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M.-C. Chen & C.-T. Su

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Optimization of machining conditions for turning cylindrical stocks into continuous finished profiles

M.-C. CHEN† and C.-T. SU†*

The optimization problem involving straight turning operations for bar components with constant finished diameters has received extensive attention. In the real-world turning processes on a computer numerical control (CNC) machine, the cylindrical stocks are frequently machined into their desired continuous profiles. Such turning processes require not only straight turning but also face turning, taper turning and circular turning to complete the removal of stock. However, machining models proposed in the literature do not simultaneously consider these turning operations to optimize machining conditions (parameters). This study formulates a machining model to optimize the machining conditions for turning cylindrical stocks into continuous finished profiles, thereby extending the applications of machining optimization. The usefulness of the developed machining model is finally established through a test example.

1. Introduction

Machining conditions (parameters) can be designed using optimization techniques. The widespread industrial applications of CNC machines have made optimizing machining conditions an important task in the process planning of machining operations. Appropriately determining the machining conditions not only extensively increases the machining economics, but also refines the product quality. The machining optimization problem, which is generally formulated as a constrained nonlinear programming model, has a high degree of computational complexity.

Turning is a conventional metal cutting process. Many researchers have derived mathematical models to optimize the turning conditions (Bhattacharyya *et al.* 1970, Boothroyd and Rusek 1976, Ermer 1971, Hati and Rao 1976, Sundaram 1978, Philipson and Ravindran 1979, Ermer and Kromodihardjo 1981, Agapiou 1992, Shin and Joo 1992, Narang and Fischer 1993, Gupta *et al.* 1994, 1995, Kee 1994, 1995, 1996, Mesquita *et al.* 1995, Tan and Creese 1995, Yeo 1995, Chen and Tsai 1996). However, the above machining optimization models are developed for straight-surface turning which has a rectangular cutter path pattern. The straight turning involves cutting a workpiece in the longitudinal (Z) direction to produce a constant stock diameter. In the real-world turning process, the finished part profile on a CNC turning machine frequently has a continuous form. The continuous part profile is bounded by straight lines, facing lines, tapers and circular arcs. Furthermore, the raw material used in turning machines is normally a cylindrical stock. Machining such a stock into a continuous part profile on a CNC turning machine often requires straight turning, taper turning, facing and circular turning. A taper is a uniform reduction in diameter measured along the axis of the workpiece; in

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† Department of Industrial Engineering and Management, National Chiao Tung University, Hsinchu, Taiwan.

* To whom correspondence should be addressed.

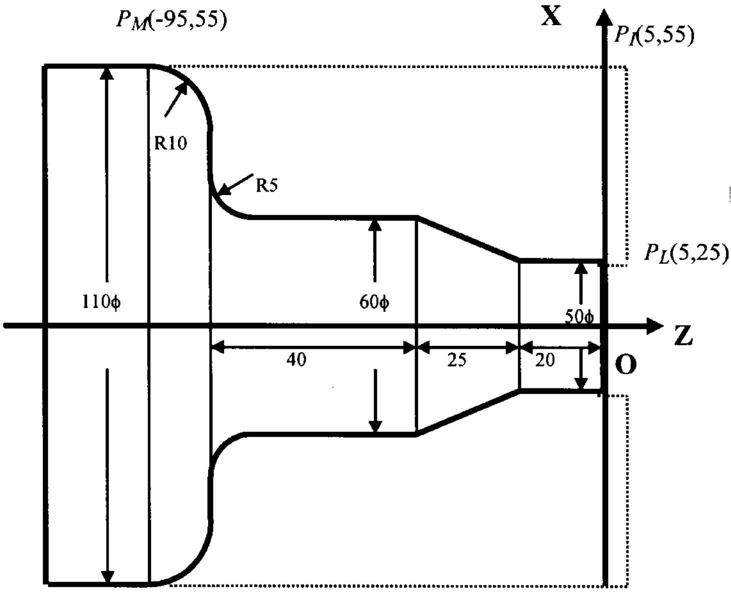

Figure 1. A cylindrical stock is turned to a continuous part profile.

addition, the linear interpolation is used to cut the taper. The facing involves removing material as the tool moves in the X direction, i.e. perpendicular to the Z axis. The circular turning is typically used to machine concave or convex circular shapes. Figure 1 presents a cylindrical stock being turned into a continuous part profile. Such a turning process requires several rough passes and a finish pass to accomplish the stock removal function.

This paper attempts to determine the optimal machining conditions (parameters) of the multi-pass turning operations. A machining optimization model is formulated for turning cylindrical stocks into continuous part profiles. In the developed machining model, straight turning, taper turning, facing and circular turning are simultaneously considered. The selected machining conditions can be stored in a CAD/CAM database to automatically generate the CNC part program, thereby reducing the manufacturing cost in turning operations.

The rest of this paper is organized as follows. Section 2 introduces the formulation of the problem. Section 3 discusses the expressions of cutting time for various turning operations. In § 4, calculations are presented for the cutting time of the turned part with a continuous finished profile. Section 5 formulates the machining optimization model. In § 6, a numerical example is described and the computational results are summarized. Concluding remarks are finally made in § 7.

2. Formulation of the problem

Machining cylindrical stocks into continuous finished profiles on a CNC turning machine frequently requires a set of cutter paths to remove the stock. The cutter paths consist of multiple rough passes and a single finish pass. The rough cut is used to rough the part to a size that is only slightly larger than its desired size in preparation for the finish cut (Lin 1994). The finish cut can be regarded as single-pass contouring, which is normally used after the stock removal in the rough cut.

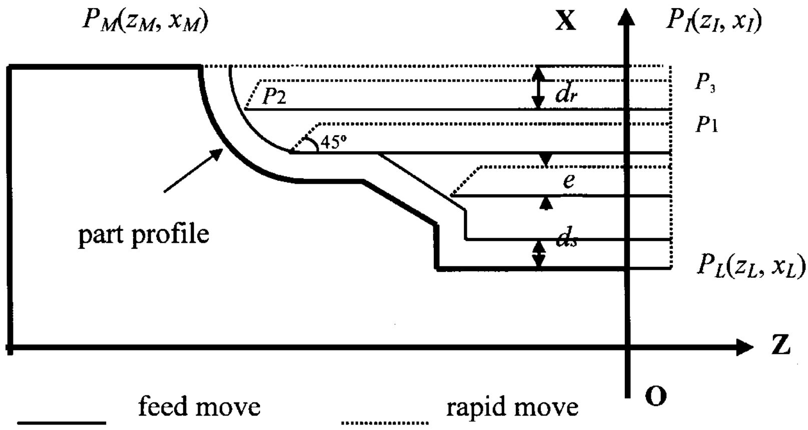

Figure 2. Cutter path for roughing and finishing.

Figure 2 illustrates a typical cutter path pattern for turning a cylindrical stock into a continuous part profile. The first rough pass has the following cutter path:

(1) Advance from initial point, $P_I(z_I, x_I)$, in the X direction by a distance d_r (depth of each rough cut) to point P_1.
(2) Feed in the Z direction to point P_2, which is located a distance of d_s (depth of finish cut) from part profile in the X or Z direction.
(3) Retract rapidly at 45° toward the initial point until the escape distance, e, is reached.
(4) Move rapidly in the Z direction to point P_3, which has the same Z coordinate as point $P_I(z_I, x_I)$.

The cutter path for roughing includes n passes, in which the second to $(n - 1)$th passes resemble that of the first pass. The roughing for the first to $(n - 1)$th passes is the first roughing stage. The last pass of roughing (second roughing stage) cuts along the part profile while leaving a specified amount of stock, d_s, for the finishing stage. The cutter path pattern for the first roughing stage is the straight turning (see figure 2). The cutter path patterns for the second roughing stage and the finishing stage, which move along the part profile, include straight turning, facing, taper turning and circular turning with respect to the part profile.

As mentioned earlier, two roughing stages and a finishing stage are required to cut cylindrical stocks into their desired continuous profiles and sizes. The machining parameters in such multi-pass turning operations consist of cutting speed, feed, depth of cut for roughing and finishing and number of rough passes. The performance of machining operations can be measured by the production cost. In this study, we determine the optimal machining parameters with respect to the minimum unit production cost criterion and a set of practical constraints. The machining constraints normally consider the CNC machine specifications, CNC machine dynamics, cutting tool dynamics and machined part design specifications.

3. Formulas for cutting time

For turned parts having continuous forms, turning processes include linear interpolation and/or circular interpolation. The linear interpolation can be classified into three operations: straight turning, facing and taper turning. Previous studies developed turning models for bar components, in which the cutting time is only calculated for straight turning. Therefore, previous machining models cannot be employed to optimize turning operations for parts having continuous profiles since computing the cutting time for each type of turning process is different.

In turning operations, the spindle speed N (rpm) can be represented as the cutting speed V (m/min) divided by the circumference length of the given diameter D (mm). For the straight turning, the distance between the tool and the workpiece centre is a constant. Thus, a constant cutting speed V can be obtained if the spindle speed N does not vary. However, the diameter D in cases involving facing, taper turning and circular turning is not a constant. Therefore, the cutting speed V cannot be constant due to the instantaneously changing diameter D. It is desirable to maintain a constant cutting speed even though the effective diameter of the cutting changes during machining (Lin 1994). To provide a constant cutting speed under such a circumstance, the CNC controller monitors the changing distance between the tool and the workpiece centre, then adjusts the spindle speed. For linear and circular interpolations, the cutting times can be calculated by the formulas derived by Lee (1988).

For the linear turning between any two points $P_1(z_1, x_1)$ and $P_2(z_2, x_2)$, the cutting time can be calculated by (Lee 1988):

$$T = \frac{\pi}{1000\,Vf}\left|\frac{x_2^2 - x_1^2}{\sin\theta}\right|, \tag{1}$$

where f represents feed (mm/rev); and

$$\theta = \tan^{-1}\left(\frac{x_2 - x_1}{z_2 - z_1}\right); \quad 0 < \theta < \pi \text{ or } \pi < \theta < 2\pi$$

If θ is not equal to 0, $\pi/2$, π, or $3\pi/2$, the linear turning can be classified as taper turning.

In the case of straight turning, where θ in equation (1) is 0 or π, equation (1) becomes

$$T = \frac{\pi|x(z_2 - z_1)|}{500\,Vf}, \tag{2}$$

where $x = x_1 = x_2$; and $|z_2 - z_1|$ is the length of straight turning.

In the case of facing, where θ in equation (1) is $\pi/2$ or $3\pi/2 (\sin\theta = \pm 1)$, the cutting time T becomes

$$T = \frac{\pi}{1000\,Vf}|x_2^2 - x_1^2|. \tag{3}$$

For the circular turning between any two points and $P_1(z_1, x_1)$ and $P_2(z_2, x_2)$, the cutting time can be calculated by

$$T = \frac{\pi r_a}{500\,Vf}|x_c(\theta_2 - \theta_1) - r_a(\cos\theta_2 - \cos\theta_1)|, \tag{4}$$

where

$$\theta_1 = \tan^{-1}\left(\frac{x_1 - x_c}{z_1 - z_c}\right); \quad \theta_2 = \tan^{-1}\left(\frac{x_2 - x_c}{z_2 - z_c}\right); \quad 0 \le \theta_1 \le 2\pi, \quad 0 \le \theta_2 \le 2\pi,$$

r_a represents the radius of the circular arc at a point $P(z, x)$ between P_1 and P_2; and $P_c(z_c, x_c)$ represents the centre of the circular arc.

4. Cutting time of the continuous profile

Machining models developed previously cannot be applied to optimize the turning process for cutting the cylindrical stocks into the desired continuous profile. Herein, such a turning process is divided into first roughing stage, second roughing stage and finishing stage. The straight turning, face turning, taper turning and circular turning are required to complete the removal of stock. Next, we derive the formulas of cutting time for the multi-pass turning problem. In the derived formulas, the cutting time is computed on the basis of the analytical formulas presented in § 3.

4.1. *Cutting time of the first roughing stage*

As figure 2 illustrates, the first roughing stage consists of $n - 1$ straight turning segments (first to $(n - 1)$th passes). Ahead of calculating the cutting time, the cutter path for each pass is initially obtained. The length of each pass in the first roughing stage is computed by the following steps.

Step 1. Construct the lines of the cutting paths in the first roughing stage. Let L_g: $x - (x_I - gd_r) = 0$ be the straight line of the gth pass, where $g = 1, 2, \ldots, n - 1$; n is the number of rough passes; x_I is the X-coordinate of the initial point, $P_I(z_I, x_I)$ (refer to figure 2); and d_r is the depth of each rough pass.

Step 2. Compute the intersecting point, $P_{N(g)}(z_{N(g)}, x_{N(g)})$, of each cutting path and part profile.

2.1. If the intersecting point is located at the taper segment between $P_{1t}(z_{1t}, x_{1t})$ and $P_{2t}(z_{2t}, x_{2t})$, the intersecting point can be obtained by

$$x_{N(g)} = x_I - gd_r, z_{N(g)} = z_{1t} + \frac{z_{2t} - z_{1t}}{x_{2t} - x_{1t}}\left[(d_t - gd_r) - x_{1t}\right]$$

2.2 If the intersecting point is located at the facing segment between $P_{1v}(z_{1v}, x_{1v})$ and $P_{2v}(z_{2v}, x_{2v})$ the intersecting point can be obtained by

$$x_{N(g)} = x_I - gd_r, z_{N(g)} = z_{1v} \text{ or } z_{N(g)} = z_{2v}\,(z_{1v} = z_{2v}).$$

2.3 If the intersecting point is located at the circular arc between $P_{1c}(z_{1c}, x_{1c})$ and $P_{2c}(z_{2c}, x_{2c})$ the intersecting point can be obtained by

$$x_{N(g)} = x_I - gd_r, z_{N(g)} = z_c + \sqrt{r_a^2 - \left[(d_t - gd_r) - x_c\right]^2}\ \text{(convex arc)};$$

$$x_{N(g)} = x_I - gd_r, z_{N(g)} = z_c - \sqrt{r_a^2 - \left[(d_t - gd_r) - x_c\right]^2}\ \text{(concave arc)};$$

where $P_c(z_c, x_c)$ is the centre of the circular arc and r_a is the arc radius.

2.4 If the intersecting point is located at any path critical point where the part features change (intersecting point between two geometric entities), this path critical point is the intersecting point.

Step 3. Compute the cutting length of each pass (straight turning), $H_{(g)}$, in the first roughing stage.

$$H_{(g)} = \left|z_{N(g)} - z_I\right| - d_s,$$

where z_I is the Z-coordinate of the initial point, $P_I(z_I, x_I)$ and d_s is the depth of finish cut.

Next, the cutting time of each pass (straight turning) in the first roughing stage can be calculated by using equation (2). The cutting time of the gth pass $T_{hr(g)}$ can be obtained by

$$T_{hr(g)} = \frac{\pi\left|x_{(g)} H_{(g)}\right|}{500\, V_r f_r}, \tag{5}$$

where $H_{(g)} = \left|z_{N(g)} - z_I\right| - d_s$; $x_{(g)} = x_{N(g)} = x_I - gd_r$; V_r and f_r are the cutting speed (m/min) and feed (mm/rev) for roughing, respectively. The cutting time of the first roughing stage S_{r1} can thus be expressed as

$$S_{r1} = \sum_{g=1}^{n-1} T_{tr(g)}. \tag{6}$$

4.2. *Cutting time of the second roughing stage*

The second roughing stage is the last pass (nth pass) of rough cut. This pass cuts along the part profile while leaving an amount of stock, d_s (depth of finish cut), for the finishing stage. A part may consist of several straight turning segments, facing segments, tapers and circular arcs.

The start and end points of the ith straight turning segment at the part profile are $P_{1(i)}(z_{1(i)}, x_{1(i)})$ and $P_{2(i)}(z_{2(i)}, x_{2(i)})$, respectively. For the nth rough pass of the ith straight turning segment, the cutting time of $T_{hr(i)(n)}$ (min) can be calculated by using equation (2). It is expressed as

$$T_{hr(i)(n)} = \frac{\pi\left|x_{(i)(n)}(z_{2(i)} - z_{1(i)})\right|}{500\, V_r f_r}, \tag{7}$$

where $x_{(i)(n)}$ represents the X-coordinate of the nth rough pass and takes the form

$$x_{(i)(n)} + \frac{D_{(i)}}{2} + d_s. \tag{8}$$

In equation (7), $z_{1(i)}$ and $z_{2(i)}$ are the Z-coordinates of $P_{1(i)}$ and $P_{2(i)}$; and $D_{(i)}$ is the diameter (mm) of the finished part for the ith straight turning segment. By substituting $x_{(i)(n)}$ in equation (7) for equation (8), $T_{hr(i)(n)}$ can be rewritten as

$$T_{hr(i)(n)} = \frac{\pi\left|\left(\frac{D_{(i)}}{2} + d_s\right)(z_{2(i)} - z_{1(i)}\right|}{500\, V_r f_r}. \tag{9}$$

The cutting time of the last rough pass for all straight turning segments $S_{hr(n)}$ can be expressed as

$$S_{hr(n)} = \sum_{i=1}^{n_h} T_{hr(i)(n)}, \tag{10}$$

where n_h is the number of straight turning segments in the turned part.

For the jth taper between two points $P_{1(j)}(z_{1(j)}, x_{1(j)})$ and $P_{2(j)}(z_{2(j)}, x_{2(j)})$, the X-ordinates of $P_{1(j)(n)}$ and $P_{2(j)(n)}$ of the nth rough pass can be represented as

$$x_{1(j)(n)} = \frac{D_{1(j)}}{2} + d_s, x_{2(j)(n)} = \frac{D_{2(j)}}{2} + d_s, \tag{11}$$

where $D_{1(j)}$ and $D_{2(j)}$ are the diameters of the finished part at points $P_{1(j)}(z_{1(j)}, x_{1(j)})$ and $P_{2(j)}(z_{2(j)}, x_{2(j)})$, respectively. By substituting x_1 and x_2 in equation (1) for

$x_{1(j)(n)}$ and $x_{2(j)(n)}$ using equation (11), the cutting time of the nth rough pass for taper turning $T_{tr(j)(n)}$ can be obtained as

$$T_{tr(j)(n)} = \frac{\pi}{1000\, V_f\, f_r}\left|\frac{\frac{1}{4}(D_{2(j)}^2 - D_{1(j)}^2) + (D_{2(j)} - D_{1(j)})d_s}{\sin\theta_{(j)}}\right|, \tag{12}$$

where

$$\theta_{(j)} = \tan^{-1}\left(\frac{x_{2(j)} - x_{1(j)}}{z_{2(j)} - z_{1(j)}}\right) = \tan^{-1}\left(\frac{D_{2(j)} - D_{1(j)}}{2(z_{2(j)} - z_{1(j)})}\right); \quad 0 < \theta_{(j)} < \pi, \pi < \theta_{(j)} < 2\pi$$

The cutting time of the last rough pass for all tapers $S_{tr(n)}$ can be expressed as

$$S_{tr(n)} = \sum_{j=1}^{n_t} T_{tr(i)(n)}, \tag{13}$$

where n_t is the number of tapers in the turned part.

The cutting time of the nth rough pass for all facing segments $s_{vr(n)}$ can be obtained from equation (12) by setting $\sin\theta = \pm 1$. It is expressed as

$$S_{vr(n)} = \sum_{k=1}^{n_v} \frac{\pi}{1000\, V_r\, f_r}\left|\tfrac{1}{4}(D_{2(k)}^2 - D_{1(k)}^2) + (D_{2(k)} - D_{1(k)})d_s\right|, \tag{14}$$

where n_v is the number of facing segments in the turned part, and $D_{1(k)}$ and $D_{2(k)}$ are the diameters of the finished part for the kth facing segment at points $P_{1(k)}(z_{1(k)}, x_{1(k)})$ and $P_{2(k)}(z_{2(k)}, x_{2(k)})$, respectively.

For the lth circular arc between two points $P_{1(l)}(z_{1(l)}, x_{1(l)})$ and $P_{2(l)}(z_{2(l)}, x_{2(l)})$, the arc radius $r_{a(l)(n)}$ in the nth rough pass can be defined as

$$r_{a(l)(n)} = R_{(l)} + d_s, \tag{15}$$

where $R_{(l)}$ is the radius of the finished part for the lth circular arc. By replacing r_a in equation (4) by $r_{a(l)(n)}$ using equation (15), the cutting time of the nth rough pass for the lth circular arc $T_{cr(l)(n)}$ can be obtained as

$$T_{cr(l)(n)} = \frac{\pi(R_{(l)} + d_s)}{500\, V_r\, f_r}\left|x_{c(l)}(\theta_{2(l)} - \theta_{1(l)}) - (R_{(l)} + d_s)(\cos\theta_{2(l)} - \cos\theta_{1(l)})\right|, \tag{16}$$

where

$$\theta_{1(l)} = \tan^{-1}\left(\frac{x_{1(l)} - x_{c(l)}}{z_{1(l)} - z_{c(l)}}\right) = \tan^{-1}\left(\frac{\frac{D_{1(l)}}{2} - x_{c(l)}}{z_{1(l)} - z_{c(l)}}\right);$$

$$\theta_{2(l)} = \tan^{-1}\left(\frac{x_{2(l)} - x_{c(l)}}{z_{2(l)} - z_{c(l)}}\right) = \tan^{-1}\left(\frac{\frac{D_{2(l)}}{2} - x_{c(l)}}{z_{2(l)} - z_{c(l)}}\right);$$

$D_{1(l)}$ and $D_{2(l)}$ are the diameters of the finished part for the lth circular arc at points $P_{1(l)}$ and $P_{2(l)}$, respectively; and $P_{c(l)}(z_{c(l)}, x_{c(l)})$ is the centre of the lth circular arc. The cutting time of the last rough cut for all circular arcs $S_{cr(n)}$ can be expressed as

$$S_{cr(n)} = \sum_{l=1}^{n_c} T_{cr(i)(n)}, \tag{17}$$

where n_c is the number of circular arcs in the turned part.

The total cutting time of the second roughing stage S_{r2} is the sum of $S_{hr(n)}$, $S_{tr(n)}$, $S_{vr(n)}$ and $S_{cr(n)}$, and it becomes

$$S_{r2} = S_{hr(n)} + S_{tr(n)} + S_{vr(n)} + S_{cr(n)}. \tag{18}$$

4.3. *Cutting time of the finishing stage*

The finish pass also cuts along the part profile. The computation of cutting time resembles that of the second roughing stage. For the finish cut of the ith straight turning segment between two points $P_{1(i)}(z_{1(i)}, x_{1(i)})$ and $P_{2(i)}(z_{2(i)}, x_{2(i)})$, The cutting time $T_{hs(i)}$ (min) can be calculated by using equation (2), and expressed as

$$T_{hs(i)} = \frac{\pi |x_{s(i)}(z_{2(i)} - z_{1(i)})|}{500 V_s f_s} = \frac{\pi |D_{(i)}(z_{2(i)} - z_{1(i)})|}{1000 V_s f_s}, \tag{19}$$

where $x_{s(i)} = D_{(i)}/2$ represents the X-coordinate of the ith straight turning segment for the finish cut; and V_s and f_s are the cutting speed and feed for finishing, respectively.

For all straight turning segments, the cutting time of finishing S_{hs} becomes

$$S_{hs} = \sum_{i=1}^{n_h} T_{hs(i)}. \tag{20}$$

For the single-pass finishing of the jth taper, the X-coordinates of $P_{1(j)}$ and $P_{2(j)}$ of the finish cut, $x_{1s(j)}$ and $x_{2s(j)}$, can be represented as

$$x_{1s(j)} = \frac{D_{1(j)}}{2}, x_{2s(j)} = \frac{D_{2(j)}}{2}. \tag{21}$$

By substituting x_1 and x_2 in equation (1) for $x_{1s(j)}$ and $x_{2s(j)}$ using equation (21), the cutting time of the finish cut for the jth taper $T_{ts(j)}$ can be expressed as

$$T_{ts(j)} = \frac{\pi}{1000 V_s f_s} \left| \frac{(D_{2(j)}^2 - D_{1(j)}^2)}{4 \sin \theta_{(j)}} \right|. \tag{22}$$

The cutting time of all taper turning segments S_{ts} can be expressed as

$$S_{ts} = \sum_{j=1}^{n_t} T_{hs(j)}. \tag{23}$$

Similarly, the cutting time of the finish cut for the kth facing segment $T_{ts(j)}$ can be obtained from equation (22) and expressed as

$$T_{vs(k)} = \frac{\pi}{1000 V_s f_s} \left| \tfrac{1}{4}(D_{2(k)}^2 - D_{1(k)}^2) \right|. \tag{24}$$

The cutting time of all facing segments S_{vs} can be defined as

$$S_{vs} = \sum_{k=1}^{n_v} T_{vs(k)}. \tag{25}$$

For the single-pass finishing of the lth circular arc, the cutting time of the finish cut $T_{cs(l)}$ can be expressed as

$$T_{cs(l)} = \frac{\pi R_{(l)}}{500\, V_s f_s} \left| x_{c(l)}(\theta_{2(l)} - \theta_{1(l)}) - R_{(l)}(\cos\theta_{2(l)} - \cos\theta_{1(l)}) \right|. \tag{26}$$

The cutting time of all circular arcs S_{cs} can be expressed as

$$S_{cs} + \sum_{l=1}^{n_c} T_{cs(l)}. \tag{27}$$

The total cutting time of the finishing stage S_f is the sum of S_{hs}, S_{ts}, S_{vs} and S_{ts}. Consequently, it becomes

$$S_f = S_{hs} + S_{ts} + S_{vs} + S_{cs}. \tag{28}$$

5. Machining optimization model

The unit production cost is utilized as the criterion to measure the optimality of machining conditions. Many practical cutting constraints are considered, including parameter bounds, cutting force constraint, power constraint, surface finish constraint, tool-life constraint, stable cutting region constraint, chip-tool interface temperature constraint and several miscellaneous constraints.

5.1. *Objective function*

The economic criterion considered herein is the minimum unit production cost which includes the cutting cost by actual time, the machine idling cost due to loading and unloading operations and idling tool motion, the tool replacement cost and the tool cost. The cutting cost C_M ($/piece) can be expressed as

$$C_M = k_o T_M, \tag{29}$$

where k_o is the sum of direct labour cost and overhead ($/min) and T_M is the actual cutting time as calculated by summing up the cutting time of first roughing state S_{r1}, the cutting time of second roughing stage S_{r2} and the cutting time of finishing stage S_f. Hence, the total cutting time T_M becomes

$$T_M = S_{r1} + S_{r2} + S_f. \tag{30}$$

The machine idling cost C_I ($/piece) can be expressed as

$$C_I = k_o T_I, \tag{31}$$

where T_I is the machine idling time (min). It is divided into a constant term (t_c) due to loading and unloading operations and a variable term due to idling tool motion. The variable term, idling tool motion time t_v (min), can be represented as the distance of tool rapid traverse l_a (mm) divided by the rapid speed V_a (mm/min). Thus,

$$t_v = \frac{l_a}{V_a}. \tag{32}$$

The distance of rapid traverse l_a can be defined as (refer to figure 2):

$$l_a = \sum_{g=1}^{n-1} H_g + \sqrt{2}(n-1)e + 2\overline{P_I P_M} + 2\overline{P_I P_L} - 2d_s, \tag{33}$$

where P_I is the initial point; P_M is the end point; and P_L is the lowest cutting point. The machine idling time T_I can be expressed as

$$T_I = t_c + t_v = t_c + \frac{l_a}{V_a}. \tag{34}$$

Hence, the machine idling cost C_I becomes

$$C_I = k_o T_I = k_o\left[t_c + \frac{\sum_{g=1}^{n-1} H_g + \sqrt{2}(n-1)e + 2\overline{P_1 P_M} + 2\overline{P_I P_L} - 2d_s}{V_a}\right]. \tag{35}$$

The tool replacement cost C_R (\$/piece) can be expressed as

$$C_R = k_o T_R, \tag{36}$$

where T_R is the tool replacement time (min). The tool replacement time can be written in terms of tool life t_l (min), time required to exchange a tool t_e (min) and cutting time T_M. It is given by

$$T_R = t_e \frac{T_M}{t_l}. \tag{37}$$

The Taylor tool-life equation is given by Armarego and Brown (1969):

$$t = \frac{C_0}{V^{\alpha} f^{\beta} d^{\gamma}}, \tag{38}$$

where α, β, γ and C_o are constants of the Taylor tool-life equation. The same tool is assumed here to be used for the entire machining process of both roughing and finishing. The wear rate of tools normally differs between roughing and finishing because the machining condition is different. Under such a circumstance, the tool life t_l can be expressed as (Shin and Joo 1992):

$$t_l = wt_r + (1 - w)t_s, \tag{39}$$

where

$$t_r = \frac{C_0}{V_r^{\alpha} f_r^{\beta} d_r^{\gamma}}$$

represents tool life for rough machining;

$$t_s = \frac{C_0}{V_s^{\alpha} f_s^{\beta} d_s^{\gamma}}$$

represents tool life for finish machining; w is a weight of tool-life equation; and $0 < w < 1$. This weight (w) is difficult to predict analytically, so that it can be selected by an empirical manner in actual manufacturing.

Hence, the tool replacement cost C_R can be expressed as

$$C_R = k_o T_R = k_o t_e \frac{T_M}{t_l}. \tag{40}$$

The tool cost C_T (\$/piece) can be obtained by

$$C_T = k_t \frac{T_M}{t_l}, \tag{41}$$

where k_t is the cutting edge cost (\$/edge).

Finally, by using the above mathematical manipulations, the unit production cost UC ($/piece) can be obtained as

$$UC = C_M + C_I + C_R + C_T$$
$$= k_o T_M + k_o T_I + k_o\left(t_e \frac{T_M}{t_l}\right) + k_t\left(\frac{T_M}{t_l}\right). \tag{42}$$

5.2. *Machining constraints*

The practical constraints imposed during the roughing and finishing operations are stated as follows.

5.2.1. *Rough machining*

Parameter bounds:

Bounds on cutting speed: $$V_{rL} \leq V_r \leq V_{rU}, \tag{43}$$

where V_{rL} and V_{rU} are the lower and upper bounds of cutting speed in roughing, respectively.

Bounds on feed: $$f_{rL} \leq f_r \leq f_{rU}, \tag{44}$$

where f_{rL} and f_{rU} are the lower and upper bounds of feed in roughing, respectively.

Bounds on depth of cut: $$d_{rL} \leq d_r \leq d_{rU}, \tag{45}$$

where d_{rL} and d_{rU} are the lower and upper bounds of depth of cut in roughing, respectively.

Tool-life constraint: The constraint on the tool life is taken as

$$T_L \leq t_r \leq T_U, \tag{46}$$

where T_L and T_U are the lower and upper bounds of tool life, respectively.

Cutting force constraint: The expression of cutting force constraint is given by (Shin and Joo 1992)

$$F_r = k_f f_r^{\mu} d_r^{\upsilon} \leq F_U, \tag{47}$$

where F_r is the cutting force during rough machining (kgf); k_f, μ and υ are constants pertaining to specific tool-workpiece combination; and F_U is the maximum allowable cutting force (kgf).

Power constraint: The power constraint is given by (Shin and Joo 1992):

$$P_r = \frac{k_f f_r^{\mu} d_r^{\upsilon} V_r}{6120\eta} \leq P_U, \tag{48}$$

where P_r is the cutting power during rough machining (kW); η is the power efficiency; and P_U is the maximum allowable cutting power (kW).

Stable cutting region constraint: This constraint is expressed as (Narang and Fischer 1993):

$$V_r^{\lambda} f_r d_r^{\upsilon} \geq S_L, \tag{49}$$

where λ and υ are constants pertaining to specific tool–workpiece combination; and S_L is the limit of the stable cutting region.

Chip-tool interface temperature constraint: This constraint can be expressed as (Hati and Rao 1976):

$$Q_r = k_q V_r^{\tau} f_r^{\phi} d_r^{\delta} \leq Q_U, \tag{50}$$

where Q_r is the temperature during roughing (°C); k_q, τ, ϕ and δ are constants related to equation of chip–tool interface temperature; and Q_U is the maximum allowable temperature (°C).

5.2.2. *Finish machining*

All the constraints other than the surface finish constraint are similar for rough and finish machining.

Parameter bounds:

Bounds on cutting speed: $$V_{sL} \leq V_s \leq V_{sU}, \tag{51}$$

where V_{sL} and V_{sU} are the lower and upper bounds of cutting speed in finishing, respectively.

Bounds on feed: $$f_{sL} \leq f_s \leq f_{sU}, \tag{52}$$

where f_{sL} and f_{sU} are the lower and upper bounds of feed in finishing, respectively.

Bounds on depth of cut: $$d_{sL} \leq d_s \leq d_{sU}, \tag{53}$$

where d_{sL} and d_{dU} are the lower and upper bounds of depth of cut in finishing, respectively.

Tool-life constraint: $$T_L \leq t_s \leq T_U. \tag{54}$$

Cutting force constraint: $$F_s = k_f f_s^{\mu} d_s^{\upsilon} \leq F_U, \tag{55}$$

where F_s is the cutting force during finish maching (kgf).

Power constraint: $$P_s = \frac{k_f f_s^{\mu} d_s^{\upsilon} V_s}{6120\eta} \leq P_U, \tag{56}$$

where P_s is the cutting power during finish machine (kW).

Stable cutting region constraint: $$V_s^{\lambda} f_s d_s^{\upsilon} \geq S_L. \tag{57}$$

Chip- tool interface temperature constraint: $$Q_s = k_q V_s^{\tau} f_s^{\phi} d_s^{\delta} \leq Q_U, \tag{58}$$

where Q_s is the temperature during finishing (°C).

Surface finish constraint: This constraint takes the form (Narang and Fischer 1993):

$$\frac{f_s^2}{8R_n} \leq R_a, \tag{59}$$

where R_n is the nose radius of cutting tool (mm) and R_a is the maximum allowable surface roughness (μm).

5.2.3. *Miscellaneous constraints*

Relations between roughing and finishing parameters: These relations can be expressed as (Chang *et al.* 1991):

$$V_s > k_1 V_r, \tag{60}$$

$$f_r > k_2 f_s, \tag{61}$$

$$d_r > k_3 d_s, \tag{62}$$

where k_1, k_2, k_3 are relationship coefficients and k_1, k_2, $k_3 \geq 1$.

Total depth of cut constraint: The depth of finish cut (d_s) should equal the maximum depth of metal to be removed $(d_t, d_t = \overline{P_I P_L})$ minus the maximum depth of metal removed in rough cut (nd_r). Therefore, this equality equation can be defined as

$$d_s = d_t - nd_r. \tag{63}$$

Bounds on number of rough passes: The bounds on number of rough passes can be expressed as follows:

$$N_L \leq n \leq N_U, \tag{64}$$

where

$$N_L = \frac{d_t - d_{sU}}{d_{rU}}, \quad N_U = \frac{d_r - d_{sL}}{d_{rL}}$$

are the lower and upper bounds of number of rough passes, respectively.

For each possible depth of finish cuts (d_s) the corresponding depth of one rough pass (d_r) can be computed, respectively. Equation (63) can be rewritten as

$$d_r = \frac{d_t - d_s}{n}. \tag{65}$$

By using the above mathematical manipulation, d_r and the equality constraint, equation (63), can be eliminated in the optimization algorithm.

6. Numerical illustration

6.1. *The problem*

According to figure 1, a cylindrical stock is turned into its desired continuous profile and size. This turned part has two straight turning segments, one facing segment, one taper and two circular arcs. The machining opimization model is established with respect to the formulation developed in § 5. The set of decision variables in the optimization algorithm is $\{n, d_r, f_r, V_r, d_s, f_s, V_s\}$ (d_r can be obtained by equation (65)). Table 1 summarizes the data for the objective function (unit production cost) and machining constraints. This test example is a constrained nonlinear optimization problem having a high degree of computational complexity.

6.2. *Solution approach*

Many solution approaches have been used to optimize turning operations, e.g. the calculus differential approach (Philipson and Ravindran 1979), geometric programming (Ermer 1971, Philipson and Ravindran 1979, Narang and Fischer 1993), the Lagrangian optimization method (Bhattacharyya *et al.* 1970, Kee 1994), goal programming (Sundaram 1978, Philipson and Ravindran 1979), sequential unconstrained minimization technique (SUMT) (Hati and Rao 1976), sequential quadratic programming (Yeo 1995), the linear approximation method (Tan and Creese 1995), direct search combining random search and Hooke-Jeeves pattern

$V_{rU} = 550$ m/min	$V_{rL} = 50$ m/min	$f_{rU} = 1.0$ mm/rev
$f_{rL} = 0.2$ mm/rev	$d_{rU} = 3.0$ mm	$d_{rL} = 1.0$ mm
$V_{sU} = 550$ m/min	$V_{sL} = 50$ m/min	$f_{sU} = 1.0$ mm/rev
$f_{sL} = 0.2$ mm/rev	$d_{sU} = 3.0$ mm	$d_{sL} = 1.0$ mm
$\alpha = 5$	$\beta = 1.75$	$\gamma = 0.75$
$k_f = 108$	$\mu = 0.75$	$\upsilon = 0.95$
$\eta = 0.85$	$\lambda = 2$	$\upsilon = -1$
$k_q = 132$	$\tau = 0.4$	$\phi = 0.2$
$\delta = 0.105$	$R_n = 1.2$ mm	$k_o = 2.0$ \$/min
$k_t = 15$ \$/edge	$V_a = 5 \times 10^4$ mm/min	$t_e = 1.5$ min/edge
$C_0 = 6 \times 10^{11}$	$t_p = 2.5$	$F_U = 5.0$ kgf
$T_L = 25$ min	$T_U = 45$ min	$Q_U = 1000^{\circ}$C
$P_U = 200$ kW	$R_a = 10\,\mu$m	$S_L = 140$
$k_1 = 1.2$	$k_2 = 1.5$	$k_3 = 2.0$
$e = 1.5$ (mm)	$d_t = 30$ mm	$w = 0.8$

Table 1. Data of the test example.

The highest cost (\$) in 50 runs	14.852
Machining parameters $\{n^*, d_r^*, f_r^*, V_r^*, d_s^*, f_s^*, V_s^*\}$	{10, 2.8850, 0.5935, 119.0844, 1.1498, 0.2442, 177.8120}
The lowest cost (\$) in 50 runs	14.578
Machining parameters $\{n^*, d_r^*, f_r^*, V_r^*, d_s^*, f_s^*, V_s^*\}$	{10, 2.8619, 0.6002, 121.4768, 1.3809, 0.3090, 152.2143}
Average final cost (\$/piece)	14.664
Standard deviation	0.0606
Average search points	29 918.9
Average CPU time (seconds/ run)	10.3

Table 2. Computational results of the test example.

search (Mesquita *et al.* 1995), a combination of dynamic programming and simplex search (Agapiou 1992), a combination of dynamic programming and Fibonacci search (Shin and Joo 1992), and a combination of geometric and linear programming (Ermer and Kromodihardjo 1981, Gupta *et al.* 1994). The above methods are only useful for a specific problem or are inclined to obtain a local optimal solution. This study applies a simulated annealing (SA) based optimization algorithm (Chen and Tsai 1996) to resolve the machining optimization problem addressed herein. The algorithm can resolve large turning optimization problems for actual manufacturing applications, as well as deliver high quality heuristic solutions with reasonable computational requirements. The superiority of this algorithm has been justified through the experimental study performed by Chen and Tsai.

Herein, the optimization algorithm is run 50 times with different initial solutions. The initial solutions are randomly selected within the parameter bounds. The test example is run on an IBM PC 586 compatible computer using C. Table 2 summarizes the computational results. The values of the algorithm-specific parameters for the SA-based optimization algorithm may influence the obtained solutions, and they are listed in table 3. The final solution of each run may vary due to the stochastic nature of the SA approach. However, table 2 reveals that the final solutions only slightly vary from one another.

$\boldsymbol{U}^o$	vector of initial step sizes, $\boldsymbol{U}^o = \{u_1^o, u_2^o, \ldots, u_m^o\}$, $\boldsymbol{U}^o = \{1, 0.01, 0.5, 0.01, 0.01, 0.5\}$
I	maximum number of step size increments allowed, $I = 10$
r_I	increasing rate of step sizes, $r_I > 1, r_I = 1.47$
r_D	decreasing rate of step sizes, $0 < r_D < 1, r_D = 0.77$
M	specified number of search points at a temperature level, $M = 45$
K	specified maximum number of n_K, $K = 25$
N	maximum number of iterations for considering no change in objective function ($\Delta E = 0$) as improvement, $N = 3000$
c	cooling ratio, a constant, $c = 0.95$
Γ^0	initial control temperature of the optimization algorithm, $\Gamma^0 = 1000$

Table 3. The algorithm-specific parameters of the SA-based optimization algorithm.

7. Conclusions

In the real-world turning processes on a CNC machine, cylindrical stocks are frequently machined into their desired continuous profiles. Such turning processes require not only straight turning but also facing, taper turning and circular turning to complete the removal of stock. This paper develops a machining model to optimize the multi-pass turning operations for turning cylindrical stocks into their desired continuous profiles and sizes. Based on the computational results presented herein, it may be concluded that the formulated machining model presents a significant enhancement of previous models. Owing to the high complexity of this machining optimization problem, a simulated annealing based optimization algorithm is applied to resolve this problem. The machining conditions obtained can be easily applied in multi-pass turning operations on CNC lathes. In addition, the machining model proposed herein can be integrated into a CAD/CAM system for determining the optimal machining parameters, thereby reducing the manufacturing cost in metal machining.

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