Some Topological Properties of Bitonic Sorters

Tsern-Huei Lee, *Senior Member*, *IEEE*, and Jin-Jye Chou, *Member*, *IEEE*

Abstract—This paper proves some topological properties of bitonic sorters, which have found applications in constructing, along with banyan networks, internally nonblocking switching fabrics in future broadband networks. The states of all the sorting elements of an $N \times N$ bitonic sorter are studied for four different input sequences $\{a_i\}_{i=1}^N$, $\{b_i\}_{i=1}^N$, $\{c_i\}_{i=1}^N$, and $\{d_i\}_{i=1}^N$, where $a_i = i - 1$, $b_i = N - i$, and the binary representations of c_i and d_i are the bit reverse of those of a_i and b_i, respectively. An application of these topological properties is to help design efficient fault diagnosis procedures. We present an example for detecting and locating single faulty sorting element under a simple fault model where all sorting elements are always in the straight state or the cross state.

Index Terms—Bitonic sorter, topological property, switching fabrics, monotonic sequence, fault diagnosis.

1 Introduction

Bitonic sorters [1] have recently been proposed to construct, along with banyan networks, internally nonblocking switching fabrics in future broadband networks [2]. However, a single fault in bitonic sorter or banyan network may become disastrous to the switching system. Therefore, before using it, one has to apply an effective fault diagnosis procedure to make sure both bitonic sorter and banyan network are fault free. Topological properties are often very useful in designing efficient fault diagnosis procedures.

In [3], an effective and efficient diagnosis procedure was proposed to detect and locate single logical faults in a banyan network. As expected, the topological properties of banyan networks are the bases of fault diagnosis. In this paper, we prove some topological properties of bitonic sorters. Although fault diagnosis strongly depends on fault model resulting from particular circuit design, these properties are expected to be useful in developing efficient fault diagnosis procedures under various fault models. In fact, with these properties, one can improve circuit design to facilitate fault diagnosis.

The rest of this paper is organized as follows: In Section 2, we review the bitonic sorters studied in this paper. The significance of the sequences to be applied to the inputs of a bitonic sorter are also discussed in this section. In Section 3, we sketchily prove some topological properties of bitonic sorters. An application example for detecting and locating single faulty sorting element (SE) is presented in Section 4. Conclusions are finally drawn in Section 5.

- *T.-H. Lee is with the Department of Communication, National Chiao Tung University, Hsinchu, Taiwan, Republic of China. E-mail: thlee@atm.cm.nctu.edu.tw.*
- *J.-J. Chou is with the Computer and Communication Research Laboratories, Industrial Technology Research Institute, Chutung, Hsinchu, Taiwan, Republic of China. E-mail: choujj@n300.ccl.itri.org.tw.*

Manuscript revised 23 May 1997.
For information on obtaining reprints of this article, please send e-mail to: tc@computer.org, and reference IEEECS Log Number 105205.

2 The Bitonic Sorters

A bitonic sorter is constructed from 2×2 SEs. There are two types of SEs in a bitonic sorter: the up SE (indicated by an upward arrow) and down SE (indicated by a downward arrow). The state of an SE is controlled by the numbers applied to both inputs. Fig. 1 illustrates the operations of the two types of SEs. In Fig. 1, x and y represent the integers applied to the upper and the lower inputs, respectively. Let S_0 and S_1 denote, respectively, the straight state and the cross state. A down (or up) SE will be in state S_0 if $x \le y$ (respectively, $x \ge y$) or state S_1 if $x > y$ (respectively, $x < y$).

Consider a bitonic sorter with N inputs/outputs where N is a power of 2. Such a sorter consists of $n = log_2 N$ levels of subsorters and, thus, will be referred to as an *n-level* bitonic sorter. Fig. 2 illustrates a three-*level* bitonic sorter. There are 2^{n-i} *level-i* sub-sorters and each *level-i* sub-sorter is similar to a banyan network with 2^i inputs/outputs. Therefore, for convenience, a subsorter is also referred to as a banyan sorter (BS). A sorter is called an ascending (or descending) sorter if it sorts the inputs in the ascending (respectively, descending) order. In this paper, we consider *n-level* ascending sorters. Descending sorters can be dealt with similarly.

We use binary representations for integers where the leftmost bit is the most significant bit and always count from the left or the top. For example, we may refer to the ith bit (from the left) of the binary representation of an integer or the jth (from the top) *level-i* BS. The quadruple (i, j, k, l) is used to represent the lth SE in the kth stage of the jth *level-i* BS. It is not hard to see that, in an *n-level* bitonic sorter, i, j, k, l satisfy $1 \le i \le n$, $1 \le j \le 2^{n-i}$, $1 \le k \le i$, and $1 \le l \le 2^{i-1}$. Fig. 3 shows an example of the numbering scheme in a three-*level* bitonic sorter.

Notice that, up to the *level-i* BSs in an *n-level* bitonic sorter, there are 2^{n-i} *i-level* bitonic sorters. The jth *i-level* bitonic sorter is an ascending sorter if j is odd or a descending sorter if j is even. Moreover, SE (i, j, k, l) is a down SE if j is odd or an up SE if j is even. As a consequence, a BS contains only down SEs or only up SEs. For convenience, a BS which contains only down (or up) SEs is referred to as

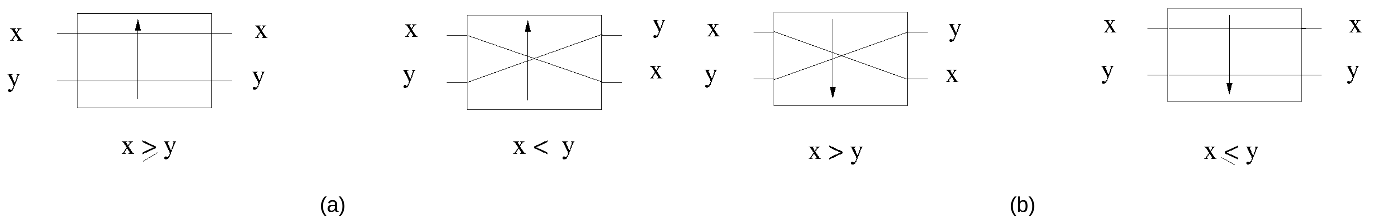


Fig. 1. Operations of sorting elements: x: integer applied to the upper limit, y: integer applied to the lower input.

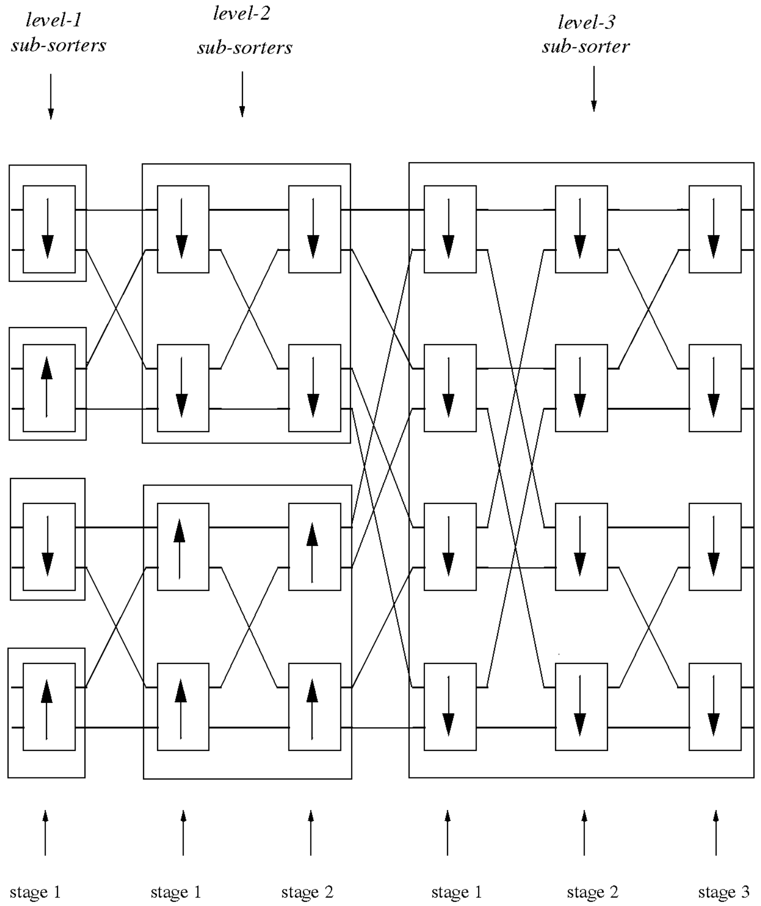


Fig. 2. A three-level bitonic sorter.

down (respectively, up) BS. From stage k of a *level-i* BS, there are 2^{k-1} sub-banyan sorters (sub-BSs) such that each sub-BS is exactly the same as a *level*-$(i - k + 1)$ BS. The state of SE (i, j, k, l) will be denoted by $S(i, j, k, l)$.

We shall study the states of all SEs when one of the four different sequences $\{a_i\}_{i=1}^N$, $\{b_i\}_{i=1}^N$, $\{c_i\}_{i=1}^N$, and $\{d_i\}_{i=1}^N$ is applied to the inputs. The sequence $\{a_i\}_{i=1}^N$ is monotonic increasing with $a_i = i - 1$ and $\{b_i\}_{i=1}^N$ is monotonic decreasing with $b_i = N - i$. The binary representations of c_i and d_i are the bit reverse of those of a_i and b_i, respectively.

When a sequence, say $\{a_i\}_{i=1}^N$, is partitioned into k (k is a divisor of N) groups $G_1, G_2, \cdots$, and G_k, we mean that all the groups are of equal size and G_1 contains the first $\frac{N}{k}$ elements of $\{a_i\}_{i=1}^N$, G_2 contains the second $\frac{N}{k}$ elements of $\{a_i\}_{i=1}^N$, and so on. Further, the elements of G_i are arranged in ascending order. We use G_i' to denote a group whose elements are identical to those in G_i but may not be arranged in ascending order. Also, $\overline{G_i'}$ is used to denote the group which contains the same elements as G_i' but in the reversed order. For example, if $\{a_i\}_{i=1}^8$ is partitioned into two groups G_1 and G_2, then $G_1 = [a_1a_2a_3a_4]$ and $G_2 = [a_5a_6a_7a_8]$. A G_1' can be $[a_1a_3a_4a_2]$ and $\overline{G_1'} = [a_2a_4a_3a_1]$. When a sequence, say $\{a_i\}_{i=1}^N$, is applied to the inputs (of a bitonic sorter), we mean that a_j is applied to the jth input. Further, when a sequence is said to be the input of a BS or a sub-BS, we mean the sequence before shuffle permutation. For any two

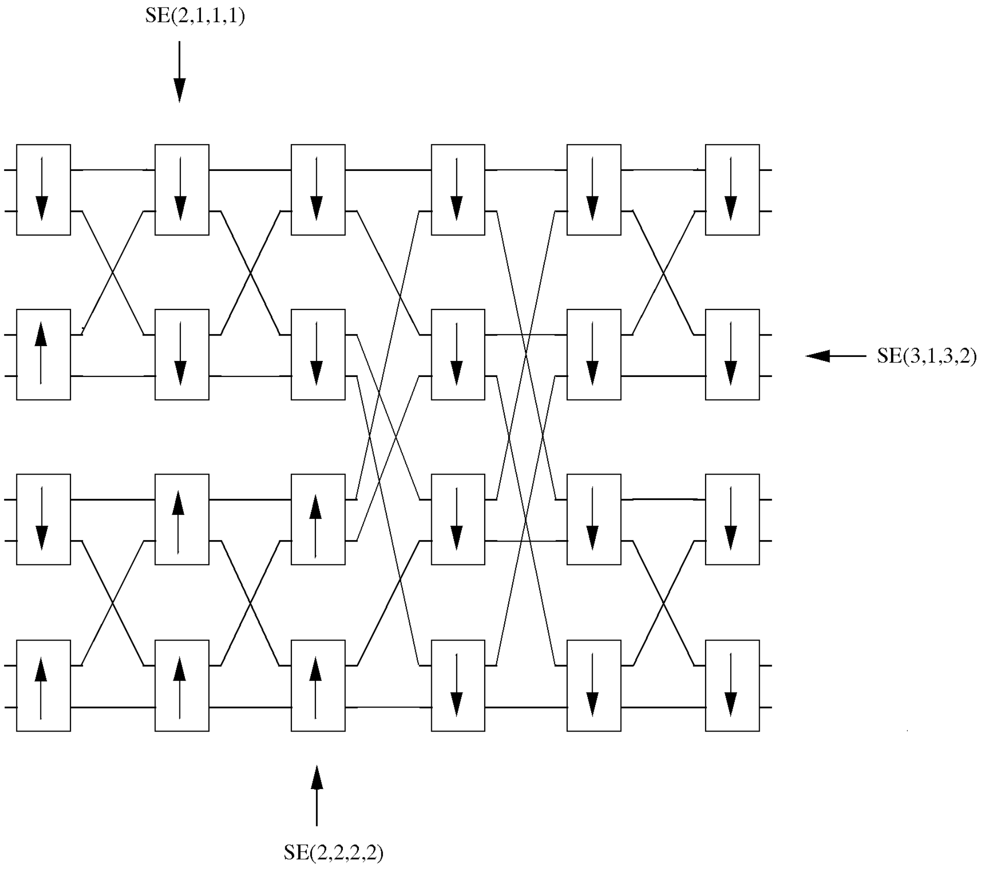


Fig. 3. Numbering scheme of a three-level bitonic sorter.

groups X and Y, define $X \oplus Y$ to be the group obtained by adding the elements of group Y to the end of group X. Also, define $X \ominus Y$ to be the group obtained by removing the elements of group Y from group X. For the rest of this paper, $\lceil x \rceil$ represents the smallest integer greater than or equal to x and $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x.

3 Some Topological Properties

In this section, we state and sketch the proofs for some topological properties of bitonic sorters. In most properties, only one case is selected to prove. One can easily generalize the proof to other cases. When two numbers x and y are said to enter into an SE, we mean that x and y enter into the SE from the upper input and the lower input, respectively. For references, the results of applying $\{a_i\}_{i=1}^N$, $\{b_i\}_{i=1}^N$, $\{c_i\}_{i=1}^N$, and $\{d_i\}_{i=1}^N$ to a bitonic sorter with $N = 16$ are illustrated in Figs. 4, 5, 6, and 7, respectively.

Property 1. *If $\{a_i\}_{i=1}^N$ is applied to the inputs, then the state of SE (i, j, k, l) is given by*

$$S(i,j,k,l) = \begin{cases} S_0 & \text{if } k = 1, j \text{ odd or } k \geq 2, l \leq 2^{i-2}, \\ S_1 & \text{if } k = 1, j \text{ even or } k \geq 2, l > 2^{i-2}. \end{cases}$$

Proof. We prove this property for $j = 1$. The BS for $j = 1$ is a down BS and the input sequence to this BS is $H_1 = a_1 a_2 \ldots a_{2^{i-1}} a_{2^i} a_{2^i - 1} \ldots a_{2^{i-1}+1}$. If $k = 1$, then the two numbers entering into SE $(i, 1, 1, l)$ are a_l and $a_{2^i - l + 1}$ which result in $S(i, 1, 1, l) = S_0$ because $a_l < a_{2^i - l+1}$. Suppose that $k \geq 2$. Partition H_1 into 2^k groups $G_1, G_2, \cdots,$ and G_{2^k}. If $l \leq 2^{i-2}$, then the two numbers x and y entering into SE $(i, 1, k, l)$ satisfy $x \in G_{2m-1}$ and $y \in G_{2m}$, where $m = \left\lfloor \frac{l-1}{2^{i-k}} \right\rfloor + 1$. Thus, we have $S(i, 1, k, l) = S_0$ because $x < y$. On the other hand, if $l > 2^{i-2}$, then the two numbers entering into $(i, 1, k, l)$ satisfy $x \in G_{2m}$ and $y \in G_{2m-1}$ which result in $S(i, 1, k, l) = S_1$ because $x > y$. □

Property 2. *If $\{b_i\}_{i=1}^N$ is applied to the inputs, then the state of SE (i, j, k, l) is given by*

$$S(i,j,k,l) = \begin{cases} S_1 & \text{if } k = 1, j \text{ odd or } k \geq 2, l \leq 2^{i-2}, \\ S_0 & \text{if } k = 1, j \text{ even or } k \geq 2, l > 2^{i-2}. \end{cases}$$

Proof. The proof for Property 2 is similar to that for Property 1 and is omitted. □

Property 3. *There is at most one common link shared by a path when $\{a_i\}_{i=1}^N$ is applied and another path when $\{b_i\}_{i=1}^N$ is applied.*

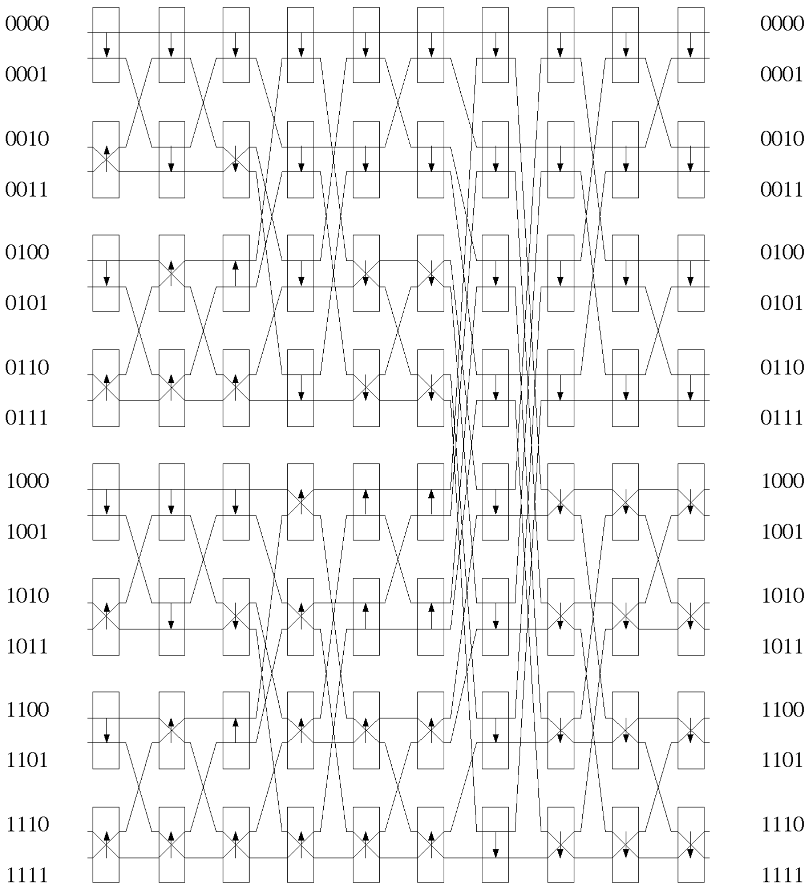


Fig. 4. Result of applying $\{a_i\}_{i=1}^{N}$ to the inputs for N = 16.

PROOF. Consider a path, called path 1, when $\{a_i\}_{i=1}^{N}$ is applied and another path, called path 2, when $\{b_i\}_{i=1}^{N}$ is applied. There is nothing to prove if path 1 and path 2 do not share any common link. Suppose that paths 1 and 2 share some common links. Consider the common link which is closest to the output. Assume that this link belongs to the jth *level-i* BS (excluding the input links but including the output links). Again, we prove the case for $j = 1$.

By the unique path property of banyan network, path 1 and path 2 do not share any more links in the *level-i* BS. Let path 1 be the route of number x when $\{a_i\}_{i=1}^{N}$ is applied and path 2 the route of number y when $\{b_i\}_{i=1}^{N}$ is applied. As a result, we have

$$x \in \left\{a_1, a_2, \ldots, a_{2^i}\right\}$$

and

$$y \in \left\{a_{N-1}, a_{N-2}, \ldots, a_{N-2^i}\right\}.$$

Property 3 is true because, up to *level*-($i-1$) BSs, the route for x when $\{a_i\}_{i=1}^{N}$ is applied and the route for y when $\{b_i\}_{i=1}^{N}$ is applied are in different ($i-1$)-*level* bitonic sorters and, thus, cannot share any link. □

PROPERTY 4. *Let $\{a_i\}_{i=1}^{N}$ or $\{b_i\}_{i=1}^{N}$ be applied to the inputs. The binary representations of the two numbers entering into an* SE *in the first stage of a level-i* BS *differ in exactly i bits, the rightmost i bits.*

PROOF. Consider the first *level-i* BS and assume $\{a_i\}_{i=1}^{N}$ is applied to the inputs. The two numbers entering into SE (i, 1, 1, l) are a_l and a_{2^i-l-1}. Since

$$a_l + a_{2^i-l+1} = l - 1 + 2^i - l = 2^i - 1,$$

we know that the binary representations of a_l and a_{2^i-l-1} differ in exactly the rightmost i bits. □

PROPERTY 5. *Let $\{a_i\}_{i=1}^{N}$ or $\{b_i\}_{i=1}^{N}$ be applied to the inputs. If two paths meet at an* SE *in stage k (k > 1) of a level-i BS, then they meet again at an* SE *in stage k + 1 of a level-(i + 1)* BS.

PROOF. We prove this property assuming that $\{a_i\}_{i=1}^{N}$ is applied to the inputs. Suppose that two paths, called path 1 and path 2, meet at an SE (called SE*) in stage k ($k > 1$) of the first *level-i* BS. Further, assume that SE* is in the mth sub-BS from stage k of the first *level-i* BS. According to the operation of bitonic sorters, the input sequence to the mth sub-BS from stage k of the first *level-i* BS is the same as the input sequence to the

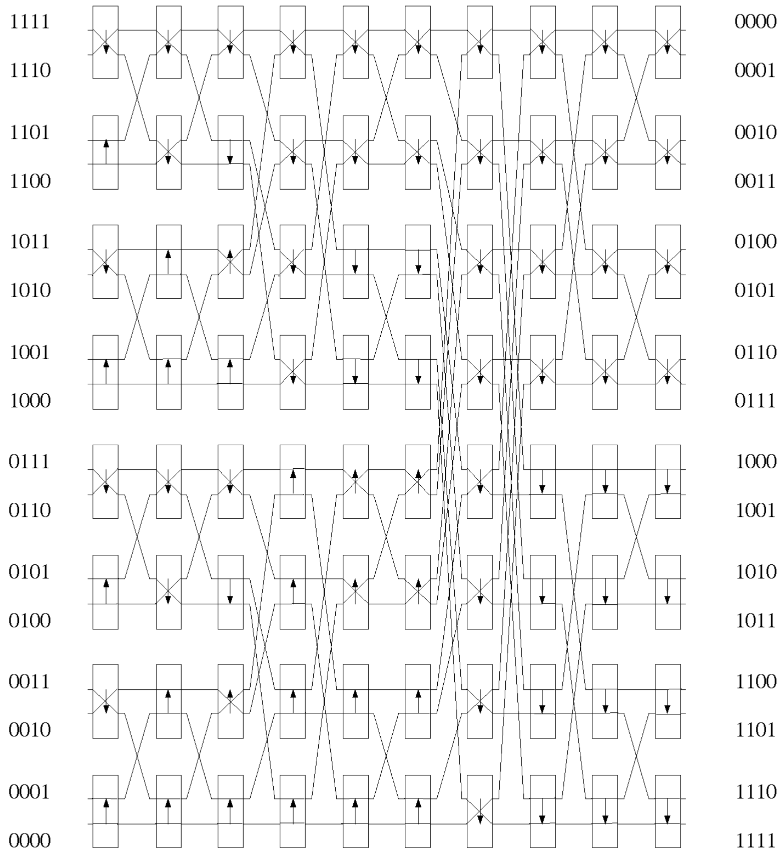


Fig. 5. Result of applying $\{b_i\}_{i=1}^{N}$ to the inputs for N = 16.

mth sub-BS from stage $(k + 1)$ of the first *level*-$(i + 1)$ BS and thus the property is true. □

PROPERTY 6. *Let* $\{a_i\}_{i=1}^{N}$ *or* $\{b_i\}_{i=1}^{N}$ *be applied to the inputs. The binary representations of the two numbers entering into an* SE *in stage* k $(k > 1)$ *of a level-*i BS *differ in exactly one bit, the* $(n - i + k)$th *bit.*

PROOF. Assume that $\{a_i\}_{i=1}^{N}$ is applied to the inputs and consider the *level-n* BS. Partition $\{a_i\}_{i=1}^{N}$ into 2^k groups $G_1, G_2, \cdots,$ and G_{2^k}. The input sequence to the *level-n* BS is $G_1G_2 \ldots G_{2^{k-1}}\overline{G_{2^k}} \ldots \overline{G_{2^{k-1}+1}}$. Consider SE $(n, 1, k, l)$ and let $m = \left\lfloor \frac{l-1}{2^{n-k}} \right\rfloor + 1$. The input sequence to the mth sub-BS from stage k is $G_{2m-1}G_{2m}$ if $m \le 2^{k-2}$ or $\overline{G_{2m}}\,\overline{G_{2m-1}}$ if $m > 2^{k-2}$. Hence, the two numbers x and y entering into SE $(n, 1, k, l)$ are, respectively, the lth elements of G_{2m-1} and G_{2m} if $m \le 2^{k-2}$ or $\overline{G_{2m}}$ and $\overline{G_{2m-1}}$ if $m > 2^{k-2}$. Since there are 2^{n-k} elements in each group, we have $|x - y| = 2^{n-k}$ which implies the binary representations of x and y differ in exactly one bit, the kth bit.

From Property 5, we know that if two numbers meet in stage k of a *level-i* BS, then they meet again in stage $(k + n - i)$ of the *level-n* BS. Thus, from the above results, they differ in exactly one bit, the $(n - i + k)$th bit. □

PROPERTY 7. *Let* $\{a_i\}_{i=1}^{N}$ *or* $\{b_i\}_{i=1}^{N}$ *be applied to the inputs. If two paths meet at an* SE *in stage 1 of a level-*i $(i > 1)$ BS, *then they meet only once.*

PROOF. Suppose that two numbers x and y meet at an SE in the first stage of a *level-i* BS. From Property 4, the binary representations of these two numbers differ in at least two bits if $i > 1$. Thus, according to Property 6, if these two numbers meet again, they must meet at an SE in the first stage of a BS, which is obviously impossible when $\{a_i\}_{i=1}^{N}$ or $\{b_i\}_{i=1}^{N}$ is applied to the inputs. □

PROPERTY 8. *Let* $\{a_i\}_{i=1}^{N}$ *be applied to the inputs. Suppose that the state of* SE (i, j, k, l) *is changed to* S_0 *if* $S(i, j, k, l) = S_1$ *or* S_1 *if* $S(i, j, k, l) = S_0$. *Then, the output sequence is correct (i.e., an ascending sequence) if* $k = i < n$ *or becomes* $G_1G_2 \ldots G^*_{2m-1}G^*_{2m} \ldots G_{2^{n-i+k}}$, *where*

$$m = \begin{cases} (j-1)2^{k-1} + \left\lfloor \frac{l-1}{2^{i-k}} \right\rfloor & \text{if } j \text{ is odd} \\ j \cdot 2^{k-1} + 1 - \left\lfloor \frac{l-1}{2^{i-k}} \right\rfloor & \text{if } j \text{ is even,} \end{cases}$$

$G^*_{2m-1} = \left(G'_{2m-1} \ominus [x]\right) \oplus [y]$ *and* $G^*_{2m} = [x] \oplus \left(G'_{2m} \ominus [y]\right)$ *for some* $x \in G_{2m-1}$ *and* $y \in G_{2m}$.

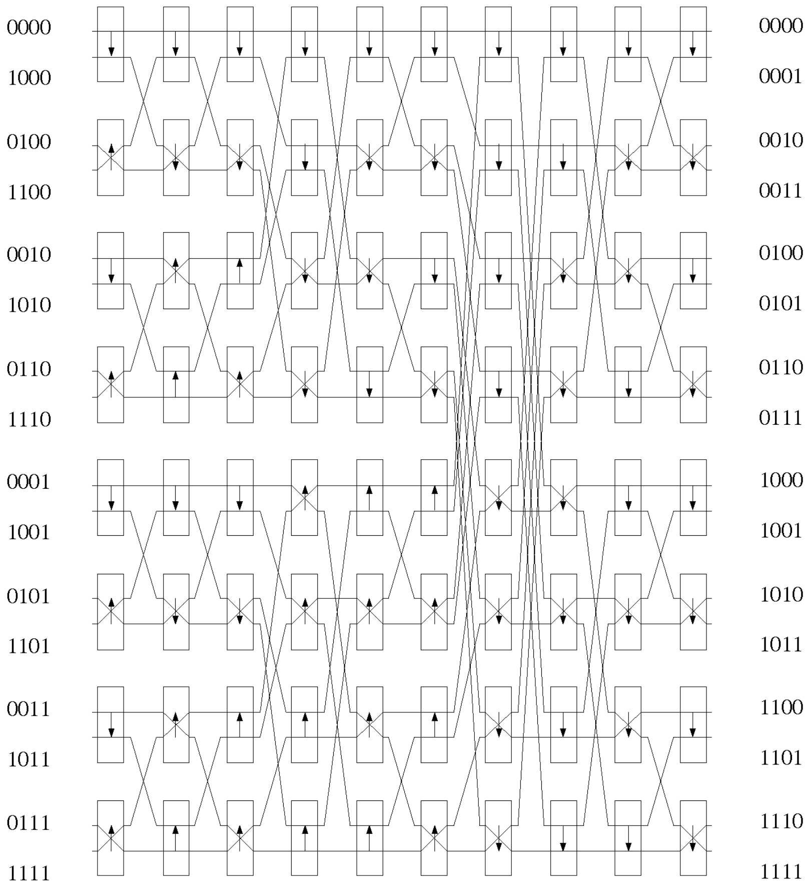


Fig. 6. Results of applying $\{c_i\}_{i=1}^{N}$ to the inputs for N = 16.

PROOF. The property can be easily verified to be true for $i = n$. Assume that $i < n$. Again, consider the case for $j = 1$. In this case, we have $m = \left\lfloor \frac{l-1}{2^{i-k}} \right\rfloor$. If $k = i$, then the two numbers entering into SE (i, j, i, l) are a_{2l-1} and a_{2l} if $l \le 2^{i-2}$ or a_{2l} and a_{2l-1} if $l > 2^{i-2}$. Since a_{2l-1} and a_{2l} are consecutive integers, Property 5 still holds even if the state of SE $(i, 1, i, l)$ is changed. In other words, a_{2l-1} and a_{2l} meet again at an SE in the last stage of the first *level*-$(i + 1)$ BS and, thus, the output sequence becomes correct.

Consider now the case $k < i < n$. If the state of SE $(i, 1, i, l)$ is changed, then the sequence at the output of the first *level*-i BS is $G_1 G_2 \ \dots \ G^*_{2m-1} G^*_{2m} \ \dots \ G_{2^k}$, where $G^*_{2m-1} = \left(G'_{2m-1} \ominus [x]\right) \oplus [y]$ and $G^*_{2m} = [x] \oplus \left(G'_{2m} \ominus [y]\right)$ for some $x \in G_{2m-1}$ and $y \in G_{2m}$. Moreover, the input sequence to the first *level*-$(i + 1)$ BS is

$$G_1 G_2 \ \dots \ G^*_{2m-1} G^*_{2m} \ \dots \ G_{2^k} \overline{G_{2^{k+1}}}\, \overline{G_{2^{k+1}-1}} \ \dots \ \overline{G_{2^k-1}}.$$

The property is proved if one can show that the sequence at the output of the first *level*-$(i + 1)$ BS is

$$G_1 G_2 \ \dots G^{**}_{2m-1} G^{**}_{2m} \ \dots \ G_{2^{k+1}},$$

where

$$G^{**}_{2m-1} = \left(G'_{2m-1} \ominus [x']\right) \oplus [y']$$

and

$$G^{**}_{2m} = [x'] \oplus \left(G'_{2m} \ominus [y']\right),$$

for some $x' \in G_{2m-1}$ and $y' \in G_{2m}$, because the same arguments can be applied to the sequences at the output of the first *level*-$(i + 2)$ BS, the first *level*-$(i + 3)$ BS, $\cdots$, and the *level*-n BS.

It can be verified that the input sequence to the mth sub-BS from stage $k + 1$ of the first *level*-$(i + 1)$ BS is $G^*_{2m-1} G^*_{2m}$. Since y is the largest number in G^*_{2m-1}, it will meet with z, the largest number in G^*_{2m} (which is also in G_{2m}). The result is either y or z enters into the $(2m - 1)$th sub-BS from stage $k + 2$. Similarly, either x or w, the smallest number in G^*_{2m-1}, enters into the $(2m)$th sub-BS from stage $k + 2$. The input sequence to the tth $(t \ne m)$ sub-BS from stage $k + 1$ is $G_{2t-1} G_{2t}$. Therefore, the property is true because the sequence at the output of the first *level*-$(i + 1)$ BS is

$$G_1 G_2 \ \dots \ G^{**}_{2m-1} G^{**}_{2m} \ \dots \ G_{2^k},$$

where

$$G^{**}_{2m-1} = \left(G'_{2m-1} \ominus [x']\right) \oplus [y']$$

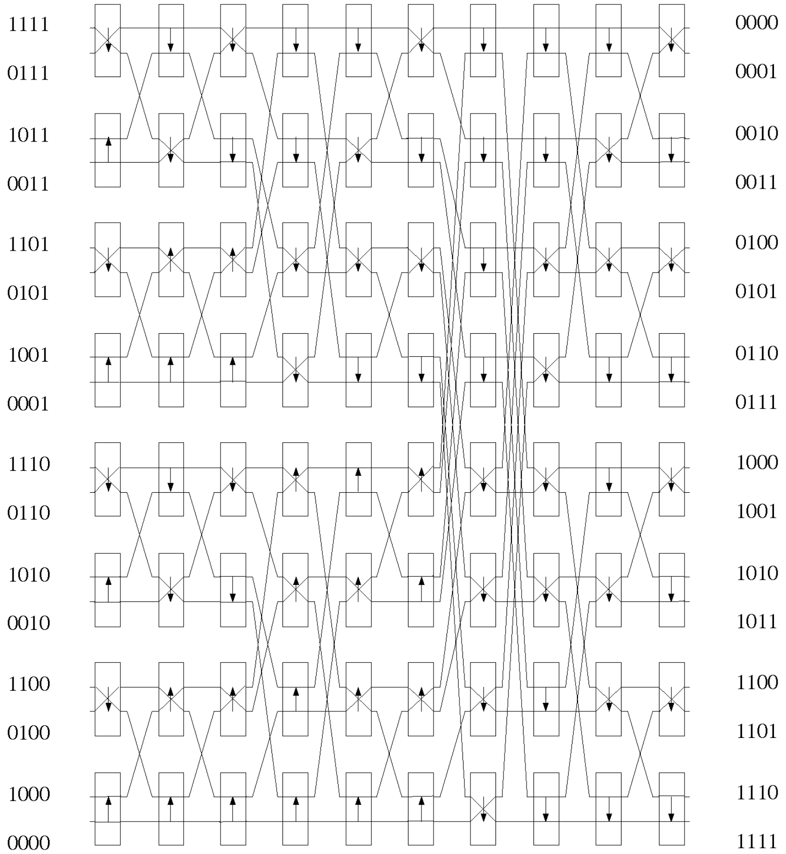


Fig. 7. Results of applying $\{d_i\}_{i=1}^{N}$ to the inputs for N = 16.

and

$$G_{2m}^{**} = [x'] \oplus \left(G'_{2m} \ominus [y']\right)$$

for some $x' \in G_{2m-1}$ and $y' \in G_{2m}$. □

PROPERTY 9. *Let $\{b_i\}_{i=1}^{N}$ be applied to the inputs. Suppose that the state of* SE (i, j, k, l) *is changed to S_0 if $S(i, j, k, l) = S_1$ or S_1 if $S(i, j, k, l) = S_0$. Then, the output sequence is correct (i.e., an ascending sequence) if $k = i < n$ or becomes $G_1 G_2 \ldots G_{2m-1}^{*} G_{2m}^{*} \ldots G_{2^{n-i+k}}$*, where

$$m = \begin{cases} 2^{n-i+k-1} - j \cdot 2^{k-1} + \left\lfloor \frac{l-1}{2^{i-k}} \right\rfloor & \text{if } j \text{ is odd} \\ 2^{n-i+k-1} - (j-1) \cdot 2^{k-1} - \left\lfloor \frac{l-1}{2^{i-k}} \right\rfloor + 1 & \text{if } j \text{ is even,} \end{cases}$$

$G_{2m-1}^{*} = \left(G'_{2m-1} \ominus [x]\right) \oplus [y]$ *and* $G_{2m}^{*} = [x] \oplus \left(G'_{2m} \ominus [y]\right)$ *for some $x \in G_{2m-1}$ and $y \in G_{2m}$.*

PROOF. The proof for Property 9 is similar to that for Property 8 and is omitted. □

PROPERTY 10. *If $\{c_i\}_{i=1}^{N}$ is applied to the inputs, then the state of* SE (i, j, k, l) *is given by*

$$S(i,j,k,l) = \begin{cases} S_0 & \text{if } m \text{ is odd, } k < i, \text{ and } r \le 2^{i-k-1} \\ & \text{or } m \text{ is even, } k < i, \text{ and } r > 2^{i-k-1} \\ & \text{or } m \text{ is odd and } k = i \\ S_1 & \text{if } m \text{ is even, } k < i, \text{ and } r \le 2^{i-k-1} \\ & \text{or } m \text{ is odd, } k < i, \text{ and } r > 2^{i-k-1} \\ & \text{or } m \text{ is even and } k = i, \end{cases}$$

where $m = \left\lfloor \frac{l-1}{2^{i-k}} \right\rfloor + 1$ *and* $r = (l-1) \bmod 2^{i-k} + 1$.

PROOF. For $i = 1$, we have $k = l = 1$ and the property can be easily verified to be true because the two numbers entering into SE $(1, j, 1, 1)$ are c_{2j-1} and c_{2j} which satisfy $c_{2j} > c_{2j-1}$. Suppose that $i > 1$ and $k < i$. Consider the case for $j = 1$. The input sequence to the first *level-i* BS is $E\overline{O}$, where $E = [e_1 e_2 \ldots e_{2^{i-1}}]$ with $e_f = (2f-2) \cdot 2^{n-i}$ and $O = [o_1 o_2 \ldots o_{2^{i-1}}]$ with $o_f = (2f-1) \cdot 2^{n-i}$. Since the common factor 2^{n-i} can be neglected, we assume that $E = [0\ 2\ 4 \cdots 2^i - 2]$ and $O = [1\ 3\ 5 \cdots 2^i - 1]$.

For $m = \left\lfloor \frac{l-1}{2^{i-k}} \right\rfloor + 1$ and $r = (l-1) \bmod 2^{i-k} + 1$, we have $(m-1)2^{i-k} + r = l$. In other words, SE (i, j, k, l) is the rth SE of the mth sub-BS from stage k. Partition E and O into 2^k groups denoted by $E_1, E_2, \ldots, E_{2^k}$ and

$O_1, O_2, \ldots, O_{2^k}$, respectively. If m is odd, the input sequence to the mth sub-BS from stage k is $E_m\overline{O_m}$ and the two numbers x and y entering into SE $(i, 1, k, l)$ are the rth elements of E_m and $\overline{O_m}$, respectively. Since $x > y$ if $r \le 2^{i-k-1}$ and $x < y$ if $r > 2^{i-k-1}$, we have $S(i, 1, k, l) = S_0$ if $r \le 2^{i-k-1}$ or S_1 if $r > 2^{i-k-1}$. Similarly, if m is even, then the input sequence to the mth sub-BS is $\overline{O_m}E_m$ and the two numbers x and y entering into SE $(i, 1, k, l)$ are the rth elements of $\overline{O_m}$ and E_m, respectively. As a result, we have $S(i, 1, k, l) = S_1$ if $r \le 2^{i-k-1}$ or S_0 if $r > 2^{i-k-1}$.

If $k = i > 1$, then $m = l$, $r = 1$, and the two numbers entering into SE $(i, 1, i, l)$ are e_l and o_l if m is odd or o_l and e_l if m is even. As a result, we have $S(i, 1, i, l) = S_0$ if m is odd or S_1 if m is even. □

PROPERTY 11. *If* $\{d_i\}_{i=1}^N$ *is applied to the inputs, then the state of* SE (i, j, k, l) *is given by*

$$S(i,j,k,l) = \begin{cases} S_0 & \text{if } m \text{ is odd, } k < i, \text{ and } r \le 2^{i-k-1} \\ & \text{or } m \text{ is even, } k < i, \text{ and } r > 2^{i-k-1} \\ & \text{or } m \text{ is even and } k = i \\ S_1 & \text{if } m \text{ is even, } k < i, \text{ and } r \le 2^{i-k-1} \\ & \text{or } m \text{ is odd, } k < i, \text{ and } r > 2^{i-k-1} \\ & \text{or } m \text{ is odd and } k = i, \end{cases}$$

where $m = \left\lfloor \frac{l-1}{2^{i-k}} \right\rfloor + 1$ *and* $r = (l-1) \bmod 2^{i-k} + 1$.

PROOF. The proof for Property 11 is similar to that for Property 10 and is omitted. □

PROPERTY 12. Let $\{c_i\}_{i=1}^N$ or $\{d_i\}_{i=1}^N$ be applied to the inputs. Suppose that the state of SE (i, j, i, l) is changed to S_0 if $S(i, j, i, l) = S_1$ or S_1 if $S(i, j, i, l) = S_0$. Then, the output becomes $G_1G_2 \ldots G^*_{2m-1}G^*_{2m} \ldots G_{2^i}$, where

$$m = \begin{cases} l & \text{if } j \text{ is odd} \\ 2^{i-1} - l + 1 & \text{if } j \text{ is even,} \end{cases}$$

$$G^*_{2m-1} = (G_{2m-1} \ominus [x]) \oplus [y]$$

and

$$G^*_{2m} = [x] \oplus (G_{2m} \ominus [y]),$$

where x is the largest number in G_{2m-1} and y is the smallest number in G_{2m}.

PROOF. Consider the case for $j = 1$ and assume that $\{c_i\}_{i=1}^N$ is applied to the inputs. As in the proof of Property 10, we neglect the common factor 2^{n-i} and assume that $E\overline{O}$ is the input sequence to the first *level-i* BS, where $E = [0\ 2\ 4 \ldots 2^i - 2]$ and $O = [1\ 3\ 5 \ldots 2^i - 1]$. As a result, the sequence at the output of the first *level-i* BS is $H = [0\ 1\ 2 \ldots 2^i - 2]$. Let E and O be partitioned into 2^k $(k \le i - 1)$ groups denoted by $E_1, E_2, \ldots, E_{2^k}$ and $O_1, O_2, \ldots, O_{2^k}$, respectively. Also, let H be partitioned into 2^k groups denoted by H_1, H_2, …, and H_{2^k}. Clearly, H_j contains all the elements of E_j and O_j. Let x and y be the largest number in E_{2m-1} and the smallest number in E_{2m}, respectively. Also, let w and z be the largest number in O_{2m-1} and the smallest number in O_{2m}, respectively. It is obvious that w is the largest number in H_{2m-1} and y is the smallest number in H_{2m}. Besides, we have $x = w - 1$, $w = y - 1$, and $y = z - 1$. This property is proved if one can show that if the input sequence to the first *level-i* BS is

$$E_1E_2 \ldots E^*_{2m-1}E^*_{sm} \ldots E_{2^k}\overline{O_{2^k}} \ldots \overline{O_1},$$

where

$$E^*_{2m-1} = (E_{2m-1} \ominus [x]) \oplus [y]$$

and

$$E^*_{2m} = [x] \oplus (E_{2m} \ominus [y]),$$

or

$$E_1E_2 \ldots E_{2^k}\overline{O_{2^k}} \ldots \overline{O^*_{2m}}\,\overline{O^*_{2m-1}} \ldots \overline{O_1},$$

where

$$O^*_{2m-1} = (O_{2m-1} \ominus [w]) \oplus [z]$$

and

$$O^*_{2m} = [w] \oplus (O_{2m} \ominus [z]),$$

then the output sequence becomes

$$H_1H_2 \ldots H^*_{2m-1}H^*_{2m} \ldots H_{2^k},$$

where

$$H^*_{2m-1} = (H_{2m-1} \ominus [w]) \oplus [y]$$

and

$$H^*_{2m} = [w] \oplus (H_{2m} \ominus [y]).$$

We shall prove this assuming that the input sequence is $E_1E_2 \ldots E^*_{2m-1}E^*_{2m} \ldots E_{2^k}\overline{O_{2^k}} \ldots \overline{O_1}$ (the case for $j = 1$).

It can be verified that the sequence at the output of the tth $(t \ne m)$ sub-BS from stage k is $H_{2t-1}H_{2t}$. Besides, the input sequence to the mth sub-BS from stage k is $E^*_{2m-1}E^*_{2m}\overline{O_{2m}}\,\overline{O_{2m-1}}$ if m is odd or $\overline{O_{2m}}\,\overline{O_{2m-1}}E^*_{2m-1}E^*_{2m}$ if m is even. Suppose that m is odd. (The case when m is even can be proved similarly.) In stage k of the first *level-i* BS, y meets with z and x meets with w. As a result, the input sequence to the $(2m - 1)$th sub-BS from stage $k + 1$ is $E^*_{2m-1}\overline{O^*_{2m-1}}$, where $\overline{O^*_{2m-1}} = [x] \oplus (\overline{O_{2m-1}} \ominus [w])$. Notice that $E^*_{2m-1}\overline{O^*_{2m-1}}$ is a bitonic sequence. Therefore, the sequence at the output of the $(2m - 1)$th sub-BS from stage $k + 1$ is $H^*_{2m-1} = (H_{2m-1} \ominus [w]) \oplus [y]$. Similarly, the input sequence to the $(2m)$th sub-BS from stage $k + 1$ is $\overline{O_{2m}}E^{**}_{2m}$, where $E^{**}_{2m} = [w] \oplus \left(E^{*}_{2m} \ominus [x]\right)$. Again, $\overline{O_{2m}}E^{**}_{2m}$ is a bitonic sequence and, therefore, the sequence at the output of the $(2m)$th sub-BS from stage $k + 1$ is $H^{*}_{2m} = [w] \oplus \left(H_{2m} \ominus [y]\right)$. Therefore, Property 12 is true for $j = 1$. □

In the remaining properties, we use subscript to denote the state of an SE when a particular sequence is applied. For example, $S_a(i, j, k, l)$ represents the state of SE (i, j, k, l) when $\{a_i\}_{i=1}^{N}$ is applied to the inputs.

PROPERTY 13. *Partition* $\{a_i\}_{i=1}^{N}$ *into two groups* G_1 *and* G_2. *If* G_2G_1 *is applied to the inputs, then we have*

$$S(i, j, k, l) = \begin{cases} S_a(i, j, k, l) & \text{if } i < n \\ S_b(i, j, k, l) & \text{if } i = n. \end{cases}$$

PROOF. Since the numbers in both G_1 and G_2 are in ascending order, $S(i, j, k, l)$ is identical to $S_a(i, j, k, l)$ if $i < n$. Moreover, the input sequence to the *level-n* BS is the same as that when $\{b_i\}_{i=1}^{N}$ is applied to the inputs and, thus, $S(n, j, k, l) = S_b(n, j, k, l)$. □

PROPERTY 14. *Partition* $\{b_i\}_{i=1}^{N}$ *into two groups* G_1 *and* G_2. *If* $\overline{G_2}\,\overline{G_1}$ *is applied to the inputs, then we have*

$$S(i, j, k, l) = \begin{cases} S_b(i, j, k, l) & \text{if } i < n \\ S_a(i, j, k, l) & \text{if } i = n. \end{cases}$$

PROOF. The proof for Property 14 is similar to that for Property 13 and is omitted. □

PROPERTY 15. *Partition* $\{a_i\}_{i=1}^{N}$ *into* 2^k *groups* $G_1, G_2, \cdots$, *and* G_{2^k}. *Consider* SE $(n, 1, k, l)$.

CASE 1. $m = \left\lfloor \frac{l-1}{2^{n-k}} \right\rfloor + 1 \le 2^{k-2}$.

Let

$$G'_{2m-1} = \left(G_{2m-1} \ominus \left[x_1x_2 \ \ldots \ x_p\right]\right) \oplus \left[y_1y_2 \ \ldots y_p\right]$$

and

$$G'_{2^k} = \left[x_1x_2 \ \ldots \ x_p\right] \oplus \left(G_{2^k} \ominus \left[y_1y_2 \ \ldots \ y_p\right]\right),$$

where $x_1 < x_2 < \cdots < x_p$ *are the smallest p numbers in* G_{2m-1} *and* $y_1 < y_2 < \cdots < y_p$ *are the largest p numbers in* G_{2^k}. *If* $G_1G_2 \ \ldots \ G'_{2m-1}G_{2m} \ \ldots \ G'_{2^k}$ *is applied to the inputs, then we have*

$$S(n, 1, k, l) = S_b(n, 1, k, l) \text{ for all } l,\ m \cdot 2^{n-k} - p + 1 \le l \le m \cdot 2^{n-k}.$$

CASE 2. $m = \left\lfloor \frac{l-1}{2^{n-k}} \right\rfloor + 1 > 2^{k-2}$.

Let

$$G'_1 = \left(G_1 \ominus \left[w_1w_2 \ \ldots \ w_p\right]\right) \oplus \left[z_1z_2 \ \ldots \ z_p\right]$$

and

$$G'_{2m} = \left[w_1w_2 \ \ldots \ w_p\right] \oplus \left(G_{2m} - \left[z_1z_2 \ \ldots \ z_p\right]\right),$$

where $z_1 < z_2 < \cdots < z_p$ *are the largest p numbers in* G_{2m} *and* $w_1 < w_2 < \cdots < w_p$ *are the smallest p numbers in* G_1. *If* $G'_1G_2 \ \ldots \ G'_{2m} \ \ldots \ G_{2^k}$ *is applied to the inputs, then we have*

$$S(n, 1, k, l) = S_b(n, 1, k, l) \text{ for all } l,\ m \cdot \{2^{n-k}\} - p + 1 \le l \le m \cdot 2^{n-k}.$$

PROOF. Consider the case for $m \le 2^{k-2}$. The input sequence to the *level-n* BS is $H_1\overline{H_2}$, where

$$H_1 = \left(G_1G_2 \ \ldots \ G_{2^{k-1}}\right) \ominus \left[x_1x_2 \ \ldots \ x_p\right] \oplus \left[y_1y_2 \ \ldots \ y_p\right]$$

and

$$H_2 = \left[x_1x_2 \ \ldots \ x_p\right] \oplus \left(G_{2^{k-1}+1} \ \ldots \ G_{2^k}\right) \ominus \left[y_1y_2 \ \ldots \ y_p\right].$$

The two numbers x and y entering into SE $(n, 1, k, l)$ satisfy

$$x \in G_1 \oplus G_2 \oplus \cdots \oplus G_{2^{k-1}} \ominus \left[x_1x_2 \ \ldots \ x_p\right]$$

and $y \in [x_1, x_2, \cdots, x_p]$ if $m \cdot 2^{n-k} - p + 1 \le l \le m \cdot 2^{n-k}$, the same situation when $\{b_i\}_{i=1}^{N}$ is applied to the inputs. Therefore, we have

$$S(n, 1, k, l) = S_b(n, 1, k, l) \text{ for all } l,\ m \cdot 2^{n-k} - p + 1 \le l \le m \cdot 2^{n-k}.$$

□

PROPERTY 16. *Partition* $\{b_i\}_{i=1}^{N}$ *into* 2^k *groups* $G_1, G_2, \cdots$, *and* G_{2^k}. *Consider* SE $(n, 1, k, l)$.

CASE 1. $m = \left\lfloor \frac{l-1}{2^{n-k}} \right\rfloor + 1 \le 2^{k-2}$.

Let

$$G'_{2m-1} = \left[y_1y_2 \ \ldots \ y_p\right] \oplus \left(G_{2m-1} \ominus \left[x_1x_2 \ \ldots \ x_p\right]\right)$$

and

$$G'_{2^k} = \left(G_{2^k} \ominus \left[y_1y_2 \ \ldots \ y_p\right]\right) \oplus \left[x_1x_2 \ \ldots \ x_p\right],$$

where $x_1 < x_2 < \cdots < x_p$ *are the largest p numbers in* G_{2m-1} *and* $y_1 < y_2 < \cdots < y_p$ *are the smallest p numbers in* G_{2^k}. *If* $\overline{G_1}\,\overline{G_2} \ \ldots \ \overline{G'_{2m-1}}\,\overline{G_{2m}} \ \ldots \ \overline{G'_{2^k}}$ *is applied to the inputs, then we have*

$$S(n, 1, k, l) = S_a(n, 1, k, l) \text{ for all } l,$$
$$(m - 1)2^{n-k} + 1 \le l \le (m - 1)2^{n-k} + p + 1.$$

CASE 2. $m = \left\lfloor \frac{l-1}{2^{n-k}} \right\rfloor + 1 > 2^{k-2}$.

Let

$$G'_1 = \left[z_1z_2 \ \ldots \ z_p\right] \oplus \left(G_1 - \left[w_1w_2 \ \ldots \ w_p\right]\right)$$

and

$$G'_{2m} = \left(G_{2m} - \left[z_1z_2 \ \ldots \ z_p\right]\right) \oplus \left[w_1w_2 \ \ldots \ w_p\right],$$

where $z_1 < z_2 < \cdots < z_p$ *are the smallest p numbers in* G_{2m} *and* $w_1 < w_2 < \cdots < w_p$ *are the largest p numbers in* G_1. *If* $G_1'G_2 \ \ldots\ G_{2m}' \ \ldots\ G_{2^k}$ *is applied to the inputs, then we have*

$$S(n, 1, k, l) = S_a(n, 1, k, l) \text{ for all } l,$$
$$(m-1)\cdot 2^{n-k} + 1 \le l \le (m-1)2^{n-k} + p + 1.$$

PROOF. The proof for Property 16 is similar to that for Property 15 and is omitted. □

4 AN APPLICATION: DIAGNOSIS OF SINGLE SE FAULTS

As mentioned before, fault diagnosis strongly depends on fault model resulting from particular circuit design. The procedure presented here assumes that every SE is always in either state S_0 or state S_1. Besides, we consider only single SE faults. Our goal is to locate the faulty SE (i, j, k, l) in an n-*level* bitonic sorter.

Before describing the fault diagnosis procedure, we state some consequences of the topological properties presented in the last section.

1) Properties 1 and 2 imply that $\{a_i\}_{i=1}^N$ and $\{b_i\}_{i=1}^N$ can be used to verify the two states S_0 and S_1 for all SEs.
2) Assume that SE (i, j, k, l) is faulty and its state is changed from S_0 to S_1 or from S_1 to S_0. Properties 8 and 9 imply that the fault can be detected unless $k = i < n$. Suppose that the fault can be detected. SE (i, j, k, l) and SE (i, j, k, l') result in the same value of m if (and only if) $\left\lfloor \frac{l-1}{2^{i-k}} \right\rfloor = \left\lfloor \frac{l'-1}{2^{i-k}} \right\rfloor$. In other words, there are 2^{i-k} SEs in the same BS that can result in the same value of m. Besides, there are 2^{i-k} SEs in the $\lceil j/2 \rceil$th *level*-$(i+1)$ BS which can result in the same value of m as SE (i, j, k, l) does. Therefore, there is an ambiguity set to be resolved to locate the faulty SE.
3) Properties 10-12 imply that $\{c_i\}_{i=1}^N$ and $\{d_i\}_{i=1}^N$ can be used to detect the fault if it cannot be detected using $\{a_i\}_{i=1}^N$ and $\{b_i\}_{i=1}^N$. According to Property 12, if j is odd, then SE (i, j, i, l), SE $(i, j+1, i, 2^{i-1} - l + 1)$, and SE $(i, j+2, i, l)$ all result in the same value of m.
4) Properties 13 and 14 can be generalized to change the states of all SEs in the jth *level*-i BS, the $\lceil j/2 \rceil$th *level*-$(i+1)$ BS, the $\lceil j/4 \rceil$th *level*-$(i+2)$ BS, ..., and the *level*-n BS. For example, if $\{a_i\}_{i=1}^N$ is partitioned into four groups G_1, G_2, G_3, G_4 and $G_3G_4G_2G_1$ is applied to the inputs, then $S(i, j, k, l) = S_b(i, j, k, l)$ if and only if $i = n$ or $i = n - 1$ and $j = 2$. The two properties can be used to design binary searches to identify the BS which contains the faulty SE. An example is given in the next section.
5) Properties 15 and 16 can be easily modified to change the states of p consecutive (from the top or bottom) SEs in the same sub-BS from stage k of a *level*-i BS. They are used to design binary searches to locate the faulty SE. An example is also given in the next section.
6) Properties 3-7 are useful for diagnosing link faults and/or SE faults under other fault models. Interested readers are referred to [5].

The diagnosis procedure consists of two phases. In Phase I and Phase II, sequences $\{a_i\}_{i=1}^N$ and $\{b_i\}_{i=1}^N$ are applied to the inputs, respectively. There are four cases. For convenience, an output is said to be faulty if it is not in ascending order.

Case 1. Both Phase I and Phase II result in faulty outputs.

Since Phase I results in faulty output, if $i = n$, then $j = 1$ and, according to Property 8, one knows the value of k, denoted by k_d, by observing the output. Besides, the range of l can be determined. As a consequence, the faulty SE is in an ambiguity set which contains SEs in k_d BSs, BS_1, BS_2, $\cdots$, and BS_{k_d}, where BS_x is a *level*-$(x + n - k_d)$ BS. The ambiguity set contains 2^{i-k} SEs of BS_x for all x, $1 \le x \le k_d$. Since Phase II also results in faulty output, one obtains another ambiguity set. Denote the BSs obtained in phase II as $BS_1', BS_2', \ldots$, and BS_{k_d}'. Obviously, $BS_{i+k_d-n} = BS_{i+k_d-n}'$ because SE (i, j, k, l) is the only faulty SE. Moreover, Properties 8 and 9 imply $BS_y = BS_y'$ for all $y \ge i + k_d - n$. If $i + k_d - n > 1$, then SE (i, j, k, l) is not in the first stage (i.e., $k > 1$) of the jth *level*-i BS. In this case, if j is odd, BS_{i+k_d-n-1} is the $(2j-1)$th *level*-$(i-1)$ BS if $l \le 2^{i-1}$ or the $(2j)$th *level*-$(i-1)$ BS if $l > 2^{i-1}$ (a consequence of Property 8). On the other hand, BS_{i+k_d-n-1}' is the $(2j)$th *level*-$(i-1)$ BS if $l \le 2^{i-1}$ or the $(2j-1)$th BS if $l > 2^{i-1}$. Consequently, BS_{i+k_d-n-1} is different from BS_{i+k_d-n-1}'. The same conclusion holds for j even. Therefore, the faulty SE is in BS_x where x is the smallest integer such that $BS_y = BS_y'$ for all $y \ge x$ and the size of the ambiguity set can be reduced to 2^{i-k} SEs. Figs. 8a and 8b show an example for $n = 4$ and $(i, j, k, l) = (3, 2, 2, 2)$.

To locate the faulty SE, one can slightly modify the design of SEs by adding one bit to the numbers applied to the inputs. The extra bit is not used in determining the states of SEs. The faulty SE can be located with the following binary search. Let (i, j, k, l_1), (i, j, k, l_2), $\ldots$, and $(i, j, k, l_{2^{n-i}})$ denote the 2^{n-i} SEs in the ambiguity set. Apply $\{f_i\}_{i=1}^N$ to the inputs where $f_m = 0$ for all m and the bit added to f_m is set to 1 if and only if $m = (j-1)2^i + x$, where $x = 1, 3, \ldots$, and $2^{n-i} - 1$. All SEs are supposed to be in state S_0 when $\{f_i\}_{i=1}^N$ is applied. If there exists a y such that the added bit received by output $2y - 1$ is 1 and that received by output $2y$ is 0, then $l = y - (j-1)2^{i-1}$ and the faulty SE is located. Otherwise, apply $\{f_i\}_{i=1}^N$ to the inputs again with the added bit of f_m set to one for $m = (j-1)2^i + 2x$, where $x = 1, 3, \ldots$, and $2^{n-i-1} - 1$. Again, by observing the added bits received by the outputs, one can either locate the faulty SE or reduce the ambiguity set and repeat the process until the faulty SE is located.

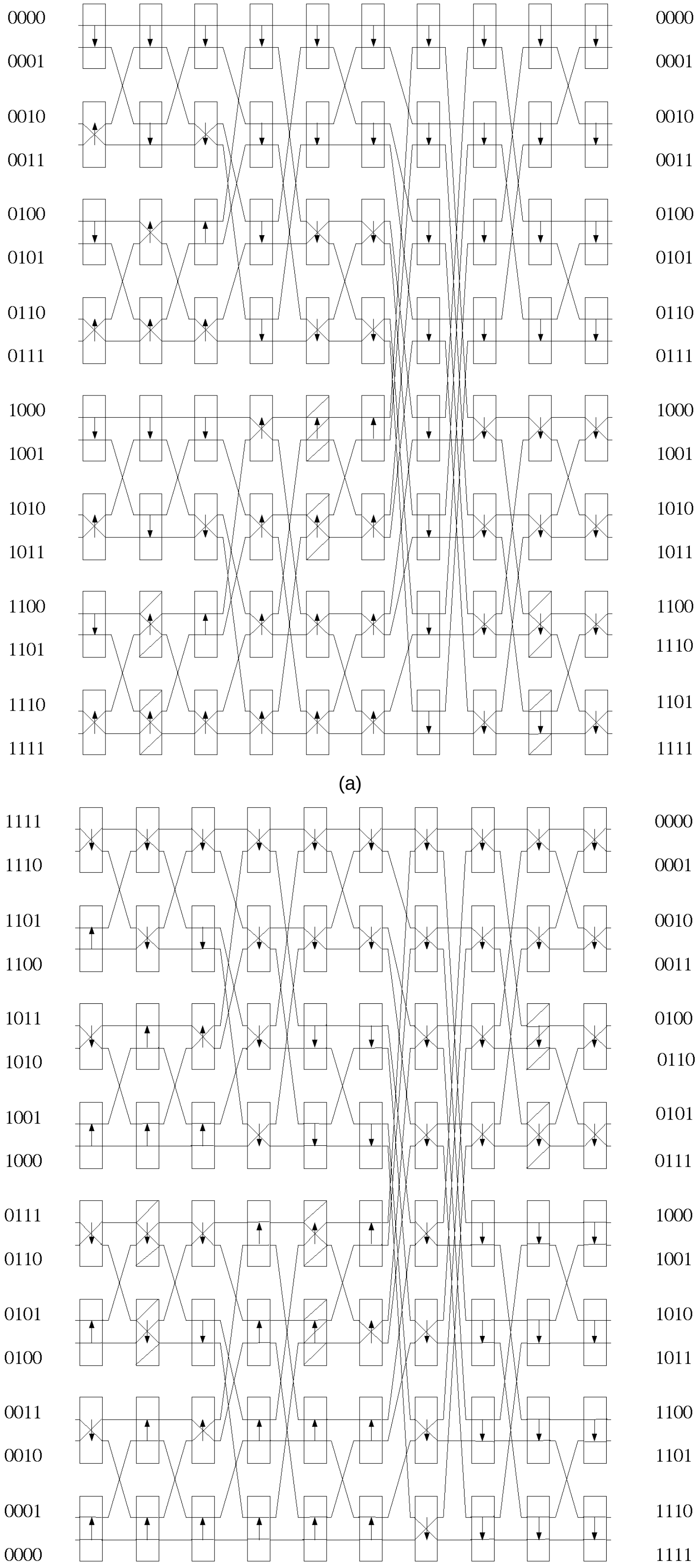


Fig. 8. Ambiguity set (the shaded SEs) in (a) Phase I and (b) Phase II diagnosis assuming SE(3, 2, 2, 2) is faulty. The value of *m* is equal to 4 in Phase 1 and 2 in Phase II.

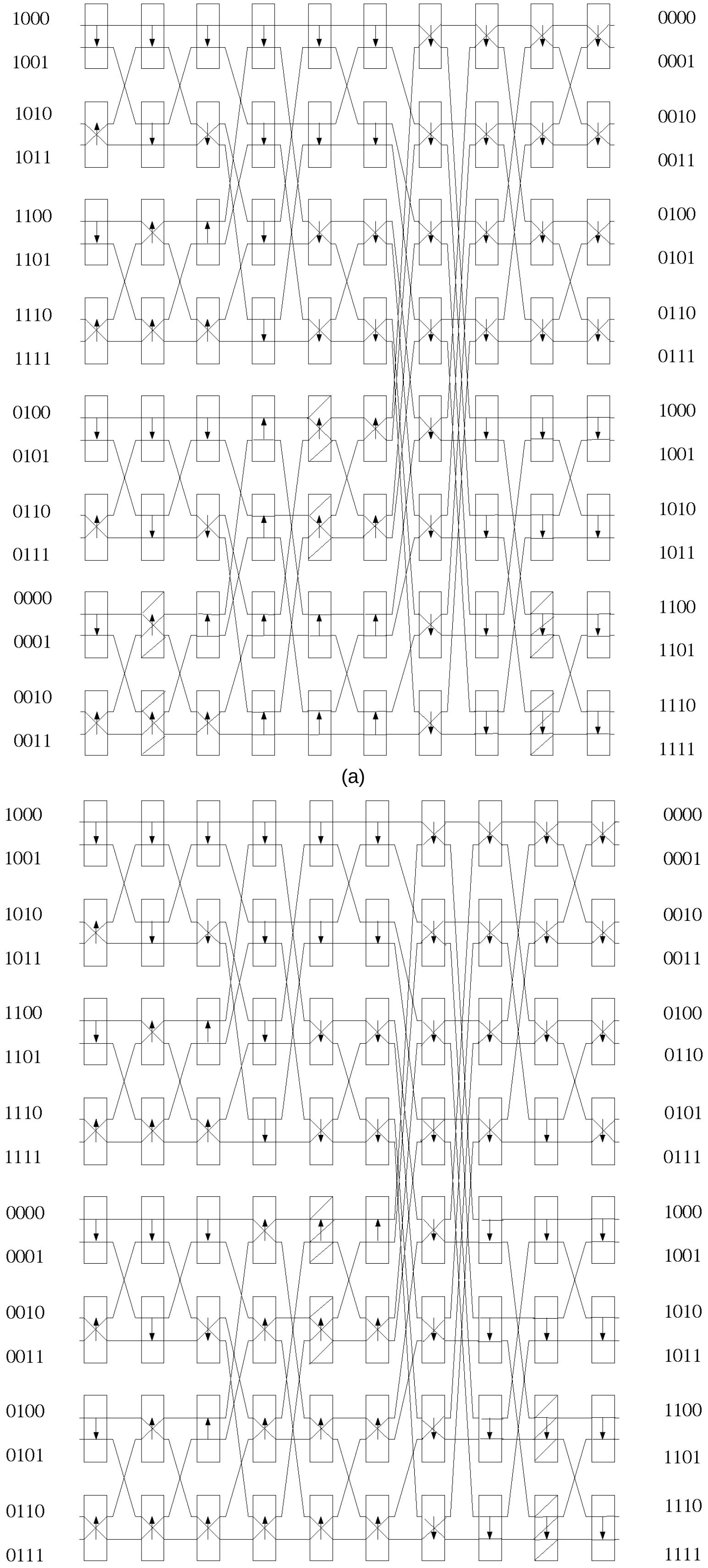


Fig. 9. Binary search to identify the BS containing the faulty SE (3, 2, 2, 2). (a) The states of all the SEs in the *level*-4 BS and the second *level*-3 BS are changed and the output is correct. (b) Only the states of all the SEs in the *level*-4 BS are changed and the output is faulty. Therefore, we get $i = 3$.

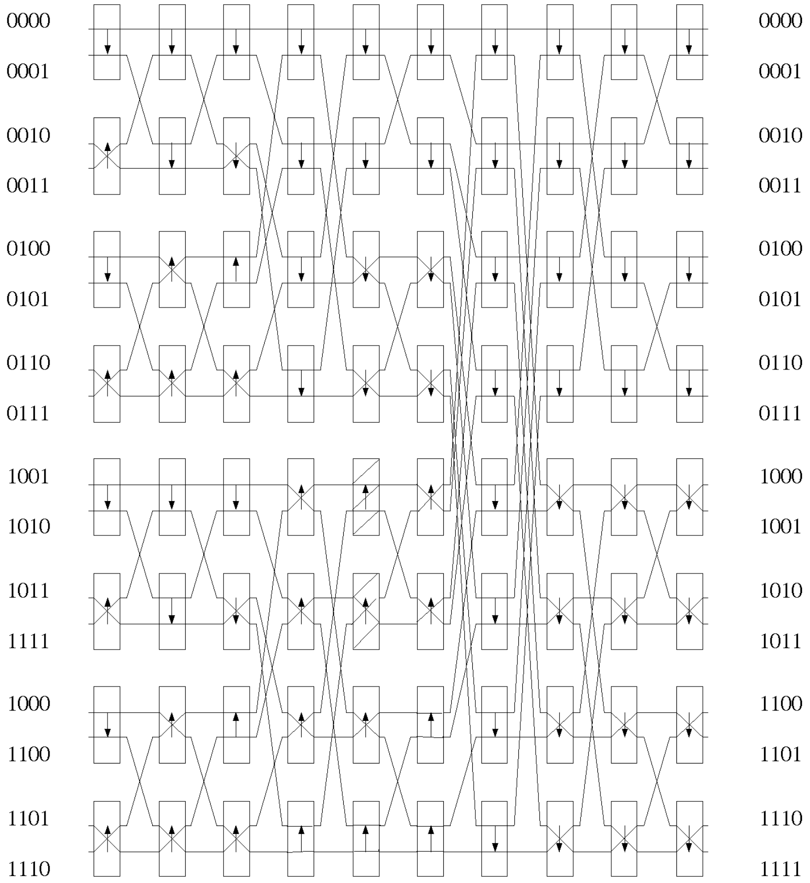


Fig. 10. Binary search to locate the faulty SE (3, 2, 2, 2). The state of the lowest SE in the ambiguity set is changed and the output is correct. Therefore, we get $l = 2$.

Case 2. Only Phase I results in faulty output.

Similar to Case 1, one can obtain an ambiguity set of SEs in BSs BS_1, BS_2, $\cdots$, and BS_{k_d} from Phase I diagnosis. The BS which contains the faulty SE can be determined using a binary search based on Property 13. An example for $n = 4$ and $(i, j, k, l) = (3, 2, 2, 2)$ is illustrated in Figs. 9a and 9b.

The faulty SE can be located with a binary search based on Property 15. Results for the same example used in Fig. 9 are shown in Fig. 10. In Fig. 10, we change the state of SE (3, 2, 2, 2) using Property 15 for $p = 1$.

Case 3. Only Phase II results in faulty output.

The procedure to locate the faulty SE is similar (use binary searches based on Properties 14 and 16) to that for Case 2 and is omitted.

Case 4. Both phases result in correct outputs.

If both Phase I and Phase II result in correct outputs, then the faulty SE must be in the last stage of a *level-i* BS for some $i < n$ (see Properties 8 and 9). In this case, we further apply $\{c_i\}_{i=1}^{N}$ and $\{d_i\}_{i=1}^{N}$ to the inputs. If applying $\{c_i\}_{i=1}^{N}$ and $\{d_i\}_{i=1}^{N}$ both result in faulty outputs, then the technique used in Case 1 (i.e., add an extra bit to the numbers) can be used to locate the faulty SE. If the output is faulty when $\{c_i\}_{i=1}^{N}$ is applied and correct when $\{d_i\}_{i=1}^{N}$ is applied, then, according to Property 12, we can obtain an ambiguity set which contains 2^{n-i} SEs. Notice that no two SEs in the ambiguity set are in the same *level-i* BS. Therefore, the faulty SE can be located with a binary search based on fact that $S_c(i, j, i, l) \neq S_d(i, j, i, l)$. An example is shown in Figs. 11a and 11b for $n = 4$ and $(i, j, k, l) = (2, 2, 2, 2)$. Finally, if the output is correct when $\{c_i\}_{i=1}^{N}$ is applied and faulty when $\{d_i\}_{i=1}^{N}$ is applied, then one can obtain an ambiguity set containing 2^{n-i} elements and use a similar binary search to locate the faulty SE.

5 CONCLUSION

We have studied in this paper the states of all the SEs in an ascending bitonic sorter for some special input sequences. We found that monotonic sequences seem to be excellent choices for diagnosing faults in a bitonic sorter. Detecting and locating single faulty SE under a simple fault model is presented as an application example. Design of SEs may need to be slightly modified to facilitate locating the faulty SE. The topological properties presented in this paper are also useful in developing efficient fault diagnosis procedures under various other fault models [5].

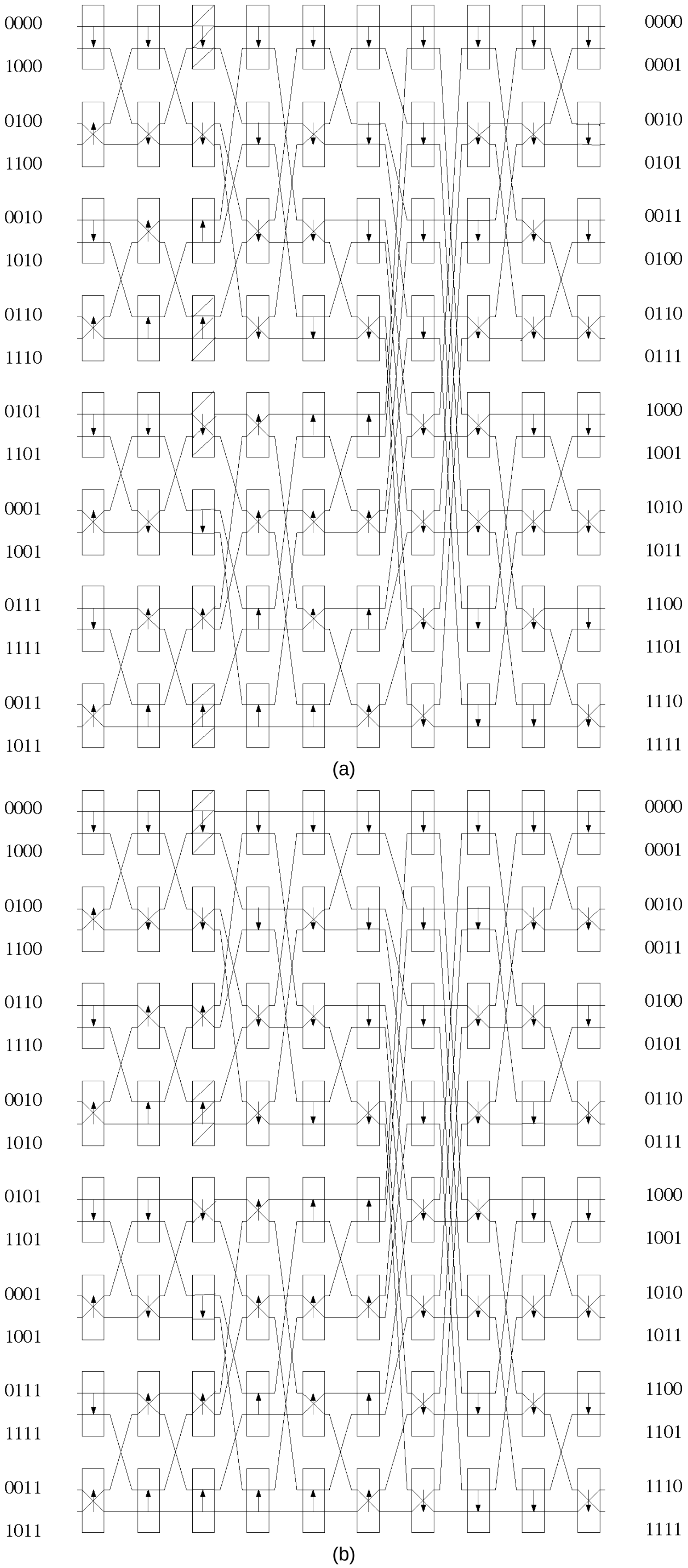


Fig. 11. Binary search to locate the faulty SE (2, 2, 2, 2). (a) The states of all the SEs in the lower two *level*-2 BSs are changed and the output is faulty, which implies $j = 1$ or 2. (b) The states of all the SEs in the lower three *level*-2 BSs are changed and the output is correct. Therefore, we get $j = 2$.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Council, Republic of China, under contract number NSC 84-2213-E0009-053, and by the Ministry of Economic Affairs, Republic of China, under contract number MOEA87-EC-2-A-17-0146.

REFERENCES

[1] K.E. Batcher, "Sorting Networks and Their Applications," *Proc. AFIPS 1968 SJCC*, pp. 307-314, 1968.

[2] J.Y. Hui and E. Authurs, "A Broadband Packet Switch for Integrated Transport," *IEEE J. Selected Areas in Comm.*, vol. 5, pp. 1,264-1,273, Oct. 1987.

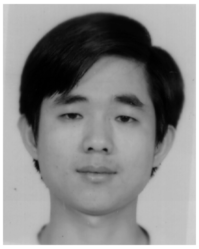
[3] W.Y-P. Lim, "A Test Strategy for Packet Switching Network," *Proc. IEEE Int'l Conf. Fault Tolerance*, pp. 96-98, 1982.

[4] C.L. Wu, and T.Y. Feng, "Fault-Diagnosis for a Class of Multistage Interconnection Networks," *IEEE Trans. Computers*, vol. 30, no. 10, pp. 743-758, Oct. 1981.

[5] J.J. Chou, "Fault Diagnosis and Tolerance of Banyan Networks and Bitonic Sorters," PhD dissertation, National Chiao Tung Univ., 1994.

Tsern-Huei Lee received the BS degree from National Taiwan University, Taipei, Taiwan, Republic of China, in 1981, the MS degree from the University of California, Santa Barbara, in 1984, and the PhD degree from the University of Southern California, Los Angeles, in 1987, all in electrical engineering.

Since 1987, he has been a member of the faculty of National Chiao Tung University, Hsinchu, Taiwan, Republic of China, where he is a professor in the Department of Communication Engineering and a member of the Center for Telecommunications Research. He received an Outstanding Paper Award from the Institute of Chinese Engineers in 1991. His current research interests are in communication protocols, broadband switching systems, network traffic management, and wireless communications. He is a senior member of the IEEE.



Jin-Jye Chou received the BS and MS degrees in communication engineering from the National Chiao Tung University, Hsinchu, Taiwan, Republic of China, in 1987 and 1989, respectively. He also received the PhD degree in electronic engineering from the National Chiao Tung University in 1994. From 1994 to 1995, he was a postdoctoral fellow at the Chiao Tung University, Republic of China.

Currently, he is at the Computer and Communication Research Laboratories, Industrial Technology Research Institute, Hsinchu, Taiwan, Republic of China, where he is active in research involving multilayer switches. His principal areas of interest include ATM switch architecture, flow control and fault tolerance of multistage interconnection networks, multilayer switching architecture, and the QOS of communication systems. He is a member of the IEEE.