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Publisher: Taylor & Francis

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International Journal of Production Research

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/tprs20>

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Published online: 15 Nov 2010.

To cite this article: C.-T. Su & C.-M. Hsu (1998) Multi-objective machine-part cell formation through parallel simulated annealing, International Journal of Production Research, 36:8, 2185-2207, DOI: [10.1080/002075498192841](https://doi.org/10.1080/002075498192841)

To link to this article: <http://dx.doi.org/10.1080/002075498192841>

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Multi-objective machine-part cell formation through parallel simulated annealing

C.-T. SU†* and C.-M. HSU†

Group technology (GT) is a manufacturing philosophy which identifies and exploits the similarity of parts and processes in design and manufacturing. A specific application of GT is cellular manufacturing (CM). The first step in the preliminary stage of cellular manufacturing system (CMS) design is cell formation, generally known as a machine-part cell formation (MPCF) or a machine-component grouping (MCG) problem. Simulated annealing (SA) is not only a highly effective and general random search method to obtain near-global optimal solutions for optimization problems, but also quite appropriate for the MPCF problem which is an NP complete, complex problem. In this study, we introduce modified SA with the merits of a genetic algorithm (GA), call parallel SA (PSA), and propose a PSA-based procedure to solve the MPCF problem. More specifically, this study aims to minimize (1) total cost which includes intercell and intracell part transportation cost and machine investment cost, (2) intracell machine loading unbalance and (3) intercell machine loading unbalance under many realistic considerations. The illustrative example, comparisons and analysis demonstrate the effectiveness of this procedure. The proposed procedure is extremely adaptive, flexible, efficient and can be used to solve real MPCF problems in factories by providing a robust manufacturing cell formation in a short execution time.

1. Introduction

New technologies are rapidly developing in today's competitive manufacturing environment and, consequently, customers' preferences constantly fluctuate. Hence, managers seek new production approaches having more flexibility and productivity. One such approach is the widely adopted group technology (GT). GT is a manufacturing philosophy which identifies and exploits the similarity of parts and processes in design and manufacturing. It is a philosophy with broad applicability, potentially affecting all areas of a manufacturing organization (Hyer and Wemmerlov 1984). One specific application of GT is cellular manufacturing (CM). CM strives to attain the merits of a product-oriented layout for medium variety, medium volume production environments by processing a family of parts in a group of machines. CM attempts to reduce setup time, throughput time and material transportation cost so that inventories and market response time can be reduced (Kinney and McGinnis 1987). The preliminary stage of cellular manufacturing system (CMS) design is a cell design which involves three basic steps: cell formation, machine layout within cells and cell arrangement. Cell formation is the first and most difficult step in CMS design, which includes identifying parts with similar processing

Received August 1997.

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requirements (part family) and the set of machines that can process the corresponding family of parts (machine group). This is known as a machine-part cell formation (MPCF) problem.

In 1963, Burbidge formally defined the MPCF problem (Burbidge 1963). Many different algorithms have been developed for the MPCF problem. Production flow analysis (PFA) is one of the most discussed approaches used. Solving MPCF problems based on PFA is a combinatorial optimization problem. Many mathematical models for the MPCF problems have been provided (e.g. Purcheck 1974, Han and Ham 1986, Kusiak 1987, Irani *et al.* 1992, Ben-Arieh and Chang 1994). The optimization algorithms may yield a global optimal solution in a possibly prohibitive computation time. Hence, a number of heuristics were proposed (e.g. McCormick *et al.* 1972, King 1980). These heuristics yield an approximate solution in an acceptable computation time; however, they might be sensitive to the initial solution, the groupability of the input machine-part matrix and the number of cells specified. Notably, most conventional PFA approaches attempt to find clusters of parts and machines based on a binary machine-part incidence matrix (e.g. De Witte 1980, King and Nakornchai 1982, Kusiak 1987, Singh 1993, Offodile *et al.* 1994, Rath *et al.* 1995, Burke and Kamal 1995, Balakrishnan 1996). The matrix only contains information regarding technological variables. Some important information, e.g. operation sequence, operation time, production quantity of the part and the machine capacity is lost, thereby limiting the modelling accuracy of a realistic environment. Moreover, several practically important considerations such as machine duplication and investment cost, intercell and intracell part transportation cost, set up time, total investment cost, intercell and intracell machine loading balance are not addressed in most research thus far (Adil *et al.* 1993, Perrego *et al.* 1995, Rath *et al.* 1995, Balakrishnan 1996, Gupta *et al.* 1996, Kamal and Burke 1996).

Ballakur and Steudel (1987) showed that under fairly restrictive conditions, the MPCF problem is NP complete. Hence, optimizing a large-scale MPCF problem is relatively difficult when using an optimization approach such as the integer programming model. Therefore, attaining a more feasible approach to resolve MPCF problems is highly desirable. The key to dealing with such problems is to go a step beyond the direct application of expert skill and knowledge and make resource to special procedures which monitor and direct the use of this skill and knowledge (Singh and Rajamani 1996). Simulated annealing (SA) is one such alternative.

SA is a highly effective and general random search method to find near-global optimal solutions for optimization problems. In particular, SA is quite appropriate for certain NP complete problems in combinatorial optimization. Hence, SA is capable of resolving the MPCF problem which has a complex solution space and is not easily optimized. Venugopal and Narendran (1992) proposed an SA-based methodology which attempts to minimize machine loading variance so that the MPCF problem can be resolved. Chen *et al.* (1995) applied an SA-based heuristic to minimize the number of intercell moves to form manufacturing cells. Boctor (1996) explicitly considered the main elements of manufacturing cost in designing a CMS and developed an algorithm based on SA to solve the MPCF problem. These researches consider some different factors among machine capacity, part demand, machine duplication cost, material handling cost, and operation time. However, some other issues, e.g. operation sequences of parts, intercell machine loading balance, and the impact of cell layout, were not addressed, thereby limiting the implementation of their approaches.

In this study, a modified SA with the merits of a genetic algorithm (GA), called parallel SA (PSA) is proposed. The MPCF problem is first constructed based on PFA under realistic considerations. A multi-objective mathematical programming model which aims to minimize (1) total cost which includes intercell and intracell part transportation cost and machine investment cost, (2) intracell machine loading unbalance and (3) intercell machine loading unbalance is then formulated. A PSA-based procedure is then introduced to solve the MPCF problem. The illustrative example, comparisons and experimental analysis demonstrate the procedure's effectiveness. The proposed procedure is extremely adaptive, flexible, efficient and can be used to solve real MPCF problems in factories by providing a robust manufacturing cell formation in a short execution time.

2. Parallel simulated annealing

In this section, the authors propose a hybrid algorithm with the merits of SA and GA to deal with the MPCF problem. Simulated annealing is first briefly introduced. Then the genetic algorithm, the inspiration of this proposed algorithm, is shortly discussed which is followed by the proposed parallel simulated annealing (PSA).

2.1. Simulated annealing

Kirkpatrick *et al.* (1983) first introduced simulated annealing based on the work of Metropolis *et al.* (1953). The SA algorithm is a general optimization technique used to solve difficult combinatorial problems through controlled randomization. SA emulates the annealing process which attempts to force a system to its lowest energy through controlled cooling. In general, the annealing process is as follows: (1) the temperature is raised to a sufficient level, (2) the temperature is maintained in each level for sufficient time, and (3) the temperature is allowed to cool under controlled conditions until the desired energy is reached. It incorporates a number of aspects related to iterative important algorithms. Applying an iterative improvement algorithm requires defining a solution's configuration, an objective (energy) function, a generation mechanism, and the annealing schedule. For each configuration, the generation mechanism defines a neighbourhood which consists of all configurations that can be reached from the initial configuration in one transition. Iterative improvement is also known as local search or neighbourhood search. The annealing schedule consists of (1) the initial temperature, (2) a cooling function, (3) the number of iterations to be performed at each temperature and (4) a stopping criterion to terminate the algorithm. Lundy and Mees (1986) proved that the SA algorithm converges with a probability close to one to the global optimum under certain assumptions. SA poses several advantages over other sophisticated combinatorial optimization approaches, e.g. relatively easy and fast implementation, flexibility, and transparency. SA has been successfully applied to difficult problems. For instance, adequate results have been attained when applying SA toward various combinatorial problems (Kirkpatrick *et al.* 1983, Bonomi and Lutton 1984, Aarts and Van Laarhoven 1985, Wilhelm and Ward 1987, Alfa *et al.* 1991).

2.2. Genetic algorithm

Darwin (1895) first introduced the concept of natural and biological evolution which subsequently inspired a class of algorithms known as the genetic algorithm (GA). To apply genetic evolutionary concepts to a specific problem, two issues must be addressed: the encoding of a potential solution and the fitness function (objective

function) to be optimized. A solution's genetic representation is a vector composed of several components (genes), called a chromosome. The initial population of chromosomes is generated according to some principles or else randomly selected. The evaluation is performed to measure the quality (fitness) of potential solutions. Optimization is made by (a) selecting pairs of chromosomes with probabilities proportional to their fitness and (b) matching them to create new offspring. Besides matching (crossover), little mutation occurs in new offspring. The replacement of bad solutions with new ones is based on some fixed strategies. The evaluation, optimization and replacement of solutions are repeated until the stopping criteria are satisfied. The basic functions required in GA are given as follows:

(1) Choice function. This function generates a matching pool which is processed by the crossover function later to create offspring. The probability of each chromosome being selected into a matching pool is proportional to its fitness function.

(2) Crossover function. This function helps the chromosomes exchange information with other paired chromosomes, thereby accelerating the process of reaching an optimal solution. Offspring (some new potential solutions) can be obtained by defining one or several crossover points and rules of exchanging genes for each paired chromosomes in the matching pool.

(3) Mutation function. This function provides the opportunity to leave the local optimum. For each chromosome gene, whether it should mutate randomly according to the prerequisite probability of mutation. If it should mutate, the gene is randomly changed to a feasible value.

2.3. *Parallel simulated annealing*

The main disadvantage of SA is its high execution time. Some investigators attempted to enhance SA's computational performance through applying a better mechanism to generate a neighbourhood (Greene and Supovit 1986, Yao 1991), parallelization strategies based on the mathematical model of SA, and specially assigned computer architectures (Abramson and Dang 1993). In this study, the authors adopt the merits of the genetic algorithm (GA) to construct a hybrid SA algorithm, called parallel simulated annealing (PSA). In the PSA, the crossover and mutation functions of GA function as the generation mechanism of SA. The basic structure of the authors' proposed PSA is as follows.

Parallel simulated annealing

```
{
  Given a set of initial solutions  $X^0$  which is in the feasible solution space  $S$ 
  randomly or systematically;
  Given an initial temperature  $T^0 > 0$ ;
  Let the set of initial solutions  $X^0$  be the set of current solutions (i.e.  $X = X^0$ ).
  Let the initial temperature  $T^0$  be the current temperature (i.e.  $T = T^0$ );
  While (The satisfactory solution has not been obtained.)
  {
    While (The equilibrium has not been reached under the current temperature  $T$ .)
    {
      Generate a matching pool  $P$  by applying the choice function of GA to the set
      of current solutions  $X$ ;
      Generate a set of candidate solutions  $X'$  by applying crossover and mutation
      functions of GA to the matching pool  $P$ ;
```

Set the solution x^* to be the optimum solution among the set of current solutions X ;

Calculate the energy function E of the optimum solution x^* (i.e. $E = f(x^*)$);

Let the set of selected (surviving) solutions be empty (i.e. $X^* = \emptyset$);

While the total number of the selected (surviving) solutions $|X^*|$ is less than the total number of the initial solutions $|X^0|$ (i.e. $|X^*| < |X^0|$)

{
For each solution x_i in the sets of current and candidate solutions $X \cup X'$

{
Calculate the energy function E' of the feasible solution x_i (i.e. $E' = f(x_i)$);

Calculate $\Delta E = f(x_i) - f(x^*)$;

If the feasible solution x_i is better than the optimum solution x^* (i.e. $\Delta E < 0$), then

the feasible solution x_i is selected into the set of selected (surviving) solutions X^*

else

the feasible solution x_i obtains an acceptable probability $p_i = e^{-\Delta E/T}$;

}

}

If the total number of the selected (surviving) solutions $|X^*|$ is less than the total number of the initial solutions $|X^0|$ (i.e. $|X^*| < |X^0|$)

{

Sequentially select an unselected feasible solution x_i into the set of selected (surviving) solutions X^* with probability p_i until the total number of the selected (surviving) solutions $|X^*|$ is equal to the total number of the initial solutions $|X^0|$ (i.e. $|X^*| = |X^0|$);

}

Set the set of selected (surviving) solutions X^* to be the set of current solutions X (i.e. $X = X^*$).

}

Cooling the current temperature T through a cooling function CF (e.g. $T = CF(T) = 0.95 \times T$).

}

}

3. The model of a cell formation problem

Designing a cellular manufacturing system may require considering some different criteria, e.g. maximizing machine utilization and scheduling flexibility, balancing machine loading, as well as minimizing total cost and intercell part flow. Such objectives generally conflict with each other. Therefore, simultaneously optimizing several different objectives is a relatively difficult task. In this study, minimizing the total cost and machine loading unbalance are of primary concern. The former prioritizes minimizing total cost which involves machine investment cost, intercell and intracell part transportation cost. The latter objective concentrates on balancing intracell and intercell machine loading. Furthermore, the impact of cell layout is also involved. Hence, our studied cell formation problem can be formulated as follows.

Minimize

- (1) Total cost (machine investment cost and part transportation cost),
- (2) Intracell machine loading unbalance,
- (3) Intercell machine loading unbalance,

subject to

- (1) Each machine is assigned exactly to one manufacturing cell.
- (2) Each part's operation is performed exactly on one machine.
- (3) Each manufacturing cell is assigned exactly to one cell in the physical cell layout.
- (4) The constraint of machine capacity.
- (5) The limitation of total number of machines in each cell.

In this model, three objectives with different scales and units are to be simultaneously optimized. To combine above three objectives into one function, the following notations are given:

$TH_{m,j}$ = Machine type m hours demanded by part type i

C_m = Capacity of machine type m

$N_{\min,m}$ = Minimum required number of machine type $m = \text{MININT}(\sum_{i \in \text{Parts}} TH_{m,i} / C_m)$ where $\text{MININT}(a)$ rounds up a to the nearest integer.

I_m = Investment cost of machine type m

MIC_{\min} = Minimum machine investment cost = $\sum_{m \in \text{Machine types}} N_{\min,m} \times I_m$

O_i = Total number of operations of part type i

$IRTC_i$ = Intercell transportation cost of part type i (dollar/unit distance)

$IATC_i$ = Intracell transportation cost of part type i (dollar/unit distance)

$BDS_{k,k'}$ = Distance between cell k and k'

WDS_k = Distance within cell k

NC = Total number of cells

ABD = Average distance between two cells = $\sum_{k \in \text{Cells}} \sum_{k' \in \text{Cells}, k' \neq k} BDS_{k,k'} / (NC \times (NC - 1))$

AWD = Average distance within a cell = $\sum_{k \in \text{Cells}} WDS_k / NC$

TC_{\max} = Maximum average cost of part transportation

$$= \sum_{i \in \text{Parts}} IRTC_i \times (O_i - 1) \times ABD$$

TC_{\min} = Minimum average cost of part transportation

$$= \sum_{i \in \text{Parts}} IATC_i \times (O_i - 1) \times AWD$$

PC_{\max} = Maximum cost = $TC_{\max} + MIC_{\min}$

PC_{\min} = Minimum cost = $TC_{\min} + MIC_{\min}$

$H_{j,i}$ = Machine j hours demanded of part type i

CAP_j = Capacity of machine $j = C_m$, where machine j belongs to machine type m

U_j = Machine j utilization = $\sum_{i \in \text{Parts}} H_{j,i} / CAP_j$

N_k = Total number of machines in cell k

CU_k = Cell k utilization = $\sum_{j \in \text{cell } k} U_j / N_k$

LU_k = Machine loading unbalance of cell $k = \sum_{j \in \text{cell } k} (|U_j - CU_k|) / N_k$

$IALU$ = Intracell machine loading unbalance = $\sum_{k \in \text{cells}} LU_k / NC$

$IRLU$ = Intercell machine loading unbalance

$$= \text{Max} \{ |CU_1 - CU_2|, |CU_1 - CU_3|, |CU_1 - CU_4|, \dots, |CU_{k-1} - CU_k| \}$$

DN_m = Total number of machine type m in the final result of grouping machines

DUP_m = Total number of duplications of machine type m in the final result of grouping machines = $DN_m - N_{\min,m}$

MIC_{dup} = Total investment cost of machine duplications = $\sum_{m \in \text{Machine types}} I_m \times DUP_m$

MIC_{total} = Total machine investment cost = $MIC_{min} + MIC_{dup}$

$T_{i,l} = 1$, if the l -th and $(l + 1)$ th operations of part type i are operated in different cells.

= 0, otherwise

$TD_{i,l} = BDS_{k,k'}$, if the l th and $(l + 1)$ th operations of part type i are operated in cells k and k' , respectively.

= WDS_k , if the l th and $(l + 1)$ th operations of part type i are operated in cell k

TC_{total} = Total transportation cost of parts

= $\sum_{i \in \text{Parts}} \sum_{l=1} (T_{i,l} \times IRTC_i \times TD_{i,l} + (1 - T_{i,l}) \times IATC_i \times TD_{i,l})$

The machine duplication attempts to reduce the total cost. Hence, the sum of the total investment cost of machine duplication and the total cost of part transportation satisfies the following equation:

$$TC_{min} \leq MIC_{dup} + TC_{total} \leq TC_{max}.$$

Total cost is the sum of total machine investment cost and part transportation cost, i.e.

$$PC_{total} = \text{Total cost} = MIC_{total} + TC_{total},$$

which must vary between PC_{min} and PC_{max} . Hence, total cost can be normalized by

$$F_1 = (PC_{total} - PC_{min}) / (PC_{max} - PC_{min}), \quad (1)$$

which varies between 0 and 1.

The intracell machine loading unbalance ($IALU$) is an index which denotes the status of machine loading within cells. The smaller this index, the smoother the flow of parts inside each cell which subsequently leads to the minimization of work in process within each cell. According to its definition, $IALU$ varies between 0 and 1/2 and can be transformed by

$$F_2 = 2 \times IALU, \quad (2)$$

which varies between 0 and 1.

Similarly, intercell machine loading unbalance ($IRLU$) is an index which represents the deviation of average machine loading between cells. According to the definition, $IRLU$ varies between 0 and 1 and does not need to be normalized. Hence, the last objective function F_3 is directly defined as

$$F_3 = IRLU. \quad (3)$$

Therefore, three objective functions in our model can be combined as:

$$F = K \times \frac{w_1 \times F_1 + w_2 \times F_2 + w_3 \times F_3}{w_1 + w_2 + w_3}, \quad (4)$$

where w_1 , w_2 and w_3 are user-defined weights which allow users to flexibly determine the importance of each criterion and K is just a scalar factor for convenience. This model is a mathematical problem with NP completeness. In the next section, the authors develop a highly effective heuristic procedure based on PSA to solve this complex problem.

4. The proposed PSA-based procedure for cell formation problems

In SA, the first critical task is to define a solution's configuration. Here, the possible solution is represented by a vector of integral values, which is divided into two parts. The prior part groups machines into cells and each element in the vector represents the manufacturing cell in which the element's corresponding machine is grouped. The posterior part represents how the cells are arranged in the physical cell layout. For instance, there is a feasible solution having eight machines which are grouped into three cells. An integer-valued vector with 11 elements ($8 + 3 = 11$), e.g. (1, 2, 1, 3, 2, 1, 3, 3, 2, 3, 1), is capable of representing this solution's configuration. The first element '1' represents that the first machine is grouped into cell 1 and the second element '2' groups the second machine into cell 2. Similarly, the third and fourth elements '1' and '3' group the third and fourth machines into cells 1 and 3, respectively. By the same logic, the succeeding four elements, i.e. 2, 1, 3 and 3, group the fifth to eighth machines into cells 2, 1, 3 and 3 respectively. Finally, the last three elements arrange the three manufacturing cells into cells 2, 3 and 1 respectively, in the physical layout (see figure 1). If there exists a feasible solution having nine machines which are grouped into four cells, a vector with 13 ($9 + 4 = 13$) elements is required to represent this feasible solution. Another basic SA-related issue is the energy function which is defined as a weight normalized function as shown in equation (4). Based on the proposed PSA algorithm, a PSA-based procedure to solve the MPCF problems is developed and diagrammed in figure 2. A more detailed description is given in the following.

Step 1. Initialize PSA parameters. Calculate the minimum required number for each machine type, maximum and minimum cost.

Step 2. Create initial solutions to group the machine into manufacturing cells/ arrange cells in the layout.

Step 2.1. Calculate correlative strengths (CS) between machine types.

$CS_{m,m'}$ = Correlative strength between machine types m and m'

$$= \sum_{i \in \text{parts}} (IRT_{ci} \times ABD - IAT_{ci} \times AWD) \times TN_{i,m,m'}$$

where $TN_{i,m,m'}$ = Total number of part type i transportation from machine types m to m'

$$= \sum_{l=1} OS_{l,i,m,m'}$$

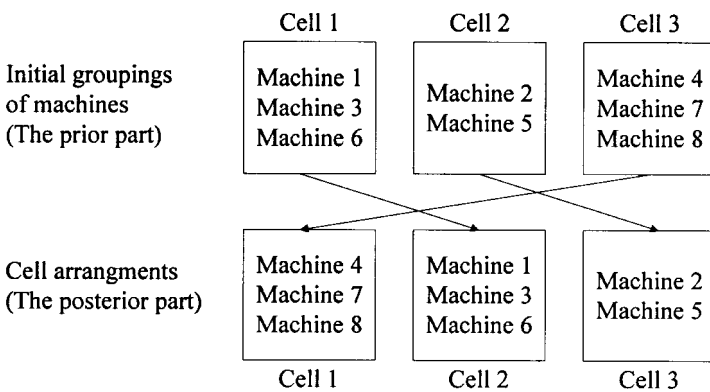


Figure 1. The grouping of machines and cell layout.

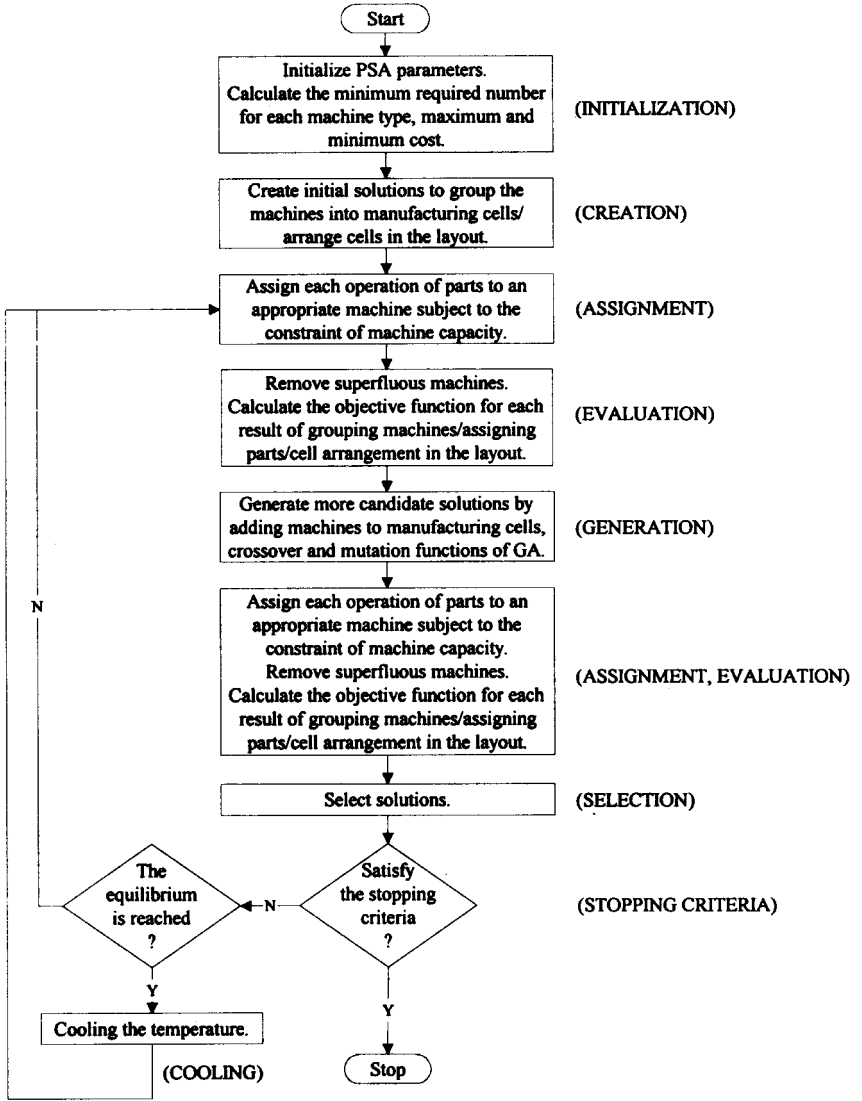


Figure 2. A flow diagram of the proposed PSA-based procedure for the cell formation problem.

$OSI_{i,l,m,m'} = 1$, if the l th operation of part type i is performed on machine type m and the $(l + 1)$ th operation is performed on machine type m'
 $= 0$, otherwise.

Step 2.2. Calculate the initial total number of machines (IN_k) for each manufacturing cell.

IN_k = Total number of initial machines in cell k

For cell $k = 1, 2, \dots, NC - 1$, IN_k must satisfy
 $MAXINT(\sum_{m \in \text{Machine types}} N_{\min,m} / NC) \leq IN_k \leq$

$MAXINT(\sum_{m \in \text{Machine types}} N_{\min,m} / NC)$

where $MAXINT(a)$ rounds down a to the nearest integer.

$IN_{NC} = \sum_{m \in \text{Machine types}} N_{\min,m} - \sum_{k=1} IN_k$ must satisfy the limitation of total number of machines in each cell.

Step 2.3. Let all machines be unassigned.

WHILE (There is any empty manufacturing cell.)

{

Assign an unassigned machine to the current manufacturing cell.

WHILE (The current manufacturing cell is not full.)

{

Find an unassigned machine which has the largest correlative strength (CS) with the last machine assigned to the current manufacturing cell.

Assign this unassigned machine to the current manufacturing cell.

}

}

Step 2.4. Randomly arrange manufacturing cells in the layout.

Step 2.5. Represent the initial solution of grouping machines/arranging cells by a vector of integral values. Duplicate this initial solution and increase the diversity of initial solutions by the mutation function of GA to form an initial population of solutions with pre-determined size.

Step 3. Assign each operation of parts to an appropriate machine subject to the constraint of machine capacity.

For each part:

DO {

List all possible combinations of assignments subject to the constraint of machine capacity.

Allow all combinations of assignments with the same least sum of intercell and intracell part transportation cost to be the candidates.

Select the candidate which inspires the least 'shock' (SK)* to the machine loading unbalance.

}

*For part type i , the shock that a possible combination of assignments inspires is defined as follows:

$$SK = \sum_{l \in \text{Operations}} SOH_{i,l} / AC_j$$

where

$SOH_{i,l}$ = The sum of setup time and operation time of the l -th operation of part type i

AC_j = Currently available capacity of machine j which performs the l -th operation of part type i .

Step 4. Remove superfluous machines. Calculate the objective function for each result of grouping machines/assigning parts/cell arrangement in the layout.

Step 4.1. For each result, remove the machine with zero machine loading.

Step 4.2. Calculate the objective function for each result of grouping machines/assigning parts/cell arrangement in the layout.

$$\text{Objective function } F = K \times (w_1 \times F_1 + w_2 \times F_2 + w_3 \times F_3) / (w_1 + w_2 + w_3) + \text{penalty}$$

where F_1 , F_2 , F_3 , w_1 , w_2 and w_3 are defined as those in the previous section; and *penalty* refers to an extremely large positive number for the violation of the limitation of total number of machines in each cell.

Step 4.3. Find the optimum solution from the current population of solutions.

Step 5. Generate more candidate solutions by adding machines to manufacturing cells, the crossover and mutation functions of GA.

For each result of grouping machines/assigning parts/cell arrangement in the layout, perform steps 5.1 and 5.2.

Step 5.1. For each machine type, calculate its potential to reduce total cost if this machine type duplicates one more machine.

Where the potential of machine type m is defined as ‘the reduced intercell part transportation costs – the increased intracell part transportation costs – the increased investment cost of the duplicated machine, if machine type m duplicates one more machine under current result of grouping machines/assigning parts/cell arrangement in the physical layout.’

Step 5.2. Duplicate one more machine with the optimum potential for the original result to generate one more candidate solution.

Step 5.3. Generate a matching pool by applying the choice function of GA to the original solutions.

Step 5.4. Generate more candidate solutions by performing the crossover and mutation functions to the matching pool.

Step 6. Assign each operation of parts to an appropriate machine subject to the constraints of machine capacities. Remove superfluous machines. Calculate an objective function for each result of grouping machines/assigning parts/cell arrangement in the layout.

Step 7. Select solutions.

Step 7.1. For each solution:

DO {

Calculate $\Delta E =$ The objective function of this solution –
The objective function of the current
optimum solution.

If $\Delta E \leq 0$, then the current solution is selected

else

the current solution obtains an acceptable probability

$$P_x = e^{-\Delta E / \text{Temperature}}$$

}

If the total number of selected solutions reaches the pre-determined size of the population of solutions, go to step 8.

Step 7.2. Sequentially select an unselected solution with probability p_x until the total number of solutions reaches the pre-determined size of the population of solutions.

- Step 8.* Let the selected solutions be the current population of solutions. Go to step 10 if the stopping criteria are satisfied.
- Step 9.* If the equilibrium is reached, cool the temperature
 Temperature = Cooling function (Temperature)
 and go to step 3. Otherwise, go to step 3.
- Step 10.* Stop.

5. Numerical illustration

In this section, a numerical example to demonstrate the effectiveness of the proposed PSA-based procedure for cell formation problems is presented. Tables 1 and 2 summarize the cell formulation problem. Table 1 lists the preliminary information of the problem including part transportation cost of a unit distance, operation sequences, operation and setup time in a production cycle. Table 2 denotes the machine types, capacity available and investment cost in a production cycle. Figure 3 displays the basic layout of manufacturing cells and the assumed distances between and within cells. The total number of cells is five and the total number of machines in each cell varies from four to eight. The initial temperature is set to 100, and 20 repetitions under each level of temperature are allowed until the equilibrium is reached. Solutions of a total number ten are simultaneously searched. The combination of weights in the objective function is set to $w_1 = 5$, $w_2 = 1$ and $w_3 = 1$, and the scalar factor $K = 1000$. The feasible result of grouping machines/assigning part/cell arrangement in the layout can be achieved by going sequentially with the proposed procedure which was coded in C language and implemented on a Pentium PC. Figure 4 shows the final machine-part incidence matrix including operation sequences of parts and the cell arrangement in the physical layout. Notably, machine types 10 and 15 duplicate one more machine (10 - 3) and (15 - 3), respectively, in the final result and the total cost is 21 604. Table 3 reveals the status of the machine and cell utilization. Figure 5 shows the development of the objective function F and the temperature T along with cycles. Moreover, a detailed stepwise illustration can be found in the Appendix.

6. Discussion

The superior strengths of the proposed procedure are illustrated by using this PSA-based procedure to solve two MPCF problems from the previous literature. The first problem originates from the problem (Logendran 1991) modified from Tabucanon and Ojha (1987). The second one originates from the hypothetical example of Gupta *et al.* (1996). Table 4 presents the basic workstation-part matrices of the two problems. Figure 6 displays the basic cell layouts of two different types with an equal distance between two continuous cells (Logendran 1991). Notably, each machine's capacity is 8 (hours/day) and cannot be duplicated in the original studies. Hence, each machine's investment cost is set to an extremely large number, e.g. 1000. Moreover, all function-identical machines in each workstation must be grouped in the same manufacturing cell. Hence, each workstation is treated as a machine type with the capacity which is the sum of the capacity of all machines in that workstation. Tables 5 and 6 compare the authors' results with those of previous work. According to that comparison, this proposed procedure can not only adapt well for problems of different scales, but also yields satisfactory results.

The numerical example is re-run by combining the total number of cells and two types of cell layouts. Table 7 summarizes those results. Simultaneously optimizing

Part	Part transportation cost (\$/unit distance)	Operation sequence	Operation time	Setup time
1	100	2, 4, 12, 2, 13, 10	1, 4, 2, 4, 5, 3	0.2, 0.3, 0.1, 0.2, 0.3
2	110	14, 7, 17, 15, 13	2, 2, 3, 2, 3	0.3, 0.1, 0.3, 0.2, 0.3
3	90	18, 10, 8, 16, 3	1, 1, 1, 2, 1	0.2, 0.2, 0.2, 0.2, 0.1
4	80	15, 1, 5, 11	2, 2, 4, 2	0.3, 0.2, 0.3, 0.2
5	120	15, 9, 6	5, 2, 2	0.4, 0.2, 0.3
6	100	9, 15, 10, 15	5, 2, 2, 2	0.3, 0.2, 0.3, 0.1
7	80	10, 3, 16	1, 2, 3	0.2, 0.1, 0.2
8	90	16, 10, 3, 18, 10	3, 1, 3, 2, 2	0.2, 0.1, 0.1, 0.1, 0.2
9	80	12, 10, 15, 4, 6, 10	1, 1, 2, 3, 1, 2	0.1, 0.3, 0.1, 0.2, 0.2, 0.2
10	80	13, 2, 12, 6, 2	1, 2, 3, 2, 2	0.1, 0.1, 0.2, 0.2, 0.1
11	80	6, 2, 4, 13, 6	1, 1, 3, 3, 3	0.2, 0.1, 0.2, 0.3, 0.2
12	80	4, 2, 6, 12, 7, 13	1, 3, 3, 3, 2, 3	0.1, 0.3, 0.5, 0.4, 0.1, 0.1
13	80	15, 7, 17, 7, 14	3, 2, 3, 3, 1	0.1, 0.2, 0.1, 0.2, 0.1
14	80	10, 18, 8, 18, 3, 16	1, 2, 3, 3, 4, 4	0.3, 0.2, 0.2, 0.1, 0.2, 0.2
15	90	8, 18, 16, 10	1, 2, 3, 2	0.1, 0.3, 0.3, 0.2
16	80	5, 11, 1, 5, 15	1, 4, 2, 1, 2	0.1, 0.2, 0.2, 0.1, 0.2
17	110	11, 1, 15, 5, 15	2, 2, 1, 5, 1	0.1, 0.3, 0.1, 0.3, 0.2
18	100	15, 9, 6, 9	4, 2, 2, 3	0.3, 0.2, 0.3, 0.1
19	90	15, 5, 11, 1	2, 3, 2, 1	0.2, 0.3, 0.2, 0.1
20	120	8, 16, 8, 18	1, 2, 2, 2	0.2, 0.2, 0.2, 0.2
21	80	6, 10, 15, 9, 10	3, 1, 1, 3, 2	0.4, 0.2, 0.1, 0.2, 0.2
22	90	7, 14, 13, 15, 14, 3	2, 2, 5, 2, 4, 4	0.2, 0.2, 0.3, 0.2, 0.1, 0.3
23	100	6, 4, 2, 12, 2, 13	1, 1, 3, 4, 1, 2	0.2, 0.1, 0.1, 0.3, 0.1, 0.3
24	90	12, 4, 10, 6, 13, 4	2, 1, 2, 2, 2, 2	0.3, 0.1, 0.3, 0.4, 0.1, 0.1
25	80	3, 7, 3, 17, 13	5, 3, 4, 2, 4	0.3, 0.3, 0.1, 0.3, 0.2
26	70	7, 14, 3, 17, 13, 7	1, 3, 3, 2, 4, 1	0.1, 0.2, 0.3, 0.1, 0.5, 0.1
27	80	3, 15, 8, 18, 16, 8	2, 1, 4, 2, 4, 1	0.2, 0.1, 0.2, 0.1, 0.3, 0.1
28	110	11, 1, 15, 1	3, 2, 1, 3	0.2, 0.2, 0.1, 0.2
29	120	1, 15, 1, 11	2, 1, 5, 1	0.2, 0.2, 0.1, 0.1
30	80	10, 6, 15, 9, 17	3, 1, 1, 2, 2	0.3, 0.1, 0.1, 0.1, 0.3
31	80	15, 10, 9, 18, 6	4, 2, 3, 3, 1	0.3, 0.3, 0.3, 0.1, 0.1
32	80	18, 10, 3, 10, 8, 16	1, 2, 2, 2, 3, 1	0.1, 0.1, 0.3, 0.1, 0.3, 0.1
33	100	17, 13, 15, 7, 14, 7	1, 2, 1, 1, 3, 2	0.2, 0.2, 0.4, 0.2, 0.2, 0.1
34	110	2, 13, 4, 12, 10	3, 1, 4, 1, 3	0.3, 0.1, 0.4, 0.1, 0.4
35	90	14, 17, 7, 3, 17, 15	1, 2, 3, 2, 2, 1	0.1, 0.2, 0.4, 0.3, 0.1, 0.3

Table 1. Parts' information.

multiple objectives is relatively difficult and, therefore, some tradeoffs must be made. Some flexible different results are given and a decision can be made according to one's own requirements. Table 8 lists the execution time of 10 experiments. The mean execution time is short and the standard deviation is acceptable. It provides good support for users to find a feasible solution for the cell formation problem. Although trial-and-error is necessary, it still costs much less than the final cell formation result which may be an important decision in the future.

7. Conclusion

In this work, a procedure based on PSA to solve the MPCF problem has been presented. The proposed procedure attempts to simultaneously minimize total cost, intracell and intercell machine loading unbalance. The study also considers many

	Machine type																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Capacity available	25	24	22	25	22	22	26	26	25	22	22	22	22	20	25	28	21	27
Investment cost	800	650	700	700	800	800	750	550	750	600	700	600	700	700	550	700	800	900

Table 2. Machines' information.

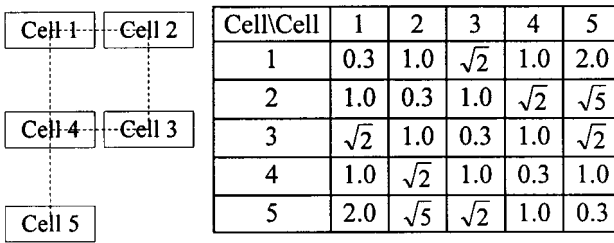


Figure 3. The basic layout of manufacturing cells.

realistic aspects such as operation sequences, setup time, operation time, intercell and intracell transportation cost of a part. Important factors regarding the investment cost, duplication and capacity of a machine are involved. The impact of the layout of manufacturing cells is also included. The MPCF problem is initially formulated into a multi-criteria mathematical programming model. Next, three objectives of different scales and units are normalized and combined into one weighted objective function. Parallel simulated annealing (PSA) with the merits of GA is then introduced and a procedure based on PSA, which is quite appropriate for the MPCF problems, is developed to solve this problem. A sample example of 35 part types and 18 machine types is then solved to test the procedure; a satisfactory result is subsequently yielded. Also, in implementing this procedure, decision makers have flexibility in determining the priorities of three difference objectives. Two different scaled problems from the literature are also considered and the numerical results are compared with those of the original authors. According to that comparison, this proposed procedure is better in most situations. Examining the execution time assures us of the proposed procedure's efficiency. The procedure is extremely adaptive, flexible, efficient and can be used to solve real MPCF problems in factories by providing a robust manufacturing cell formation in a short execution time. Furthermore, the procedure is a highly effective tool in assisting manufacturers to develop more efficient production plans under competitive environments.

Appendix. A detailed stepwise illustration for the example in section 5

- (1) Machine hours demanded can be found in table 9. Table 10 presents the minimum required number of each machine type. Therefore, we have $PC_{max} = 33\ 124$ and $PC_{min} = 19\ 889$. Table 11 displays the initial total number of machines in each cell.
- (2) The correlative strengths (CS) between machine types are calculated and shown in table 12. Hence, one initial solution of grouping machines is

P/M	6-1	9	10-2	15-3	3-2	7	13-2	14	15-1	17	3-1	8	10-1	16	18	1	5	11	15-2	2	4	6-2	10-3	12	13-1	
5	3	2		1																						
6		1	3	2,4																						
18	3	2,4		1																						
21	1	4	2,5	3																						
30	2	4	1	3						5																
31	5	3	2	1											4											
2						2	5	1	4	3																
13						2,4		5	1	3																
22						6	1	3	2,5	4																
25						1,3	2	5			4															
26						3	1,6	5	2		4															
33						4,6		5	3	1															2	
35						4	3		1	6	2,5															
3												5	3	2	4	1										
7												2		1	3											
8												3		2,5	1	4										
14												5	3	1	6	2,4										
15													1	4	3	2										
20													1,3		2	4										
27						1				2			3,6		5	4										
32													3	5	2,4	6	1									
4																	2	3	4	1						
16																	3	1,4	2	5						
17																	2	4	1	3,5						
19																	4	2	3	1						
28																		2,4	1	3						
29																		1,3		4	2					
1																				1,4	2		6	3	5	
9																			3		4	5	2,6	1		
10																				2,5		4		3	1	
11																				2	3	1,5			4	
12						5	6													2	1	3			4	
23																				3,5	2	1			4	
24																					2,6	4	3	1	5	
34																					1	3		5	4	2

Cell 1
Machines
1, 5, 11, 15-2

Cell 2
Machines
2, 4, 6-2, 10-3,
12, 13-1

Cell 4
Machines
3-1, 8, 10-1, 16, 18

Cell 3
Machines
3-2, 7, 13-2, 14
15-1, 17

Cell 5
Machines
6-1, 9, 10-2, 15-3

Figure 4. Machine-part incidence matrix (including operation layout sequences of parts) and the cell arrangement in the physical layout.

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Cell	Machine	Machine utilization	Cell utilization
1	1	82.00%	68.10%
	5	68.64%	
	11	68.18%	
	15-2	53.60%	
2	2	89.58%	75.79%
	4	82.00%	
	6-2	67.73%	
	10-3	56.82%	
	12	79.55%	
	13-1	79.09%	
3	3-1	58.18%	83.76%
	8	67.31%	
	10-1	60.91%	
	16	84.64%	
4	18	71.85%	68.58%
	3-2	97.73%	
	7	92.31%	
	14	86.00%	
	13-2	92.73%	
	15-1	45.20%	
5	17	88.57%	66.33%
	6-1	46.36%	
	8	85.60%	
	10-2	51.36%	
	15-3	82.00%	

Table 3. The utilization of machines and cells.

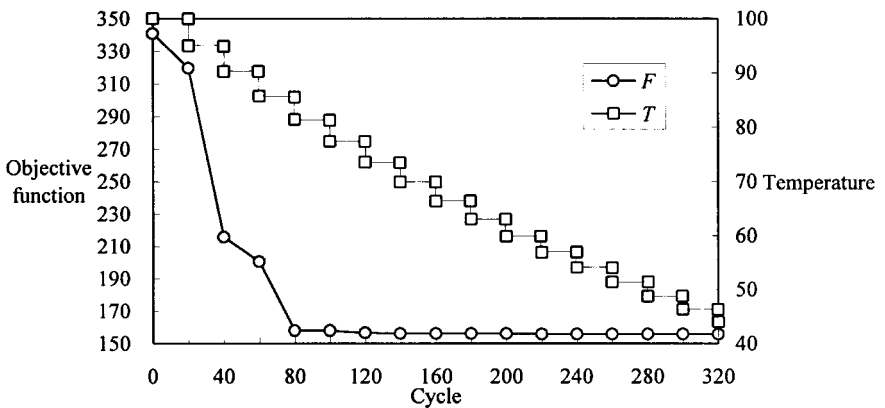


Figure 5. The developments of the objective function F and the temperature T along with searching cycles.

found and represented by a vector (1, 1, 2, 3, 2, 4, 1, 4, 5, 3, 1, 3, 4, 4, 5, 2, 5, 5, 1, 4, 3, 2, 3, 5, 1, 2, 4, 3). The first element '1' represents that machine type one is grouped into cell 1 and the second element '1' groups machine type two into cell 1. Similarly, the third and fourth elements '2' and '3' group the first and second machines of machine type three into cells 2 and 3, respectively.

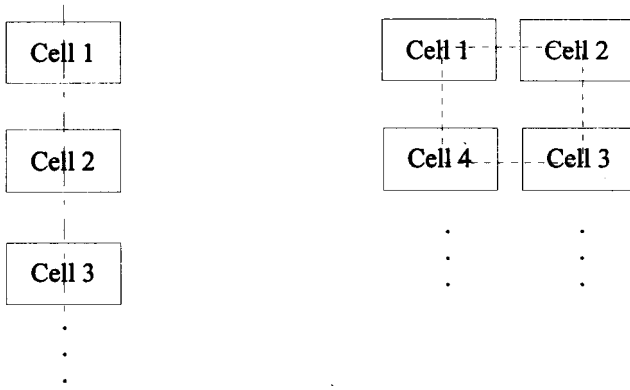
Workstation	Parts														Total workload on workstation (h)	Number of machines
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14		
W1	0.69				2.42		2.44	2.48					2.72		10.75	2
W2	0.50			0.61	0.90	2.09	1.35								5.45	1
W3								2.50	3.03	0.71	1.61				7.85	1
W4			3.10				1.35	1.03	0.58	0.99					7.05	1
W5		1.22					4.45					3.84			9.51	2
W6	0.50					4.55	2.26								7.31	1
W7	0.55			4.74	3.61	1.47		3.87	4.68						18.92	3

(a)

Workstation	Parts																												Work station/Total workload on workstation (h)/Number of machines			
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24	P25	P26	P27	P28		P29	P30	
0.1	0.8	0.1	0.8	1.2	1.2	0.5	1.7	1.8	2.0																							W1/8.5/2
0.9	0.8	0.4	1.8	0.7	1.8	0.7	1.3	0.5	0.8	1.7	0.2	0.8	0.3	0.3	0.5	1.1	1.9	1.4	1.4	1.9	0.3	0.3	1.7	1.1	1.1	1.1	1.9	1.9	1.7	1.7	W2/8.6/2	
1.2	1.1		0.9		0.9	1.5	0.9	0.1	0.8	0.6	0.6	0.5	0.5	0.5	1.2	1.9	1.3	1.3	1.3	1.2	0.6	1.2	1.2	1.3	1.3	0.9	1.4	1.4	1.9	0.7	1.9	W3/8.6/2
0.1	0.6	1.2	1.7	1.3	0.2	1.0	1.4	2.0	0.6	1.9	0.4	1.9	2.0	2.0	1.9	1.9	1.8	1.8	1.0	1.8	1.0	1.0	1.8	1.3	1.5	1.5	1.5	1.3	1.8	1.8	1.3	W4/8.2/2
0.6	1.2					0.9	0.4	1.3	0.8	1.2	0.8	1.2	1.8	1.8	0.7	0.7	0.7	0.7	0.2	1.7	1.7	1.7	1.7	1.8	1.8	0.9	1.4	1.4	1.4	1.4	1.8	W5/7.0/1
0.1	0.6	1.2	1.7	1.3	0.2	1.0	1.4	2.0	0.6	1.9	0.4	1.9	2.0	2.0	1.9	1.9	1.8	1.8	1.0	1.8	1.0	1.0	1.7	1.8	1.5	1.5	1.5	1.3	1.8	1.8	1.8	W6/9.5/2
0.3	0.4					0.9	0.4	1.3	0.8	1.2	0.8	1.2	1.8	1.8	0.7	0.7	0.7	0.7	0.2	1.7	1.7	1.7	1.7	1.8	1.5	1.5	1.5	1.3	1.8	1.8	1.8	W7/5.3/1
1.6						0.3	0.4	1.5	1.4	1.5	1.4	1.5	1.5	1.5	1.0	1.0	1.3	1.2	0.4	0.4	0.7	0.1	0.1	1.4	1.5	1.5	1.5	1.3	1.8	1.8	1.8	W8/9.7/2
						0.3	0.4	1.3	1.3	0.3	0.5	0.3	0.5	0.5	1.0	1.0	1.3	1.2	0.4	0.4	0.7	0.1	0.1	1.4	1.5	1.5	1.5	1.3	1.8	1.8	1.8	W9/10.1/2
						0.3	0.4	1.3	1.3	0.3	0.5	0.3	0.5	0.5	1.0	1.0	1.3	1.2	0.4	0.4	0.7	0.1	0.1	1.4	1.5	1.5	1.5	1.3	1.8	1.8	1.8	W10/9.5/2
						0.3	0.4	1.3	1.3	0.3	0.5	0.3	0.5	0.5	1.0	1.0	1.3	1.2	0.4	0.4	0.7	0.1	0.1	1.4	1.5	1.5	1.5	1.3	1.8	1.8	1.8	W11/7.0/1
						0.3	0.4	1.3	1.3	0.3	0.5	0.3	0.5	0.5	1.0	1.0	1.3	1.2	0.4	0.4	0.7	0.1	0.1	1.4	1.5	1.5	1.5	1.3	1.8	1.8	1.8	W12/7.6/1
						0.3	0.4	1.3	1.3	0.3	0.5	0.3	0.5	0.5	1.0	1.0	1.3	1.2	0.4	0.4	0.7	0.1	0.1	1.4	1.5	1.5	1.5	1.3	1.8	1.8	1.8	W13/10.6/2
						0.3	0.4	1.3	1.3	0.3	0.5	0.3	0.5	0.5	1.0	1.0	1.3	1.2	0.4	0.4	0.7	0.1	0.1	1.4	1.5	1.5	1.5	1.3	1.8	1.8	1.8	W14/5.8/1
						0.3	0.4	1.3	1.3	0.3	0.5	0.3	0.5	0.5	1.0	1.0	1.3	1.2	0.4	0.4	0.7	0.1	0.1	1.4	1.5	1.5	1.5	1.3	1.8	1.8	1.8	W15/6.7/1

(b)

Table 4. The basic workstation-part matrices: (a) Problem 1; (b) Problem 2.



(a) Linear single-row cellular layout (b) Linear double-row cellular layout.

Figure 6. The basic cell layouts of two different types (Logendran 1991).

Total # of cells	Layout type	Total move	Workstations in cell <i>i</i>			
			Cell 1	Cell 2	Cell 3	Cell 4
2	1	6.30	2,3,4,6,7	1,5		
	2	6.30	1,5	2,3,4,6,7		
3	1	7.50*	3	2,4,6,7	1,5	
	2	7.50*	1,5	2,4,6,7	3	
4	1	9.40*	3	2,4,6,7	1	5
	2	8.88*	2,6,7	1,5	3	4

*The proposed PSA-based procedure is better than Logendran's (1991).

Table 5. The result of minimizing total cost (move) (Problem 1).

The last five elements arrange five manufacturing cells into cells 5, 1, 2, 4 and 3, respectively, in the physical layout. Duplicate this initial solution and perform mutation function to form an initial population of solutions which are represented by vectors in table 13.

- (3) For the first solution of grouping machines/cell arrangement, table 14 presents all feasible combinations of assigning operations to machines that have the least part transportation cost. This table also contains the shock (*SK*) to the machine loading unbalance of each combination. Hence, the first part is assigned to be performed sequentially on machines 2, 4, 12, 2, 13-2, 10-1. In fact, the final operation can be performed on either machine 10-1 or 10-2 because of the same *SK* value that these two combinations inspire. Similarly, the same task can be performed on all other parts and all solutions of grouping machines/cell arrangement.
- (4) Calculate the machine loading of each solution according to its optimum combinations of assigning operations to machines. Now, no machines have zero machine loading, and no superfluous machines must be removed. Next, the normalized total cost (F_1), intracell machine loading unbalance (F_2), intercell machine loading unbalance (F_3) and objective function (F) are

Total # of cells	Layout type	Total move	Workstations in cell <i>i</i>						
			Cell 1	Cell 2	Cell 3	Cell 4	Cell 5	Cell 6	
4	1	37.60*	14	11	1, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13		2		
		34.67*	2	1	3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15		11		
5	1	44.20*	14	11	3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 15		1	2	
		37.54*	14	11	2	3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 15		1	
6	1	53.30*	1	2	3, 4, 5, 6, 7, 8, 9, 10, 12, 13		15	11	14
		40.92*	14	15	3, 4, 5, 6, 7, 8, 9, 10, 12, 13		11	2	1

*The proposed PSA-based procedure is better than Gupta *et al.*'s (1996).

Table 6. The result of minimizing total cost (move) (Problem 2).

Total # of cells	Type 1 layout					Type 2 layout				
	Total cost	F_1	F_2	F_3	F	Total cost	F_1	F_2	F_3	F
3	20 606	0.0549	0.3839	0.1414	114.3	20 419	0.0501	0.3923	0.1288	110.2
	20 512	0.0477	0.3485	0.2365	117.7	20 419	0.0501	0.3933	0.1350	111.3
4	21 142	0.1185	0.2936	0.1325	145.5	20 828	0.0888	0.2484	0.1440	118.2
	21 097	0.1141	0.3026	0.1608	147.7	20 899	0.0954	0.2528	0.1350	123.6
5	21 397	0.1425	0.2520	0.1194	154.9	21 021	0.1069	0.2398	0.1637	134.0
	21 585	0.1283	0.2449	0.1592	149.3	21 604	0.1296	0.2688	0.1742	155.9
5	21 607	0.1299	0.2598	0.1775	155.2	21 603	0.1296	0.2674	0.1915	158.9
	21 660	0.1338	0.2091	0.1535	159.3	21 627	0.1313	0.2791	0.1842	160.0

Table 7. Results of different combinations of the total number of cells and layout types.

Experiment	1	2	3	4	5	6	7	8	9	10	Mean	Standard deviation
Time (sec)	218	179	196	211	193	215	189	232	218	204	205.5	15.36

Table 8. The execution time of 10 experiments.

Machine type	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Minimum hours demanded	20.521.5	34.3	20.5	15.1	25.1	24.0	17.5	21.4	37.2	15.0	17.5	37.8	17.2	45.2	23.7	18.6	19.4	

Table 9. Machine hours demanded.

Machine type	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Minimum required number	1	1	2	1	1	2	1	1	1	2	1	1	2	1	2	1	1	1

Table 10. The minimum required number of each machine type.

Cell	1	2	3	4	5
Initial total number of machines	5	4	5	5	4

Table 11. The initial total number of machines in each cell.

M/M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
1					167.66						125.74				356.28				
2				188.62		83.83						188.62	324.84						
3							83.83			83.83						83.83	167.66	251.49	94.31
4		188.62				83.83				94.31		220.05	83.83						
5											261.97				199.10				
6		167.66		104.79					104.79	167.66		83.83	94.31		83.83				
7			178.14										83.83	356.28				199.10	
8																303.88		387.71	
9						230.53				83.83					104.79		83.83	83.83	
10						178.14		178.14	83.83						272.45			83.83	
11	408.67																		
12		209.57		94.31		83.83	83.83			199.10									
13		83.83		209.57		83.83	73.35			104.79					199.10				
14			167.66				220.05						94.31					94.31	
15	324.84			83.83	209.57		188.62	83.83	398.19	188.62			115.27	94.31					
16				94.31				209.57		188.62									
17							178.14						261.97		209.57				
18			83.83			83.83		83.83		272.45						178.14			

Table 12. The correlative strengths (CS) between machine types.

Initial solution	Vector
1	(1, 1, 2, 3, 2, 4, 1, 4, 5, 3, 1, 3, 4, 4, 5, 2, 5, 5, 1, 4, 3, 2, 3, 5, 1, 2, 4, 3)
2	(5, 5, 2, 3, 1, 4, 1, 4, 1, 3, 1, 2, 4, 4, 5, 2, 2, 5, 1, 5, 2, 2, 3, 5, 4, 1, 2, 3)
3	(1, 1, 2, 3, 2, 4, 1, 4, 5, 3, 1, 3, 4, 4, 5, 2, 5, 5, 1, 4, 3, 2, 3, 5, 1, 2, 4, 3)
4	(1, 3, 2, 3, 3, 4, 1, 4, 3, 3, 1, 2, 4, 2, 5, 2, 5, 3, 1, 4, 3, 2, 3, 4, 3, 2, 1, 5)
5	(1, 1, 2, 3, 2, 4, 1, 4, 5, 3, 1, 3, 4, 4, 5, 2, 5, 5, 1, 4, 3, 2, 3, 2, 4, 5, 1, 3)
6	(1, 1, 2, 3, 2, 4, 1, 4, 5, 3, 1, 3, 4, 4, 5, 2, 5, 5, 1, 4, 3, 2, 3, 5, 1, 2, 4, 3)
7	(1, 1, 2, 3, 4, 4, 1, 4, 5, 3, 1, 3, 4, 4, 4, 2, 5, 5, 1, 4, 3, 2, 3, 5, 1, 2, 3, 4)
8	(1, 1, 2, 5, 2, 4, 1, 4, 5, 5, 1, 3, 4, 2, 5, 2, 5, 5, 2, 4, 5, 5, 3, 3, 2, 4, 5, 1)
9	(1, 1, 2, 3, 2, 4, 1, 4, 5, 3, 1, 3, 4, 4, 5, 2, 5, 5, 1, 4, 3, 2, 3, 3, 2, 1, 4, 5)
10	(4, 1, 2, 3, 2, 4, 1, 4, 5, 3, 1, 3, 3, 1, 5, 5, 5, 5, 4, 1, 3, 2, 3, 3, 5, 2, 1, 4)

Table 13. The initial population of solutions.

calculated and shown in table 15. The objective function of the optimum solution is 307.5.

- (5) In the first solution of grouping machines/assigning parts/cell arrangement, the potential of machine type one as defined before is calculated and shown in table 16. Similarly, the potential of other machine types can be found and

Combination	Assigning operations to machines	SK
1	2, 4, 12, 2, 13-2, 10-1	0.8834
2	2, 4, 12, 2, 13-2, 10-2	0.8834

Table 14. The feasible combinations of assigning operations to machines that have the least part transportation cost (for the first part type in the first solution of grouping machines/cell arrangement).

Solution	F_1	F_2	F_3	Penalty	F
1	0.4445	0.2067	0.1025	0	361.7
2	0.6598	0.1392	0.2690	0	529.6
3	0.4445	0.2067	0.1025	0	361.7
4	0.8191	0.2460	0.1533	0	642.1
5	0.3533	0.2130	0.1734	0	307.5
6	0.4445	0.2067	0.1025	0	361.7
7	0.7129	0.2118	0.1819	0	565.4
8	0.8422	0.2428	0.2357	0	669.9
9	0.4927	0.2100	0.1073	0	397.3
10	0.5830	0.2089	0.1521	0	468.0

Table 15. Objective functions.

Cell	1	2	3	4	5
Potential	0.00	0.00	0.00	546.00	0.00

Table 16. The potential of machine type one (for the first solution of grouping machines/assigning parts/cell arrangement).

it is possible to determine which machine type will duplicate one more machine and which cell this machine is grouped in. Duplicate one more machine for the machine type which has the optimum potential to form one more candidate solution (vector). Similarly, the same task can be performed on all other solutions of grouping machines/assigning parts/cell arrangement.

- (6) Based on the choice function, some solutions are selected to form a matching pool. By applying the crossover and mutation functions of GA, more candidate solutions of grouping machines/cell arrangement can be found.
- (7) All similar tasks are performed to (a) assign each operation of parts to an appropriate machine subject to the constraint of machine capacity, (b) remove superfluous machines and (c) calculate the objective function for each result of grouping machines/assigning parts/cell arrangement.
- (8) The fifth solution in table 15 is selected to form the population of solutions because of $\Delta E = 307.5 - 307.5 \leq 0$. However, the first solution is not selected and obtains an acceptable probability $e^{-(361.7-307.5)/100}$. The same task can be performed for each candidate solution. Hence, the population of the pre-determined total number of solutions can be found according to the acceptable probability and the random choice.

- (9) Lower the temperature when the equilibrium is reached. By continuously applying the proposed procedure, a satisfactory solution of grouping machines/assigning parts/cell arrangement will be found.

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