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Magnetic properties for anisotropic $d_{x^2-y^2}$-wave superconductivity

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Abstract

The single vortex structure is derived in the presence of applied magnetic field. According to our results, the winding number of the s-wave does not change regardless of the vector potential. The generic London equation for a $d_{x^2-y^2}$-wave superconductor with mass anisotropy is expressed. By neglecting higher order terms, this work analyzes the magnetic-field distribution with and without a vortex. The interaction force between two parallel vortices is derived as well. Our results further reveal the presence of a torque between vortices irrespective of *s*- or *d*-wave order parameter, which is expected to vanish for isotropic cases. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Recently, microscopic theory and experimental phenomenology of high-T_c superconductors have demonstrated that high-temperature cuprate superconductors favor an order parameter with an unconventional system [1–7]. The symmetry of the order parameter provides further insight into mechanism of superconductivity. Accordance with experimental observations regarding the phase of the order parameter [8–10], the symmetry belongs to the $d_{x^2-y^2}$ symmetry with a line of nodes along the $|k_x| = |k_y|$ directions. Some experimental measurements, Photoemission studies [11,12], Josephson interference [13] and *c*-axis Josephson tunneling experiment [14], can not be clarified within a pure *d*-wave order parameter. For instance, Sun et al. [14] studied the Josephson tunneling between a conventional superconductor (Pb) and a series of high-T_c cuprates ($YBa_2Cu_3O_{7-\delta}$ and the alloys $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$) and observed the nonzero tunneling current. This observation contradicts the assumption that $YBa_2Cu_3O_{7-\delta}$ is a pure $d_{x^2-y^2}$ superconductor. Theoreticians suggest that there are two gaps in YBCO. The main gap is caused by the CuO_2 planes and an induced smaller one results from the CuO chains. Therefore, the structures of YBCO can not be confined in tetragonal crystal symmetry due to the existence of CuO chains. Moreover, YBCO exhibits a large anisotropy between the a and b

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directions, as evidenced by the measurements of the penetration depth [15] and the vortex structure by scanning tunneling microscopy (STM) [16].

The vortex structure in d-wave superconductors has received increasing attention owing to the different nature of vortex from that of the conventional one. Soininen et al. solved the Bogoliubov–de Gennes (BdG) equations for electrons on a square lattice with attractive nearest-neighbor interaction [17]. They found an admixture of the s-wave component to the d-wave order parameter in the vicinity of the vortex core. Berlinsky et al. [18], Franz et al. [19] and Ren et al. [20] analyzed the structure of the admixtured s-wave order parameter using the Ginzburg–Landau (GL) theory. In a related work, Heeb et al. [21] studied effects of orthorhombic distortions for $YBa_2Cu_3O_7$ from group-theory arguments. These investigators accounted for nonvanishing Josephson tunneling currents between s-wave superconductors and orthorhombically distorted $d_{x^2-y^2}$-wave superconductors by introducing the coupling terms. Xu et al. [22] developed a GL theory for a d-wave superconductor with mass anisotropy in the presence of a magnetic field. Later, Han and Zhang [23] investigated microscopically the GL theory of a $d+s$-wave superconductor with orthorhombic distortion in a magnetic field applied to the c axis. They all showed the single vortex structure in a similar manner. In addition, both the s- and d-wave order parameters were reduced to twofold symmetry, as expected to be fourfold one for isotropic or tetragonal superconductivity.

The lower critical field (H_{c1}), the upper critical field (H_{c2}), the coherence length, and the magnetic penetration depth involve the superconducting mechanism of cuprate. The angular dependence of H_{c2} has been calculated and presented some important note [24]. However, near H_{c1}, the GL equations approximately lead to the nonlinear London equation, which profoundly influences in high-T_c superconductors. The corresponding London free energy offers a simple method of investigating the vortex lattice in a type-II superconductor. Affleck et al. [25] derived the generic London model to study the vortex lattice structure and investigated neutron scattering, scanning tunneling microscopy, Bitter decoration, and muon-spin-rotation experiments. In this work, we derive the London equation for a d-wave superconductor with mass anisotropy. Based on the Ginzburg-Landau theory, derived by Xu et al. [22], the linear and gradient terms of d-wave component in GL equations replace the s-wave component. The single vortex structure is obtained by solving GL equations. The winding number of the s-wave does not change, whether the vector potential exists or not. By varying the London free energy with respect to the vector potential and restricting magnetic field $\vec{B}$ along the z direction, the corresponding London equation was derived. As generally known, the problem involving the field of an isolated vortex is difficult to resolve due to nonlinearity of the London equation. By neglecting higher order terms, some specific solutions can be calculated in the absence and presence of vortex. Some interesting consequences worth mentioning include the dependence of the London penetration depth upon the temperature, as well as the magnetic field. Of course, we derived the vortex line energy to determine the lower critical field. This work also demonstrates that the interaction force between two vortices is not only in a repulsion but in a torque for anisotropic $d_{x^2-y^2}$-wave superconductors. A summary of our results and some detailed discussion are finally made.

2. The structure of d-wave superconductor with mass anisotropy

The GL theory for a $d_{x^2-y^2}$-wave superconductor with mass anisotropy has been derived completely by Xu et al. [22]. By assuming that a d-wave pairing interacts with a repulsive on site Coulomb interaction, the GL free energy was obtained. For convenience, we cast the GL free energy density into a dimensionless form. At first we introduce the units $(\alpha_s/\gamma_d\alpha)^{1/2}$ for the s-wave order parameter, $\left[\left(1+\sqrt{M_a}\right)^4|\alpha_d|/\gamma_d\alpha(M_a+1)\left(1+\sqrt{M_a}+M_a\right)\right]^{1/2}$ for the d-wave order parameter, $\xi_d^a=\left[\gamma_d\alpha\mu\left(1-\sqrt{M_a}+3M_a+M_a\sqrt{M_a}\right)/\left(1+\sqrt{M_a}\right)^3 m_x|\alpha_d|\right]^{1/2}$ for the length, $(e\ \xi_d^a)^{-1}$ for the $\boldsymbol{A}$ field, and

$\left[\alpha_d^2\left(1+\sqrt{M_a}\right)^4/\gamma_d\alpha(M_a+1)\left(1+\sqrt{M_a}+M_a\right)\right]$ for the energy density. After tedious calculations, we obtain the following:

$$\begin{aligned}\varepsilon_{\text{density}} &= \tilde{\alpha}_s|\psi_s|^2+\tilde{\alpha}_s|\psi_s|^4-|\psi_d|^2+|\psi_d|^4-\beta_1(\psi_s*\psi_d+\psi_d*\psi_s)+4\beta_2|\psi_d|^2|\psi_s|^2\\ &\quad+\beta_2\left(\psi_s^{*2}\psi_d^2+\psi_s^2\psi_d^{*2}\right)+\beta_3|\psi_s|^2(\psi_s*\psi_d+\psi_s\psi_d*)+\beta_4|\psi_d|^2(\psi_s^*\psi_d+\psi_s\psi_d^*)\\ &\quad+\gamma_1\left(|\Pi_x\psi_s|^2+M_a|\Pi_y\psi_s|^2\right)+|\Pi_x\psi_d|^2+\gamma_2|\Pi_y\psi_d|^2\\ &\quad+\left(\gamma_3\Pi_x\psi_s\Pi_x*\psi_d*-\gamma_4\Pi_y\psi_s\Pi_y*\psi_d*+\text{h.c.}\right)+\ell^2(\nabla\times\boldsymbol{A})^2 \end{aligned}\tag{1}$$

where

$$\tilde{\alpha}_s=\frac{(M_a+1)\left(1+\sqrt{M_a}+M_a\right)\alpha_s^2}{\left(1+\sqrt{M_a}\right)^4\alpha_d^2},\qquad \beta_1=\frac{(M_a-1)}{\left(1+\sqrt{M_a}\right)^4|\alpha_d|}\sqrt{\frac{\left(1+\sqrt{M_a}+M_a\right)\alpha_s}{(M_a+1)|\alpha_d|}},$$

$$\beta_2=\frac{(M_a+1)\alpha_s}{\left(1+\sqrt{M_a}\right)^2|\alpha_d|},\qquad \beta_3=\frac{2\left(\sqrt{M_a}-1\right)\alpha_s^{3/2}}{\left(1+\sqrt{M_a}\right)^3|\alpha_d|^{3/2}}\sqrt{(M_a+1)\left(1+\sqrt{M_a}+M_a\right)},$$

$$\beta_4=\frac{2\left(M_a\sqrt{M_a}-1\right)}{\left(1+\sqrt{M_a}\right)}\sqrt{\frac{\alpha_s}{(M_a+1)\left(1+\sqrt{M_a}+M_a\right)|\alpha_d|}},$$

$$\gamma_1=\frac{(M_a+1)\left(1+\sqrt{M_a}+M_a\right)\alpha_s}{\left(1-\sqrt{M_a}+3M_a+M_a\sqrt{M_a}\right)\left(1+\sqrt{M_a}\right)|\alpha_d|},$$

$$\gamma_2=\frac{M_a\left(1+3\sqrt{M_a}-M_a+M_a\sqrt{M_a}\right)}{1-\sqrt{M_a}+3M_a+M_a\sqrt{M_a}},$$

$$\gamma_3=\frac{\left(M_a+2\sqrt{M_a}-1\right)}{\left(1-\sqrt{M_a}+3M_a+M_a\sqrt{M_a}\right)}\sqrt{\frac{(M_a+1)\left(1+\sqrt{M_a}+M_a\right)\alpha_s}{|\alpha_d|\left(1+\sqrt{M_a}\right)}},$$

$$\gamma_4=\frac{M_a\left(2\sqrt{M_a}+1-M_a\right)}{\left(1-\sqrt{M_a}+3M_a+M_a\sqrt{M_a}\right)}\sqrt{\frac{(M_a+1)\left(1+\sqrt{M_a}+M_a\right)\alpha_s}{|\alpha_d|\left(1+\sqrt{M_a}\right)}},$$

$$\alpha_d=\frac{1}{\alpha_s}\left(\frac{M_a-1}{M_a+1}\right)^2-\frac{2(1+M_a)}{\left(1+\sqrt{M_a}\right)^2}\gamma_d\ln\frac{T_c}{T},\ \text{and}\ \ \ell^2=\frac{\lambda_{\text{sc}}^2}{(\xi_d^a)^2}.\tag{2}$$

Here we introduce an anisotropic mass parameter $M_a=m_x/m_y$, $\Pi_\alpha\equiv-i\nabla_\alpha-2\mathrm{A}$, the screening length $\lambda_{\text{sc}}=\left[\mu(M_a+1)\left(1+\sqrt{M_a}+M_a\right)/8\pi e^2m_x|\alpha_d|\left(1+\sqrt{M_a}\right)\left(1-\sqrt{M_a}+3M_a+M_a\sqrt{M_a}\right)\right]^{1/2}$ and the coherence length ξ_d^a for the anisotropic d-wave superconductivity. In comparison with the isotropic free energy density [20], the extra terms are second-order coupling terms $\beta_1(\psi_s^*\psi_d+\psi_s\psi_d^*)$ and fourth-order terms $\beta_3\ |\psi_s|^2(\psi_s^*\psi_d+\psi_s\psi_d^*)$. These terms play an important role in bringing about an s-wave order parameter in the bulk. Different from isotropic system, in which s-wave component can only be nucleated from the mixed gradient terms in a nonuniform system, these new terms induce a nonzero s-wave component even in a uniform case. For $M_a > 1$, the coefficient of the second-order coupling terms must be negative and we obtain the lower free energy. These extra terms due to mass anisotropy are similar to those caused by orthorhombic distortion [23]. By varying the GL free energy with respect to the order parameter $\psi_d *$ and $\psi_s *$, the corresponding GL equations are obtained as

$$\psi_d = \left(\Pi_x^2 + \gamma_2 \Pi_y^2 \right) \psi_d + \left(\gamma_3 \Pi_x^2 - \gamma_4 \Pi_y^2 \right) \psi_s + 2|\psi_d|^2 \psi_d - \beta_1 \psi_s + 4\beta_2 |\psi_s|^2 \psi_d + 2\beta_2 \psi_s^2 \psi_d^* + \beta_3 |\psi_s|^2 \psi_s + \beta_4 \left(\psi_s^* \psi_d^2 + 2|\psi_d|^2 \psi_s \right), \tag{3}$$

$$\psi_s = -2|\psi_s|^2 \psi_s + \frac{\beta_1}{\tilde{\alpha}_s} \psi_d - \frac{4\beta_2}{\tilde{\alpha}_s} |\psi_d|^2 \psi_s - \frac{2\beta_2}{\tilde{\alpha}_s} \psi_s^* \psi_d^2 - \frac{2\beta_3}{\tilde{\alpha}_s} |\psi_s|^2 \psi_d - \frac{\beta_3}{\tilde{\alpha}_s} \psi_s^2 \psi_d^* - \frac{\gamma_1}{\tilde{\alpha}_s} \left(\Pi_x^2 + M_a \Pi_y^2 \right) \psi_s - \frac{\gamma_3}{\tilde{\alpha}_s} \Pi_x^2 \psi_d - \frac{\gamma_4}{\tilde{\alpha}_s} \Pi_y^2 \psi_d - \frac{\beta_4}{\tilde{\alpha}_s} |\psi_d|^2 \psi_d. \tag{4}$$

For a bulk system in the absence of the magnetic field, the gradient terms in the two GL equations are equal to zero. Xu et al. indicated that the bulk system in the mixed $s+d$ system is stable [22]. Neglecting the higher term $|\psi_d|^2\ \psi_d$, ψ_s and ψ_d can be expressed as

$$\psi_s \approx -\frac{\beta_1}{\tilde{\alpha}_s} \psi_d \tag{5}$$

and

$$\psi_d = \Gamma(M_a) \tag{6}$$

with

$$\Gamma(M_a)^2 = \left(1 + \frac{\beta_1^2}{\tilde{\alpha}_s} \right) \left(2 + \frac{6\beta_1^2 \beta_2}{\tilde{\alpha}_s} - \frac{\beta_1^3 \beta_3}{\tilde{\alpha}_s} - \frac{2\beta_1 \beta_4}{\tilde{\alpha}_s} \right)^{-1}.$$

The mixed gradient terms in the d-wave component on the right hand side of Eq. (4) plays an important role in determining the behavior of an induced s-wave component [21] and $|\psi_d|^2 \psi_d$ is negligible small compared with the linear term ψ_d. Obviously, it will be convenient to work in the cylindrical gauge expressed in the usual polar coordinates $\vec{r} = (r,\theta)$ to determine the vortex structure. In the limit for $|\psi_s| \ll |\psi_d|$, we can neglect the contribution of the s-wave component and assume the zero-order solution for ψ_d and $\mathbf{A}$. The single vortex solution derived belongs to the pure d-wave case. Using the $\psi_d = d(r) e^{i\theta}$ allows us to obtain the source term:

$$\psi_s \simeq \frac{\gamma_3}{\tilde{\alpha}_s} \Pi_x^2 \psi_d + \frac{\gamma_4}{\tilde{\alpha}_s} \Pi_y^2 \psi_d + \frac{\beta_1}{\tilde{\alpha}_s} \psi_d \tag{7}$$

$$= -\frac{1}{2} \left(\frac{\gamma_3}{\tilde{\alpha}_s} \right) \left(1 + \frac{\gamma_4}{\gamma_3} \right) \left\{ e^{3i\theta} \left(-\frac{1}{2} d'' + \frac{3}{2r} d' - \frac{3}{2r^2} d - 2eAd' + \frac{3eA}{r} d - eA'd - 2e^2 A^2 d \right) + e^{-i\theta} \left(-\frac{1}{2} d'' - \frac{1}{2r} d' + \frac{1}{2r^2} d + 2eAd' + \frac{eA}{r} d + eA'd - 2e^2 A^2 d \right) \right\} + e^{i\theta} \left\{ -\frac{1}{2} \left(\frac{\gamma_3}{\tilde{\alpha}_s} \right) \left(\frac{\gamma_4}{\gamma_3} - 1 \right) \times \left(d'' + \frac{1}{r} d' - \frac{1}{r^2} d + \frac{4eAd}{r} \right) + \frac{\beta_1}{\tilde{\alpha}_s} d \right\} \equiv s_1(r) e^{3i\theta} + s_2(r) e^{-i\theta} + s_3(r) e^{i\theta}, \tag{8}$$

where $\beta_1/\tilde{\alpha}_s\psi_d$ is due to anisotropic and appears here only in the form of $e^{i\theta}$. The vector potential is expressed as

$$A = A(r)\hat{\theta}. \tag{9}$$

The asymptotic solutions near the center of the vortex and outside the core are easily solved. Different from the work of Xu et al. [22], we don't limit the anisotropic mass parameter value. Furthermore, the winding number of the s-wave does not change regardless of the vector potential **A**. Meanwhile, we assume that two different order parameters correspond to the same transition temperature for a d-wave superconductor, proposed by Müller [26]. We also show that the coefficient of cos 2θ term in $|\psi_s|^2$ exists when $M_a \neq 1$, i.e., there is a two-fold symmetry for an anisotropic d-wave superconductor.

In the case of $|\psi_s| \sim |\psi_d|$, the feedback of s-wave can not be neglected. Above considerations are no longer valid. This problem is much more complex and solutions can be obtained only numerically.

3. The London equation

Affleck et al. [25] have derived the generalized isotropic London model, starting from the GL free energy density with both ψ_d and ψ_s order parameters. Here, the generalized anisotropic London model is presented. By substituting Eq. (7) into $\varepsilon_{\text{density}}$, the leading derivative terms in ψ_d of the form can be expressed as

$$\varepsilon_{\text{density}} = c_1|\Pi_x\psi_d|^2 + c_2|\Pi_y\psi_d|^2 - \epsilon_a(\xi_d^a)^2|\left(\Pi_x^2\right) - \frac{\gamma_4}{\gamma_3}\Pi_y^2)\psi_d|^2 + \cdots. \tag{10}$$

with

$$c_1 = \frac{\beta_1}{\tilde{\alpha}_s}(\gamma_1 + 2\gamma_3) + 1, \;\; c_2 = \frac{\beta_1}{\tilde{\alpha}_s}(\gamma_1 M_a - 2\gamma_4) + \gamma_2.$$

and

$$\epsilon_a = \frac{\left(1+\sqrt{M_a}\right)^5\left(M_a + 2\sqrt{M_a} - 1\right)^2\alpha_d^2}{\dfrac{\gamma_d\alpha\mu}{m_x}\left(1-\sqrt{M_a} + 3M_a + M_a\sqrt{M_a}\right)^3\alpha_s}.$$

The above equation does not consider higher order terms because they are tedious in our problem. The parameter ϵ_a determines how the s-wave component couples with the d-wave component. To avoid complex mathematical calculation, we follow the example of $\epsilon_a \ll 1$. Namely, the strength of the s-d coupling is rather weak. Assuming that the London penetration depth $\gg \xi_d^a$ leads to

$$\psi_d \simeq d_o e^{i\theta(x,y)}. \tag{11}$$

In this region, the vector potential effect can be negligible. We set $A_x \simeq A_y \simeq A_R$. By doing so, the London free energy density ε_{L} can be written as

$$\varepsilon_{\text{L}} = d_o^2\left\{\left(c_1 v_x^2 + c_2 v_y^2\right) - \epsilon_a(\xi_d^a)^2\left[\left(v_x^2 - \frac{\gamma_4}{\gamma_3}v_y^2\right)^2 + \left(\partial_x v_x - \frac{\gamma_4}{\gamma_3}\partial_y v_y\right)^2\right]\right\} + \ell^2 B^2, \tag{12}$$

expressed in terms of the superfluid velocity

$$\vec{v} \equiv \vec{\nabla}\theta - 2A$$

and θ is the phase of ψ_d. The corresponding London equation can be derived by varying ε_{L} with respect to the vector potential A and written as:

$$\frac{\ell^2}{2d_0^2}\left(\vec{\nabla}\times\vec{B}\right)=\left(c_1 v_x\hat{x}+c_2 v_y\hat{y}\right)-2\epsilon_a\left(\xi_d^a\right)^2 \times\left[\left(v_x\hat{x}-\frac{\gamma_4}{\gamma_3}v_y\hat{y}\right)\left(v_x^2-\frac{\gamma_4}{\gamma_3}v_y^2\right)-\frac{1}{2}\left(\hat{x}\partial_x-\hat{y}\partial_y\right)\left(\partial_x v_x-\frac{\gamma_4}{\gamma_3}\partial_y v_y\right)\right]. \tag{13}$$

The supercurrent j_{s} can be obtained from the Maxwell equation $\vec{\nabla}\times\vec{B}=4\pi j_{\mathrm{s}}$. The second term of Eq. (13) in the square brackets is 'nonlocal' owing to spatial derivatives. This term is the dominant one in determining the single vortex structure [25]. The superfluid velocity can be written in terms of $\vec{B}$ and its derivative. For $\epsilon_a=0$, we have the following form:

$$v_x^{(0)}=\frac{\left(\vec{\nabla}\times\vec{B}\right)_x}{B_{0x}},\quad v_y^{(0)}=\frac{\left(\vec{\nabla}\times\vec{B}\right)_y}{B_{0y}}=\frac{\left(\vec{\nabla}\times\vec{B}\right)_y}{c_{12}B_{0x}} \tag{14}$$

with $B_{0x}=2c_1 d_0^2/\ell^2$ and $c_{12}=c_2/c_1$. Substituting the expressions for $v^{(0)}$ into the London free energy density, we get:

$$\varepsilon_{\mathrm{L}}^{(0)}=\ell^2\left\{B^2+\lambda_0^2\left[c_{12}\left(\vec{\nabla}\times\vec{B}\right)_x^2+\left(\vec{\nabla}\times\vec{B}\right)_y^2\right]\right\}. \tag{15}$$

So, the London penetration depth of the magnetic field for $\epsilon_a=0$ is

$$\lambda_0^2=\frac{1}{2c_{12}B_{0x}}=\frac{\ell^2}{4c_2 d_0^2}. \tag{16}$$

Deriving the London equation in closed form for $\vec{v}$ as a function of $\vec{B}$ for $\epsilon_a\neq 0$ is extremely difficult. Next, the small ϵ_a is considered for convenience and the perturbative method is used. The first order corrections are

$$v_x^{(1)}=\frac{2\epsilon_a\left(\xi_d^a\right)^2}{c_1}\left\{\frac{1}{B_{0x}^3}\left[\left(\partial_y B\right)^3-\left(\frac{\gamma_4}{c_{12}^2\gamma_3}\right)\left(\partial_y B\right)\left(\partial_x B\right)^2\right]-\frac{1}{2B_{0x}}\left(1+\frac{\gamma_4}{c_{12}\gamma_3}\right)\left(\partial_x^2\partial_y B\right)\right\}$$

and

$$v_y^{(1)}=\frac{2\epsilon_a\left(\xi_d^a\right)^2}{c_2}\left\{-\frac{\gamma_4}{c_{12}\gamma_3 B_{0x}^3}\left[\left(\partial_x B\right)\left(\partial_y B\right)^2-\left(\frac{\gamma_4}{c_{12}^2\gamma_3}\right)\left(\partial_x B\right)^3\right]-\frac{1}{2B_{0x}}\left(1+\frac{\gamma_4}{c_{12}\gamma_3}\right)\left(\partial_x\partial_y^2 B\right)\right\}. \tag{17}$$

This subsequently leads to the London free energy density

$$\varepsilon_{\mathrm{L}}=\varepsilon_{\mathrm{L}}^{(0)}+\epsilon_a\left(\xi_d^a\right)^2\ell^2\lambda_0^2\left(\frac{c_{12}}{c_1}\right)\left\{\frac{3}{B_{0x}^2}\left[\left(\partial_y B\right)^2-\frac{\gamma_4}{c_{12}^2\gamma_3}\left(\partial_x B\right)^2\right]^2-\left(1+\frac{\gamma_4}{c_{12}\gamma_3}\right)^2\left(\partial_x\partial_y B\right)^2\right\} +O\left(\epsilon_a^2\right)+\cdots. \tag{18}$$

In the above equation, the free energy is expressed in terms of magnetic field $\vec{B}$ and its derivatives. The coefficients of the fourth-order terms B^4 and higher-order gradient terms ∇^4 in the above equation are not important [25]. The same result can also be obtained by considering the generation of quasiparticles near gap nodes in a d-wave superconductor [27], in a range of temperature and field where the supercurrent can be Taylor expanded in the superfluid velocity [25]. The corresponding London equation is derived by varying the free energy ϵ_{L} with respect to $\vec{B}$ to have a minima ϵ_{L}. For $B = B\hat{z}$, we obtain

$$\left[1-\lambda_0^2\left(\partial_x^2+c_{12}\partial_y^2\right)+\epsilon_a(\xi_d^a)^2\lambda_0^2\left(\frac{c_{12}}{c_1}\right)\left(1+\frac{\gamma_4}{c_{12}\gamma_3}\right)^2(\partial_x\partial_y)^2\right]B-\frac{6\epsilon_a(\xi_d^a)^2\lambda_0^2}{B_{0x}^2}\left(\frac{c_{12}}{c_1}\right)$$
$$\times\left[\left(\partial_y^2-\frac{\gamma_4}{c_{12}^2\gamma_3}\partial_x^2\right)B+\left(\partial_y B\partial_y-\frac{\gamma_4}{c_{12}^2\gamma_3}\partial_x B\partial_x\right)\right]\left[(\partial_y B)^2-\frac{\gamma_4}{c_{12}^2\gamma_3}(\partial_x B)^2\right]=0. \tag{19}$$

The final term is nonlinear due to the first term in brace in Eq. (18).

In the local field approximation, this final term of Eq. (18) is discarded. To solve the London equation, we give an exponentially decaying field

$$B \propto e^{-r/\lambda_a}, \tag{20}$$

where λ_a depends on the angle θ. For the specific case, we assume a weak field which depends only on x or else only on $(x+y)$. The solutions of the linearized London equation correspond to λ_a are $\lambda_a = \lambda_0$ for variation along the x axis and

$$\lambda_a=\lambda_0\sqrt{(1+c_{12})/4+\left\{(1+c_{12})^2/4-\epsilon_a(\xi_d^a)^2(c_{12}/c_1)\left[1+\gamma_4/(c_{12}\gamma_3)\right]^2/\lambda_0^2\right\}^{1/2}/2}$$

for variation at $\pi/4$ to the crystal axis. The London penetration depth depends not only on the temperature, but also on the direction of applied field. In the weak-coupling limit of a $d_{x^2-y^2}$-wave superconductor the slope of the penetration depth curve near T_c is much lower than in the corresponding s-wave isotropic case.

To calculate the linearized London free energy, or vortex-line energy, the magnetic field $\vec{B}$ can be expressed in terms of Fourier components

$$\vec{B}_{\vec{k}}=\int\vec{B}(x,y)\exp\left(-i\vec{k}\cdot\vec{r}\right)\mathrm{d}x\,\mathrm{d}y. \tag{21}$$

As long as the vortex spacing is large compared to λ_a, there will be negligible overlap or interaction of the vortices, so that each can be treated in isolation. Next, the Fourier Transformation (FT) is used to evaluate the total London free energy E_{L} for a single vortex. The magnetic field can be written in FT as

$$B_{\vec{k}}=\frac{\phi_0}{1+\lambda_0^2\left(k_x^2+c_{12}k_y^2\right)+\epsilon_a(\xi_d^a)^2\lambda_0^2\left(\frac{c_{12}}{c_1}\right)\left(1+\frac{\gamma_4}{c_{12}\gamma_3}\right)^2k_x^2k_y^2}. \tag{22}$$

Then we obtain the energy:

$$\begin{aligned}E_{\mathrm{single}}&=\frac{1}{(2\pi)^2}\int\varepsilon_L\mathrm{d}k_x\mathrm{d}k_y\\&=\frac{\phi_0^2\ell^2}{(2\pi)^2}\int\left\{\mathrm{d}k_x\mathrm{d}k_y\left[1+\lambda_0^2\left(k_x^2+c_{12}k_y^2\right)-\epsilon_a(\xi_d^a)^2\lambda_0^2\left(\frac{c_{12}}{c_1}\right)\left(1+\frac{\gamma_4}{c_{12}\gamma_3}\right)^2k_x^2k_y^2\right]\right.\\&\quad\left.\times\left[1+\lambda_0^2\left(k_x^2+c_{12}k_y^2\right)+\epsilon_a(\xi_d^a)^2\lambda_0^2\left(\frac{c_{12}}{c_1}\right)\left(1+\frac{\gamma_4}{c_{12}\gamma_3}\right)^2k_x^2k_y^2\right]^{-2}\right\}.\end{aligned} \tag{23}$$

To avoid the integral divergent at $k \to \infty$, the cutoff at k's of the order of $1/\xi_d^a$ is employed. Choosing the integration domain in $\vec{k}$ plane over the circle of the radius $1/\xi_d^a$ is more reliable than that over the ellipse with semiaxes $k_x = 1/\xi_d^a$ and $k_y = c_{12}^{-1/2}/\xi_d^a$, respectively. Kogan [28] made a similar argument for anisotropic type-II superconductors. Finally, the asymptotic London free energy is obtained as

$$E_{\text{single}} \simeq \frac{\phi_0^2 c_{12}^{-1/2} \ell^2}{4\pi\lambda_0^2} \left\{ \ln\left(m_1^{-1}\kappa_2\right) + \frac{3\epsilon_a(\xi_d^a)^2}{4\lambda_0^2 c_1}\left(1 + \frac{\gamma_4}{c_{12}\gamma_3}\right)^2 \left[\ln\left(m_1^{-1}\kappa^2\right) + \text{const.}\right] \right\} + O\left(\epsilon_a^2\right) + \cdots. \tag{24}$$

where the coherence length ξ_d^a can be simplified as

$$\xi_d^a = \sqrt{m_1}\, \xi_d^i$$

with $\xi_d^i = \xi_d^a(M_a = 1)$ and $m_1^2 = 2[1 - 4\sqrt{M_a}/(1+\sqrt{M_a})^3]$. Here we neglect the 1's with respect to $m_1^{-1}\kappa^2$. The Ginzburg–Landau parameter, by defining $\kappa \equiv \lambda_0/\xi_d^a$, is introduced. During calculation, the perturbed method is applied owing to $\epsilon_a(\xi_d^a)^2\lambda_0^2 k^4/(1+\lambda_0 k^2) \sim \epsilon_a \ll 1$ within the limits of integration from $k = 0$ to $k = 1/\xi_d^a$. When $M_a = 1$ and $\epsilon_a = 0$, the London free energy becomes

$$E_L \sim \ln\left(\frac{\gamma_0}{\xi_d^a}\right) \sim \ln \kappa. \tag{25}$$

This is the same as that for conventional superconductors. Calculating the line energy is an attempt to determine the lower critical field H_{c1}, at which flux first penetrates the sample. By definition, at $H = H_{c1}$, the Gibbs free energy must have the same value regardless of whether the first vortex is in or out of the sample. Thus,

$$H_{c1} = \frac{4\pi}{\phi_0} E_{\text{single}}. \tag{26}$$

Next, the asymptotic behavior of $B(x,y)$is calculated. We return to $B(x,y)$ in real space by the inverse FT of $B_{\vec{k}}$ given by Eq. (22). The inverse Fourier transform provides $\vec{B}(x,y)$

$$\vec{B}(x,y) = \frac{1}{(2\pi)^2}\int B_{\vec{k}} \exp\left(i\vec{k}\cdot\vec{r}\right) \mathrm{d}^2 k. \tag{27}$$

Therefore, we have

$$\begin{aligned} B(x,y) &= \frac{1}{(2\pi)^2}\int \frac{\phi_0 e^{i\vec{k}\cdot\vec{r}} \mathrm{d}k_x \mathrm{d}k_y}{1 + \lambda_0^2\left(k_x^2 + c_{12}k_y^2\right) + \epsilon_a(\xi_d^a)^2\lambda_0^2 \dfrac{c_{12}}{c_1}\left(1 + \dfrac{\gamma_4}{c_{12}\gamma_3}\right)^2 k_x^2 k_y^2} \\ &\simeq \frac{\phi_0}{(2\pi)^2}\int \frac{e^{i\vec{k}\cdot\vec{r}}\mathrm{d}k_x \mathrm{d}k_y}{1 + \lambda_0^2\left(k_x + c_{12}k_y^2\right)} - \frac{\phi_0 \epsilon_a(\xi_d^a)^2\lambda_0^2}{(2\pi)^2}\left(\frac{c_{12}}{c_1}\right)\left(1 + \frac{\gamma_4}{c_{12}\gamma_3}\right)^2 \\ &\quad \times \int \frac{e^{i\vec{k}\cdot\vec{r}} k_x^2 k_y^2 \mathrm{d}k_x \mathrm{d}k_y}{\left[1 + \lambda_0^2\left(k_x^2 + c_{12}k_y^2\right)\right]^2} + O\left(\epsilon_a^2\right) + \cdots \\ &= \frac{\phi_0 c_{12}^{-1/2}}{2\pi\lambda_0^2}\left\{ K_0\left(\frac{\rho}{\lambda_0}\right) + \frac{\epsilon_a(\xi_d^a)^2}{8c_1\lambda_0^2}\left(1 + \frac{\gamma_4}{c_{12}\gamma_3}\right)^2 \left[2K_0\left(\frac{\rho}{\lambda_0}\right) + \frac{\rho}{4\lambda_0}K_1\left(\frac{\rho}{\lambda_0}\right) + \frac{\rho}{4\lambda_0}K_3\left(\frac{\rho}{\lambda_0}\right)\right]\right\} \\ &\quad + \cdots, \end{aligned} \tag{28}$$

where $\rho^2 = x^2 + c_{12} y^2$ and K_0 is a zero-order Hankel function of imaginary argument. Qualitatively, $K_0(\rho/\lambda_0)$ cuts off as $e^{-\rho/\lambda_0}$ at large distances and diverges logarithmically as $\ln(\lambda_0/\rho)$ at $\rho \to 0$. Notably, some useful mathematical integral relations [29] are applied while deriving Eq. (28). From the expression $B(x,y)$, the anisotropic effect introduces the second power of (ξ_d^a/λ_0).

4. Interaction between two vertices

Next, we calculate the interaction energy between two parallel vortices A at $\rho = 0$ and B at $\rho = R$. The interaction energy can be expressed as

$$\begin{aligned}
E_{\text{int}} &= E - 2E_{\text{single}} \\
&= \frac{\phi_0^2 \ell^2}{(2\pi)^2} \int \frac{\mathrm{d}k_x \mathrm{d}k_y \left[1 + \lambda_0^2\left(k_x^2 + c_{12}k_y^2\right) - \epsilon_a(\xi_d^a)^2 \lambda_0^2 \left(\frac{c_{12}}{c_1}\right)\left(1 + \frac{\gamma_4}{c_{12}\gamma_3}\right)^2 k_x^2 k_y^2\right]}{\left[1 + \lambda_0^2\left(k_x^2 + c_{12}k_y^2\right) + \epsilon_a(\xi_d^a)^2 \lambda_0^2 \left(\frac{c_{12}}{c_1}\right)\left(1 + \frac{\gamma_4}{c_{12}\gamma_3}\right)^2 k_x^2 k_y^2\right]^2} \cos(k \cdot R) \\
&= 2\phi_0 \ell^2 B_A(R) + \frac{\phi_0^2 \epsilon_a(\xi_d^a)^2 c_{12}^{-1/2} \ell^2}{8\pi\lambda_0^4} \left(\frac{1}{c_{12}c_1}\right)^{1/2} \left(1 + \frac{\gamma_4}{c_{12}\gamma_3}\right)^2 \\
&\quad \times \left[2K_0\left(\frac{R_1}{\lambda_0}\right) + \frac{R_1}{4\lambda_0^2} K_1\left(\frac{R_1}{\lambda_0}\right) + \frac{R_1}{4\lambda_0} K_3\left(\frac{R_1}{\lambda_0}\right)\right] + O\left(\epsilon_a^2\right) + \cdots .
\end{aligned} \tag{29}$$

Here $B_A(R)$ is the field of vortex A at point R and $R_1^2 = \mathrm{X}^2 + c_{12}Y^2$. The interaction showing in Eq. (29) is repulsive. The force arises from this interaction by taking a derivative of E_{int}.

$$\begin{aligned}
\vec{f} &= -\frac{\partial E_{\text{int}}}{\partial R} \hat{R} \\
&= -\frac{\phi_0^2 c_{12}^{-1/2} \ell^2}{\pi\lambda_0^3} \left(X\hat{x} + c_{12}Y\hat{y}\right) \left\{ \frac{1}{R_1} K_1\left(\frac{R_1}{\lambda_0}\right) \right. \\
&\quad \left. + \frac{\epsilon_a(\xi_d^a)^2}{8c_1\lambda_0^2} \left(1 + \frac{\gamma_4}{c_{12}\gamma_3}\right)^2 \left[\frac{2}{R_1} K_1\left(\frac{R_1}{\lambda_0}\right) - \frac{1}{4\lambda_0} K_0\left(\frac{R_1}{\lambda_0}\right)\right] + \cdots \right\}.
\end{aligned} \tag{30}$$

It proves that the direction of this force between two vortices for anisotropic d-wave superconductor is not consistent with vec $\vec{R} = X\hat{x} + Y\hat{y}$, which connects the interacting vortices. Therefore, a torque of the system exists and can be expressed as

$$\begin{aligned}
\vec{\tau} = \vec{R} \times \vec{f} \quad & \frac{\phi_0^2 c_{12}^{-1/2} \ell^2}{\pi\lambda_0^3} \left(\frac{c_2^2 - c_1^2}{c_1 c_2}\right)^{1/2} XY\hat{z} \left\{ \frac{1}{R_1} K_1\left(\frac{R_1}{\lambda_0}\right) \right. \\
& \left. + \frac{\epsilon_a(\xi_d^a)^2}{8c_1\lambda_0^2} \left(1 + \frac{\gamma_4}{c_{12}\gamma_3}\right)^2 \left[\frac{2}{R_1} K_1\left(\frac{R_1}{\lambda_0}\right) - \frac{1}{4\lambda_0} K_0\left(\frac{R_1}{\lambda_0}\right)\right] + \cdots \right\}.
\end{aligned} \tag{31}$$

It is worthy noting $\vec{\tau}$ disappears if both vortices are limited to either x or y axes and the system reduces to isotropic $d_{x^2-y^2}$-wave superconductivity with $M_a = 1$. In the anisotropic s-wave superconductor, a torque between two vortices also exists [30].

5. Summary and discussion

Substituting the linear and gradient terms of d-wave component for the s-wave component in the Ginzburg–Landau free energy allows us to determine the vortex structure in anisotropic form. The effect of the twofold symmetry of a single structure is small but remains far from the core. Moreover, our results reveal the presence of r-component of the supercurrent as $r \to 0$ for anisotropic d-wave superconductor in the Appendix. No r-component of the supercurrent as $r \to 0$ exists for isotropic d-wave superconductors.

We also derive the generic London model and calculate the London free energy to determine the lower critical field H_{c1}. Using this model allows us to express the magnetic field in the absence and presence of the vortex. The London penetration depth λ_a depends not only on temperature but also on the direction of an applied weak field. As mentioned earlier, this work has demonstrated not only how the weak s–d coupling influences the London penetration depth, the London free energy, the lower critical field, and interacting force, but also how c_{12} affects the mass anisotropy dependence of magnetic properties. Hence, the interaction between two vortices and stability of the vortex lattice is changed. The vortex–vortex interacting force can be used to determine the stable structure of the vortex lattice. This force is a contributing factor to the vortex motion, subsequently leading to energy dissipation and a longitudinal resistive voltage. It provides further insight into the dynamics of flux penetration and the mechanisms of vortex pinning.

The vortex lattice problem is also worth mentioning. In the vicinity of the upper critical field H_{c2}, the linearized GL equation corresponding to d-wave component, by neglecting the nonlinear terms and substituting Eq. (7) for s wave, can be written as

$$(H_0 - 1)\psi_d + H_1\psi_d = 0, \tag{32}$$

where

$$H_0 = \left(\Pi_x^2 + \gamma_2 \Pi_y^2\right) - \frac{\gamma_3^2}{\tilde{\alpha}_3}\left(\Pi_x^2 - \frac{\gamma_4}{\gamma_3}\Pi_y^2\right)^2$$

and

$$H_1 = \beta_1\left[\frac{B_1}{\tilde{\alpha}_s}\psi_d + \frac{2\gamma_3}{\tilde{\alpha}_s}\left(\Pi_x^2 - \frac{2\gamma_3}{\tilde{\alpha}_s}\Pi_x^2\right)\right].$$

Chang et al. [31] derived the perturbative solution of the isotropic linearized GL equations. The term $H_1\psi_d$, involving the second order coupling terms, shifts the H_{c2} value but does not change the power of temperature. Also, the vortex structure would be changed because including the term $H_1\psi_d$ breaks the fourfold symmetry into twofold symmetry.

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Appendix A. Supercurrent

In this Appendix, we investigate the distribution of supercurrent around a d-wave vortex with mass anisotropy. In the limit of $\xi_d^a \ll \lambda_a$, the supercurrent can be expressed as

$$\vec{j}= \frac{1}{4\pi\ell^2}\Big\{\hat{x}\big[\psi_d^*(\Pi_x\psi_d)+\gamma_1\psi_s^*(\Pi_x\psi_s)+\gamma_3\big(\psi_s(\Pi_x\psi_d)^*+\psi_d^*(\Pi_x\psi_s)\big)+\text{h.c.}\big]$$
$$+\hat{y}\big[\gamma_2\psi_d^*(\Pi_y\psi_d)+\gamma_1 M_a\psi_s^*(\Pi_y\psi_s)-\gamma_4\big(\psi_s(\Pi_y\psi_d)^*+\psi_d^*(\Pi\psi_s)\big)+\text{h.c.}\big]\Big\}. \tag{A1}$$

The solutions of order parameters ψ_s and ψ_d, neglecting the magnetic field effect, can be estimated simply. Far away from the vortex, we set

$$d(r)=1 \tag{A2}$$

and obtain

$$s_1=\left(1+\frac{\gamma_4}{\gamma_3}\right)\left(\frac{3\gamma_3}{4\tilde{\alpha}_s r^2}\right)\equiv\frac{3a_0}{r^2},\quad s_2=-\left(1+\frac{\gamma_4}{\gamma_3}\right)\left(\frac{\gamma_3}{4\tilde{\alpha}_s r^2}\right)\equiv-\frac{a_0}{r^2},$$
$$s_3=\left(\frac{\gamma_4}{\gamma_3}-1\right)\left(\frac{\gamma_3}{2\tilde{\alpha}_s r^2}\right)+\frac{\beta_1}{\tilde{\alpha}_s}\equiv\frac{b_0}{r^2}+\frac{\beta_1}{\tilde{\alpha}_s}. \tag{A3}$$

The supercurrent is

$$\vec{j}=\frac{1}{4\pi\ell^2 r}\Bigg\{\hat{r}\Bigg[(\gamma_2-1)\sin 2\theta-\frac{\gamma_1(1-M_a)\beta_1}{\tilde{\alpha}_s}\sin 2\theta\left(\frac{\beta_1}{\tilde{\alpha}_s}+\frac{2b_0}{r^2}\right)-\frac{8(1+M_a)\gamma_1\beta_1 a_0}{\tilde{a}_s r^2}\sin 2\theta$$
$$-\frac{10(1-M_a)\gamma_1\beta_1 a_0}{\tilde{\alpha}_s r^2}\sin 4\theta-(\gamma_3+\gamma_4)\left(\frac{2\beta_1}{\tilde{\alpha}_s}+\frac{2b_0}{r^2}\right)\sin 2\theta-\frac{8(\gamma_3-\gamma_4)a_0}{r^2}\sin 2\theta$$
$$-\frac{25(\gamma_3+\gamma_4)a_0}{4r^2}\sin 4\theta\Bigg]+\hat{\theta}\Bigg[(\gamma_2+1)+(\gamma_2-1)\cos 2\theta-\frac{8(M_a-1)\gamma_1\beta_1 a_0}{\tilde{\alpha}_s r^2}\sin^2 2\theta$$
$$-\frac{\gamma_4\beta_1}{\tilde{\alpha}_s}\left(\frac{\gamma_1(1+M_a)}{\tilde{a}_s}-\frac{\gamma_1(1+M_a)}{\tilde{\alpha}_s}\cos 2\theta-\gamma_4\right)\left(\frac{\beta_1}{\tilde{\alpha}_s}+\frac{12a_0}{r^2}\cos 2\theta+\frac{2b_0}{r^2}\right)$$
$$+\frac{8(\gamma_4-\gamma_3)a_0}{r^2}\sin^2 2\theta\Bigg]\Bigg\}+O\left(\frac{1}{r^5}\right). \tag{A4}$$

Both r- and θ-components of the supercurrent have the same leading order $1/r$. Near the vortex core,

$$\psi_d \sim c_0 r e^{i\theta}$$

and

$$\psi_s \sim \frac{c_0\beta_1}{\tilde{\alpha}_s} r e^{i\theta}. \tag{A5}$$

The supercurrent can be written as

$$\vec{j}=\frac{1}{4\pi\ell^2}\left\{c_0^2 r\sin 2\theta\left[(\gamma_2-1)-\frac{\gamma_1(1-M_a)\beta_1^2}{\tilde{\alpha}_s^2}-\frac{2(\gamma_3+\gamma_4)\beta_1}{\tilde{\alpha}_s}\right]\hat{r}\right.$$
$$+c_0^2 r\left[(\gamma_2+1)+(\gamma_2-1)\cos 2\theta+\frac{\gamma_1(1+M_a)\beta_1^2}{\tilde{\alpha}_s^2}-\frac{(1-M_a)\beta_1^2}{\tilde{\alpha}_s^2}-\frac{2(\gamma_3+\gamma_4)\beta_1}{\tilde{\alpha}_s}\right.$$
$$\left.\left.+\frac{2(\gamma_3+\gamma_4)\beta_1}{\tilde{\alpha}_s}\cos 2\theta\right]\hat{\theta}\right\}. \tag{A6}$$

We find an interesting result that there exists an r-component in the supercurrent as $r \to 0$ for a d-wave superconductor with mass anisotropy which vanishes for isotropic case. Namely, the current does not flow around the vortex uniformly along the tangent line.

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