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ORIGINAL PAPER

Fuzzy decision maps: a generalization of the DEMATEL methods

Gwo-Hshiung Tzeng · Wen-Hsien Chen · Rachung Yu · Meng-Lin Shih

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Abstract The Decision making trial and evaluation laboratory (DEMATEL) method is used to build and analyze a structural model with causal relationships between different criteria. In this paper, it shows that DEMATEL is the specific case of fuzzy decision maps (FDM) when the threshold function is linear. Both FDM and DEMATEL have the same direct and indirect influence matrix. FDM incorporates the eigenvalue method, the fuzzy cognitive maps, and the weighting equation. In addition two numerical examples are illustrated to demonstrate the proposed results. On the basis of the mathematical proof and numerical results, we can conclude that FDM is a generalization of DEMATEL method.

G.-H. Tzeng
Department of Business Administration,
Kainan University, Taoyuan, Taiwan

G.-H. Tzeng
Institute of Management of Technology,
National Chiao Tung University, Hsinchu, Taiwan

W.-H. Chen
Department of Management Information System,
Takming University of Science and Technology,
Taipei, Taiwan

R. Yu (✉)
Department of Information Technology,
Ching Kuo Institute of Management and Health,
Keelung, Taiwan
e-mail: ray@ems.cku.edu.tw

M.-L. Shih
Department of Information Management,
National Chung Cheng University,
Ming-Hsiung, Taiwan

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1 Introduction

Making decisions is the part of our daily lives. The major consideration is that almost all decision problems involved multiple, usually conflicting, and interactive criteria. Many methods in multiple criteria decision making (MCDM) had developed to solve the problems (Chen and Hwang 1992). These methods are based on multiple attribute utility theory (MAUT), have been proposed (e.g., the weighted sum and the weighted product methods) to deal with the MCDM problems. The concept of MAUT is to aggregate all criteria to a specific unique-dimension which is called utility function to evaluate alternatives. Although many papers have been proposed to discuss the aggregation operator of MAUT (Fishburn 1970), the main problem of MAUT is the assumption of preferential independence (Grabisch 1995; Hillier 2001).

Utility independence or utility separability is usually the basic assumption of the multiple attribute decision making (MADM) methods for employing the additive function to represent the preferences of decision-makers. However, in the realistic problems, the assumption of utility independence or utility separability seems to be irrational. Therefore, it is interesting to clarify the structure among criteria, and then we can determine the appropriate MADM methods based on the results of structural models.

Both Decision Making Trial and Evaluation Laboratory (DEMATEL) method and fuzzy decision map (FDM) could clarify the structure among criteria (Chih et al. 2006; Huang and Tzeng 2007; Liou James et al. 2007; Yu and

Tzeng 2006) for solving MCDM problems in preferential weights by using the AHP (independence) or ANP (dependence and feedback) or fuzzy integral (inter-dependent/relation). The assumption of the DEMATEL method is that it's the direct and indirect influence matrix is generating by linear transformation. Comparing to the DEMATEL, the FDM will have better fit in the real world situation due to the flexible threshold function. In this paper, it uses mathematical proof to show that FDM is a generalization method of the DEMATAL. Two numerical examples are illustrated to demonstrate the proposed results.

The rest part of this paper is organized as follows. In Sect. 2, we describe the contents of the DEMATEL method. FDMs and mathematical proof are proposed in Sect. 3. Two numerical examples, which are used here to demonstrate the proposed results, are in Sect. 4. Discussions are presented in Sect. 5 and conclusions are in last section.

2 The DEMATEL method

The DEMATEL method, developed by the Science and Human Affairs Program of the Battelle Memorial Institute of Geneva between 1972 and 1976, was used for researching and solving the complicated and intertwined problem group (Fontela and Gabus 1976; Gabus and Fontela 1972; Warfield 1976). DEMATEL was developed in the belief that pioneering and appropriate use of scientific research methods could improve understanding of the specific problematic, the cluster of intertwined problems, and contribute to identification of workable solutions by a hierarchical structure. The methodology, according to the concrete characteristics of objective affairs, can confirm the interdependence among the variables/attributes and restrict the relation that reflects the characteristics with an essential system and development trend (Hori and Shimizu 1999; Tzeng et al. 2007). Using the DEMATEL method to size and process individual subjective perceptions, brief and impressionistic human insights into problem complexity can be gained. Following the DEMATEL process the end product of the analysis is a visual representation, an individual map of the mind, according to which the respondent organizes his own action in the world, if he is to keep internally coherent to respect his implicit priorities and to reach his secret goals.

The steps of the DEMATEL method can be described as follows:

2.1 Step 1: calculate the average matrix by scores

Respondents are asked to indicate the direct influence that they believe each factor/element i exerts on each factor/element j of the others, as indicated by a_{ij}, using an integer scale ranging, for example from 0 to 4 (going from "No influence (0)," to "Extreme strong influence (4)"). The higher score indicates that the respondent has expressed that the insufficient involvement in problem of factor i exerts the stronger possible direct influence on the inability of factor j, or, in positive terms, that greater improvement i is required to improve j.

From any group of direct matrices of respondents it is possible to derive an average matrix $\boldsymbol{A}$. Each element of this average matrix will be in this case the mean of the same elements in the different direct matrices of the respondents.

2.2 Step 2: calculate the initial direct influence matrix

The initial direct influence matrix $\boldsymbol{D}$ can be obtained by normalizing the average matrix $\boldsymbol{A}$, in which all principal diagonal elements are equal to zero. Based on matrix $\boldsymbol{D}$, the initial influence which a factor dispatches to and receives from another is shown.

The elements of matrix $\boldsymbol{D}$ portrays a contextual relation among the elements of the system and can be converted into a visible structural model, an impact- digraph-map, of the system with respect to that relation. For example, as in Fig. 1, the respondents are asked to indicate only direct links. In the directed digraph graph represented here, factor i affects directly only factors j and k; indirectly, it also affects first l, m and n and, secondly, o and q.

2.3 Step 3: derive the full direct/indirect influence matrix

A continuous decrease of the indirect effects of problems along the powers of matrix $\boldsymbol{D}$, e.g., D^2, D^3 ,…, D^∞, and therefore guarantees convergent solutions to matrix inversion. In a configuration such as Fig. 1, the influence exerted by factor i on factor p will be smaller than the one that it exerts on factor m, and again smaller than the one exerted

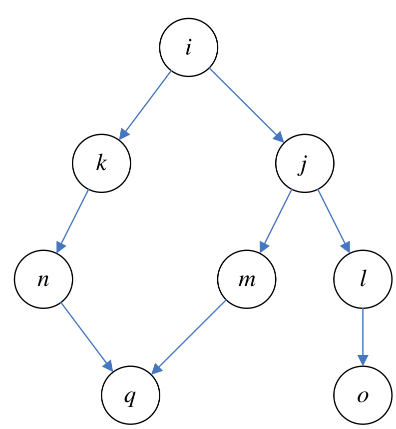

Fig. 1 An example of direct graph

on factor j. This being so, the infinite series of direct and indirect effects can be illustrated. Let the (i, j) element of matrix $\boldsymbol{A}$ is denoted by a_{ij}, the matrix can be gained following Eqs. 1–4.

$$\boldsymbol{D} = s \cdot \boldsymbol{A}, \quad s > 0 \tag{1}$$

or

$$[d_{ij}]_{n\times n} = s \cdot [a_{ij}]_{n\times n}, \quad s > 0, \quad i, j \in \{1, 2, \ldots, n\} \tag{2}$$

where

$$s = \mathrm{Min}\left[\frac{1}{\max_{1\le i\le n}\sum_{j=1}^{n}|a_{ij}|}, \frac{1}{\max_{1\le i\le n}\sum_{i=1}^{n}|a_{ij}|}\right] \tag{3}$$

and

$$\lim_{m\to\infty} \boldsymbol{D}^m = [0]_{n\times n}, \quad \text{where } \boldsymbol{D} = [d_{ij}]_{n x n}, \quad 0 \le d_{ij} < 1 \tag{4}$$

where $0 \le \left\{\sum_{j=1}^{n} d_{ij} \text{ or } \sum_{i=1}^{n} d_{ij}\right\} < 1$ and at least one $\sum_{j=1}^{n} d_{ij}$ or $\sum_{i=1}^{n} d_{ij}$ equal 1, $\forall i, j \in \{1, 2, \ldots, n\}$, then $\lim_{m\to\infty} D^m = [0]_{n\times n}$.

The full direct/indirect influence matrix $\boldsymbol{F}$, the infinite series of direct and indirect effects of each factor, can be obtained by the matrix operation of $\boldsymbol{D}$. The matrix $\boldsymbol{F}$ can show the final structure of factors after the continuous process (see Eq. 5). Let $W_i(f)$ denote the normalized ith row sum of matrix $\boldsymbol{F}$, thus, the $W_i(f)$ value means the sum of influence dispatching from factor i to the other factors both directly and indirectly. The $V_i(f)$, the normalized ith column sum of matrix $\boldsymbol{F}$, means the sum of influence that factor i receives from the other factors.

The total-influence matrix $\boldsymbol{T}$ can be obtained by using Eq. 5 where $\boldsymbol{I}$ is denoted as the identity matrix.

$$\boldsymbol{F} = \boldsymbol{D} + \boldsymbol{D}^2 + \cdots + \boldsymbol{D}^m = \sum_{i=1}^{m} \boldsymbol{D}^i = \boldsymbol{D}(\boldsymbol{I} - \boldsymbol{D})^{-1}, \quad \text{when } m \to \infty \tag{5}$$

2.4 Step 4: set threshold value and obtain the impact-digraph- map

Setting a threshold value, p, to filter the *obvious* effects denoted by the elements of matrix $\boldsymbol{F}$, is necessary to explain the structure of factors. Base on the matrix $\boldsymbol{F}$, each element, f_{ij}, of matrix $\boldsymbol{F}$ provides the information about a factor i dispatches influence to factor j, or, in another word, factor j receives influence from factor i. If all the information from matrix $\boldsymbol{F}$ converts to the impact-digraph-map, it will be too complex to show the necessary information for decision-making. In order to obtain an appropriate impact-digraph-map, setting a threshold value of the influence level is necessary for the decision maker. Only some elements, whose influence level in matrix $\boldsymbol{F}$ higher than the threshold value, can be chosen and converted into the impact-digraph-map.

The threshold value is decided by the decision makers or, in this paper, experts through discussions. Like matrix $\boldsymbol{D}$, contextual relation among the elements of matrix $\boldsymbol{F}$ can also be converted into a digraph map. If the threshold value is too low, the map will be too complex to show the necessary information for decision-making. If the threshold value is too high, many factors will be presented as independent factors without relations to another factor. Each time the threshold value increases, some factors or relationship will be removed from the map. An appropriate threshold value is necessary to obtain a suitable impact-digraph-map and proper information for the further analysis and decision-making.

After threshold value and relative impact-digraph-map are decided, the final influence result can be shown. For example, the impact-digraph-map of a factor is the same as Fig. 1 and eight elements exist in this map. Because of continuous direct/indirect effects between them, finally, the effectiveness of these eight elements could be considered to be represented by two independent *final affected elements*: o and q. The other components, not shown in the impact-digraph-map, of a factor can be considered as independent elements because no obvious interrelation with others exists.

3 Fuzzy decision maps

In order to deal with the problem of dependence and feedback among criteria, we first depict the FCM as shown in Fig. 2 to illustrate the situation of decision making. In Fig. 2, e_{ij} denotes the interaction effect from the ith criterion to the jth criterion, and e_{ii} indicates the compound effect of the ith criterion by self-relation. As we know, due to the problem of compound and interaction effects, it is hard for decision makers to make a good decision using the simple weighted method.

One way to overcome the problems above is to obtain the information of influences among criteria and then to derive the finial weights by considering the influences among criteria. However, since these criteria may have loop or feedback relationships, it is hard to derive the influences among criteria. Next, we first employ the FCM to derive the influence among criteria and then obtain the finial weights by using the weighted formulation.

FCM, which was first proposed by Koska (1988) and Sekitani and Takahashi (2001), extends the original cognitive maps of political elite (Axelrod 1976) by incorporating fuzzy measures to provide a flexible and realistic method for extracting the fuzzy relationships among objects

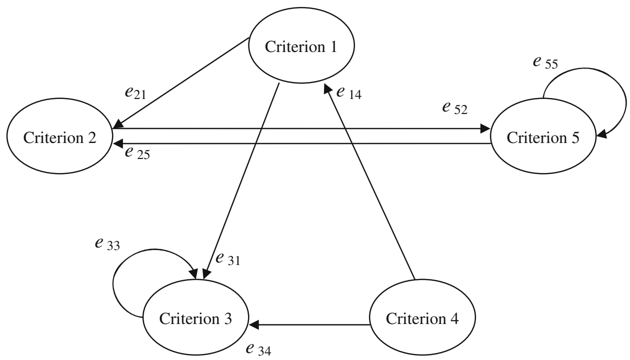

Fig. 2 The problem of a decision map

in a complex system. Therefore, recently, FCM have been widely employed in the applications of political decision-making, business management, industrial analysis, and system control (Andreou et al. 2005; Pagageorgiou and Groumpos 2005; Stylios and Groumpos 2004), except for the area of MCDM. The concepts of FCM can be described as follows.

Given a 4-tuple $(N, \boldsymbol{E}, \boldsymbol{C}, f)$ where $N = \{N_1, N_2, \ldots, N_n\}$ denotes the set of n objects, E denotes the connection matrix (initial direct-influence matrix) which is composed of the weights between objects, C is the state matrix, where $C^{(0)}$ is the initial matrix and $C^{(t)}$ is the transition-state matrix at certain iteration t, and f is a threshold function, which indicates the weighting relationship between $C^{(t)}$ and $C^{(t+1)}$. Several formulas have been used as threshold functions such as

$$f(x) = \begin{cases} 1 & \text{if} \quad x \geq 1 \\ 0 & \text{if} \quad x < 1 \end{cases}, \quad \text{(Hard line function)}$$

$$f(x) = \tanh(x) = (1 - e^{-x})/(1 + e^{-x}), \quad \text{(Hyperbolic} - \text{tangent function)}$$

and

$$f(x) = 1/(1 + e^{-x}). \quad \text{(Logistic function)}$$

The different threshold functions will get the discriminative numerical results and a sensitive analysis was proposed by Chen et al. (2008).

The influence of the specific criterion to other criteria can be calculated using the following updating equation:

$$\boldsymbol{C}^{(t+1)} = f(\boldsymbol{C}^{(t)}\boldsymbol{E}), \quad \boldsymbol{C}^{(0)} = \boldsymbol{I}_{n\times n} \tag{6}$$

where $\boldsymbol{I}_{n\times n}$ denotes the identity matrix.

The vector–matrix multiplication operation to derive successive FCM transition-states is iterated until it converges to a fixed point situation or a limit steady-state cycle. The steady-state vector–matrix remains unchanged for successive iterations is called a fixed point situation and the sequence of the steady-state vector–matrix keeps repeating indefinitely is called a limit state cycle. Now, we can summarize the proposed method to derive the priorities of criteria as follows:

Step 1 Compare the importance among criteria to derive the local weight vector using the eigenvalue approach;

Step 2 Depict the fuzzy cognitive map (FCM) to build the influence among criteria by the expert;

Step 3 Calculate Eq. 6 for obtaining the steady-state matrix;

Step 4 Derive the global weight vector. In order to derive the global weights, we should first normalize the local weight vector (z) and the steady-state matrix ($\boldsymbol{C}^*$) as follows:

$$z_t = \frac{1}{\lambda} z, \tag{7}$$

and

$$\boldsymbol{C}_t^* = \frac{1}{\gamma} \boldsymbol{C}^* \tag{8}$$

where λ is the largest element of z and γ is the largest row sum of $\boldsymbol{C}^*$. Then, we can obtain the global weight vector by using the following weighting equation:

$$\boldsymbol{w} = z_t + \boldsymbol{C}_t^* z_t. \tag{9}$$

3.1 Mathematical proof

In this section, we show that DEMATEL is the specific case of FDM when the threshold function is linear. Both FDM and DEMATEL have the same direct and indirect influence matrix.

Consider the following threshold function using in FDM.

$$f(x) = x \quad \text{(pure} - \text{linear function)}$$

In FDM, it uses Eq. 6 to obtain the direct and indirect influence matrix.

$\boldsymbol{C}^0 = \boldsymbol{I}_{n\times n}$; $\boldsymbol{C}^1 = \boldsymbol{C}^0\boldsymbol{E} = \boldsymbol{E}$, $\boldsymbol{E}$ is the initial direct-relation matrix.

$$\begin{aligned}
\boldsymbol{C}^t &= f(\boldsymbol{C}^{t-1}\boldsymbol{E} + \boldsymbol{C}^0) \\
&= (\boldsymbol{C}^{t-1}\boldsymbol{E} + \boldsymbol{C}^0)\boldsymbol{E} \\
&= f(\boldsymbol{C}^{t-2})\boldsymbol{E}^2 + \boldsymbol{E} \\
&= ((\boldsymbol{C}^{t-2}\boldsymbol{E} + \boldsymbol{C}^0)\boldsymbol{E}^2 + \boldsymbol{E} = f(\boldsymbol{C}^{t-3})\boldsymbol{E}^3 + \boldsymbol{E}^2 + \boldsymbol{E} \\
&= ((\boldsymbol{C}^{t-3}\boldsymbol{E} + \boldsymbol{C}^0)\boldsymbol{E}^3 + \boldsymbol{E}^2 + \boldsymbol{E} = f(\boldsymbol{C}^{t-4})\boldsymbol{E}^4 + \boldsymbol{E}^3 + \boldsymbol{E}^2 + \boldsymbol{E} \\
&\cdots \\
&= \boldsymbol{E}^t + \boldsymbol{E}^{t-1} + \cdots + \boldsymbol{E} \\
&= \boldsymbol{E}(\boldsymbol{I} + \boldsymbol{E} + \boldsymbol{E}^2 + \cdots + \boldsymbol{E}^{t-1})(\boldsymbol{I} - \boldsymbol{E})(\boldsymbol{I} - \boldsymbol{E})^{-1} \\
&= \boldsymbol{E}[(\boldsymbol{I} + \boldsymbol{E} + \boldsymbol{E}^2 + \cdots + \boldsymbol{E}^{t-1})(\boldsymbol{I} - \boldsymbol{E})](\boldsymbol{I} - \boldsymbol{E})^{-1} \\
&= \boldsymbol{E}[(\boldsymbol{I} + \boldsymbol{E} + \boldsymbol{E}^2 + \cdots + \boldsymbol{E}^{t-1}) - (\boldsymbol{E} + \boldsymbol{E}^2 + \cdots + \boldsymbol{E}^t)](\boldsymbol{I} - \boldsymbol{E})^{-1} \\
&= \boldsymbol{E}(\boldsymbol{I} - \boldsymbol{E}^t)(\boldsymbol{I} - \boldsymbol{E})^{-1}
\end{aligned}$$

and

$$\lim_{t\to\infty} \boldsymbol{E}^t = [0]_{n\times n}, \text{ where } \boldsymbol{E} = [e_{ij}]_{n\times n},$$

$$0 \le e_{ij} < 1 \text{ and } \sum_{i=1}^{n} e_{ij} \le 1.$$

Therefore, $\boldsymbol{C}^t = \lim_{t\to\infty} \sum_{i=1}^{t} \boldsymbol{E}^i = \boldsymbol{E}(\boldsymbol{I}-\boldsymbol{E})^{-1}$, then the formula of FDM equals to that of DEMETEL (Eq. 5). So, DEMATEL is a specific case of FDM.

Next it uses two numerical examples to demonstrate the proposed results in Sect. 4 and two extra examples in Appendix 3 to support our research findings.

4 Numerical examples

In this section, two numerical examples are illustrated to demonstrate the proposed results and compared the results between in the FDM and the DEMATEL. The first example is the multi-criteria decision problem about suppliers-evaluation. Then the second example is the more complicated decision problem about customers' evaluation to product purchasing. Note that in this paper we use the threshold function that is the pure-linear function in FDM to indicate the relationships among criteria. In Appendix 3, two extra examples are proposed to reinforce our research results.

Example 1 Consider a decision maker tries to select the suppliers according the following criteria including Quality (Q), Matching (M), Lead-time (L), and Costs (C). For choosing the best alternative, we should derive the influence scores of each criterion and calculate the influence scores of each supplier. Figure 3 and scoring matrix show the relationships between criteria (scale: 0 no influences; 1 low influences; 2 moderate influences; 3 strong influences; 4 extreme strong influences).

And scoring matrix as

$$\boldsymbol{S} = \begin{array}{c} \\ Q \\ C \\ L \\ M \end{array} \begin{array}{c} \begin{array}{cccc} Q & C & L & M \end{array} \\ \begin{bmatrix} 0 & 4 & 1 & 0 \\ 2 & 0 & 0 & 3 \\ 1 & 2 & 0 & 0 \\ 4 & 2 & 2 & 0 \end{bmatrix} \end{array}, \quad \max_j \sum_{i=1}^{n} e_{ij} = \max_j [7,8,3,3] = 8 \text{ and } \max_i \sum_{j=1}^{n} e_{ij} = \max_j [5,5,3,8]$$

$$k = \min_{i,j} \left[\frac{1}{\max_j \sum_{i=1}^{n} e_{ij}}, \frac{1}{\max_i \sum_{j=1}^{n} e_{ij}} \right] = \frac{1}{8}.$$

From the Fig. 3 and scoring matrix, we can calculate the grades/degrees of scoring matrix by dividing the max row sum, i.e., $\boldsymbol{E} = k\boldsymbol{S}$. The grades/degrees of direct influence matrix as follow:

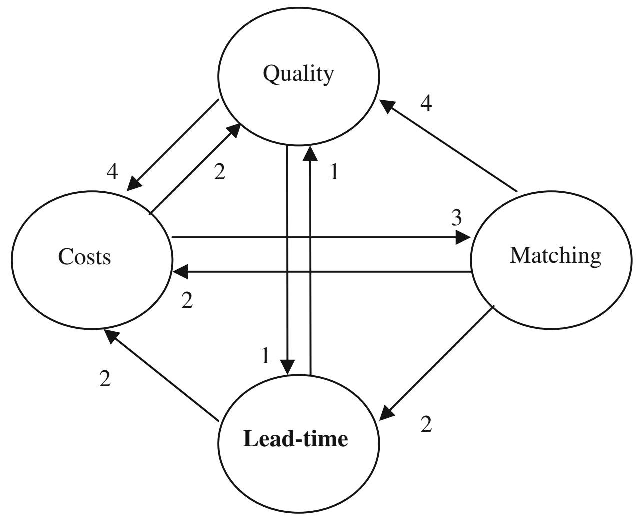

Fig. 3 The evaluation of a supplier

$$\boldsymbol{E} = \begin{array}{c} \\ Q \\ C \\ L \\ M \end{array} \begin{array}{c} \begin{array}{cccc} Q & C & L & M \end{array} \\ \begin{bmatrix} 0 & 0.50 & 0.125 & 0 \\ 0.25 & 0 & 0 & 0.375 \\ 0.125 & 0.25 & 0 & 0 \\ 0.5 & 0.25 & 0.25 & 0 \end{bmatrix} \end{array}$$

Next, we can obtain the steady-state matrix by calculating Eq. 6 in the FDM method and Eq. 5 in the DEMATEL as follows (see Appendix 1):

DEMATEL	Quality	Costs	Lead-time	Matching
Quality	0.4005	0.8428	0.2541	0.3160
Costs	0.7126	0.5616	0.2355	0.5856
Lead-time	0.3532	0.4957	0.0906	0.1859
Matching	0.9667	0.9357	0.4586	0.3509
FDM ($f(x) = x$)				
Quality	0.4004[a]	0.8427[a]	0.2541	0.3160
Costs	0.7126	0.5615[a]	0.2355	0.5856
Lead-time	0.3532	0.4957	0.0906	0.1859
Matching	0.9667	0.9357	0.4585[a]	0.3509

[a] Means the difference between FDM and DEMATEL is 0.0001

Example 2 In this example, consider a customer to purchase a product according to the following five criteria including Quality (Q), Delivery (D), Price (P), Yield (Y), and Service (S). Figure 4 and scoring matrix display the relationships between criteria (Scale: 0 no influences; 1 low influences; 2 moderate influences; 3 strong influences; 4 extreme strong influences).

And scoring matrix as

$$\boldsymbol{E} = \begin{array}{c} \\ Q \\ D \\ P \\ Y \\ S \end{array} \begin{array}{c} \begin{array}{ccccc} Q & D & P & Y & S \end{array} \\ \begin{bmatrix} 0 & 2 & 4 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 3 & 0 \end{bmatrix} \end{array},$$

$$\max_j \sum_{i=1}^{n} e_{ij} = \max_j [2, 5, 7, 4, 8] = 8$$

and

$$\max_i \sum_{j=1}^{n} e_{ij} = \max_j [9, 1, 9, 2, 5] = 9, \text{ then}$$

$$k = \min_{i,j} \left[\frac{1}{\max_j \sum_{i=1}^{n} e_{ij}}, \frac{1}{\max_i \sum_{j=1}^{n} e_{ij}} \right] = \frac{1}{9}.$$

From the Fig. 4 and scoring matrix, we can calculate the grades/degrees of scoring matrix by dividing the max row sum, i.e., $\boldsymbol{E} = k\boldsymbol{S}$. The grades/degrees of direct influence matrix as follow:

$$\boldsymbol{E} = \begin{array}{c} \\ Q \\ D \\ P \\ Y \\ S \end{array} \begin{array}{c} \begin{array}{ccccc} Q & D & P & Y & S \end{array} \\ \begin{bmatrix} 0 & 0.2222 & 0.4444 & 0.1111 & 0.2222 \\ 0 & 0 & 0.1111 & 0 & 0 \\ 0.2222 & 0.3333 & 0 & 0 & 0.4444 \\ 0 & 0 & 0 & 0 & 0.2222 \\ 0 & 0 & 0.2222 & 0.3333 & 0 \end{bmatrix} \end{array}$$

Next, we can obtain the steady-state matrix by calculating Eq. 6 in the FDM method and Eq. 5 in the DEMATEL as follows (see Appendix 2):

	Quality	Delivery	Price	Yield	Service
DEMATEL					
Quality	0.1589	0.4958	0.7150	0.3461	0.6522
Delivery	0.0334	0.0575	0.1504	0.0307	0.0811
Price	0.3007	0.5179	0.3533	0.2766	0.7297
Yield	0.0160	0.0276	0.0722	0.0947	0.2789
Service	0.0722	0.1243	0.3248	0.4263	0.2551
FDM ($f(x) = x$)					
Quality	0.1589	0.4958	0.7150	0.3461	0.6521[a]
Delivery	0.0334	0.0575	0.1503[a]	0.0307	0.0811
Price	0.3007	0.5179	0.3533	0.2766	0.7297
Yield	0.0160	0.0276	0.0722	0.0947	0.2789
Service	0.0722	0.1243	0.3247[a]	0.4263	0.2551

[a] Means the difference between FDM and DEMATEL is 0.0001

Both calculation processes of the numerical examples were collected in Appendix 1 and 2. Two extra examples in

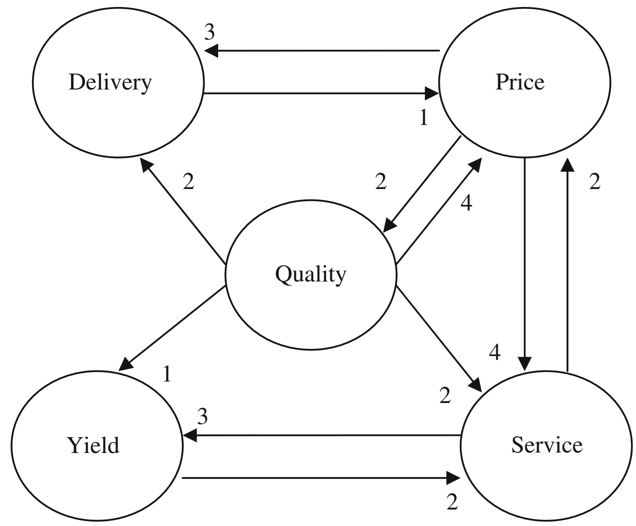

Fig. 4 The product evaluation of a customer

Appendix 3 are proposed to support our research findings. Next section, we provide the depth discussions according to the results of the numerical examples above in Sect. 5.

5 Discussions

Structural MADM problems involve determining the best options by considering the effects among criteria. In order to solve the problems above, the major point is to derive the direct and indirect influence matrix. After getting a direct and indirect influence matrix, it can obtain the impact diagraph map. Although the DEMATEL has been proposed to deal this problem, but it has the limitation that the problems influence must be interactive linearly.

In this paper, the FDMs, which combine the eigenvalue method, the FCMs, and the weighting equation, are proposed to deal with the structural MADM problems. The direct and indirect influence matrix is derived by using the updating equation. Finally, it reaches the steady-state matrix. Under the threshold function is linear, both the DEMATEL and FDM have the direct/indirect influence matrix.

From the results of two numerical examples and mathematical proof above, it exposes three main shortcomings of the DEMATEL method. First, the criteria states may not be interactive linearly in some real world situations. Second, the initial direct matrix of DEMATEL method has the characteristic that its all principal diagonal elements are equal to zero. This characteristic means the DEMATEL method can not handle the criterion which contains self-feedback loop shown in Fig. 2. Third, both the DEMATEL method and FDM method have the discrete-time process with the Markov property. The DEMATEL processes the criteria in different states with dynamical linearly.

In contrast, the benefit of the FDM methods can be summarized as follows. First, the FDM method modifies the shortcoming of the DEMATEL. We can employ the different threshold functions to indicate the various kinds of relationship among criteria. The more flexible threshold functions may have better fitness when construct the direct and indirect influence matrix. Second, the FDM method can treat the problem of compound and interaction effect of the criterion containing self feedback loop. Third, In the FDM method, it can process the dynamic criteria status and exceeds the limitation of the DEMATEL method with dynamical non-linearly.

6 Conclusions

The MCDM problems with the complicated intertwined problem group are hard for the decision-maker to make a good decision. Although the DEMATEL method has been widely used to deal with this problem, the shortcoming should be overcome for proving the satisfaction solution. In this paper, the FDM method is proposed to deal with the structural MCDM problems. Without limiting by the problems influence must be interactive linearly, it can employ the flexible threshold function to get proper solutions in real world cases. On the basis of the numerical results and mathematical proof, we can conclude that FDM is a generalization of DEMATEL method.

Appendix 1

Result of the supplier evaluation in DEMATAL for the operation processes of Example 1

$$\boldsymbol{I} = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} 0.000 & 0.500 & 0.125 & 0.000 \\ 0.250 & 0.000 & 0.000 & 0.375 \\ 0.125 & 0.250 & 0.000 & 0.000 \\ 0.500 & 0.250 & 0.250 & 0.000 \end{bmatrix}$$

$$\boldsymbol{I} - \boldsymbol{D} = \begin{bmatrix} 1.000 & -0.500 & -0.125 & 0.000 \\ -0.250 & 1.000 & 0.000 & -0.375 \\ -0.125 & -0.250 & 1.000 & 0.000 \\ -0.500 & -0.250 & -0.250 & 1.000 \end{bmatrix}$$

$$(\boldsymbol{I} - \boldsymbol{D})^{-1} = \begin{bmatrix} 1.4005 & 0.8428 & 0.2541 & 0.3160 \\ 0.7126 & 1.5616 & 0.2355 & 0.5856 \\ 0.3532 & 0.4957 & 1.0906 & 0.1859 \\ 0.9667 & 0.9357 & 0.4586 & 1.3509 \end{bmatrix}$$

$$\boldsymbol{F} = \boldsymbol{D}(\boldsymbol{I} - \boldsymbol{D})^{-1} = \begin{bmatrix} 0.4005 & 0.8428 & 0.2541 & 0.3160 \\ 0.7126 & 0.5616 & 0.2355 & 0.5856 \\ 0.3532 & 0.4957 & 0.0906 & 0.1859 \\ 0.9667 & 0.9357 & 0.4586 & 0.3509 \end{bmatrix}$$

Result of the supplier evaluation in FDM for the operation processes of Example 1

$\boldsymbol{E} = \boldsymbol{D}$

$$\boldsymbol{E} = \begin{bmatrix} 0.0000 & 0.5000 & 0.1250 & 0.0000 \\ 0.2500 & 0.0000 & 0.0000 & 0.3750 \\ 0.1250 & 0.2500 & 0.0000 & 0.0000 \\ 0.5000 & 0.2500 & 0.2500 & 0.0000 \end{bmatrix}$$

$\boldsymbol{C}^0 = \boldsymbol{I}$

$$\boldsymbol{C}^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{C}^1 = \boldsymbol{C}^0\,\boldsymbol{E} = \begin{bmatrix} 0.0000 & 0.5000 & 0.1250 & 0.0000 \\ 0.2500 & 0.0000 & 0.0000 & 0.3750 \\ 0.1250 & 0.2500 & 0.0000 & 0.0000 \\ 0.5000 & 0.2500 & 0.2500 & 0.0000 \end{bmatrix}$$

$$\boldsymbol{C}^2 = (\boldsymbol{C}^1 + \boldsymbol{C}^0)\,\boldsymbol{E} = \begin{bmatrix} 0.1406 & 0.5313 & 0.1250 & 0.1875 \\ 0.4375 & 0.2188 & 0.1250 & 0.3750 \\ 0.1875 & 0.3125 & 0.0156 & 0.0938 \\ 0.5938 & 0.5625 & 0.3125 & 0.0938 \end{bmatrix}$$

$$\boldsymbol{C}^3 = (\boldsymbol{C}^2 + \boldsymbol{C}^0)\,\boldsymbol{E} = \begin{bmatrix} 0.2422 & 0.6484 & 0.1895 & 0.1992 \\ 0.5078 & 0.3438 & 0.1484 & 0.4570 \\ 0.2520 & 0.3711 & 0.0469 & 0.1172 \\ 0.7266 & 0.6484 & 0.3477 & 0.2109 \end{bmatrix}$$

…

$$\boldsymbol{C}^{24} = (\boldsymbol{C}^{23} + \boldsymbol{C}^0)\,\boldsymbol{E} = \begin{bmatrix} 0.4004 & 0.8427 & 0.2541 & 0.3160 \\ 0.7126 & 0.5615 & 0.2355 & 0.5856 \\ 0.3532 & 0.4957 & 0.0906 & 0.1859 \\ 0.9666 & 0.9357 & 0.4585 & 0.3509 \end{bmatrix}$$

$$\boldsymbol{C}^{25} = (\boldsymbol{C}^{24} + \boldsymbol{C}^0)\,\boldsymbol{E} = \begin{bmatrix} 0.4004 & 0.8427 & 0.2541 & 0.3160 \\ 0.7126 & 0.5615 & 0.2355 & 0.5856 \\ 0.3532 & 0.4957 & 0.0906 & 0.1859 \\ 0.9667 & 0.9357 & 0.4585 & 0.3509 \end{bmatrix}$$

Appendix 2

Result of the customer evaluation in DEMATAL for the operation processes of Example 2

$$\boldsymbol{I} = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} 0.0000 & 0.2222 & 0.4444 & 0.1111 & 0.2222 \\ 0.0000 & 0.0000 & 0.1111 & 0.0000 & 0.0000 \\ 0.2222 & 0.3333 & 0.0000 & 0.0000 & 0.4444 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2222 \\ 0.0000 & 0.0000 & 0.2222 & 0.3333 & 0.0000 \end{bmatrix}$$

$$\boldsymbol{I}-\boldsymbol{D} = \begin{bmatrix} 1.0000 & -0.2222 & -0.4444 & -0.1111 & -0.2222 \\ 0.0000 & 1.0000 & -0.1111 & 0.0000 & 0.0000 \\ -0.2222 & -0.3333 & 1.0000 & 0.0000 & -0.4444 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & -0.2222 \\ 0.0000 & 0.0000 & -0.2222 & -0.3333 & 1.0000 \end{bmatrix}$$

$$(\boldsymbol{I}-\boldsymbol{D})^{-1} = \begin{bmatrix} 1.1589 & 0.4958 & 0.7150 & 0.3461 & 0.6522 \\ 0.0334 & 1.0575 & 0.1504 & 0.0307 & 0.0811 \\ 0.3007 & 0.5179 & 1.3533 & 0.2766 & 0.7297 \\ 0.0160 & 0.0276 & 0.0722 & 1.0947 & 0.2789 \\ 0.0722 & 0.1243 & 0.3248 & 0.4263 & 1.2551 \end{bmatrix}$$

$$\boldsymbol{F} = \boldsymbol{D}(\boldsymbol{I}-\boldsymbol{D})^{-1} = \begin{bmatrix} 0.1589 & 0.4958 & 0.7150 & 0.3461 & 0.6522 \\ 0.0334 & 0.0575 & 0.1504 & 0.0307 & 0.0811 \\ 0.3007 & 0.5179 & 0.3533 & 0.2766 & 0.7297 \\ 0.0160 & 0.0276 & 0.0722 & 0.0947 & 0.2789 \\ 0.0722 & 0.1243 & 0.3248 & 0.4263 & 0.2551 \end{bmatrix}$$

Result of the customer evaluation in FDM for the operation processes of Example 2

$\boldsymbol{E} = \boldsymbol{D}$

$$\boldsymbol{E} = \begin{bmatrix} 0 & 0.2222 & 0.4444 & 0.1111 & 0.2222 \\ 0 & 0 & 0.1111 & 0 & 0 \\ 0.2222 & 0.3333 & 0 & 0 & 0.4444 \\ 0 & 0 & 0 & 0 & 0.2222 \\ 0 & 0 & 0.2222 & 0.3333 & 0 \end{bmatrix}$$

$\boldsymbol{C^0} = \boldsymbol{I}$

$$\boldsymbol{C}^0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{C}^1 = \boldsymbol{C}^0 \times \boldsymbol{E} = \begin{bmatrix} 0 & 0.2222 & 0.4444 & 0.1111 & 0.2222 \\ 0 & 0 & 0.1111 & 0 & 0 \\ 0.2222 & 0.3333 & 0 & 0 & 0.4444 \\ 0 & 0 & 0 & 0 & 0.2222 \\ 0 & 0 & 0.2222 & 0.3333 & 0 \end{bmatrix}$$

$$\boldsymbol{C}^2 = (\boldsymbol{C}^1 + \boldsymbol{C}^0)\,\boldsymbol{E} = \begin{bmatrix} 0.0987 & 0.3703 & 0.5185 & 0.1852 & 0.4444 \\ 0.0247 & 0.0370 & 0.1111 & 0 & 0.0494 \\ 0.2222 & 0.3827 & 0.2345 & 0.1728 & 0.4938 \\ 0 & 0 & 0.0494 & 0.0741 & 0.2222 \\ 0.0494 & 0.0741 & 0.2222 & 0.3333 & 0.1728 \end{bmatrix}$$

$$\boldsymbol{C}^3 = (\boldsymbol{C}^2 + \boldsymbol{C}^0)\,\boldsymbol{E} = \begin{bmatrix} 0.1152 & 0.4169 & 0.6282 & 0.2702 & 0.5157 \\ 0.0247 & 0.0425 & 0.1372 & 0.0192 & 0.0549 \\ 0.2743 & 0.4608 & 0.2510 & 0.1893 & 0.6364 \\ 0.0110 & 0.0165 & 0.0494 & 0.0741 & 0.2606 \\ 0.0494 & 0.0850 & 0.2908 & 0.3964 & 0.1838 \end{bmatrix}$$

…

$$C^{17} = (C^{16} + C^0)\,E = \begin{bmatrix} 0.1589 & 0.4958 & 0.7150 & 0.3461 & 0.6521 \\ 0.0334 & 0.0575 & 0.1503 & 0.0307 & 0.0811 \\ 0.3007 & 0.5179 & 0.3533 & 0.2766 & 0.7297 \\ 0.0160 & 0.0276 & 0.0722 & 0.0947 & 0.2789 \\ 0.0722 & 0.1243 & 0.3247 & 0.4263 & 0.2551 \end{bmatrix}$$

$$C^{18} = (C^{17} + C^0)\,E = \begin{bmatrix} 0.1589 & 0.4958 & 0.7150 & 0.3461 & 0.6521 \\ 0.0334 & 0.0575 & 0.1503 & 0.0307 & 0.0811 \\ 0.3007 & 0.5179 & 0.3533 & 0.2766 & 0.7297 \\ 0.0160 & 0.0276 & 0.0722 & 0.0947 & 0.2789 \\ 0.0722 & 0.1243 & 0.3247 & 0.4263 & 0.2551 \end{bmatrix}$$

Appendix 3

Two extra examples are proposed to reinforce our research results. Example 3 shows how to evaluate the human capital. Example 4 shows how to evaluate the external structure capital.

Example 3 The human capital includes four indices: leadership (LS), turnover of professional employees (TPE), replacement cost of professional employees (RPE), and team work (TW). The grades/degrees of direct influence matrix as follow:

$$\boldsymbol{E} = \begin{array}{c} \\ \text{LS} \\ \text{TPE} \\ \text{RPE} \\ \text{TW} \end{array} \begin{array}{c} \begin{array}{cccc} \text{LS} & \text{TPE} & \text{RPE} & \text{TW} \end{array} \\ \begin{bmatrix} 0 & 0.3462 & 0.1923 & 0.4615 \\ 0.1923 & 0 & 0.1538 & 0.1154 \\ 0.1538 & 0.3462 & 0 & 0.1538 \\ 0.2692 & 0.3846 & 0.1923 & 0 \end{bmatrix} \end{array}$$

Next, we can obtain the steady-state matrix by calculating Eq. 6 in the FDM method and Eq. 5 in the DEMATEL as follows.

DEMATEL	LS	TPE	RPE	TW
Leadership (LS)	0.5844	1.1515	0.6677	0.9668
Turnover of professional employees (TPE)	0.4648	0.4714	0.4015	0.4460
Replacement cost of professional employees (RPE)	0.5128	0.8462	0.3325	0.5393
Team work (TW)	0.7039	1.0386	0.5904	0.5355

FDM ($f(x) = x$)	LS	TPE	RPE	TW
Leadership (LS)	0.5843[a]	1.1514[a]	0.6677	0.9667[a]
TPE	0.4648	0.4714	0.4014[a]	0.4460
RPE	0.5128	0.8462	0.3325	0.5392[a]
TW	0.7039	1.0386	0.5904	0.5355

[a] Means the difference between FDM and DEMATEL is 0.0001

In above table, the numerical results show the DEMATEL method and the FDM method using linear function almost the same. This finding supports the FDM is a general methods of the DEMATEL method.

Example 4 The external structure capital includes five indices: market share (MS), customer satisfaction (CS), market growth rate (MGR), brand loyalty (BL), and future prospective of product market (FP). The grades/degrees of direct influence matrix as follow:

$$\boldsymbol{E} = \begin{array}{c} \\ MS \\ CS \\ MGR \\ BL \\ FP \end{array} \begin{array}{c} \begin{array}{ccccc} MS & CS & MGR & BL & FP \end{array} \\ \begin{bmatrix} 0 & 0.2903 & 0.1935 & 0.2581 & 0.1613 \\ 0.3226 & 0 & 0.2258 & 0.3226 & 0.1290 \\ 0.1613 & 0.0645 & 0 & 0.0645 & 0.2903 \\ 0.3226 & 0.2903 & 0.1290 & 0 & 0.0968 \\ 0.0968 & 0.0323 & 0.3548 & 0.0323 & 0 \end{bmatrix} \end{array}$$

Next, we can obtain the steady-state matrix by calculating Eq. 6 in the FDM method and Eq. 5 in the DEMATEL as follows.

DEMATEL	MS	CS	MGR	BL	FP
Market share (MS)	0.7212	0.8059	0.8677	0.7831	0.7093
Customer satisfaction (CS)	1.0403	0.6438	0.9494	0.8840	0.7410
Market growth rate (MGR)	0.5287	0.3725	0.4425	0.3687	0.5878
Brand loyalty (BL)	0.9660	0.8133	0.8048	0.5845	0.6477
Future prospective of product market (FP)	0.4190	0.2895	0.6525	0.2863	0.3221

continued

FDM ($f(x) = x$)	MS	CS	MGR	BL	FP
MS	0.7212	0.8059	0.8677	0.7831	0.7093
CS	1.0403	0.6438	0.9494	0.8840	0.7410
MGR	0.5287	0.3725	0.4425	0.3687	0.5878
BL	0.9660	0.8133	0.8048	0.5845	0.6477
Future prospective of FP	0.4190	0.2895	0.6525	0.2863	0.3221

In above table, the numerical results are the same in both two methods. The DEMATEL method and the FDM ($f(x) = x$) have the same values

References

Andreou AS, Mateou NH, Zombanakis GA (2005) Soft computing for crisis management and political decision making: the use of genetically evolved fuzzy cognitive maps. Soft Comput 9(3):194–210

Axelrod R (1976) Structure of decision, the cognitive maps of political elite. Princeton University Press, London

Chen SJ, Hwang CL (1992) Fuzzy multiple attribute decision making: methods and applications. Springer, Berlin

Chen WH, Yu R, Tzeng GH (2008) Comparison between the DEMATEL methods and fuzzy decision maps: a sensitive analysis approach, International Symposium on Management Engineering 2008. Waseda University, Kitakyushu

Chih YJ, Chen HC, Syzu Joseph Z, Tzeng GH (2006) Marketing strategy based on customer behavior for the LCD-TV. Int J Manag Decis Making 7(2/3):143–165

Fishburn PC (1970) Utility theory for decision-making. Wiley, New York

Fontela E, Gabus A (1976) The DEMATEL observer. Battelle Geneva Research Centre, Geneva

Gabus A, Fontela E (1972) World problems, an invitation to further thought within the framework of DEMATEL. Battelle Geneva Research Centre, Geneva

Grabisch M (1995) Fuzzy integral in multicriteria decision making. Fuzzy Sets Syst 69(3):279–298

Hillier FS (2001) Evaluation and decision models: a critical perspective. Kluwer, Boston

Hori S, Shimizu Y (1999) Designing methods of human interface for supervisory control systems. Control Eng Pract 7(11):1413–1419

Huang CY, Tzeng GH (2007) Reconfiguring the innovation policy portfolios for Taiwan's SIP mall industry. Technovation 27(12):744–765

Kosko B (1988) Hidden patterns in combined and adaptive knowledge networks. Int J Approx Reason 2(4):377–393

Liou James JH, Tzeng GH, Chang HC (2007) Airline safety measurement using a novel hybrid model. J Air Transp Manag 13(4):243–249

Pagageorgiou EI, Groumpos PP (2005) A new hybrid method using evolutionary algorithms to train fuzzy cognitive maps. Appl Soft Comput 5(4):409–431

Sekitani K, Takahashi I (2001) A unified model and analysis for AHP and ANP. J Oper Res Soc Jpn 44(1):67–89

Stylios CD, Groumpos PP (2004) Modeling complex systems using fuzzy cognitive maps. IEEE Trans Syst Man Cybern A Syst Hum 34(1):155–162

Tzeng GH, Chiang CH, Li CW (2007) Evaluating intertwined effects in E-learning programs: a novel hybrid MCDM model based on

factor analysis and DEMATEL. Expert Syst Appl 32(4):1028–1044
Warfield JN (1976) Social systems, planning and complexity. Wiley, New York
Yu R, Tzeng GH (2006) A soft computing method for multi-criteria decision making with dependence and feedback. Appl Math Comput 180(1):63–75