On the Stability of the Hybrid-Damped Resolved-Acceleration Control

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This paper points out that the stability analysis of the hybrid-damped resolved-acceleration control in our earlier work is incomplete, since the stability was concluded directly from the fact that the joint velocities come to rest as time approaches infinity. A similar incomplete technique was also used in the work of Wampler and Leifer to prove the stability of a damped least-squares resolved-acceleration control scheme. In this paper, we use LaSalle's invariance principle rigorously to show that the solution trajectory of the hybrid-damped resolved-acceleration control will eventually come to the target without steady-state error or will stay at a kinematic singular point with some steady-state error. Discussions on the case of staying at a singular point are also given.

1. INTRODUCTION

The hybrid-damped resolved-acceleration control scheme for nonredundant manipulator control in Cartesian space was proposed in our earlier work.[1] In the stability analysis, it was shown that $\dot{\mathbf{q}} = \mathbf{0}$ as $t \to \infty$, where $\dot{\mathbf{q}}$ is the vector of joint velocities. This stability analysis is actually based on Wampler and Leifer's work.[2] Although this result implies that the manipulator eventually come to rest, it cannot be concluded that the steady-state error is necessarily zero. In the rigorous sense, such stability analysis is incomplete for showing that the target point is a globally asymptotically stable equilibrium.

This paper uses LaSalle's invariant principle rigorously to show that the target point of a step input is a globally asymptotically stable equilibrium if the manipulator does not eventually stay at a kinematic singular point. Even if the manipulator eventually stays at a kinematic singular point, it is shown that the error is along a linear combination of degenerate directions.

Furthermore, LaSalle's invariance principle is also used to study the stability problem of the

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so-called damped-acceleration resolved-acceleration control. This control scheme has the phenomenon of self-motion, which is also interpreted by LaSalle's invariance principle in this paper.

This paper is organized as follows. In the next section, the hybrid-damped resolved-acceleration control and LaSalle's invariance principle are briefly reviewed, and then the stability of this control and its proof are presented. The extension of the result to the damped-acceleration resolved-acceleration control is studied. Some discussions on the case that the manipulator will eventually stay at a singular point for a step input are also given in the next section. The conclusion is drawn in section 3.

2. ASYMPTOTICAL STABILITY

Consider the dynamic system of a six-joint manipulator

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{f}(\dot{\mathbf{q}}, \mathbf{q}) = \boldsymbol{\tau}, \tag{1}$$

where $\mathbf{M}(\mathbf{q})$ is the inertia matrix, $\mathbf{f}(\dot{\mathbf{q}}, \mathbf{q})$ is the vector comprising Coriolis, centrifugal, and gravity force, $\boldsymbol{\tau}$ is the vector of actuator forces, and $\mathbf{q}$ is the vector of joint displacements.

The so-called hybrid-damped resolved-acceleration control scheme[1] is the computed-torque scheme

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}^* + \mathbf{f}(\dot{\mathbf{q}}, \mathbf{q}) \tag{2}$$

cooperating with the hybrid-damped resolved-acceleration law

$$\ddot{\mathbf{q}}^* = (\mathbf{J}^T\mathbf{J} + \rho^2\mathbf{I})^{-1}\mathbf{J}^T\mathbf{a}^* + \rho_r\rho^2(\mathbf{J}^T\mathbf{J} + \rho^2\mathbf{I})^{-1}\dot{\mathbf{q}}, \tag{3}$$

$$\mathbf{a}^* = \begin{bmatrix} \ddot{\mathbf{r}}_d \\ \boldsymbol{\alpha}_d \end{bmatrix} + \mathbf{K}_D\begin{bmatrix} \dot{\mathbf{r}}_d - \dot{\mathbf{r}} \\ \boldsymbol{\omega}_d - \boldsymbol{\omega} \end{bmatrix} + \mathbf{K}_P\begin{bmatrix} \boldsymbol{\varepsilon}_r \\ \boldsymbol{\varepsilon}_e \end{bmatrix} - \dot{\mathbf{J}}\dot{\mathbf{q}}, \tag{4}$$

where $\mathbf{J}$ is the Jacobian, $\mathbf{I}$ is the identity matrix, ρ and $\rho_r > 0$ are damping factors, $\mathbf{r}$ is the position of the end-effector, $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ are, respectively, the angular velocity and angular acceleration of the end-effector, superscript T denotes the transpose, subscript d denotes the desired value (i.e., the input value), $\boldsymbol{\varepsilon}_r$ and $\boldsymbol{\varepsilon}_e$ are, respectively, the position and orientation errors, and $\mathbf{K}_D$ and $\mathbf{K}_P$ are the gain matrices. Note that the error $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_r^T, \boldsymbol{\varepsilon}_e^T]^T$, and $\boldsymbol{\varepsilon}_r = \mathbf{r}_d - \mathbf{r}$.

The orientation error can be defined as $\boldsymbol{\varepsilon}_e = f(\theta_e)\mathbf{u}_e$, where $\mathbf{u}_e$ and θ_e are, respectively, the unit vector of the rotational axis and the rotational angle from the current orientation to the desired one, and

$$f(\theta_e) \equiv \begin{cases} \theta_e, & \text{with } -\pi < \theta_e \le \pi, \\ \tan\dfrac{\theta_e}{2}, & \text{with } -\pi < \theta_e < \pi, \\ \sin\theta_e, & \text{with } -\pi < \theta_e < \pi, \\ \sin\dfrac{\theta_e}{2}, & \text{with } -\pi < \theta_e \le \pi. \end{cases} \tag{5}$$

The rotation representations $f(\theta_e)\mathbf{u}_e$ with $f(\theta_e)$ in (5) are called, in order, Euler angles, Rodriques parameters, parameters of Luh et al., and Euler parameters.[3] Note that $\tan\theta_e/2$ at $\theta_e = \pm\pi$ is not well defined, and $\sin\theta_e = 0$ at $\theta_e = \pm\pi$ does not reflect the orientation error. For a detailed discussion on a well-defined orientation error, the reader is referred to Lin's work.[3]

To deal with the stability of the hybrid-damped resolved-acceleration control scheme, we will use LaSalle's invariance principle,[4,5] which is cited in the following lemma.

Lemma 1 (LaSalle's invariance principle): *Consider the periodic nonlinear system* $\dot{\mathbf{x}} = \mathbf{f}[t, \mathbf{x}(t)]\ \forall t \ge 0$ *with the period* T, *i.e.*, $\mathbf{f}(t + T, \mathbf{x}) = \mathbf{f}(t, \mathbf{x})\ \forall t \ge 0$ *and* $\forall \mathbf{x} \in \Re^n$, *where* $\mathbf{x}(t) \in \Re^n$ *and* $\mathbf{f}: \Re_+ \times \Re^n \to \Re$ *is continuous. Suppose that there exists a* C^1 *function* $V: \Re_+ \times \Re^n \to \Re$ *having the same period of* T *such that*:

- **(i)** *V is a positive definite function and is radially unbounded;*
- **(ii)** $\dot{V}(t, \mathbf{x}) \le 0\ \forall t \ge 0$ *and* $\forall \mathbf{x} \in \Re^n$.

Let M be the largest invariant set of the nonlinear system contained in the set

$$R = \{\mathbf{x} \in \Re^n : \exists t \ge 0 \text{ such that } \dot{V}(t, \mathbf{x}) = 0\}. \tag{6}$$

Then all solution trajectories (denoted by $\mathbf{s}(t, t_0, \mathbf{x}_0)$ for an initial state $\mathbf{x}_0$ and an initial time t_0) globally asymptotically converge to M as $t \to \infty$, i.e.,

$$\mathbf{x}_0 \in \Re^n, \qquad t_0 \ge 0 \quad \Rightarrow \quad d[\mathbf{s}(t, t_0, \mathbf{x}_0), M] \to 0 \quad \text{as } t \to \infty, \tag{7}$$

where $d(\mathbf{s}, M) \equiv \min_{\mathbf{y} \in M}\|\mathbf{s} - \mathbf{y}\|$ denotes the distance from the point $\mathbf{s}$ to the set M.

This lemma is a global version of the invariance principle and can be straightforwardly derived from the local version.[4] The dynamic system of the manipulator (1) is autonomous and then periodic with an arbitrary period, so Lemma 1 applies to it.

Theorem 2: *Consider the overall system of the manipulator with the hybrid-damped resolved-acceleration control scheme* (1)–(4). *Suppose that the representation of the orientation error is well defined like those in* (5), *and the input is a step input, i.e.,* $\ddot{\mathbf{r}}_d = \dot{\mathbf{r}}_d = \mathbf{0}$ *and* $\boldsymbol{\alpha}_d = \boldsymbol{\omega}_d = \mathbf{0}$. *If* $\mathbf{K}_D$ *is a positive definite matrix and*

$$\mathbf{K}_P = \begin{bmatrix} k_{pr}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & k_{pe}\mathbf{I} \end{bmatrix}$$

with strictly positive k_{pr} *and* k_{pe}, *then either of the following statements holds.*

1. *The equilibrium of* $[\boldsymbol{\varepsilon}^T, \dot{\mathbf{q}}^T]^T = \mathbf{0}$ *is globally asymptotically stable.*
2. *The manipulator will eventually stay at a singular point where* $\mathbf{K}_P\boldsymbol{\varepsilon}$ *is along a linear combination of degenerate directions.*

Proof: The system of (1)–(4) is a second-order differential system. The state variables can then be $\boldsymbol{\varepsilon}$ and $\dot{\mathbf{q}}$, since $\dot{\mathbf{q}}_d = \mathbf{0}$ due to $\dot{\mathbf{r}}_d = \mathbf{0}$. Choose the Lyapunov function candidate

$$V(\boldsymbol{\varepsilon}, \dot{\mathbf{q}}) = \tfrac{1}{2}k_{pr}\boldsymbol{\varepsilon}_r^T\boldsymbol{\varepsilon}_r + k_{pe}\int_0^{\theta_e} f(\phi)\, d\phi + \tfrac{1}{2}\dot{\mathbf{q}}^T(\mathbf{J}^T\mathbf{J} + \rho^2\mathbf{I})\dot{\mathbf{q}}. \tag{8}$$

It follows from (5) that

$$\int_0^{\theta_e} f(\phi)\, d\phi = \begin{cases} \dfrac{\theta_e^2}{2}, & \text{for } f(\phi) = \phi, \\ -2\ln\left|\cos\dfrac{\theta_e}{2}\right|, & \text{for } f(\phi) = \tan\dfrac{\phi}{2}, \\ 1 - \cos\theta_e, & \text{for } f(\phi) = \sin\phi, \\ 2\left(1 - \cos\dfrac{\theta_e}{2}\right), & \text{for } f(\phi) = \sin\dfrac{\phi}{2}. \end{cases} \tag{9}$$

Thus, $V(\boldsymbol{\varepsilon}, \dot{\mathbf{q}})$ is positive definite (i.e., $V > 0$ for $[\boldsymbol{\varepsilon}^T, \dot{\mathbf{q}}^T]^T \neq \mathbf{0}$ and $V = 0$ for $[\varepsilon^T, \dot{\mathbf{q}}^T]^T = \mathbf{0}$) and radially unbounded (i.e., $V \to \infty$ as $\|[\boldsymbol{\varepsilon}^T, \dot{\mathbf{q}}^T]^T\| \to \infty$, uniformly in $\boldsymbol{\varepsilon}$ and $\dot{\mathbf{q}}$). We differentiate V along the system trajectories (1)–(4) to obtain

$$\begin{aligned} \dot{V}(\boldsymbol{\varepsilon}, \dot{\mathbf{q}}) &= k_{pr}\boldsymbol{\varepsilon}_r^T(\dot{\mathbf{r}}_d - \dot{\mathbf{r}}) + k_{pe}f(\theta_e)\mathbf{u}_e^T(\boldsymbol{\omega}_d - \boldsymbol{\omega}) \\ &\quad + \dot{\mathbf{q}}^T(\mathbf{J}^T\mathbf{J} + \rho^2\mathbf{I})\ddot{\mathbf{q}} + \dot{\mathbf{q}}^T\mathbf{J}^T\dot{\mathbf{J}}\dot{\mathbf{q}} \\ &= k_{pr}\boldsymbol{\varepsilon}_r^T\dot{\mathbf{r}}_d + k_{pe}\boldsymbol{\varepsilon}_e^T\boldsymbol{\omega}_d - \rho_r\rho^2\dot{\mathbf{q}}^T\dot{\mathbf{q}} \\ &\quad + \dot{\mathbf{q}}^T\mathbf{J}^T\begin{bmatrix} \ddot{\mathbf{r}}_d \\ \boldsymbol{\alpha}_d \end{bmatrix} + \dot{\mathbf{q}}^T\mathbf{J}^T\mathbf{K}_D\left(\begin{bmatrix} \dot{\mathbf{r}}_d \\ \boldsymbol{\omega}_d \end{bmatrix} - \mathbf{J}\dot{\mathbf{q}}\right). \end{aligned} \tag{10}$$

Note that $\dot{\theta}_e = \mathbf{u}_e^T(\boldsymbol{\omega}_d - \boldsymbol{\omega})$ and $\ddot{\mathbf{q}} = \ddot{\mathbf{q}}^*$. Substituting the assumptions of $\ddot{\mathbf{r}}_d = \dot{\mathbf{r}}_d = \mathbf{0}$ and $\boldsymbol{\alpha}_d = \boldsymbol{\omega}_d = \mathbf{0}$ into (10) yields

$$\dot{V}(\boldsymbol{\varepsilon}, \dot{\mathbf{q}}) = -\dot{\mathbf{q}}^T\mathbf{J}^T\mathbf{K}_D\mathbf{J}\dot{\mathbf{q}} - \rho_r\rho^2\dot{\mathbf{q}}^T\dot{\mathbf{q}}. \tag{11}$$

Because $\mathbf{K}_D$ is positive definite and $\rho_r > 0$, (11) implies that $\dot{V} \le 0\ \forall(\boldsymbol{\varepsilon}, \dot{\mathbf{q}}) \in \Re^6 \times \Re^6$ and that $\dot{V} = 0$ if and only if $\dot{\mathbf{q}} = \mathbf{0}$. The set R defined in (6) is then

$$R = \{(\boldsymbol{\varepsilon}, \dot{\mathbf{q}}) : \dot{\mathbf{q}} = 0\}. \tag{12}$$

The next step is to find the largest invariant set M contained in the set R. For this purpose, we let $[\boldsymbol{\varepsilon}_s(t), \dot{\mathbf{q}}_s(t)]$ be a solution trajectory that lies entirely in the set R. We then have $\dot{\mathbf{q}}_s(t) = \mathbf{0}\ \forall t \ge 0$, so then $\ddot{\mathbf{q}}_s(t) = \mathbf{0}\ \forall t \ge 0$. Examining (3) and (4), we find

$$\begin{aligned} \ddot{\mathbf{q}}^* &= (\mathbf{J}^T\mathbf{J} + \rho^2\mathbf{I})^{-1}\mathbf{J}^T\mathbf{K}_P\boldsymbol{\varepsilon} \\ &= \sum_{i=1}^{6} \frac{\sigma_i}{\sigma_i^2 + \rho^2}\mathbf{v}_i(\mathbf{u}_i^T\mathbf{K}_P\boldsymbol{\varepsilon}) \end{aligned} \tag{13}$$

when $\ddot{\mathbf{r}}_d = \dot{\mathbf{r}}_d = \mathbf{0}$, $\boldsymbol{\alpha}_d = \boldsymbol{\omega}_d = \mathbf{0}$ and $\dot{\mathbf{q}} = \mathbf{0}$, where σ_i, $i = 1, \ldots, 6$, are the singular values of $\mathbf{J}$, and $\mathbf{u}_i$ and $\mathbf{v}_i$ are, respectively, the ith left and the ith right singular vectors of $\mathbf{J}$. Recall that applying the computed-torque scheme (2) to the manipulator system (1) yields $\ddot{\mathbf{q}} = \ddot{\mathbf{q}}^*$. It then follows from (13) that $\ddot{\mathbf{q}}_s(t) = \mathbf{0}\ \forall t \ge \mathbf{0}$ if and only if $\boldsymbol{\varepsilon}_s(t) = \mathbf{0}\ \forall t \ge 0$ or $\mathbf{K}_P\boldsymbol{\varepsilon}_s(t)\ \forall t \ge 0$ is a linear combination of $\mathbf{u}_j$ for js having $\sigma_j = 0$. Note that when $\sigma_j = 0$, $\mathbf{u}_j^T\mathbf{K}_P\boldsymbol{\varepsilon}$ in (13) has no contribution to $\ddot{\mathbf{q}}^*$ and $\mathbf{u}_j$ represents a degenerate direction. Finally,

$$M = \left\{(\boldsymbol{\varepsilon}, \dot{\mathbf{q}}) : \dot{\mathbf{q}} = \mathbf{0} \text{ and either } \boldsymbol{\varepsilon} = \mathbf{0} \text{ or } \mathbf{K}_P\boldsymbol{\varepsilon} = \sum_{\{j:\, \sigma_j = 0\}} a_j\mathbf{u}_j \text{ for } a_j \in \Re\right\}. \tag{14}$$

According to Lemma 1, the solution trajectory for a given initial state and a given step input globally asymptotically converges to M as $t \to \infty$. In fact, the solution trajectory will eventually come to rest and stay at one point in M, since $\dot{\mathbf{q}} = \mathbf{0}$ for all points in M.

In the case that the solution trajectory globally asymptotically converges to $[\boldsymbol{\varepsilon}^T, \dot{\mathbf{q}}^T]^T = \mathbf{0}$ as $t \to \infty$, if the origin is also stable, then it is a globally asymptotically stable equilibrium. The stability of the origin directly follows from the basic theorem of Lyapunov's direct method (see Theorem 5.3.1 in ref. 4), since V in (8) is also a Lyapunov function in a small neighborhood of the origin. This completes the proof. ■

Remark 1: In the work of Wampler and Leifer,[2] the damped least-squares resolved-acceleration control scheme is the same as (1)–(4) with the exception of $\mathbf{K}_D = k_D\mathbf{I}$ and replacing (3) with

$$\begin{aligned}\ddot{\mathbf{q}}^* &= (\mathbf{J}^T\mathbf{J} + \rho^2\mathbf{I})^{-1}\mathbf{J}^T\mathbf{a}^* \\ &\quad + \rho^2(\mathbf{J}^T\mathbf{J} + \rho^2\mathbf{I})^{-1}[k_D\mathbf{I} + \mathbf{B}(t,\mathbf{q})]\dot{\mathbf{q}}, \quad (15)\end{aligned}$$

where $\mathbf{B}(t,\mathbf{q})$ is positive definite and symmetric, which may provide additional damping to the manipulator, although it is not necessary for stability.[2] If we set $\mathbf{B} = \mathbf{0}$, then the control scheme (15) is more restrictive than (3), since ρ_r in (3) is not required to be k_D, the gain of the velocity error. For practical applications, ρ_r should be closely related to the smallest singular value of $\mathbf{J}$.[1]

Another interesting thing is that a strictly positive ρ_r (or k_D in (15)) is inevitable for stability. This is because $\dot{\mathbf{q}} = \mathbf{0}$ is not a necessary condition for $\dot{V} = 0$ when $\rho_r = 0$. The control scheme (3) with $\rho_r = 0$ is named damped-acceleration resolved-acceleration control (DARAC),[1] for which (11) turns out to be

$$\begin{aligned}\dot{V}(\boldsymbol{\varepsilon}, \dot{\mathbf{q}}) &= -\dot{\mathbf{q}}^T\mathbf{J}^T\mathbf{K}_D\mathbf{J}\dot{\mathbf{q}} \\ &= -\left(\sum_{i=1}^{6}\sigma_i\mathbf{u}_i\mathbf{v}_i^T\dot{\mathbf{q}}\right)^T\mathbf{K}_D\left(\sum_{l=1}^{6}\sigma_l\mathbf{u}_l\mathbf{v}_l^T\dot{\mathbf{q}}\right). \quad (16)\end{aligned}$$

Thus, when $\dot{\mathbf{q}}$ is a linear combination of $\mathbf{v}_j$ that are corresponding to $\sigma_j = 0$, then $\dot{V} = 0$, too. Under such a situation, $\mathbf{r} = \mathbf{0}$ and $\boldsymbol{\omega} = \mathbf{0}$, since $\mathbf{J}\dot{\mathbf{q}} = \mathbf{0}$ at this moment. The set R in (6) for the DARAC is then

$$R = \{(\boldsymbol{\varepsilon}, \dot{\mathbf{q}}) \colon \mathbf{J}\dot{\mathbf{q}} = \mathbf{0}\}. \quad (17)$$

Let $[\boldsymbol{\varepsilon}_s(t), \dot{\mathbf{q}}_s(t)]$ be a solution trajectory that lies entirely in the set R. We then have $\dot{\mathbf{r}}_s(t) = \mathbf{0}$ and $\boldsymbol{\omega}_s(t) = \mathbf{0}$ $\forall t \geq 0$, so that $\ddot{\mathbf{r}}_s(t) = \mathbf{0}$ and $\boldsymbol{\alpha}_s(t) = \mathbf{0}$ $\forall t \geq 0$, too. From this it follows that $\mathbf{J}\ddot{\mathbf{q}}_s(t) = -\dot{\mathbf{J}}\dot{\mathbf{q}}_s(t)$ $\forall t \geq 0$. By the assumption of a step input and $\rho_r = 0$, combining (3) and (4) yields

$$\begin{aligned}\ddot{\mathbf{q}}_s^* &= (\mathbf{J}^T\mathbf{J} + \rho^2\mathbf{I})^{-1}\mathbf{J}^T\mathbf{K}_P\boldsymbol{\varepsilon}_s + (\mathbf{J}^T\mathbf{J} + \rho^2\mathbf{I})^{-1}\mathbf{J}^T\mathbf{J}\ddot{\mathbf{q}}_s \\ &= \sum_{i=1}^{6}\frac{\sigma_i}{\sigma_i^2 + \rho^2}\mathbf{v}_i\mathbf{u}_i^T\left(\mathbf{K}_P\boldsymbol{\varepsilon}_s - \dot{\mathbf{J}}\dot{\mathbf{q}}_s\right). \quad (18)\end{aligned}$$

Assume $\sigma_j = 0$ for $j = k+1, \ldots, 6$, i.e., the rank of $\mathbf{J}$ is k. Equation (18) indicates that if $(\mathbf{K}_P\boldsymbol{\varepsilon}_s - \dot{\mathbf{J}}\dot{\mathbf{q}}_s)$ is not in the span of $\mathbf{u}_j$ (i.e., $\mathbf{J}^T(\mathbf{K}_P\boldsymbol{\varepsilon}_s - \dot{\mathbf{J}}\dot{\mathbf{q}}_s) \neq \mathbf{0}$), then $\ddot{\mathbf{q}}_s^*$ has a component not in the span of $\mathbf{v}_j$, which will generate some component of $\ddot{\mathbf{q}}_s$ (and then that of $\dot{\mathbf{q}}_s$) not in the span of $\mathbf{v}_j$ at the next time. The contrapositive tells that the largest invariant set M is

$$M = \left\{(\boldsymbol{\varepsilon}, \dot{\mathbf{q}}) \colon \mathbf{J}^T\left(\mathbf{K}_P\boldsymbol{\varepsilon} - \dot{\mathbf{J}}\dot{\mathbf{q}}\right) = \mathbf{0}\right\}. \quad (19)$$

For the points in the case that $\dot{\mathbf{J}}\dot{\mathbf{q}} = \mathbf{0}$ if and only if $\dot{\mathbf{q}} = \mathbf{0}$ (and then $\dot{\mathbf{J}}\dot{\mathbf{q}} = 0$), the same stability result as that in Theorem 2 can be concluded.

Now, consider a particular point where $\boldsymbol{\varepsilon} = \mathbf{0}$ and $\dot{\mathbf{J}}\dot{\mathbf{q}} = \mathbf{0}$ when $\dot{\mathbf{J}}\dot{\mathbf{q}} = \mathbf{0}$ but $\dot{\mathbf{q}} \neq 0$. Apparently, this point is in M. It follows from (3) and (4) with $\rho_r = 0$ that $\ddot{\mathbf{q}}^* = \mathbf{0}$ and then $\ddot{\mathbf{q}} = \mathbf{0}$. This implies that $\dot{\mathbf{q}}$ is eternally constant. According to Lemma 1, the solution trajectory may asymptotically converge to this point with zero steady-state error but $\dot{\mathbf{q}} \neq 0$. Such a phenomenon is called *self-motion*, and was first demonstrated[6] for the PUMA 560 robot. The spherical wrist of the PUMA 560 robot allows the rotations of joints 4 and 6 with the same angular speed in opposite directions to hold the orientation of the end-effector stationary, when joint 5 stays in the orientation-singular configuration where joints 4 and 6 are collinear. An experiment[6] showed that self-motion of joints 4 and 6 occurs in the orientation-singular configuration when the DARAC is applied to the PUMA 560 robot for a special command. The above analysis is another interpretation of the self-motion in the DARAC in contrast to that in the earlier work.[1]

Remark 2: Consider nonredundant manipulators with type 1 geometry,[7] where the singular points are all on the boundary of the workspace. It is known that the step response of the resolved-acceleration control is a straight line coupled with a rotation of the end-effector,[3] if the solution trajectory does not pass a neighborhood of a singular point. The response of the hybrid-damped resolved-acceleration control resembles that of the original resolved-acceleration, since the former is only to remedy the problem in a neighborhood of a singular point. If the target of a step input is in the workspace and the line of the initial point to the target does not cross the boundary, then the solution trajectory of the hybrid-damped resolved-acceleration never touches a singular point. According to Theorem 2, the target is a globally asymptotically stable equilibrium.

If the target is outside the workspace such as that shown in Figure 1, the line from the initial point to the target intersects with the boundary at a singular point C. Assume that the solution trajectory passes through the point C (actually, it only passes a neighborhood of C and touches another singular point close to C). At the point C, the error has two components: one is in the degenerate direction (i.e., $\mathbf{u}_2$ in Fig. 1.); the other is normal to it. The point C is not in the set M in (14), so the solution trajectory will continue going to point B', where the component of the error along the direction of $\mathbf{u}_1$ is zero. It is apparent that point B' is in the set M and is isolated (i.e., there is a neighborhood of this point such that no other point in M is also in this neighborhood). It is then concluded from Theorem 2 that the manipulator will eventually stay at point B'. A simulation for such a case can be found in Example 2 of our earlier work.[1]

Now consider another special case that the line from the initial point to the target is collinear with a degenerate direction and intersects with the boundary at point A' (see Fig. 2). The target A is either in the workspace or outside the workspace. If the target is in the workspace [see Fig. 2(a)], point A' is unstable, although it is also in the set M. When the manipulator stays stationary at point A', a perturbation makes the component of the error along $\mathbf{u}_1$ no more zero and the manipulator moves away from point A'. On the contrary, if the target is outside the workspace [see Fig. 2(b)], although any perturbation also makes the component of the error along $\mathbf{u}_1$

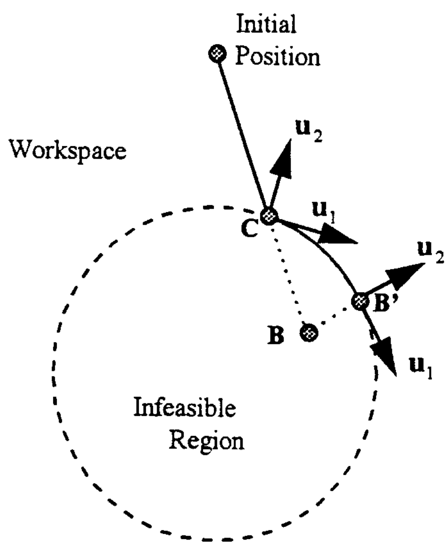

Figure 1. Target outside the workspace.

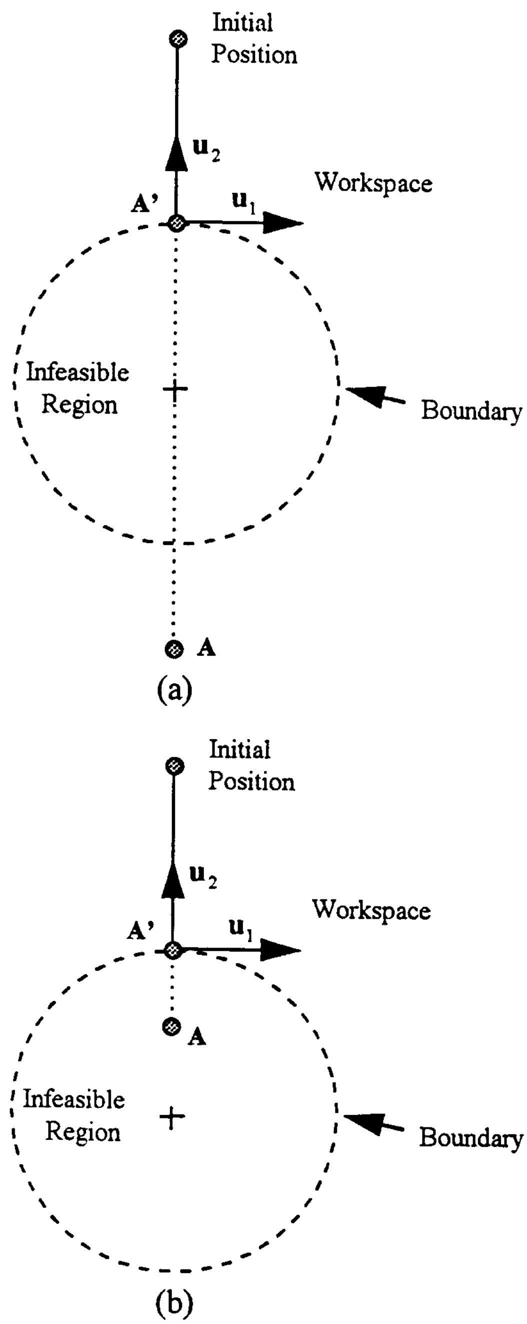

Figure 2. Collinearity of the line from the initial point to the target and a degenerate direction: (a) target in the workspace; (b) target outside the workspace.

nonzero, this nonzero component draw the manipulator back to point A'. Then point A' is a stable equilibrium for the target outside the workspace. The case of the target in the workspace [i.e., Fig. 2(a)] is demonstrated by a simulation in the following.

In the simulation on the PUMA 560 robot, the initial point is $(-0.1, 0.2, 0.8)$ and the target is $(0.1, -0.2, 0.8)$. It is apparent that the line from the initial point to the target is through the center of the infeasible circle, $(0, 0, 0.8)$, so that the line is collinear with a degenerate direction. All technical data and the control parameters are set identical to those in the simulations of our earlier work.[1] The simulation results for the DARAC and the hybrid-damped resolved-acceleration control are shown in Figure 3, which shows that the target is a globally asymptotically stable equilibrium for both control schemes. Of course, the drawback of the DARAC is the fluctuation when the manipulator touches singular points, which is also shown in Figure 3(a).

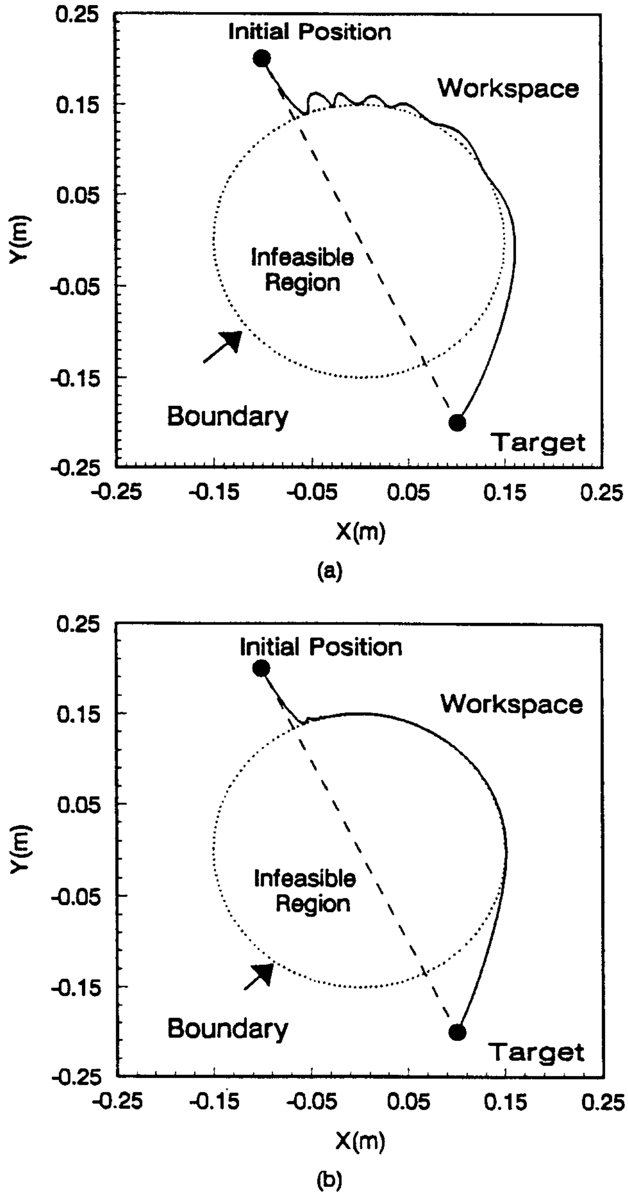

Figure 3. Simulation responses of the PUMA 560 robot: (a) damped-acceleration resolved-acceleration control; (b) hybrid-damped resolved-acceleration control.

3. CONCLUSION

This paper compensates for the incompleteness of the stability analysis in our earlier work.[1] A rigorous proof is given to show that for the hybrid-damped resolved-acceleration control with a step input, either the target is globally asymptotically stable or the manipulator will eventually stay at a singular point. The same analysis is also used to study the stability problem of the damped-acceleration resolved-acceleration control and to interpret the phenomenon of self-motion.

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