liquid Reynolds number ($Re_{LO} < 500$) than that of Abdelall et al.'s data [15] ($2578 < Re_{LO} < 3530$). The Zivi slip flow model [21] along with the Armand-type void fraction, $\alpha = 0.5J_G/(J_L + J_G)$, under predicted their 24 data up to about 30%, where J is the superficial velocity.

Ahmed et al. [17] utilized an analytical formulation for the pressure recovery of two-phase flow across a sudden expansion. Their formula covered the pressure change, Eq. (3), for the sudden enlargement from the momentum balance derived by Romie [4] with the change in void fraction across the expansion, and also included the difference between the pressure at the centerline just before the expansion (P_1) and the average pressure of the downstream face at the expansion (P_0), as well as the additional wall shear stress in the developing region downstream of the expansion. Experiments were performed using air–oil two-phase flow to evaluate the relative contribution of the individual terms to the pressure recovery for three area ratios of 0.0625, 0.25 and 0.444. The ranges of the tested mass quality and oil mass flux are 0.0007–0.67 and 20–2050 kg/m²s, respectively. Upstream and downstream flow regimes were identified by a high-speed video camera. The cross-sectional average void fractions at different locations along the test section were obtained using capacitance sensors located along the test section [16]. Four pressure taps were located on the downstream face of the sudden expansion wall to measure the local wall pressures (P_w). The pressure at the centerline of the expansion, P_1 , was obtained by extrapolating the fully developed upstream pressure gradient line to the sudden expansion section. The average wall pressure (P_0) was integrated from the measured local wall pressures (P_w).

For the model predictions of Ahmed et al. [17], the upstream and downstream flow patterns were identified using the flow pattern map of Taitel and Dukler [22]. The upstream and downstream void fractions were estimated using the appropriate correlations on the local flow pattern. The wall pressure was obtained by extrapolating the measured data to different area ratios and higher mass qualities for all the present air–oil data, while the additional wall shear stress term was estimated using the correlation of Aloui et al. [23] for bubbly flow. For annular flow, the wall pressure term rises with the mass quality, and its contribution is more significant than the additional wall shear stress in the developing region. While for the elongated bubbly and intermittent flows, the additional wall shear stress is more significant than the wall pressure. The existing literature data along with their air–oil data (192 points) were compared against the predicted values using their formulation. Most of the data were in a good agreement with the predicted values to within $\pm 35\%$ of the relative standard deviation.

However, the data of Schmidt and Friedel [12] at mass qualities higher than 0.5 were highly over predicted with a standard deviation of 50% since the mass quality for most of their data was less than 0.5. The inclusion of the wall pressure term had improved the prediction of the pressure recovery for mass qualities lower than 0.5.

Ahmed et al. [17] utilized the pipe geometry and local flow conditions at upstream and downstream to predict the flow patterns, then using the appropriate correlations related to the flow patterns to predict the void fraction. They also proposed a formula to predict the pressure recovery via the momentum recovery, the wall pressure at the expansion surface and the additional viscous shear stress in the developing length. The developing length was found to be strongly dependent on the upstream liquid Reynolds number and the sudden expansion area ratio [18] in their correlation. However, for the engineering application, this formula is tedious to use since the flow patterns at the upstream and downstream are required for the prediction of void fractions. Also, the predictions of the pressure change across the sudden expansions entail the empirical wall pressure and the developing length which are not practical for most of the cases. In addition, air and oil are used for the two-phase mixtures in their studies. However, the name of oil and the oil properties were not given in their papers, even not included in Ahmed's Ph.D. Thesis [16]. Therefore, their oil and water data were not included in this study for correlation development.

From the foregoing review of the two-phase pressure change across the sudden expansion, the information of void fraction is required except the homogeneous model, and equations of Wadle [11] and Chisholm and Sutherland [10]. The void fraction may vary in the short length of the sudden expansion due to the flow separation, velocity and geometry changes. Most of the pressure change correlations were based on the inlet conditions to give a constant void fraction. The void fraction can be estimated from measurements, predictions from conventional correlations, or by the individually developed empirical correlations. Some of the investigations included the upstream and downstream void fractions from the measurements [12,15,17]. Ahmed et al. [17] also predicted the flow patterns at the upstream and downstream, then using the appropriate correlations to predict the corresponding void fractions.

McGee [9] neglected the density change and replacing the local α by the average value at upstream and downstream, the predictions did not affect the results significantly. Abdelall et al. [15] and Chalfi et al. [1] utilized Zivi slip flow model [21] for the prediction of the averaged void fraction. There are many void fraction correlations available in the literature. Recently, Dalkilic et al. [24] had surveyed the void fraction correlations and a summary of 35 correlations. The correlations were then compared with data, and it is found that the Thom's void fraction correlation [25] was among the best. Thom obtained an empirical relationship between quality and void fraction by assuming the slip velocity to be dependent on phase viscosities and densities. Thom's correlation is given as

$$\alpha = \frac{\gamma x}{1 + x(\gamma - 1)} \quad \text{where } \gamma = Z^{1.6}, \quad Z = \left(\frac{\rho_L}{\rho_G} \right)^{0.555} \left(\frac{\mu_G}{\mu_L} \right)^{0.111} \quad (31)$$

The void fraction α is calculated by the Thom's correlation to predict the pressure change across the sudden expansions in this study. Several void fraction correlations had also been tested, but Thom's correlation gave the overall best results.

In summary of the aforementioned review, most of the proposed correlations/models are only applicable to their own database. Also, some of the correlations are not handy for the engineering application. To tackle this problem, the present study

is to provide a simplified and reliable correlation to predict the two-phase pressure change at sudden expansions with rationally based parameters from a much wider database of the literatures.

3. Results and discussion

To test the validity of the foregoing described models/correlations from the existing literatures, 90 data from Mendler [8], 64 data from McGee [9], 90 data from Schmidt and Friedel [12], 14 data from Abdellal et al. [15] and 24 data from Chalfi et al. [1] are collected and their test conditions, as well as the ranges of the momentous parameters are also listed in Table 1. The data are compared with the previously described correlations/models by Delhaye [4] for Eq. (4), homogeneous for Eq. (5), Richardson [6] for Eq. (8), Chisholm and Sutherland [10] for Eq. (10), Wadle [11] for Eq. (13), Schmidt and Friedel [12] for Eq. (15), Attou and Bolle [14] for Eq. (14) and Abdelall et al. [15] for Eq. (25). Table 2 shows the standard deviations for the predictions of correlations with a total of 282 data. The standard deviations of the relevant predictions to the total data are 54%, 143%, 63%, 74%, 92%, 69%, 66%, 230% by correlations of Delhaye [4], homogeneous flow model, Richardson [6], Chisholm and Sutherland [10], Wadle [11], Schmidt and Friedel [12], Attou and Bolle [14] and Abdelall et al. [15], respectively. Also, the mean deviations for the above predictions are obtained as 42%, 83%, 58%, 49%, 65%, 43%, 52% and 146%, respectively. None of them can accurately predict the entire database. The standard deviation is defined by:

$$\text{standard deviation} = \sqrt{\frac{\sum_{n=1}^n \left(\frac{\text{data} - \text{prediction}}{\text{data}} \right)^2}{n}} \quad (32)$$

where n is the number of data. The mean deviation is defined by:

$$\text{mean deviation} = \frac{\sum_{n=1}^n \left| \frac{\text{data} - \text{prediction}}{\text{data}} \right|}{n} \quad (33)$$

Fig. 2 shows the comparison of Delhaye [4] predictions against the database. Acceptable predictions are observed to the data sets of Mendler [8], McGee [9], and Chalfi et al. [1], but over predicts the Abdelall et al. [15] data (139% standard deviation) and the under

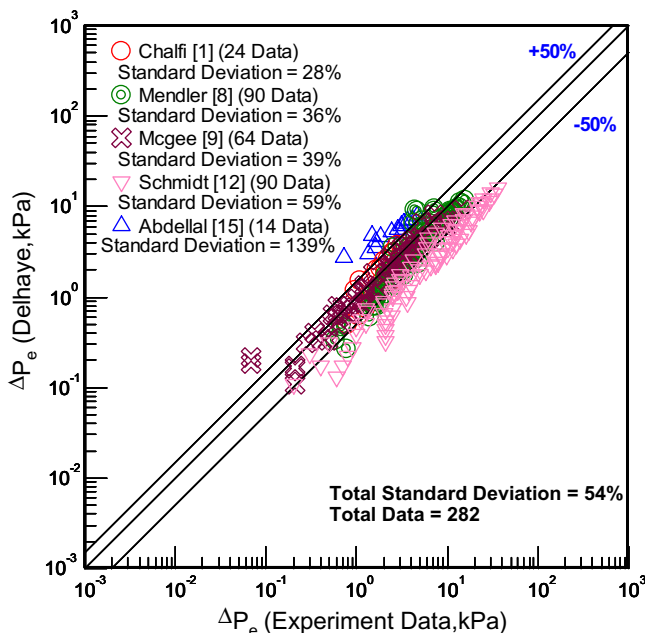


Fig. 2. Comparison of Delhaye [4] predictions, Eq. (4), with data.

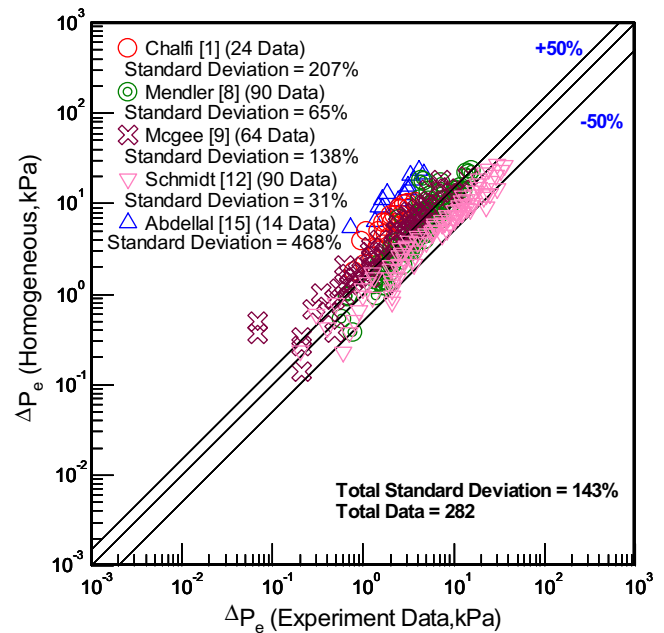


Fig. 3. Comparison of homogeneous predictions, Eq. (5), with data.

predicts the Schmidt and Friedel [12] data (59% standard deviation). Notice that Abdelall et al. [15] and Chalfi et al. [1] utilized the same mini test section. The Chalfi et al.'s data [1] fell within an even lower all liquid Reynolds number ($Re_{LO} < 500$) than that of Abdelall et al.'s data [15] ($2578 < Re_{LO} < 3530$), but the standard deviations are 28% and 139%, respectively. Fig. 3 compares the homogeneous predictions with the database, and it shows a poor predictive ability with a standard deviation of 143%. Particularly, the standard deviations for Abdelall et al. data [15] and Chalfi et al. data [1] are 468% and 207%, respectively. Despite its poor ability, homogeneous model is quite handy to use and it is easy to apply especially for engineering applications. The large discrepancy of this model may arise from surface tension in the mini test

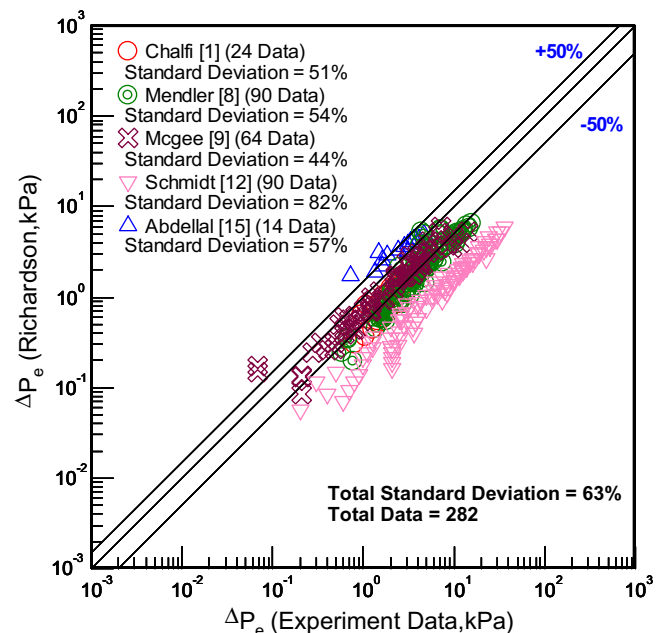


Fig. 4. Comparison of Richardson [6] predictions, Eq. (8), with data.

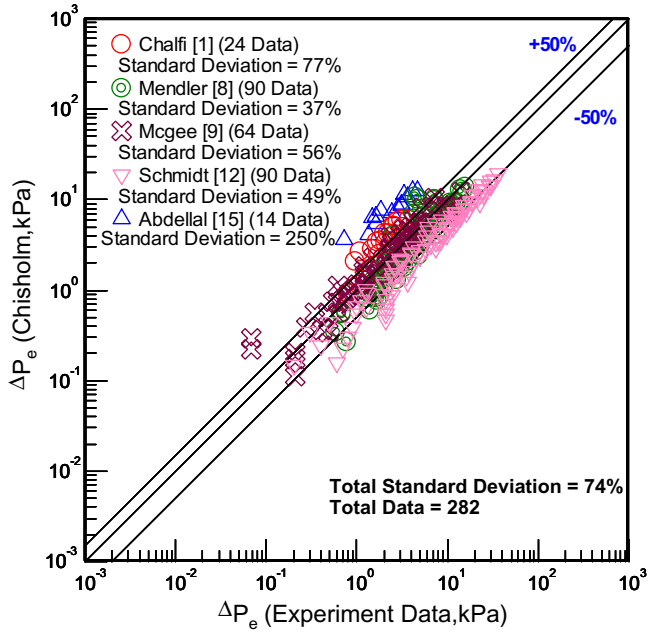


Fig. 5. Comparison of Chisholm and Sutherland [10] predictions, Eq. (10), with data.

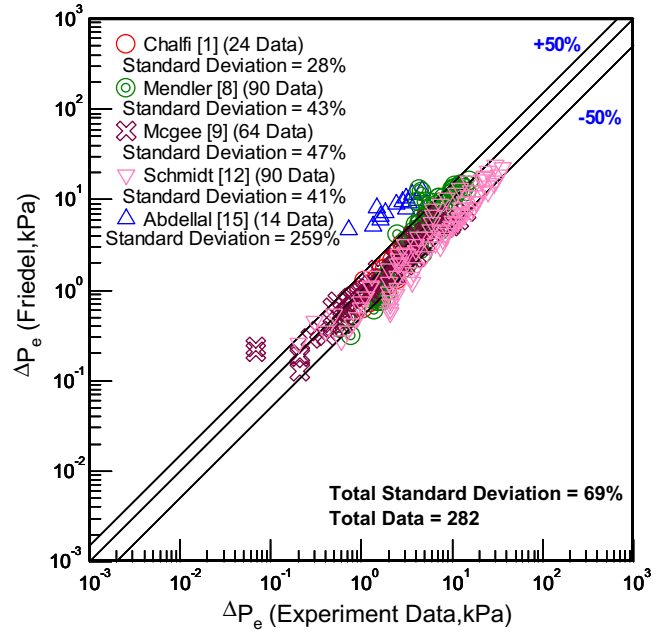


Fig. 7. Comparison of Schmidt and Friedel [12] predictions, Eq. (15), with data.

section. Fig. 4 is the comparison of Richardson [6] predictions with data. It gives a fair agreement with a standard deviation of 63%. However, the correlation significantly under predicts Schmidt and Friedel data [12] with a standard deviation of 82%. Fig. 5 shows the comparison of Chisholm and Sutherland [10] predictions to the database also with a fair standard deviation of 74% to all the data but it significantly over predicts (250%) the Abdellal et al.'s data [15]. Fig. 6 is the comparison of Wadle [11] predictions to the data. Fair predictions are observed to the data sets of Mender [8], McGee [9], and Chalfi et al. [1], but over predictions are found for the data sets of Schmidt and Friedel [12] and Abdellal et al. [15] with 123% and 178% standard deviations, respectively. Fig. 7 shows the fair predictive ability of Schmidt and Friedel [12] correlation

but it also significantly over predicts the Abdelall et al.'s data [15]. Similarly, as shown in Fig. 8, the correlation of Attou and Bolle [14] also over predicts the Abdelall et al.'s data [15] with 93% standard deviation and it also highly under predicts the Schmidt and Friedel [12] data (83%), but gives fair predictions to the other three data sets. In addition, the Abdelall et al.'s method [15] gives very poor predictions to most of the data sets as shown in Table 2. This is not surprised because this correlation is developed from only 14 data in a mini test section.

In summary of the predictive capability for the existing models/correlations from Table 2 and in Figs. 2–8, it appears that none of the existing correlations can accurately predict the entire database. Most of the correlations highly over predict the data with a mini

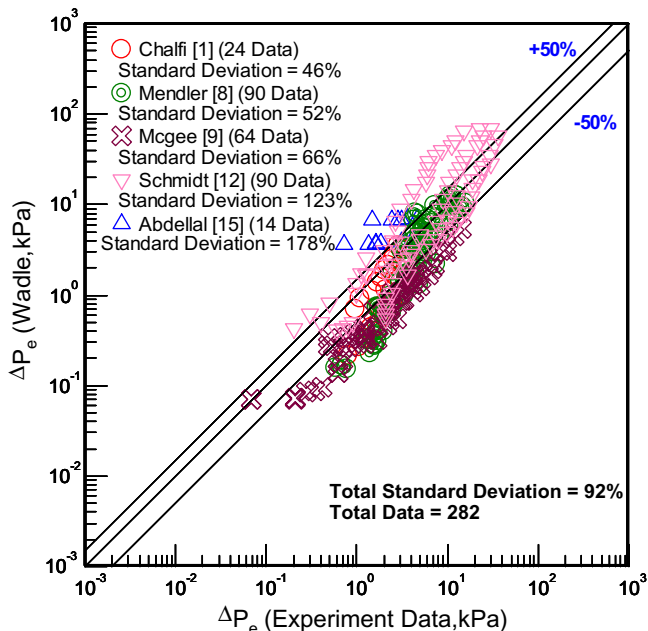


Fig. 6. Comparison of Wadle [11] predictions, Eq. (13), with data.

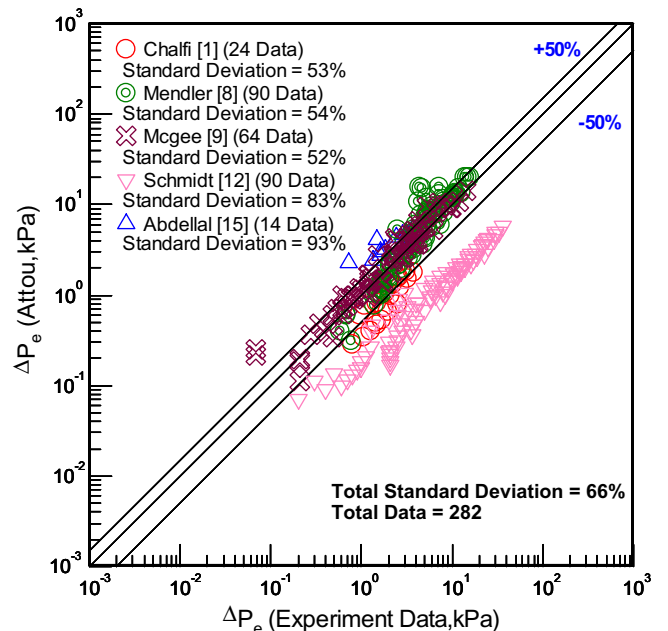


Fig. 8. Comparison of Attou and Bolle [14] predictions, Eq. (14), with data.

test section which has a Bond number being less than 0.1 in which the surface tension plays essential role. Also, some of the correlations significantly under predict the Schmidt and Friedel [12] data. This is probably because the original correlation is valid for small or medium size test section in contrast to the large test section of Schmidt and Friedel [12].

Delhaye [4] correlation seems to give the best predictions to the database with a standard deviation of 54%, but it over predicts the Abdelall et al. [15] data (139% standard deviation) and under predicts the Schmidt and Friedel [12] data (59% standard deviation). Followed by the Richardson [6] correlation with a standard deviation of 63%, but it significantly under predicts the Schmidt and Friedel [12] data (82% standard deviation). Attou and Bolle [14] correlation comes in third with a standard deviation of 66%, but it also significantly under predicts the Schmidt and Friedel [12] data (83% standard deviation). The above three correlations all entail the void fraction during calculation. This will raise uncertainty for there are so many correlations available in the literature [24,26]. It is very difficult to justify the best one. Moreover, the void fraction does not remain a constant in the short path of the sudden expansion due to the changes of velocity and flow pattern. Therefore, to include the void fraction in the prediction is inconvenient for the viewpoint of engineering applications. In addition, the correlations of Attou and Bolle [14] and Wadle [11] include an artificial factor which is varied for different two-phase flow mixtures.

The highly over predictions by the existing correlations to the Abdelall et al. data [15] could be attributed to the very small outlet section ($d_{in} = 0.84$ mm, $d_{out} = 1.6$ mm). In addition, the significantly under predictions for most of the correlations to the Schmidt and Friedel [12] data could be due to the very large test sections ($d_{in} = 17.2$ mm, $d_{out} = 72.2$ mm and $d_{in} = 19.0$ mm, $d_{out} = 56.0$ mm). For obtaining a better predictive ability for mini test sections, one should also take into account the influence of surface tension force [27]. The balance of buoyancy force and surface tension force can be represented by Bond number (Bo) as:

$$Bo = \frac{\Delta \rho g d^2}{\sigma} \quad (34)$$

When the value of Bo is near or less than 1.0, the stratified flow pattern is not able to exist in most of the two-phase flow conditions. Chen et al. [2,28] had utilized the Bond number (Bo) to balance buoyancy force and surface tension in developed the two-phase frictional pressure drop correlations in small straight tubes and sudden contractions, respectively.

Considering the effects of total mass flux and gas quality to the surface tension, Schmidt and Friedel [12,20] had proposed a Weber number to correlate the two-phase pressure change across sudden expansions and contractions. Chen et al. [29] recognized this influence and defined the Weber number as the ratio between the mixture inertia and liquid surface tension for correlating the frictional two-phase pressure drop in small U-type wavy tubes

$$We = \frac{G^2 d}{\sigma \rho_h} \quad (35)$$

The Froude number representing the ratio between the mixture inertia and the buoyancy force was utilized by Friedel [30] for the two-phase frictional pressure drop correlation in conventional straight tubes

$$Fr = \frac{G^2}{\rho_h^2 g d} \quad (36)$$

In addition, the liquid Reynolds number, $Re_{LO} = Gd/\mu_L$, was used as a significant parameter for the two-phase frictional pressure drop correlation in small tubes [31]. Ahmed et al. [18] also utilized the liquid Reynolds number for correlating the developing length at the downstream of a sudden expansion.

As discussed in previous section, homogeneous model shows a little poor than the others, but it is handy to use. Hence by taking account the influences of the significant parameters in prior discussions, the corresponding effects can be corrected by introducing the correction factors that include the influences Bond number, Weber number, Froude number, liquid Reynolds number, gas quality and area ratio to the original homogeneous model (Eq. (5)). The calculations of the relevant parameters and fluid properties are all based on the inlet conditions of the sudden expansions. The proposed modified homogeneous model takes the form as:

$$\begin{aligned} \Delta P_{Modify} &= \Delta P_{Homogeneous} \times (1 + \Omega_1 - \Omega_2) \times (1 + \Omega_3) \\ \Omega_1 &= \left(\frac{We Bo}{Re_{Lo}} \right)^2 \times \left(\frac{1-x}{x} \right)^{0.3} \times \frac{1}{Fr^{0.8}} \\ \Omega_2 &= 0.2 \times \left(\frac{\mu_G}{\mu_L} \right)^{0.4} \\ \Omega_3 &= 0.4 \times \left(\frac{x}{1-x} \right)^{0.3} + 0.3 \times e^{\frac{-1.6}{Re_{Lo}^{0.1}}} - 0.4 \times \left(\frac{\rho_L}{\rho_G} \right)^{0.2} \end{aligned} \quad (37)$$

This proposed modified homogeneous correlation considerably improves the predictive ability over existing correlations with a mean deviation of 23% and a standard deviation of 29% to all the data as shown in Fig. 9. The standard deviations for the predictions of the original homogeneous model to the data sets of Abdelall et al. [15], Chalfi et al. [1], Mendler [8], McGee [9] and Schmidt and Friedel [12] are greatly improved by the proposed modified homogeneous model, from 468%, 207%, 65%, 138% and 31% to 26%, 32%, 31%, 34% and 24%, respectively. In summary, the proposed correlation shows a very good accuracy against the existing data and is capable of handling the effects of gas quality, area ratio, mixture inertia, surface tension and buoyancy force, and is applicable for much wider ranges with $506 < G < 5642$ kg/m² s, $0.002 < x < 0.99$, $0.057 < \sigma_A < 0.607$, $0.84 < d_{in} < 19$ mm, $0.095 < Bo < 92$, $1.03E+1 < Fr < 9.19E+5$, $1.0E+2 < We < 8.3E+4$, and $4.35E+2 < Re_{LO} < 4.95E+05$.

4. Conclusion

The flow of two-phase mixtures across sudden expansions and contractions are widely encountered in typical industrial pipe lines

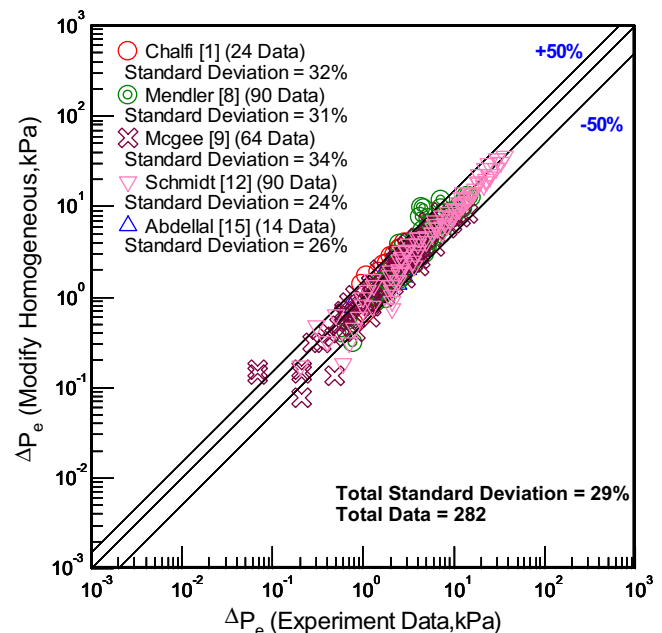


Fig. 9. Comparison of the data with modified homogeneous model, Eq. (37).

and heat exchanging devices. There had been some studies concerning this subject but mostly are applicable for larger channels. Also, most of correlations are only applicable to their data, extrapolating the correlation outside their database is usually not recommended.

In this study, the authors firstly give a thorough review of the relevant literature for two-phase flow across sudden expansions, and examine the applicability of the existing correlations. Also, 282 data available from five publications are collected. It is found that none of the existing eight correlations can accurately predict the entire database. Most of the correlations highly over predict the database with a mini test section which has a Bond number being less than 0.1 in which the effect of surface tension dominates. Some of the correlations significantly under predict the data with very large test sections. The significant parameters of Bond number, Weber number, Froude number, and liquid Reynolds number to the frictional two-phase pressure change across sudden area expansions are discussed.

Among the models/correlations being examined, the homogeneous model shows a poor predictive ability than the others, but it is handy as far as engineering application is concerned. Hence by taking account the influences of Bond number, Weber number, Froude number, liquid Reynolds number, gas quality and area ratio into the original homogeneous model for correlating with the data, a modified homogeneous correlation is proposed that considerably improves the predictive ability over existing correlations with a mean deviation of 23% and a standard deviation of 29% to all the data with much wider ranges of application.

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References

- [1] T.Y. Chalfi, Y. Toufik, S.M. Ghiaasiaan, Pressure drop caused by flow area changes in capillaries under low flow conditions, *Int. J. Multiphase Flow* 34 (2008) 2–12.
- [2] I.Y. Chen, C.Y. Tseng, Y.T. Lin, C.C. Wang, Two-phase flow pressure change subject to sudden contraction in small rectangular channels, *Int. J. Multiphase Flow* 35 (3) (2009) 297–306.
- [3] W.M. Kays, A.L. London, *Compact Heat Exchangers*, third ed., McGraw-Hill Book Company, New York, 1984. p. 112 (Chapter 5).
- [4] J.M. Delhaye, Singular pressure drops, in: A.E. Bergles (Ed.), *Two-Phase and Heat Transfer in the Power and Process Industries*, Hemisphere, Washington, DC, 1981, pp. 124–149 (Chapter 4).
- [5] F. Romie, Private Communication to P. Lottes, American Standard 466 Co., 1958 (see Lottes [7]).
- [6] B. Richardson, Some Problems in Horizontal Two-Phase, Two-Component Flow, Ph.D. Thesis, University Chicago, Chicago, IL, 1958.
- [7] P.A. Lottes, Expansion losses in two-phase flow, *Nucl. Sci. Eng.* 9 (1961) 26–31.
- [8] O.J. Mendler, Sudden Expansion Losses in Single and Two-Phase Flow, Ph.D. Thesis, University of Pittsburgh, Pennsylvania, 1963.
- [9] J.W. McGee, Two-Phase Flow through Abrupt Expansions and Contractions, Ph.D. Thesis, University of North Carolina at Raleigh, 1966.
- [10] D. Chisholm, L.A. Sutherland, Prediction of pressure gradients in pipeline system during two-phase flow, *Proc. Inst. Mech. Eng.* 184 (Pt. 3C) (1969) 24–32.
- [11] M. Wadle, A new formula for the pressure recovery in an abrupt diffuser, *Int. J. Multiphase Flow* 15 (2) (1989) 241–256.
- [12] J. Schmidt, L. Friedel, Two-phase flow pressure change across sudden expansions in duct areas, *Chem. Eng. Commun.* 141–142 (1996) 175–190.
- [13] A. Attou, M. Giot, J.M. Seynhaeve, Modelling of steady-state two-phase bubbly flow through a sudden enlargement, *Int. J. Heat Mass Transfer* 40 (14) (1997) 3375–3385.
- [14] A. Attou, L. Bolle, A new correlation for the two-phase pressure recovery downstream from a sudden enlargement, *Chem. Eng. Technol.* 20 (1997) 419–423.
- [15] E.F. Abdelall, G. Hahm, S.M. Ghiaasiaan, S.I. Abdel-Khalik, S.S. Jeter, M. Yoda, D.L. Sadowski, Pressure drop caused by abrupt flow area changes in small channels, *Exp. Thermal Fluid Sci.* 29 (2005) 425–434.
- [16] W.H. Ahmed, Two-Phase Flow through Sudden Area Expansions, Ph.D. Thesis, McMaster University, Hamilton, Ontario, Canada, 2005.
- [17] W.H. Ahmed, C.Y. Ching, M. Shoukri, Pressure recovery of two-phase flow across sudden expansions, *Int. J. Multiphase Flow* 33 (2007) 575–594.
- [18] W.H. Ahmed, C.Y. Ching, M. Shoukri, Development of two-phase flow downstream of a horizontal sudden expansion, *Int. J. Heat Fluid Flow* 29 (2008) 194–206.
- [19] L. Owen, A. Abdou-Ghani, A.M. Amini, Diffusing a homogenized two-phase flow, *Int. J. Multiphase Flow* 18 (1992) 531–540.
- [20] J. Schmidt, L. Friedel, Two-phase flow pressure drop across sudden contractions in duct areas, *Int. J. Multiphase Flow* 23 (2) (1997) 283–299.
- [21] S.M. Zivi, Estimation of steady state steam void-fraction by means of principle of minimum entropy production, *ASME Trans. C* 86 (1964) 237–252.
- [22] Y. Taitel, A.E. Dukler, A model for predicting flow regime transitions in horizontal and near horizontal gas–liquid flow, *AIChE J.* 22 (1976) 47–55.
- [23] F. Aloui, L. Doubliez, J. Legarand, M. Souhar, Bubbly flow in an axisymmetric sudden expansion: pressure drop, Void fraction, wall shear stress, bubble velocities and sizes, *Exp. Therm. Fluid Sci.* 19 (1999) 118–130.
- [24] A.S. Dalkilic, S. Laohalertdecha, S. Wongwises, Effect of void fraction models on the two-phase friction factor of R134a during condensation in vertical downward flow in a smooth tube, *Int. Commun. Heat Mass Transfer* 35 (2008) 921–927.
- [25] J.R.S. Thom, Prediction of pressure drop during forced circulation boiling of water, *Int. J. Heat Mass Transfer* 7 (1964) 709–724.
- [26] M.A. Woldeemayat, A.J. Ghajar, Comparison of void fraction correlations for different flow patterns in horizontal and upward inclined pipes, *Int. J. Multiphase Flow* 33 (2007) 347–370.
- [27] K.A. Tripplet, S.M. Ghiaasiaan, S.L. Abdel-Khalik, A. LeMouel, B.N. McCord, Gas–liquid two-phase flow in microchannels. Part II: Void fraction and pressure drop, *Int. J. Multiphase Flow* 25 (1999) 395–410.
- [28] I.Y. Chen, K.S. Yang, C.C. Wang, An empirical correlation for two-phase frictional performance in small diameter tubes, *Int. J. Heat Mass Transfer* 45 (2002) 3667–3671.
- [29] I.Y. Chen, S.Y. Lin, C.C. Wang, Measurements and correlations of frictional single-phase and two-phase pressure drops of R-410A in small U-type wavy tubes, *Int. J. Heat Mass Flow* 47 (10–11) (2004) 2241–2249.
- [30] L. Friedel, Improved friction pressure drop correlations for horizontal and vertical two-phase pipe flow, in: *European Two-Phase Group Meeting*, Ispra, Italy, Paper E2, 1979.
- [31] I.Y. Chen, K.S. Yang, Y.J. Chang, C.C. Wang, Two-phase pressure drop of air–water and R-410A in small horizontal tubes, *Int. J. Multiphase Flow* 27 (2001) 1293–1299.