Spatial variability of the poroelastic response of the heterogeneous medium

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ABSTRACT

In this work, a stochastic methodology is applied to analyze the variability of the poroelastic response of the heterogeneous medium at the field scale. To solve the problem analytically, we restrict our attention to the one-dimensional models, where fluid flow as well as deformation occurs in one direction only under a constant applied stress. Assuming statistic homogeneity, the closed-form solutions that describe the variability of fluid pressure head, and a solid's strain and displacement are developed using a spectral approach based on Fourier–Stieltjes representations for the perturbed quantities. The influence of the correlation length of the log hydraulic conductivity on these results is investigated. It is found that the variances of the solid's strain and displacement increase with the correlation length of the log hydraulic conductivity, while the correlation length of the log hydraulic conductivity plays the role in reducing the variability of the specific discharge.

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1. Introduction

The poroelasticity theory provides a means for analyzing the interaction between the fluid flow and skeletal-matrix deformation [4,11,13]. It has been developed invoking the assumption of fluid flow in an isotropic homogeneous medium. As such, it does not reflect the influence of the formation heterogeneity. However, many practical problems of subsurface flow require predictions over relative large space scale, where a wide range of formation heterogeneities are included in the flow domain. Therefore, there arises a need to incorporate the influence of natural heterogeneity into the poroelasticity theory.

Natural porous earth materials are observed to display spatial variability of their properties. Heterogeneity has been shown to play an important role in the analysis of the behavior of groundwater flow and transport of solutes at field scale [3,7,12,16]. Motivated by that, this paper attempts to assess the influence of heterogeneity, related to the random spatial variability of hydraulic conductivity, on the field-scale poroelastic response of the heterogeneous medium. Limited studies have devoted to the investigation of poroelasticity at field scale in the stochastic framework [5,6,14]. The application of spectral techniques to the analysis of the variability of the poroelastic response of the heterogeneous medium at the field scale has so far not been attempted, to our best knowledge, and this is the task undertaken here.

To solve the problem analytically, focus is placed on one-dimensional models where the flow is in the horizontal direction under a constant applied stress. These cases may not be very practical, while they do provide some basic understanding of the influence of heterogeneity on the poroelastic response of the heterogeneous medium at the field scale.

Fluctuations in pore groundwater pressure in response to the changes in imposed stresses are often encountered in many practical problems of subsurface flow. In general, the porous medium is deformable. Such interaction will cause the deformation of the solid matrix, which in turn affects the storage of groundwater in the void space. Thus, the assessment of the variability of the poroelastic response of the medium is essential for the planning and management of groundwater resources in aquifers. The results of this work may serve as rough estimates of the uncertainty of prediction of field-scale poroelastic response of the aquifer.

2. Mathematical formulation of the problem

For an isotropic, linearly elastic porous medium with incompressible grains, the equation of stress equilibrium takes the form [2,13]

$$\frac{1}{\alpha}\nabla^2\varepsilon_b - \nabla^2 P = 0 \tag{1}$$

where $\nabla^2(\ ) = \partial^2(\ )/\partial X_1^2 + \partial^2(\ )/\partial X_2^2 + \partial^2(\ )/\partial X_3^2$, $\alpha = (\lambda + 2\mu)^{-1}$, and λ and μ are macroscopic constant coefficients called Lame's coefficients for a porous medium, ε_b is the solid's volume strain $= \nabla\cdot\mathbf{W}$, $\mathbf{W}$ is the displacement vector, and P is the pressure increment. The mass balance equation for a compressible fluid phase is characterized by [2,13]

$$\nabla\cdot\left[\frac{K}{\gamma_w}\nabla P\right] = n\beta\frac{\partial P}{\partial t} + \frac{\partial\varepsilon_b}{\partial t} \tag{2}$$

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where K is the hydraulic conductivity, γ_w is the unit weight of water, n is the porosity, and β is the coefficient of fluid compressibility.

The general solution of Eq. (1) is

$$\frac{1}{\alpha}\varepsilon_b = P + \Pi(X,t) \tag{3}$$

where Π satisfies $\nabla^2\Pi = 0$. By inserting Eq. (3) into Eq. (2), we obtain

$$\nabla\cdot\left[\frac{K}{\gamma_w}\nabla P\right] = [n\beta + \alpha]\frac{\partial P}{\partial t} + \alpha\frac{\partial\Pi}{\partial t} \tag{4}$$

Verruijt [13] concluded that Π vanishes under the conditions that the fluid flow as well as deformation occur in one direction only, and the applied stress remains unchanged. Accordingly, from Eq. (3), the relationship between the solid's strain and the change in pore pressure can be expressed as

$$\varepsilon_b = \alpha P \tag{5}$$

and Eq. (4) reduces considerably to

$$\frac{\partial}{\partial X_1}\left[\frac{K}{\gamma_w}\frac{\partial P}{\partial X_1}\right] = [n\beta + \alpha]\frac{\partial P}{\partial t} \tag{6}$$

where the flow is assumed to be one dimensional in the horizontal direction.

Expanding Eq. (6) leads to

$$\frac{\partial^2}{\partial X_1^2}\frac{P}{\gamma_w} + \frac{1}{K}\frac{\partial K}{\partial X_1}\frac{\partial}{\partial X_1}\frac{P}{\gamma_w} = \frac{S_0}{K}\frac{\partial}{\partial t}\frac{P}{\gamma_w} \tag{7}$$

where S_0 (the specific storativity) $= \gamma_w(n\beta + \alpha)$. Our starting point of the stochastic analysis lies in the local-scale equation Eq. (7). It is recognized that Lame's coefficients and porosity do not vary significantly in space compared to the spatial variation of hydraulic conductivity [6,15]. Thus, the effects of spatial variations of Lame's coefficients and porosity are neglected in this analysis.

3. Spectral perturbation approach

Eq. (7) can be solved using the spectral perturbation approach. Applications of the spectral perturbation approaches to groundwater systems were presented in the works of Bakr et al. [1], Mizell et al. [9], Gelhar and Axness [8], and others. The spectral perturbation approach to groundwater flow and solute transport in saturated and unsaturated media is the focus of the book by Gelhar [7].

We consider that the flow domain is sufficiently large compared to the correlation scale of log hydraulic conductivity. The spatially random distribution of ln$K$ is assumed because of the natural heterogeneity of geological formations. The ln$K$ field is typically represented by a small perturbation expansion

$$\ln K(X_1) = \langle \ln K(X_1)\rangle + f(X_1) = F + f(X_1) \tag{8}$$

where $< >$ stands for the expected value operator, and $f(X_1)$ is a spatially correlated, statistically stationary random field with a zero mean. The spatially correlated random heterogeneity in ln$K$ parameter results in the spatially correlated random perturbations in pressure head

$$\frac{P}{\gamma_w}(X_1,t) = \left\langle \frac{P}{\gamma_w}(X_1,t)\right\rangle + \phi(X_1,t) = \Gamma(X_1,t) + \phi(X_1,t) \tag{9}$$

where $\phi(X1,t)$ is the zero-mean perturbations. Substituting Eqs. (8) and (9) into Eq. (7) and neglecting all products of perturbations, the first-order approximation of the mean pressure head equation is found as

$$\frac{\partial^2\Gamma}{\partial X_1^2} = \frac{S_0}{e^F}\frac{\partial\Gamma}{\partial t} \tag{10}$$

Subtracting this mean equation from Eq. (7) leads to a first-order perturbation equation

$$\frac{\partial^2\phi}{\partial X_1^2} + \frac{\partial f}{\partial X_1}\frac{\partial\Gamma}{\partial X_1} = \frac{S_0}{e^F}\left[\frac{\partial\phi}{\partial t} - f\frac{\partial\Gamma}{\partial t}\right] \tag{11}$$

Note that the perturbation expansion is formally limited to relatively small variance ($\sigma_f^2 \ll 1$, where σ_f^2 is the variance of ln$K$). However, Zhang and Winter [15] found it to be accurate for the head variance solutions for σ_f^2 as high as 4.38. A similar finding was reported in Gehar [7].

By regarding the $\partial\Gamma/\partial t$ term in Eq. (10) slowly varying in time, the mean pressure head is thus treated approximately as time-invariant. This implies from Eq. (10) that the mean pressure head gradient is constant (i.e., $J = -\partial\Gamma/\partial X_1 =$ constant). Hence, the pressure head perturbation Eq. (11) reduces to

$$\frac{\partial^2\phi}{\partial X_1^2} - J\frac{\partial f}{\partial X_1} = \frac{S_0}{e^F}\frac{\partial\phi}{\partial t} \tag{12}$$

Eq. (12) provides the stochastic differential equation required to develop the second moment of the pressure head in terms of the statistics of the input hydraulic parameters.

Eq. (12) may be solved using a spectral approach based on Fourier–Stieltjes representations for the perturbed quantities [1,8,9]. By using this approach, the random fluctuations f and ϕ are considered to be realizations of second-order stationary random fields and represented by the following integrals:

$$f(X_1) = \int_{-\infty}^{\infty} e^{i RX_1} dZ_f(R) \tag{13}$$

$$\phi(X_1,t) = \int_{-\infty}^{\infty} e^{i RX_1} dZ_\phi(R,t) \tag{14}$$

where $dZ_f(R)$ and $dZ_\phi(R,t)$ are the complex Fourier amplitudes of ln$K$ and pressure head processes, respectively, and R is the wave number.

Note that assuming the existence of solutions over infinite domains and statistically homogeneous random fields, the spectral representation theorem of random functions in Fourier space reduces the complexity of the stochastic groundwater flow problem and provides simple closed-form solutions. In reality, groundwater flow fields are not over infinite domains. However, an infinite-domain assumption may be reduced to a large finite-domain assumption if the correlation length of the random fields is much smaller than the domain size.

Making use of these representations in Eq. (12) and invoking the uniqueness of the spectral representations results in

$$\frac{d}{dt}dZ_\phi(R,t) + \frac{e^F R^2}{S_0}dZ_\phi(R,t) = -iJ\frac{e^F R}{S_0}dZ_f(R) \tag{15}$$

It is assumed that at $t = 0$ the head distribution is smooth, that is, $\phi(X_1, t = 0) = 0$. Thus, $dZ_\phi = 0$ at $t = 0$. With this initial condition, the solution for Eq. (15) is then

$$dZ_\phi(R,\upsilon) = -i\frac{J}{R}[1 - e^{-R^2\upsilon}]dZ_f(R) \tag{16}$$

where $v = e^F t / S_0$. Taking the expected value of the product of the Fourier amplitude $dZ_\phi(R,v)$ and its complex conjugate $dZ_\phi^*(R,v)$ gives the spectrum of fluctuations in pressure head:

$$S_{\phi\phi}(R,v)dR = \langle dZ_\phi(R,v)dZ_\phi^*(R,v)\rangle = J^2[1-e^{-R^2 v}]^2 \frac{S_{ff}(R)}{R^2}dR \quad (17)$$

where $S_{ff}(R)$ is the spectrum of $\ln K$. The Fourier transform of $S_{\phi\phi}$ yields the auto-covariance of pressure head fluctuations

$$R_\phi(\xi,v) = \int_{-\infty}^{\infty} e^{iR\xi}S_{\phi\phi}(R,v)dR = J^2 \int_{-\infty}^{\infty}\cos(R\xi)\frac{[1-e^{-R^2 v}]^2}{R^2}S_{ff}(R)dR \quad (18)$$

where ξ is the spacing separation. From Eq. (5), the auto-covariance of solid's strain can be related to that of pressure head fluctuation by

$$R_\varepsilon(\xi,v) = (\alpha\gamma_w)^2 R_\phi(\xi,v) \quad (19)$$

It may attack to the deformation of a solid from the description of the solid's displacement. Using the strain-displacement relationship

$$\frac{\partial W_1}{\partial X_1} = \varepsilon_b \quad (20)$$

the variance of solid's displacement can then be formulated as

$$\sigma_W^2(X_1,v) = \int_0^{X_1}\int_0^{X_1}\left\langle\varepsilon'(\xi')\varepsilon'(\xi'')\right\rangle d\xi' d\xi'' = \int_0^{X_1}\int_0^{X_1} R_\varepsilon(\xi'-\xi'')d\xi' d\xi'' \quad (21)$$

where $\varepsilon' = \varepsilon_b - \langle\varepsilon_b\rangle$. Introducing the Cauchy algorithm [3] into Eq. (21) yields

$$\sigma_W^2(X_1,v) = 2\int_0^{X_1} R_\varepsilon(\xi,v)[X_1-\xi]d\xi \quad (22)$$

4. Closed-form solutions

To proceed with the development of the variance of solid's displacement (Eqs. (18), (19) and (22)) one must select the form of the $\ln K$ spectrum. The random $\ln K$ perturbation field f under consideration is characterized by the following spectral density function [1]

$$S_{ff}(R) = \frac{2\sigma_f^2\eta^3 R^2}{\pi(1 + \eta^2 R^2)^2} \quad (23)$$

where σ_f^2 is the variance of $\ln K$ and η is the correlation length of $\ln K$.

The closed-form expression for the auto-covariance of pressure head fluctuations results from substituting Eq. (23) into Eq. (18) and integrating over the wave number domain as

$$R_\phi(\zeta,\tau) = J^2\sigma_f^2\eta^2\Big\{e^{-|\zeta|}(1 + |\zeta|)-e^{-\zeta^2 4\tau}\Big[4\sqrt{\tau/\pi} + e^{A_1^2}(1-2\tau-\zeta)\Psi(A_1)$$
$$+ e^{B_1^2}(1-2\tau + \zeta)\Psi(B_1)\Big] + \frac{1}{2}e^{-\zeta^2/8\tau}\Big[4\sqrt{2\tau/\pi}$$
$$+ e^{A_2^2}(1-4\tau-\zeta)\Psi(A_2) + e^{B_2^2}(1-4\tau + \zeta)\Psi(B_2)\Big]\Big\} \quad (24)$$

where $\zeta = \xi/\eta$, $\tau = v/\eta^2$, $A_1 = \tau^{0.5} + 0.5\zeta/\tau^{0.5}$, $B_1 = \tau^{0.5} - 0.5\zeta/\tau^{0.5}$, $A_2 = (2\tau)^{0.5} + 0.5\zeta/(2\tau)^{0.5}$, $B_2 = (2\tau)^{0.5} - 0.5\zeta/(2\tau)^{0.5}$, and $\Psi(-)$

denotes the complementary error function. The pressure head variance is found by taking $\zeta = 0$:

$$\sigma_\phi^2(\tau) = J^2\sigma_f^2\eta^2\left[1-4\sqrt{\frac{\tau}{\pi}} + (4\tau-2)e^\tau\Psi(\sqrt{\tau}) + 2\sqrt{\frac{2\tau}{\pi}}-(4\tau-1)e^{2\tau}\Psi(\sqrt{2\tau})\right] \quad (25)$$

With the pressure head auto-covariance determined, the auto-covariance of solid's strain is simply expressed by Eq. (19)

$$R_\varepsilon(\zeta,\tau) = J^2\sigma_f^2\eta^2(\gamma_w\alpha)^2\Big\{e^{-|\zeta|}(1 + |\zeta|)-e^{-\zeta^2/4\tau}\Big[4\sqrt{\tau/\pi} \quad (26)$$
$$+ e^{A_1^2}(1-2\tau-\zeta)\Psi(A_1) + e^{B_1^2}(1-2\tau + \zeta)\Psi(B_1)\Big]$$
$$+ \frac{1}{2}e^{-\zeta^2/8\tau}\Big[4\sqrt{2\tau/\pi} + e^{A_2^2}(1-4\tau-\zeta)\Psi(A_2)$$
$$+ e^{B_2^2}(1-4\tau + \zeta)\Psi(B_2)\Big]\Big\}$$

The resulting variance of solid's strain is given by

$$\sigma_\varepsilon^2(\tau) = J^2\sigma_f^2\eta^2(\gamma_w\alpha)^2\left[1-4\sqrt{\frac{\tau}{\pi}} + (4\tau-2)e^\tau\Psi(\sqrt{\tau}) \quad (27)\right.$$
$$\left. + 2\sqrt{\frac{2\tau}{\pi}}-(4\tau-1)e^{2\tau}\Psi(\sqrt{2\tau})\right]$$

The result of Eq. (27) is presented graphically in Fig. 1. It indicates that the variance of solid's strain increases monotonically with the correlation length of $\ln K$. Note that an increase in η produces more persistence of pressure head fluctuation which leads to larger deviations of pressure head from the mean pressure head. It is clear from Eq. (5) or Eq. (19) that the fluctuations in the solid's strain are positively correlated to those in the pressure head. Therefore, the variance of solid's strain increases with the correlation length of $\ln K$. Fig. 1 also shows that if the correlation scale of $\ln K$ remains constant, the variance of solid's strain will increase with time.

It follows from Eqs. (22) and (30) that the variance of solid's displacement can be written in the form

$$\sigma_W^2(\chi,\tau) = 2J^2\sigma_f^2\eta^4(\gamma_w\alpha)^2\Big\{\Big[-3 + 2\chi + (3 + \chi)e^{-\chi}\Big] + 12\sqrt{\tau/\pi}\Big(1-e^{-\chi^2/4\tau}\Big)$$
$$-4\chi\Phi\left(\frac{\chi}{2\sqrt{\tau}}\right)-2(2\tau-3)e^\tau\Psi(\sqrt{\tau}) + (2\tau-3-\chi)e^{\tau-\chi}\Psi\left(\frac{2\tau-\chi}{2\sqrt{\tau}}\right)$$
$$+ 2\chi\Phi\left(\frac{\chi}{2\sqrt{2\tau}}\right) + (2\tau-3 + \chi)e^{\tau + \chi}\Psi\left(\frac{2\tau + \chi}{2\sqrt{\tau}}\right)$$
$$+ 6\sqrt{\frac{2\tau}{\pi}}\Big(-1 + e^{-\chi^2/8\tau}\Big) + (4\tau-3)e^{2\tau}\Psi\left(\sqrt{2\tau}\right)$$
$$-\frac{1}{2}(4\tau-3-\chi)e^{2\tau-\chi}\Psi\left(\frac{4\tau-\chi}{2\sqrt{2\tau}}\right)-\frac{1}{2}(4\tau-3 + \chi)e^{2\tau + \chi}\Psi\left(\frac{4\tau + \chi}{2\sqrt{2\tau}}\right)$$
$$\quad (28)$$

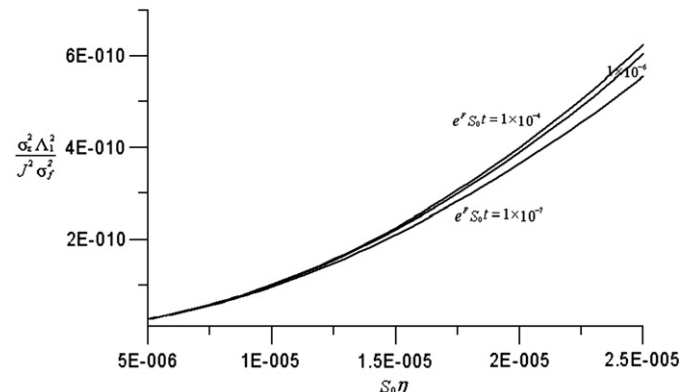where $\chi = X_1/\eta$ and Φ denotes the error function.

Fig. 1. Dimensionless variance of solid's strain as a function of dimensionless correlation length of $\ln K$ for various values of dimensionless time, where $\Lambda_1 = (\eta\beta + \alpha)/\alpha$.

The behavior of the variance of solid's displacement in Eq. (28) as a function of the correlation length of ln$K$ at various values of position is illustrated in Fig. 2. This feature is a consequence of the increase in variability of solid's strain with the correlation length of ln$K$. Larger values of correlation length of ln$K$ produce larger fluctuations in solid's strain, which in turn results in an enhanced variability of solid's displacement.

The theoretical result in this paper may be applied to the situation of changes in water level in the aquifer induced by seismic waves. Seismic waves from distant earthquakes interacting with aquifers produce changes in pore pressure and a pressure gradient in the aquifer [10]. The changes in water level are attributed to the horizontal pressure diffusion caused by the pressure gradient. The redistribution of pore pressure change in the aquifer can generate solid's deformation. The stochastic result provides a rational basis for judging reliability (variance or second moment) of the model prediction at which direct observations of dependent variables (e.g., pressure head, solid's strain) are not feasible.

5. A note on the variance of specific discharge

The variations of the specific discharge about the mean can be developed simply from the Darcy equation. The result, the first-order equation for the specific discharge perturbation, is of the form [8]

$$q'_1 = e^F \left[Jf - \frac{\partial \phi}{\partial X_1} \right] \tag{29}$$

where $q'_1 = q_1 - <q_1>$, q_1 is the specific discharge in the horizontal direction. Substituting Eqs. (13) and (14) and the Fourier–Stieltjes representations for specific discharge perturbations into Eq. (29) and invoking the uniqueness of the spectral representation gives the following relationship for the Fourier amplitude of q_1:

$$dZ_{q_1} = e^{2F} \left(JdZ_f - iRdZ_\phi \right) \tag{30}$$

where dZ_{q_1} is the complex Fourier amplitudes of the specific discharge.

With Eq. (16), a spectral relationship between the input log hydraulic conductivity variations and the resulting specific discharge variations,

$$S_{q_1 q_1}(R) = e^{2F} \left[J^2 + \left(1 - e^{-R^2 v} \right)^2 \right] S_{ff}(R) \tag{31}$$

is found by taking the expected value of the product of the Fourier amplitude dZ_{q1} and its complex conjugate. In Eq. (31), S_{q1q1} is the spectrum of specific discharge variations. The variance of specific discharge can now be computed by integrating Eq. (31) over the wave number domain, according to

$$\sigma_{q_1}^2 = \int_{-\infty}^{\infty} S_{q_1 q_1}(R) dR \tag{32}$$

Substituting Eqs. (23) and (31) into Eq. (32) and integrating over the wave number domain yields the closed-form expression for variations in specific discharge as

$$\sigma_{q_1}^2 = e^{2F} J^2 \sigma_f^2 [1 + 4\sqrt{\tau/\pi} - 2\sqrt{2\tau/\pi} - 2e^\tau (1 + 2\tau)\Psi(\sqrt{\tau}) + e^{2\tau}(1 + 4\tau)\Psi(\sqrt{2\tau})] \tag{33}$$

where $\Psi(-)$ denotes the complementary error function.

The result of Eq. (33) is shown in Fig. 3, which indicates that the variance of the specific discharge diminishes as $S_0 \eta^2$ becomes large at a specified time. In other words, soils with a larger specific storativity S_0 or correlation length of log hydraulic conductivity η exhibit less variability of the specific discharge.

6. Conclusions

The paper addresses the problem of the poroelastic response of a one-dimensional heterogeneous medium at the field scale, where fluid flow as well as deformation occurs in one direction only under a constant applied stress. On the basis of the assumption of statistic homogeneity, closed-form expressions for the auto-covariances of fluid pressure head and solid's strain, and the variance of solid's displacement are developed through the spectral perturbation approach to demonstrate the influence of the correlation length of the log hydraulic conductivity on these results. It was found that the correlation length of ln$K$ plays an important role in increasing the fluctuations in solid's strain, which enhance the variability of solid's displacement. An increase in the specific storativity S_0 or correlation length of log hydraulic conductivity η tends to reduce the variability of the specific discharge at a specified time. It is hoped that our findings will be useful in simulating further research in this area.

Notation
A_1 $= \tau^{0.5} + 0.5\zeta/\tau^{0.5}$
A_2 $= (2\tau)^{0.5} + 0.5\zeta/(2\tau)^{0.5}$
B_1 $= \tau^{0.5} - 0.5\zeta/\tau^{0.5}$
B_2 $= (2\tau)^{0.5} - 0.5\zeta/(2\tau)^{0.5}$
F Mean value of ln$K$
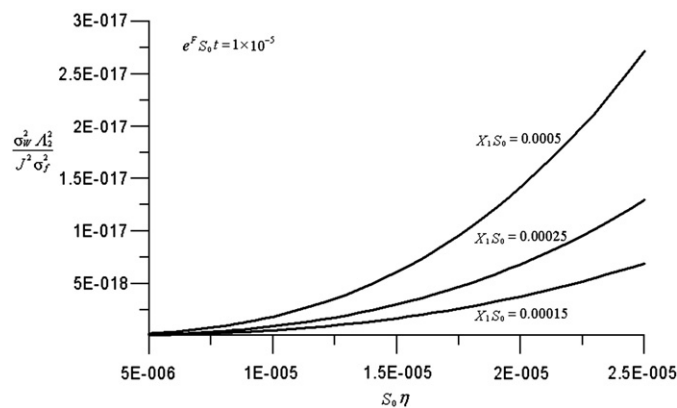J $= -\partial \Gamma/\partial X_1 = $ constant
K Hydraulic conductivity

Fig. 2. Dimensionless variance of solid's displacement as a function of dimensionless correlation length of ln$K$ at various values of dimensionless position, where $\Lambda_2 = \gamma_w (n\beta + \alpha)^2/\alpha$.
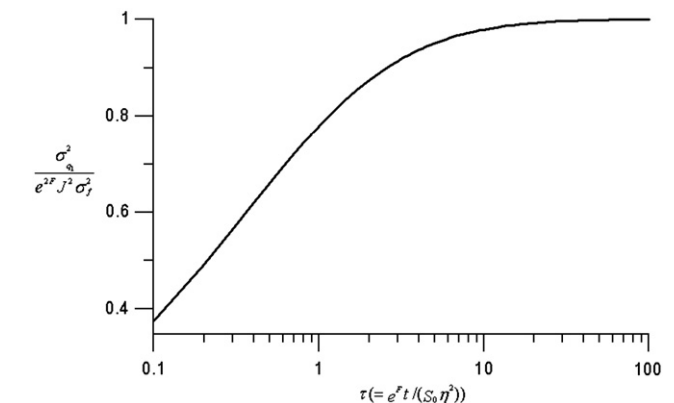

Fig. 3. The dimensionless variance of specific discharge based in Eq. (33) as a function of τ.

P	Pressure increment
R	Wave number
$R_\phi(\xi, \upsilon)$	Auto-covariance of pressure head fluctuations
$R_\varepsilon(\xi, \upsilon)$	Auto-covariance of solid's strain fluctuations
S_0	$= \gamma_w(n\beta + \alpha)$; the specific storativity
$Sff(R)$	Spectrum of $\ln K$ variations
$S_{\phi\phi}(R)$	Spectrum of pressure head variations
$S_{q1q1}(R)$	Spectrum of specific discharge variations
$\mathbf{W}$	Solid's displacement vector
W_1	Solid's horizontal displacement
X_1	Horizontal space coordinate
$dZ_f(R)$	Complex Fourier amplitudes of $\ln K$ process
$dZ_\phi(R,t)$	Complex Fourier amplitudes of pressure head process
$dZ_{q1}(R,t)$	Complex Fourier amplitudes of the specific discharge
f	Fluctuation in $\ln K$
n	Porosity
q_1	Horizontal specific discharge
q'_1	Fluctuation in horizontal specific discharge
t	Time
Λ_1	$= (n\beta + \alpha)/\alpha$
Λ_2	$= \gamma_w(n\beta + \alpha)^2/\alpha$
Π	A function of space and time
$\Phi(-)$	Error function
α	$= (\lambda + 2\mu)^{-1}$
β	Coefficient of fluid compressibility
χ	$= X1/\eta$
$\Gamma(X_1)$	Mean pressure head
$\Psi(-)$	Complementary error function
ε_b	$= \nabla \cdot \mathbf{W}$; solid's volume strain
ε'	Fluctuation in solid's volume strain
$\phi(X_1,t)$	Fluctuation in pressure head
γ_w	Unit weight of water
η	Correlation length of $\ln K$
λ, μ	Macroscopic constant coefficients called Lame's coefficients for a porous medium
σ_f^2	Variance of $\ln K$
σ_ϕ^2	Variance of pressure head
σ_ε^2	Variance of solid's strain
σ_{q1}^2	Variance of specific discharge
σ_W^2	Variance of solid's displacement
τ	$= \upsilon/\eta^2$
υ	$= e^F t/S_0$
ξ	Spacing separation
ζ	$= \xi/\eta$

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