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COMPUTER-AIDED MANUFACTURING OF SPIRAL BEVEL AND HYPOID GEARS WITH MINIMUM SURFACE-DEVIATION

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Abstract—The linear regression method to minimize the deviations of a real cut gear-tooth-surface is investigated in this paper. Based on the Gleason hypoid gear generator, a mathematical model of the tooth surface is proposed. Applying the proposed mathematical model, the sensitivities of tooth surface due to the variations of machine-tool setting are also investigated. The corrective machine-tool settings, calculated by using the sensitivity matrix and the linear regression method, are used to minimize the tooth-surface deviations. The minimization problem was solved by using the singular value decomposition (SVD) method. The result of this paper can improve the conventional development process and be also applied to different manufacturing machines and methods. Two examples are presented to demonstrate the proposed methodology.

INTRODUCTION

Manufacturing spiral bevel and hypoid gears requires high precision and state-of-the-art machinery because such gears have complex tooth-surface geometries. The fundamental geometry and characteristics of spiral bevel and hypoid gears have been investigated by many researchers [1–10]. However, the inherent tooth surface deviations caused by machine-tool setting inaccuracies, machine constant errors, machine flexibility, and other factors are unavoidable in the manufacturing process. Therefore, it is desirable to build up a methodology that can minimize the deviations of a real cut gear-tooth-surface within the permissible tolerance.

In the recent years, several computer-aided inspection systems and closed-loop manufacturing systems that combine CNC coordinate measuring machines with theoretical gear-tooth-surface data, have been developed by Gleason Workers [11], M&M Precision Systems [12], Klingelnberg Soehne [13], and Lemanski [14]. Theoretical gear-tooth-surface data can be obtained from mathematical models of bevel and hypoid gears. Krenzer [15] proposed computer-aided corrective machine-tool settings for manufacturing bevel and hypoid gear sets using first-order and second-order sensitivity matrices. Litvin *et al.* [16, 17] and Zhang and Litvin [18] proposed a series of methodologies to minimize deviations in real cut gear-tooth surfaces and to analyze the meshing and contact of real cut gear-tooth surfaces. In Litvin's model the theoretical tooth-surfaces were derived based on basic machine-tool settings instead of actual machine-tool settings. Therefore, it was necessary to convert the basic machine-tool settings into the actual machine-tool settings and this could affect the precision of the results. Besides, characteristics of gear generators were more difficult to be controlled by using the basic machine-tool settings.

In this paper, the mathematical model of spiral bevel and hypoid gears cut by using Gleason No. 463 hypoid grinder is developed in terms of actual machine-tool settings and machine constants. Since the tooth-surface geometry of spiral bevel and hypoid gears is quite complex and

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sensitive to machine-tool settings, the sensitivity matrix between machine-tool settings and tooth-surface geometry is investigated and calculated. Using the sensitivity analysis technique, characteristics of gear generators can thus be obtained and controlled. In addition, the results of sensitivity analysis can be applied to decide what type of machine-tool setting or combination should be chosen and modified to minimize surface deviations of the real cut gear-tooth surface. The theoretical tooth-surface data can be determined using the proposed gear-set mathematical model and then down-loaded to the CNC coordinate measuring machine. The real cut gear-tooth-surface data can be obtained using CNC coordinate measuring machines to measure the coordinates of sampling points on tooth surfaces. The measured data can then be compared with the theoretical data to calculate gear-tooth surface deviations. Using the measured surface deviations and the sensitivity matrix, the corrective machine-tool settings that minimize the surface deviations within tolerances can be obtained by using the linear regression method. In this paper, the problem on how to solve the corrective machine-tool settings with minimum gear-surface-deviation is accomplished by applying the singular value decomposition (SVD) [19] method.

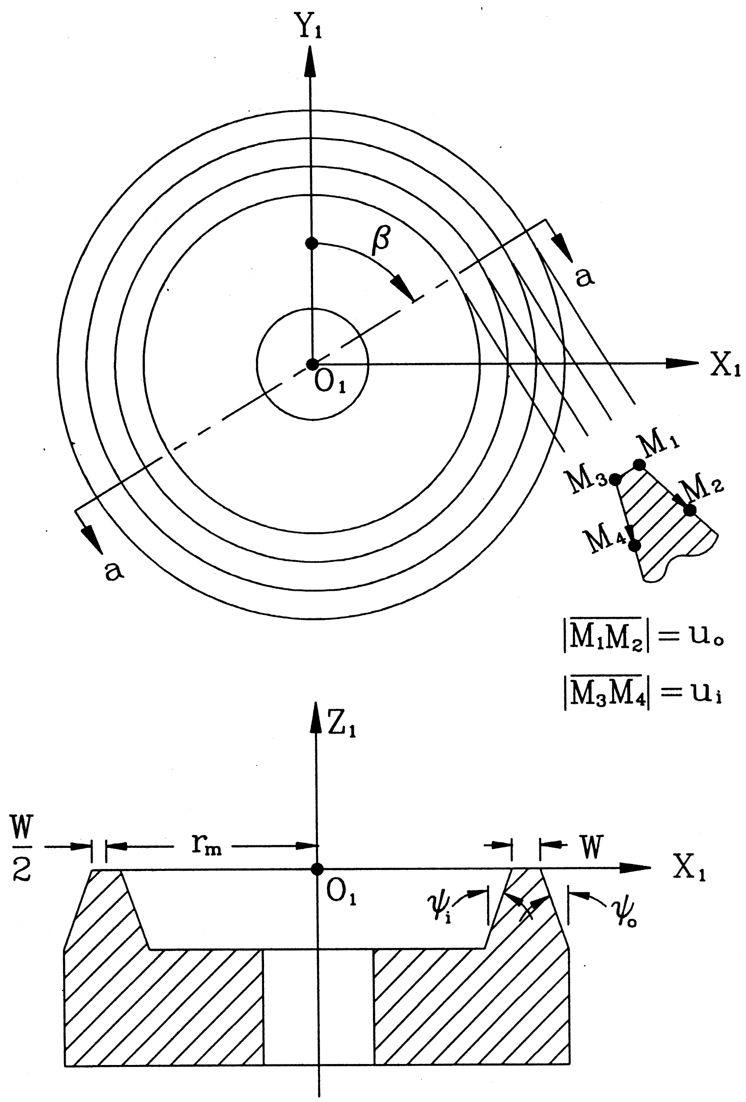

Fig. 1. Coordinate system S_1 and geometry of the cup-shaped grinding wheel.

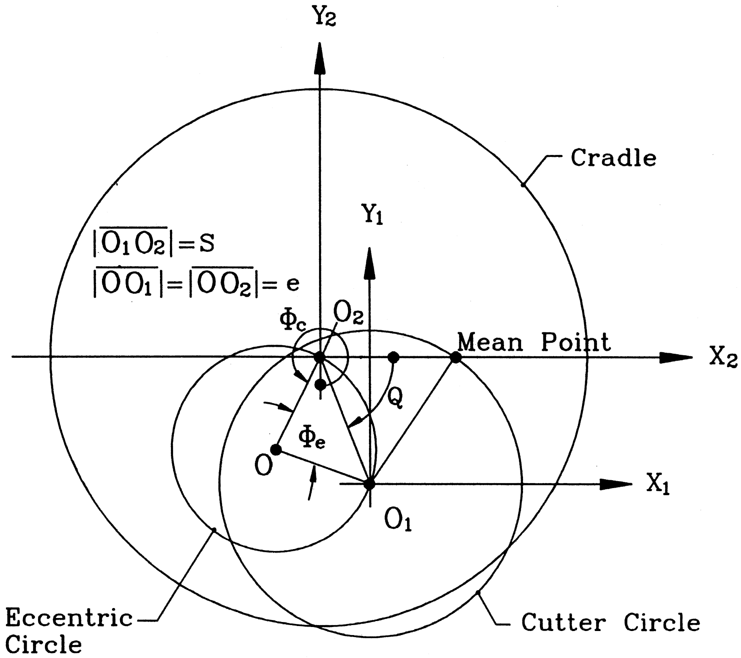

Fig. 2. Relationships among machine settings and basic cradle settings.

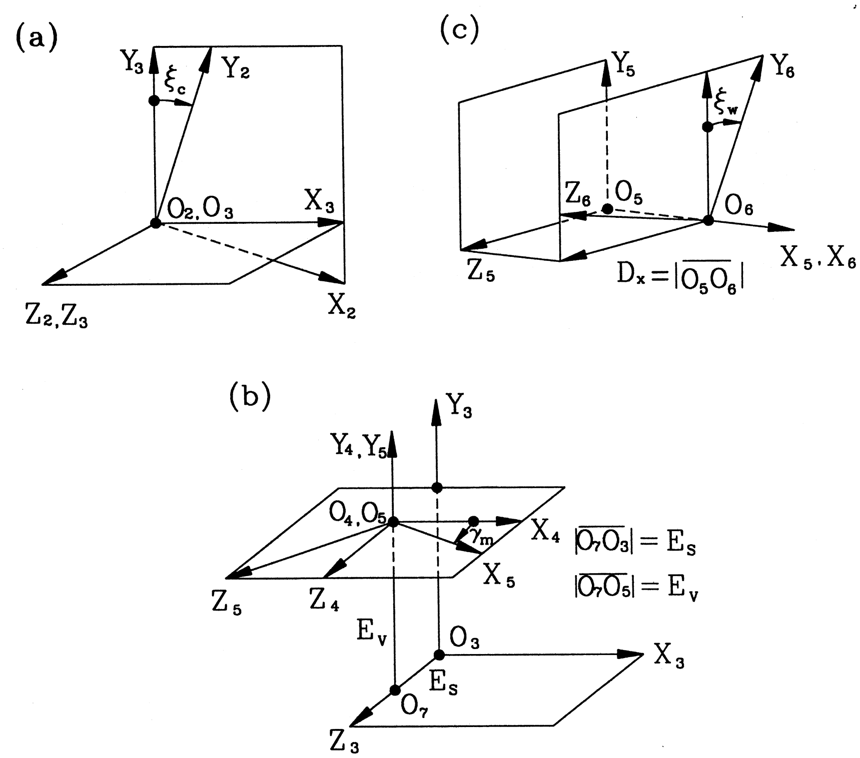

Fig. 3. Coordinate systems for the bevel gear generation mechanism.

According to the proposed approaches, an improved procedure for the development of spiral bevel and hypoid gears is suggested. Two examples are presented to illustrate the sensitivity analysis and the proposed methodology.

MATHEMATICAL MODEL OF THE GLEASON SPIRAL BEVEL AND HYPOID GEARS

In this section, the proposed mathematical model for spiral bevel and hypoid gears is developed based on the mechanism of Gleason No. 463 hypoid grinder. The Gleason No. 463 hypoid grinder is an automatic wet-type, high speed machine for grinding spiral bevel and hypoid gears. The grinding of bevel gears makes the extreme precision and heavy load capacity attainable that is required in modern gear production. The generating train of Gleason No 463 hypoid grinder is designed to perform the Modified Roll Motion by means of a special cam reciprocator mechanism. The mechanism of Gleason No. 463 hypoid grinder can be divided into four major parts: (a) cup-shaped grinding wheel, (b) special Modified Roll generating train, (c) feeding and driving mechanisms, and (d) work head assembly. The detailed description of the mechanism investigated by Lin, *et al.* [20] and the Gleason Works [21] is omitted here. The axial cross-section of the cup-shaped grinding wheel is straight edges in the a-a cross section as shown in Fig. 1, and it can be expressed in coordinate system $S_1(x_1, y_1, z_1)$ as follows:

$$\mathbf{R}_1(u_j, \beta_j) = \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \end{Bmatrix} = \begin{Bmatrix} \left[r_m \pm \left(\frac{W}{2} + u_j \sin\psi_j\right)\right]\sin\beta_j \\ \left[r_m \pm \left(\frac{W}{2} + u_j \sin\psi_j\right)\right]\cos\beta_j \\ -u_j \cos\psi_j \end{Bmatrix}, \tag{1}$$

where $j = i$ and o, and parameters u_i, β_i, u_o and β_o are the head cutter surface coordinates of inside and outside blades, respectively. Subscript "i" indicates the inside blade, and "o" represents the outside blade; the "$\pm$" sign should be regarded as "$+$" sign for the outside blade ($j = o$), and "$-$" sign for the inside blade ($j =$ i).

Coordinate systems of the Gleason No. 463 hypoid grinder mechanism are shown in Figs 2 and 3. The grinding wheel spindle is parallel to the cradle axis in the Modified Roll Method. According to the geometry shown in Fig. 2, the tooth surface of the imaginary generating gear can be represented as follows:

$$Q = 360^\circ - \phi_c + \frac{\phi_e}{2}, \tag{2}$$

$$S = 2e \sin\left(\frac{\phi_e}{2}\right), \tag{3}$$

and

$$\mathbf{R}_2 = \begin{Bmatrix} x_2 \\ y_2 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} S\cos Q + x_1 \\ -S\sin Q + y_1 \\ z_1 \end{Bmatrix}, \tag{4}$$

where ϕ_e is the cradle angle and is one of the machine settings for the Gleason hypoid grinder;

ϕ_e is the eccentric angle and is also a machine setting; e is the machine eccentric constant, and $e = 8$ inches for the Gleason No. 463 hypoid grinder; S, is the basic radial distance setting; Q, is the basic cradle angle setting; and x_1, y_1 and z_1 are the surface coordinates of the cup-shaped grinding wheel represented in Equation (1).

As shown in Fig. 3 [20], coordinate systems $S_2(x_2, y_2, z_2)$, $S_3(x_3, y_3, z_3)$, $S_4(x_4, y_4, z_4)$, $S_5(x_5, y_5, z_5)$, and $S_6(x_6, y_6, z_6)$ are rigidly attached to the cradle, machine frame, sliding base, work head, and workpiece, respectively. Therefore, the locus of the imaginary generating gear represented in the workpiece coordinate system S_6 can be obtained by applying the following coordinate transformation matrix equation:

$$\mathbf{R}_6 = [M_{65}][M_{53}][M_{32}]\mathbf{R}_2, \tag{5}$$

where

$$[M_{65}] = \begin{bmatrix} 1 & 0 & 0 & -D_x \\ 0 & \cos\xi_w & -\sin\xi_w & 0 \\ 0 & \sin\xi_w & \cos\xi_w & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$[M_{53}] = \begin{bmatrix} \cos\gamma_m & 0 & \sin\gamma_m & -E_s\sin\gamma_m \\ 0 & 1 & 0 & -E_v \\ -\sin\gamma_m & 0 & \cos\gamma_m & E_s\cos\gamma_m \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$[M_{32}] = \begin{bmatrix} \cos\xi_c & \sin\xi_c & 0 & 0 \\ -\sin\xi_c & \cos\xi_c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

ξ_c, is the cradle rotational angle; ξ_w, is the work spindle rotational angle; γ_m, is the machine root angle; D_x, is the increment of machine center to back; E_s, is the sliding base setting; and E_v, is the vertical offset setting.

The ratio between the work spindle rotation angle ξ_w and cradle angle ξ_c is called the roll ratio, and is defined as $\eta_a = \xi_w/\xi_c$. For the Gleason No. 463 hypoid grinder, η_a is not a constant and was derived by Lin, *et al.* [20].

Because the two mating surfaces are in tangency, the relative velocity of the mating surfaces lies on the common tangent plane during the generation process, and the following equation must be satisfied [22]:

$$\mathbf{N} \bullet \mathbf{V}_{12} = 0, \tag{6}$$

where $\mathbf{N}$ is the normal vector of the generating tool surface, and $\mathbf{V}_{12}$ is the relative velocity between the generating tool surface and the generated gear blank surface. Equation (6) is the so-called equation of meshing in the theory of gearing. The equation of meshing for spiral bevel and hypoid gears was developed by Fong and Tsay [7], and is given as follows:

$$AA + BB\sin\xi_c + CC\cos\xi_c = 0, \tag{7}$$

where

$$AA = \left(\sin\gamma_m - \frac{1}{\eta_a}\right)(y_2 n_{x2} - x_2 n_{y2}) + n_{z2}E_v\cos\gamma_m,$$

$$BB = -E_v n_{y2}\sin\gamma_m - [n_{x2}(z_2 - E_s) - x_2 n_{z2}]\cos\gamma_m,$$

and

$$CC = -E_v n_{x2}\sin\gamma_m + \left[n_{y2}\left(z_2 - E_s + \frac{L}{2\pi}\xi_c\right) - y_2 n_{c2}\right]\cos\gamma_m.$$

The tooth surface of the generated spiral bevel gear is determined by considering Equations (5) and (7) simultaneously. The unit normal vector $\mathbf{n}_j$ of the generated spiral bevel and hypoid gear tooth surfaces can be obtained by applying the following equation:

$$\mathbf{n}_j = \frac{\mathbf{N}_j}{|\,\mathbf{N}_j\,|}, (j = i, o), \tag{8}$$

where

$$\mathbf{N}_j = \frac{\partial \mathbf{R}_j}{\partial u_j} \times \frac{\partial \mathbf{R}_j}{\partial \beta}, \ (j = i, o),$$

and $\mathbf{R}_j$ represents the position vector of the generated spiral bevel and hypoid gear tooth surfaces, and $\mathbf{N}_j$ indicates its surface normal.

SENSITIVITY ANALYSIS

The constant sensitivity of spiral bevel and hypoid gears with respect to roll test machine settings has been studied by the engineers of Gleason Works [23], Fong and Tsay [9], and Litvin *et al.* [24]. In this paper, the sensitivity of spiral bevel and hypoid gear tooth-surfaces manufactured by the Gleason Modified Roll method, due to machine-tool setting variations is studied by using the proposed mathematical model. Using the sensitivity analysis technique, the characteristics of machine-tool settings for gear generator can be obtained. Based on the repeat characteristic of the same gear generator in manufacturing process, the surface deviation of real cut gear-tooth surfaces can be minimized by choosing appropriate corrective machine-tool settings. The machine-tool settings of the Gleason No. 463 hypoid grinder used in sensitivity analysis include the machine root angle setting γ_m, cradle angle ϕ_c, eccentric angle ϕ_e, sliding base setting E_s, increment of machine center to back D_x, vertical offset E_v, mean grinding wheel diameter D_c, cam setting δ, and cam guide angle φ. The first variation on pinion and gear tooth surfaces due to the variations of machine-tool settings is defined as

$$\delta \mathbf{R}_i = \Sigma \frac{\partial \mathbf{R}_i}{\partial \zeta_j} \delta \zeta_j \tag{9}$$

where $\delta \mathbf{R}_i$ represents the variation on the tooth-surface position vector; parameter $\delta\zeta_j$ indicates the perturbation increment of machine-tool settings.

The perturbation increment of each machine-tool setting should be chosen according to the precision limitation of the Gleason spiral bevel and hypoid gears generator: 0.01 mm for linear displacements, and 1 min for angular displacements. In sensitivity analysis of spiral bevel and hypoid gear-tooth surfaces, $m \times n$ discrete sampling points have been chosen to represent the tooth-surface geometric characteristics, as shown in Fig. 4. The values of m and n depend on tooth-surface geometry, sampling accuracy, machine precision as well as product requirements. The surface sampling points are numbered ascendant from the root to tip, and the heel to toe. The sensitivity coefficient S_{ij} is defined as the surface displacement variation along the normal direction of tooth-surface point due to the perturbation of each machine-tool setting $\delta\zeta_j$. Therefore, the system equation for the variation on the tooth-surface position vector at each sampling point can be rewritten as

$$\{\delta \mathbf{R}_i\} = [S_{ij}]\{\delta\zeta_j\},$$

and

$$\mathbf{S}_{ij} = \frac{\partial \mathbf{R}_i}{\partial \zeta_j} (i = 1, \ldots, p; \quad \text{and} \quad j = 1, \ldots, q), \tag{10}$$

where $p = m \times n$ is the number of sampling points; q is the number of machine-tool settings. The sensitivity matrix $[S_{ij}]$ of spiral bevel and hypoid gears can be used to calculate the corrective machine-tool settings in the gear manufacture process.

Example 1

A spiral bevel gear set cut on Gleason No. 463 hypoid grinder using the Modified Roll method is chosen as an example to investigate the tooth-surface sensitivity due to machine-tool setting variations. The gear blank dimensions and grinding wheel specifications are shown in Table 1 while the machine-tool settings are listed in Table 2.

In this study, 45 surface sampling points have been chosen to represent the pinion and gear tooth-surface geometric characteristics, as shown in Fig. 4. The maximum displacement perturbations in each column of the sensitivity matrix $\{\mathbf{S}_{ij}\}$ for the pinion is calculated by applying

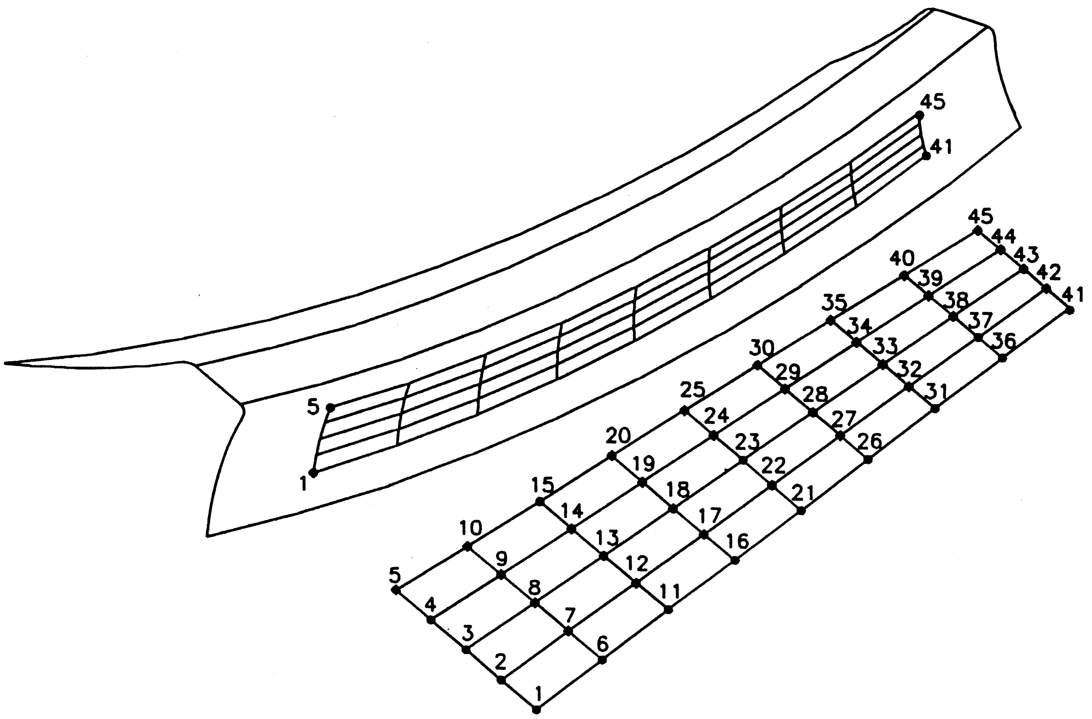

Fig. 4. Sampling surface points on the spiral bevel gear convex side.

Equation (10), and shown in Fig. 5. The abbreviation I.B. denotes the convex side of the pinion tooth-surface which is ground by the inside blade of the grinding wheel while O.B. denotes the concave side of the pinion tooth-surface. It is found that the maximum perturbation displacement is very sensitive to the variations of the machine root angle setting γ_m, eccentric angle ϕ_e, sliding base setting E_s, and cam setting δ. Therefore, when installing the machine-tool settings, these four machine settings should be carefully measured due to their high sensitivities to the real cut tooth-surface geometry.

The pinion surface perturbative diagrams of the machine root angle setting γ_m, eccentric angle ϕ_e, sliding base setting E_s, and cam setting δ are investigated and shown in Figs. 6–9, respectively. The rigid-body-rotation is extracted from the displacement variation by overlapping the sampling surface point No. 1 for each perturbation of machine-tool settings. The vertical displacements in Figs. 6–9 represent the surface perturbation of each sampling point measured along the surface normal direction. The scale is shown at the upper right corner of each diagram. The results of sensitivity analysis can be applied to decide what type of machine-tool setting or com-

Table 1. Gear blank and grinding wheel used in the example

Items	Pinion	Gear
	Blank data	
Number of teeth	27	28
Face width	25.00 mm	25.00 mm
Pitch angle	22° 4′	22° 56′
Outside diameter	128.73 mm	132.91 mm
Pitch apex to crown	148.38 mm	147.45 mm
	Grinding wheel specifications	
Mean diameter of grinding wheel	—	250.00 mm
Point diameter of grinding wheel (I.B.)	250.95 mm	—
Point diameter of grinding wheel (O.B.)	251.46 mm	—
Inside blade angle	21° 0′	21° 0′
Outside blade angle	19° 0′	19° 0′
Point width	—	2.41 mm
Tip fillet	1.09 mm	0.05 mm

Table 2. Machine-tool settings used in the examples

Machine-tool Settings	Pinion I.B.	Pinion O.B.	Gear
Machine root angle	20° 26′	20° 26′	21° 15′
Machine center to back	−9.01 mm	−8.46 mm	−2.40 mm
Sliding base	3.15 mm	2.96 mm	0.87 mm
Vertical offset	−11.64 mm	−11.28 mm	2.96 mm
Cam setting	101.60 mm	101.60 mm	76.20 mm
Eccentric angle	32° 49′	32° 38′	35° 44′
Cradle angle	67° 3′	71° 45′	324° 59′
Cam guide angle	5° 10′	0° 0′	−0° 40′
Feed cam setting	6° 0′	0° 0′	0° 0′
Generating cam no.	#23	#23	#29
Index interval	10	10	9
Index gears	48/80 × 50/81	48/80 × 50/81	42/70 × 45/84

I.B.: Convex side of pinion tooth-surface ground by inside blade of grinding wheel
O.B.: Concave side of pinion tooth-surface ground by outside blade of grinding wheel

bination should be chosen and modified to minimize surface deviations of the real cut gear-tooth surface in the spiral bevel gear development process.

CORRECTIVE MACHINE-TOOL SETTINGS FOR GEAR SET MANUFACTURING

Deviations between the theoretical gear-tooth surfaces and the real cut gear-tooth surfaces may exist for a number of reasons, such as the machine-tool setting inaccuracies, machine constant errors, machine flexibility, and so on. Whatever the reason, the corrective machine-tool settings are required to minimize tooth-surface deviations within a permissible level. Conventionally, the rolling test development was used to obtain the corrective machine-tool settings and compensate the tooth-surface deviations [25]. However, it is a time-consuming and inefficient process for the manufacture development of spiral bevel and hypoid gears.

In this section, a linear regression method that calculates the corrective machine-tool settings to minimize the surface deviations is applied to reduce the shop time during the development process. Using the proposed gear-set mathematical model, theoretical tooth surfaces can be represented by the meshed sampling points, as shown in Fig. 4. The coordinates of the sampling points $\mathbf{R}_i$ are down-loaded to the CNC coordinate measuring machine, then measuring the corresponding sampling points on the real cut gear-tooth surface and recording the measured coordinates $\mathbf{R}_i^*$ on a data diskette. The measurement data are then compared with the theoretical data, and surface deviations for each sampling point measured along its normal direction can be

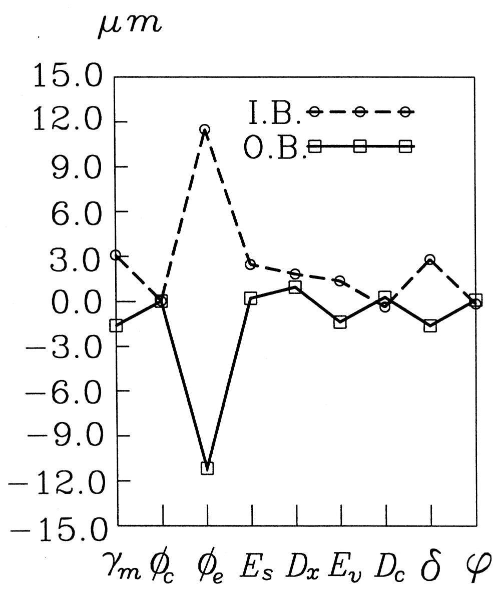

Fig. 5. Pinion surface perturbations due to machine-tool setting variations.

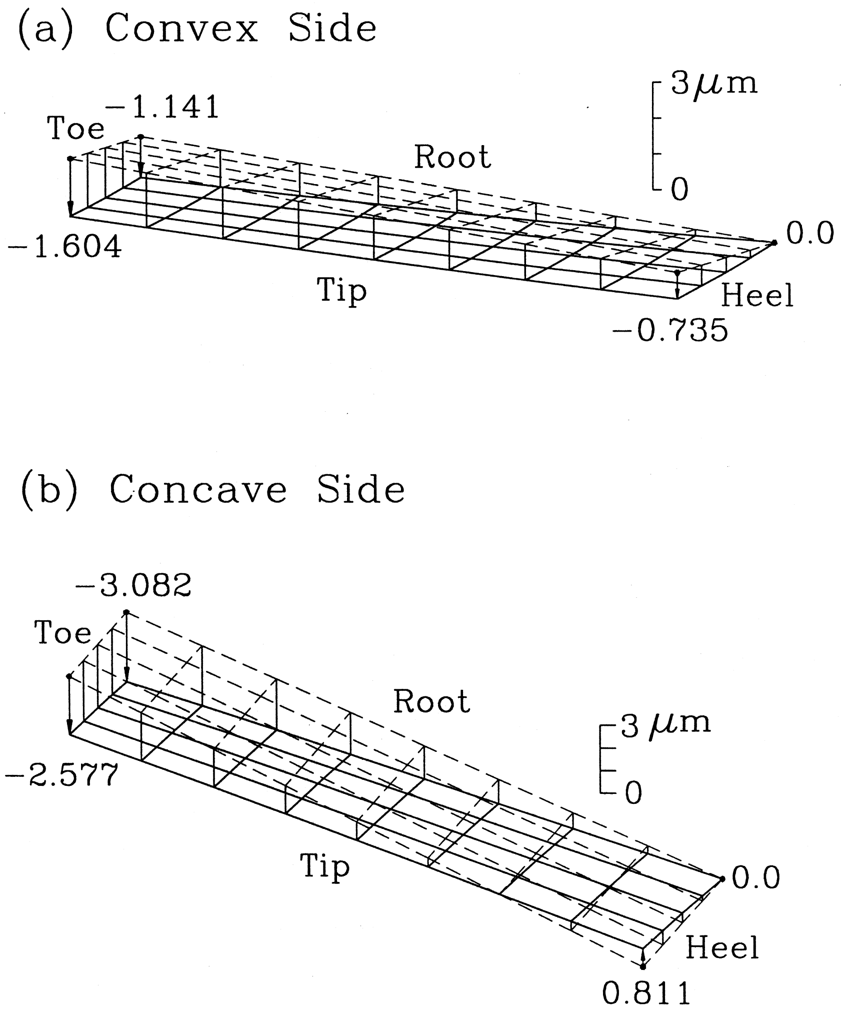

Fig. 6. Surface perturbations on the pinion convex and concave sides due to the variation of machine root angle γ_m.

determined according to the following equation:

$$\Delta R_i = (\mathbf{R}_i - \mathbf{R}_i^*) \cdot \mathbf{n}_i, \tag{11}$$

where subscript i designates the number of sampling points; $\mathbf{R}_i$ and $\mathbf{n}_i$ represent the theoretical position vector and unit normal vector of sampling points on the pinion and gear tooth-surface, respectively.

Based on the calculated surface deviations ΔR_i and the sensitivity matrix $[S_{ij}]$ of the generator, the governing equation used to minimize surface deviations in the real cut pinion and gear tooth-surface can be written as follows:

$$\begin{Bmatrix} \Delta R_1 \\ \Delta R_2 \\ \cdots \\ \cdots \\ \Delta R_p \end{Bmatrix} = \begin{bmatrix} \frac{\partial R_1}{\partial \zeta_1} & \cdots & \frac{\partial R_1}{\partial \zeta_q} \\ \frac{\partial R_2}{\partial \zeta_1} & \cdots & \frac{\partial R_2}{\partial \zeta_q} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \frac{\partial R_p}{\partial \zeta_1} & \cdots & \frac{\partial R_p}{\partial \zeta_q} \end{bmatrix} \begin{Bmatrix} \Delta \zeta_1 \\ \Delta \zeta_2 \\ \cdots \\ \cdots \\ \Delta \zeta_q \end{Bmatrix}, \tag{12}$$

or

$$\{\Delta R_i\} = [S_{ij}]\{\Delta \zeta_j\}(i = 1, \ldots, p; \quad \text{and} \quad j = 1, \ldots, q), \tag{13}$$

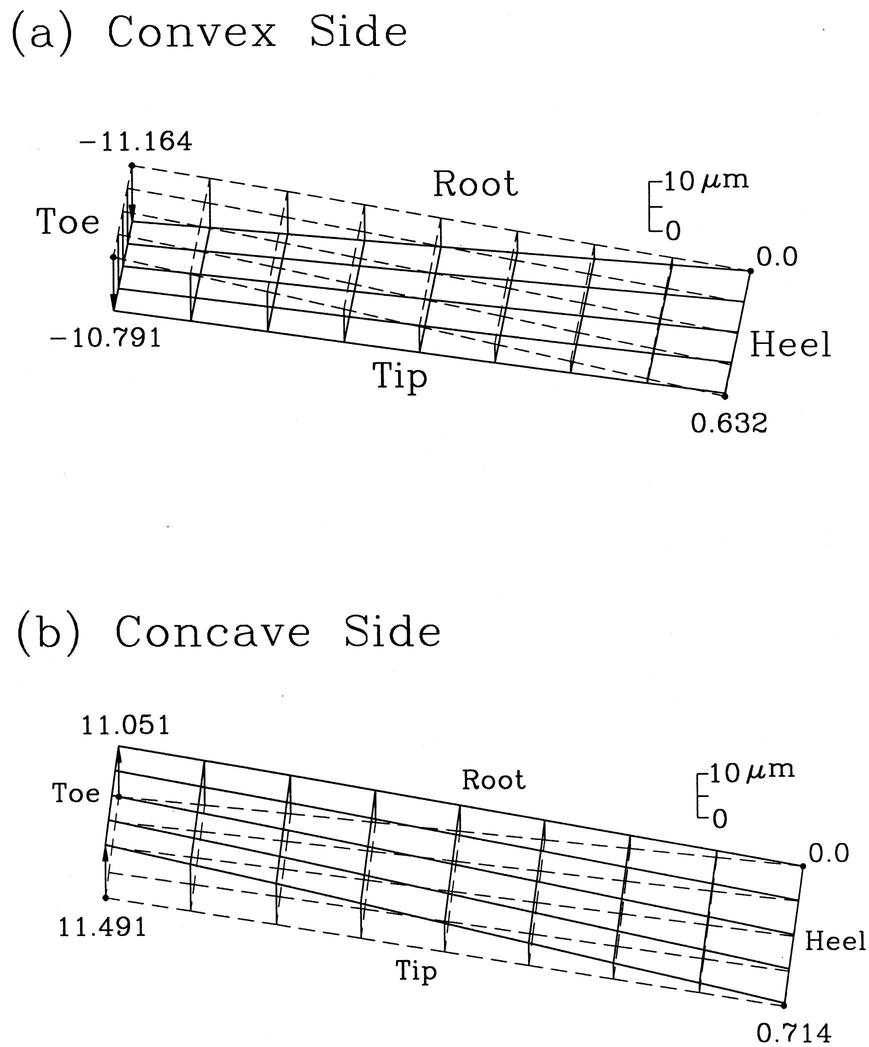

Fig. 7. Surface perturbations on the pinion convex and concave sides due to the variation of eccentric angle ϕ_e.

where $\{\Delta R_i\}$ represents the surface deviation of sampling points, $[S_{ij}]$ is the sensitivity matrix of partial derivatives $\frac{\partial R_i}{\partial \zeta_1}$, and $\{\Delta \zeta_j\}$ represents the corrective machine-tool settings.

The system Equation (12) is overdetermined since $p \gg q$. An overdetermined system equations will generally not have a solution. For that reason, we seek a solution $\Delta \zeta_j^*$ that possesses the closest possible solution in the least square sense. In other words, we solve Equation (12) and find

$$\Delta \zeta_j^* \text{ which minimizes } \| [S_{ij}][\Delta \zeta_j^*] - [\Delta R_i] \|_2 . \tag{14}$$

The problem was solved by the singular value decomposition (SVD) method because of the SVD dealing with system equations that are either singular or else numerically very close to singular. The SVD method will not only diagnose the problem, it will also solve the problem and give a useful numerical solution. The details on how to derive and solve equations are discussed by Press *et al.* [19] and are omitted here.

Example 2

The spiral bevel gear set cut by Gleason No. 463 hypoid grinder using the Modified Roll method is used as an example to demonstrate the proposed development procedure. The initial

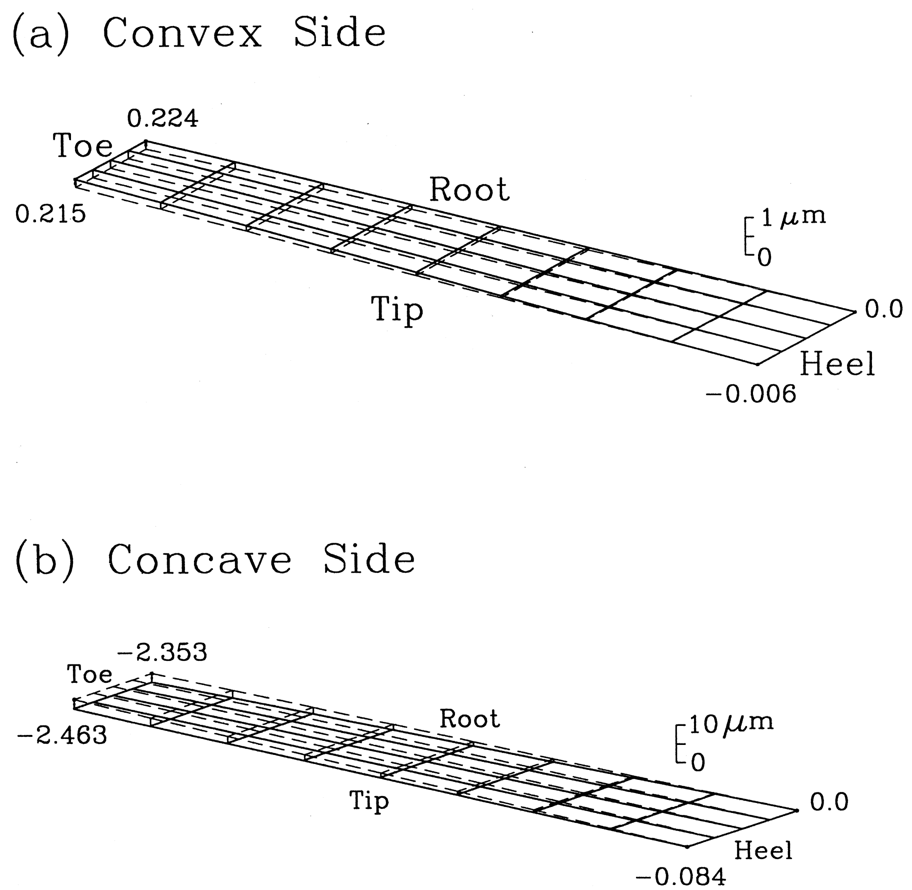

Fig. 8. Surface perturbations on the pinion convex and concave sides due to the variation of sliding base setting E_s.

design values for this example are the same as those listed in Tables 1 and 2. The gear was finished using the duplex grind (i.e. Spread blade) process while the pinion was ground side-by-side using different machine-tool settings (i.e. Fixed settings).

Again, 45 surface sampling points have been chosen to represent the pinion and gear tooth-surface geometric characteristics. The sample gear was cut using the primary machine-tool settings shown in Table 2 and the coordinates of the tooth-surface sampling points on the real cut gear-tooth surfaces were measured using CNC coordinate measuring machines. The measured data were then compared with the theoretical data obtained from the proposed gear-tooth mathematical model. For considerations of precision and minimization of runout errors, four actual teeth were measured and the average measurement values were taken as the actual surface data. Surface deviations at each sampling point on the real cut gear surface are shown in Fig. 10. The maximum deviation on the real cut gear-tooth surface was 0.0045 mm, and occurred at sampling point A1 on the convex side; the tooth thickness deviation at the basic reference point E3 was 0.1578 mm. Using the proposed development procedure and the developed computer simulation programs, modifications or changes in machine-tool settings were calculated, and are listed in Table 3. Based on these corrective machine-tool settings, a spiral bevel gear was cut using the same Gleason No. 463 hypoid grinder. The surface deviations at sampling points on the real cut gear-tooth surface are shown in Fig. 11. The tooth thickness deviation at the basic reference point E3 was reduced to 0.0231 mm, and the maximum surface deviation of about 0.005 mm was occurred at sampling point I5 on the convex side. These surface deviations are within the permissible tolerances and no further gear development is required.

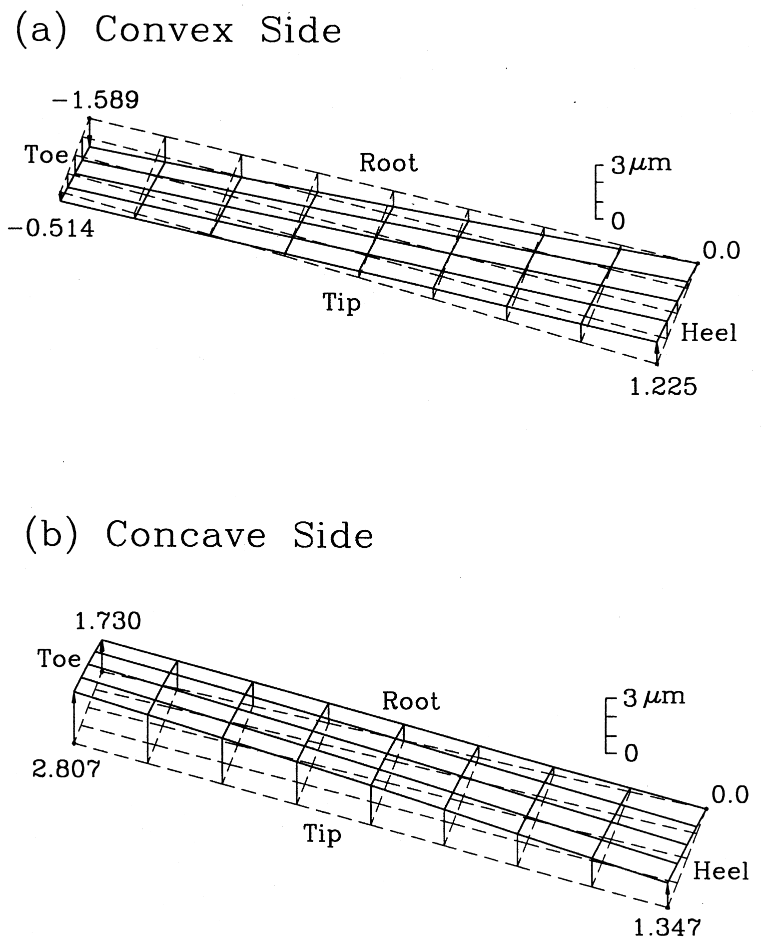

Fig. 9. Surface perturbations on the pinion convex and concave sides due to the variation of cam setting δ.

On the other hand, the pinion was cut according to the primary machine-tool settings shown in Table 2 and pinion-surface coordinates of the sampling points were also measured. Since the convex and concave pinion sides were ground side-by-side with different machine-tool settings. The surface deviations at sampling points on the convex and concave sides of the real cut pinion-surfaces are shown in Figs. 12 and 13, respectively. The maximum surface deviations were 0.033 mm on the convex side and 0.081 mm on the concave side, respectively. Using the proposed development procedure and the calculated machine-tool setting changes shown in Table 3, a pinion was cut and its surface sampling point deviations are shown in Fig. 14 and Fig. 15. The maximum surface deviations were substantially reduced to 0.013 mm on the convex

Table 3. Machine-tool setting changes

Items	Pinion I.B.	Pinion O.B.	Gear
Machine root angle	0° 26′	–0° 23′	–0° 15′
Machine center to back	–0.42 mm	0.16 mm	—
Sliding base	—	—	0.75 mm
Vertical offset	–0.59 mm	–0.78 mm	—
Cam setting	0.24 mm	0.24 mm	–0.27 mm
Eccentric angle	—	—	0° 9′
Cam guide angle	1° 16′	–1° 35′	3° 13′

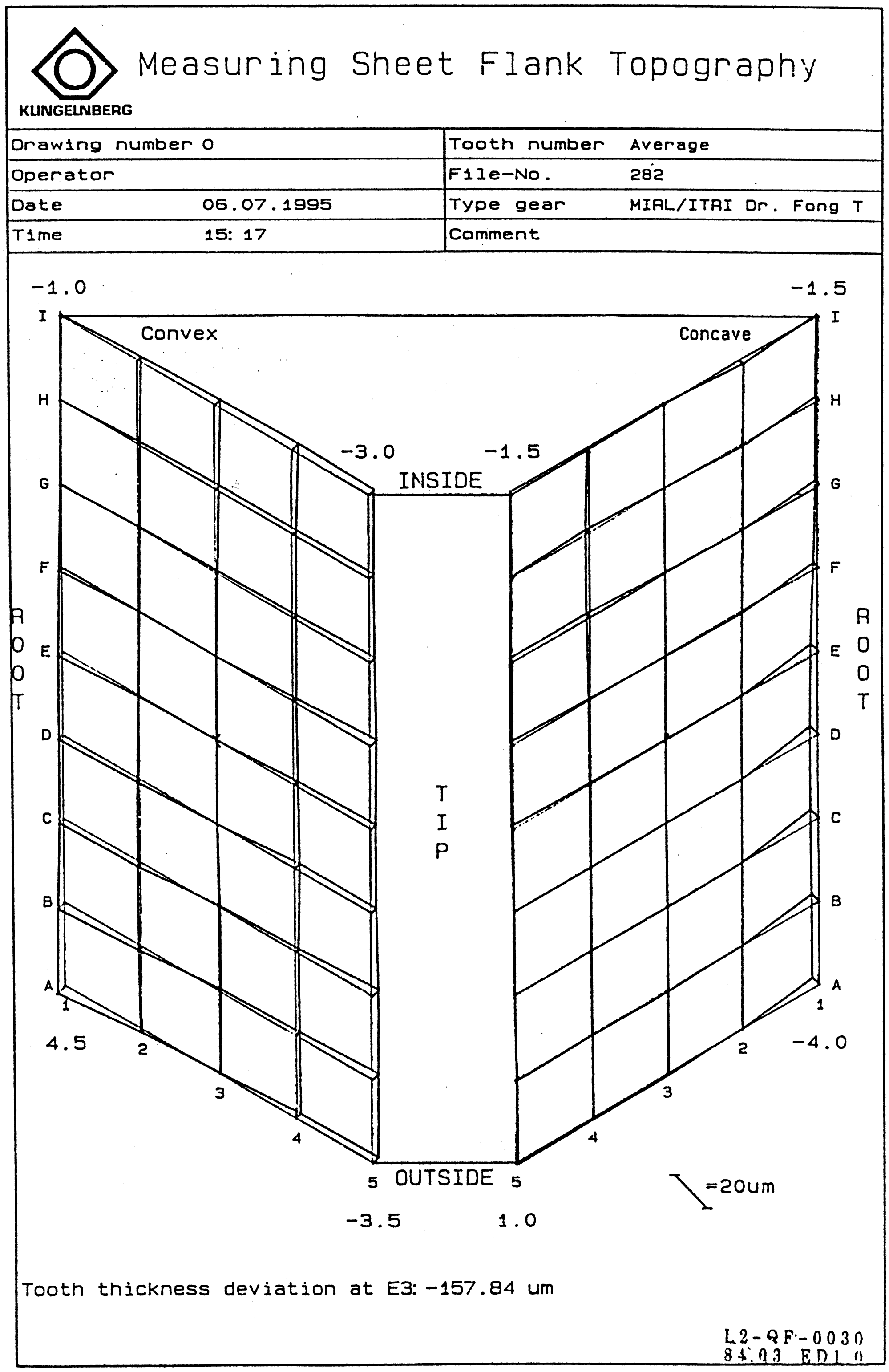

Fig. 10. Gear surface deviations cut by using primary machine settings.

side and 0.023 mm on the concave side, respectively. Therefore, the proposed method for obtaining corrective machine-tool settings to minimize surface deviations of the real cut pinion and gear has proved to be very useful. This indicates that the sensitivity analysis and linear regression method were successfully applied in the proposed methodology.

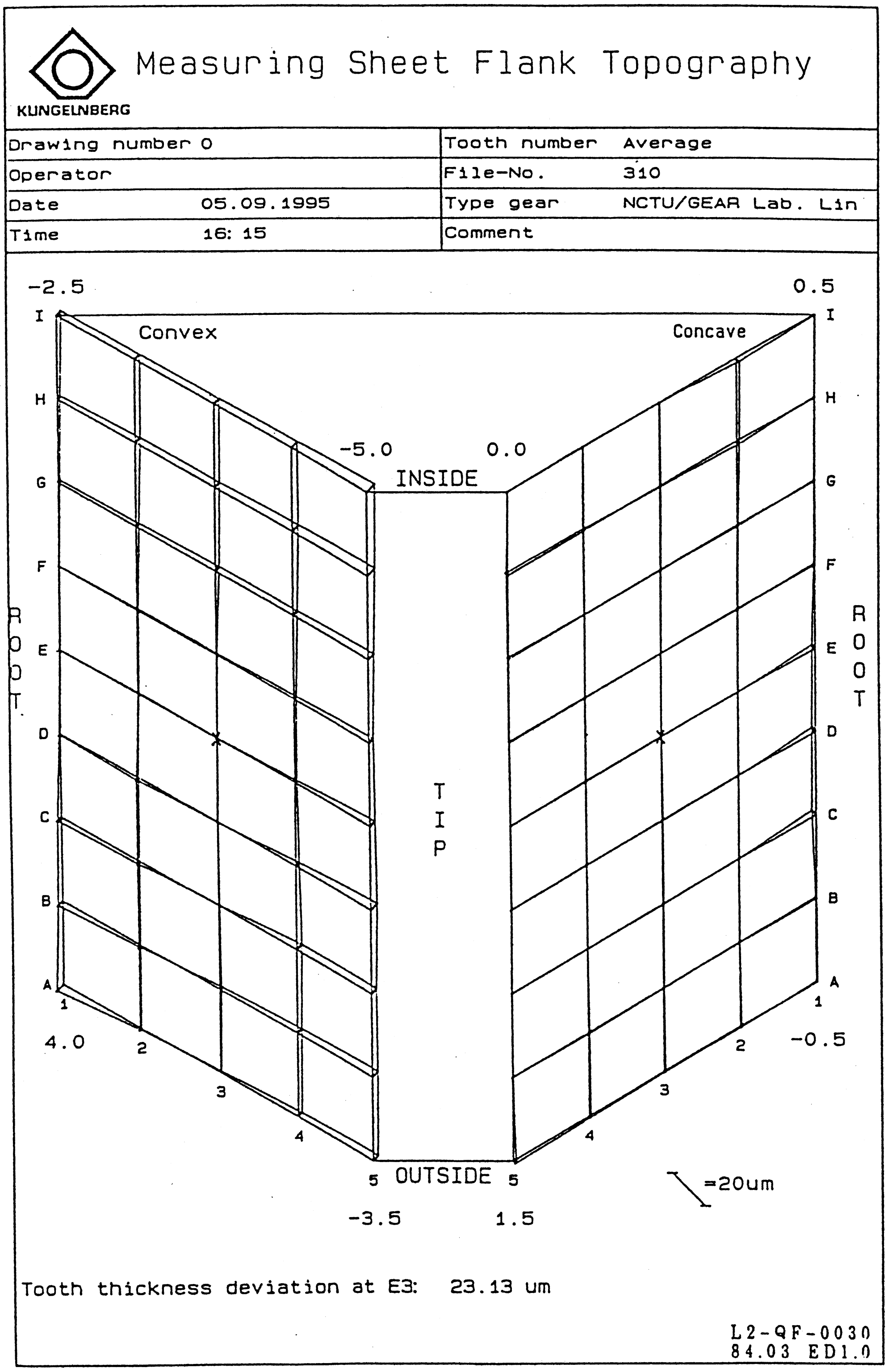

Fig. 11. Gear surface deviations cut by using corrective machine-tool settings.

CONCLUSION

By applying the developed mathematical model of Gleason spiral bevel gear generator, the sensitivity of tooth surfaces due to manufacture errors has been investigated. Based on the results of sensitivity analysis and using a CNC coordinate measurement machine, the linear re-

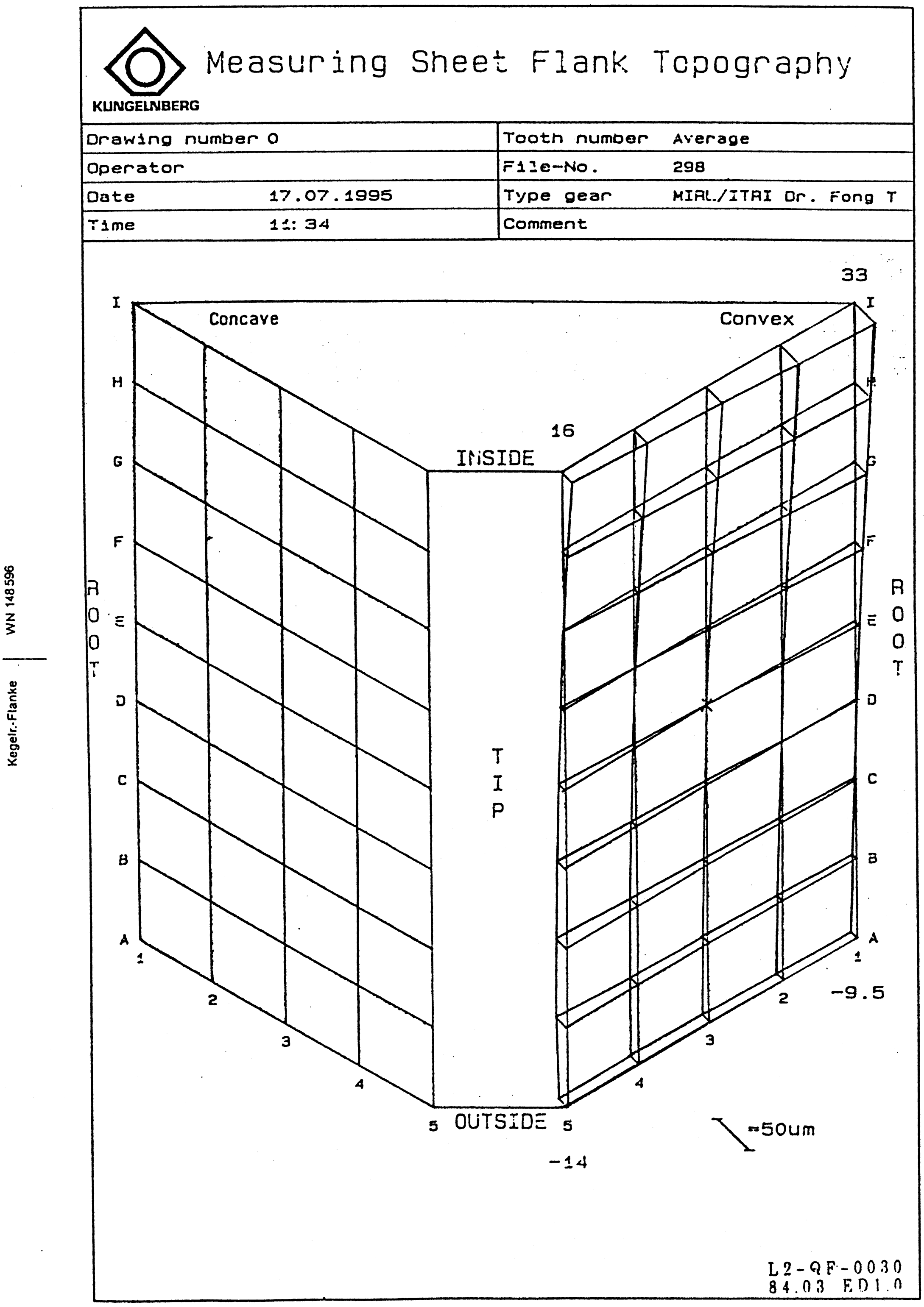

Fig. 12. Pinion surface deviations on convex side cut by using primary machine-tool settings.

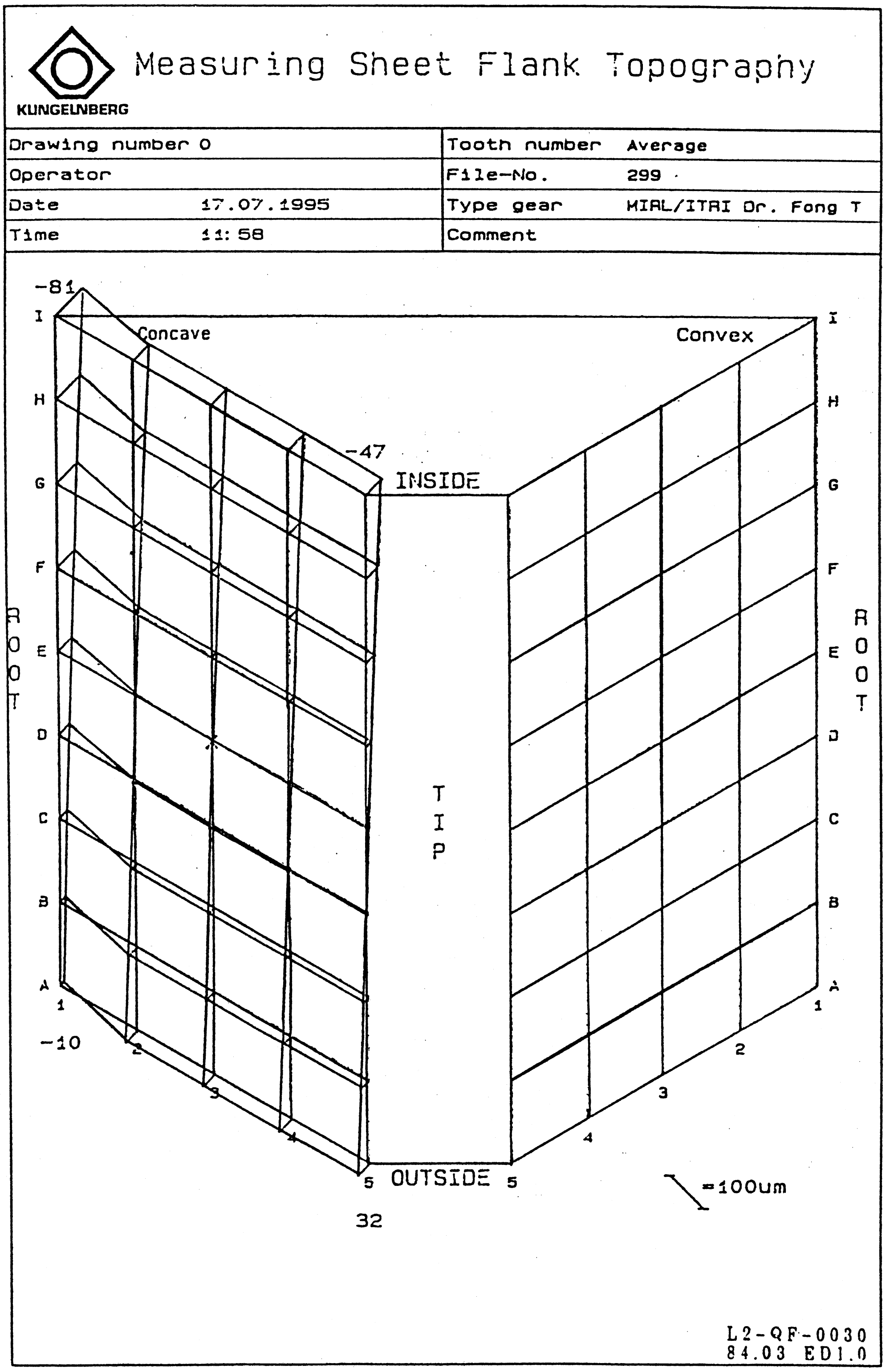

Fig. 13. Pinion surface deviations on concave side cut by using primary machine-tool settings.

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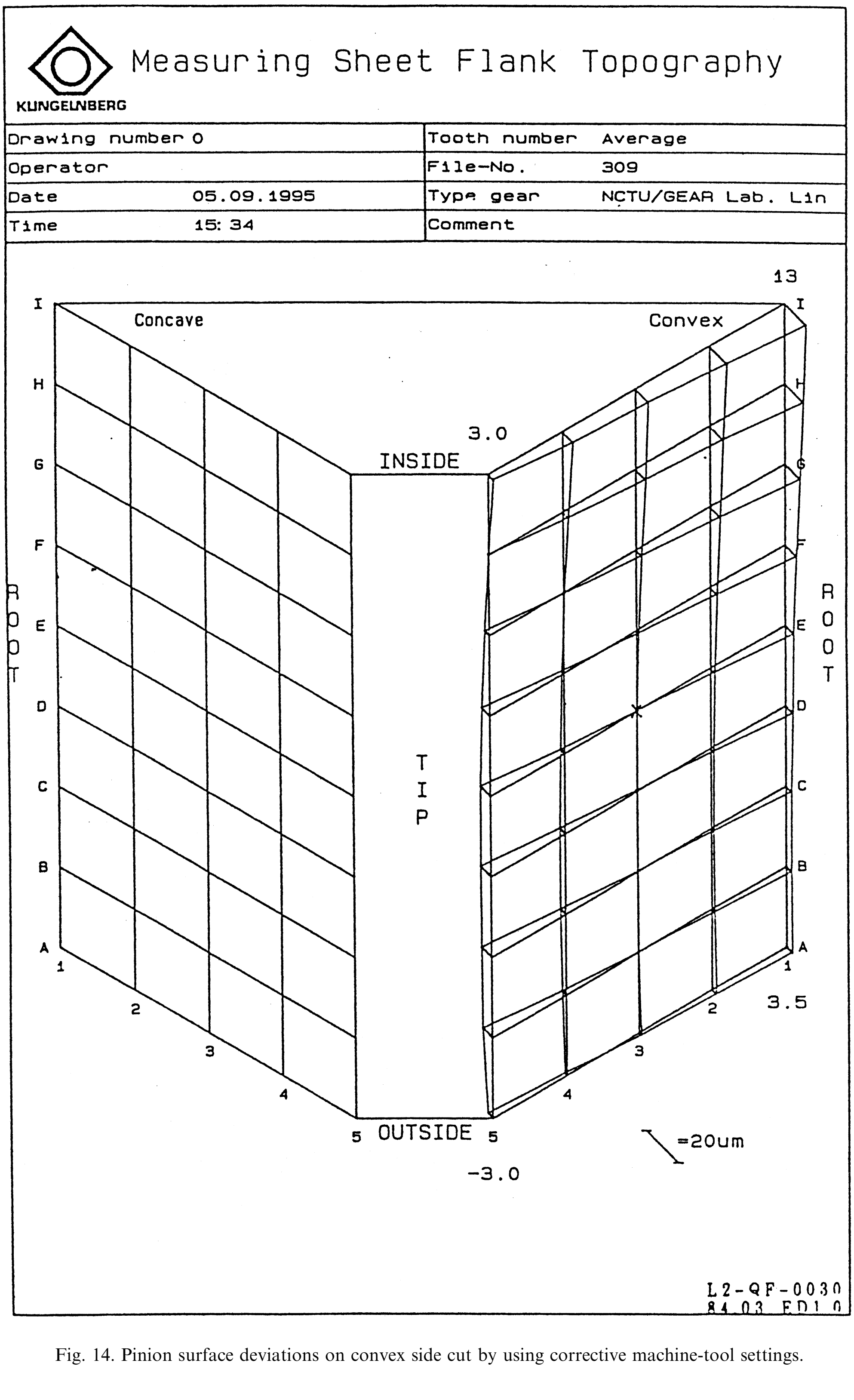

Fig. 14. Pinion surface deviations on convex side cut by using corrective machine-tool settings.

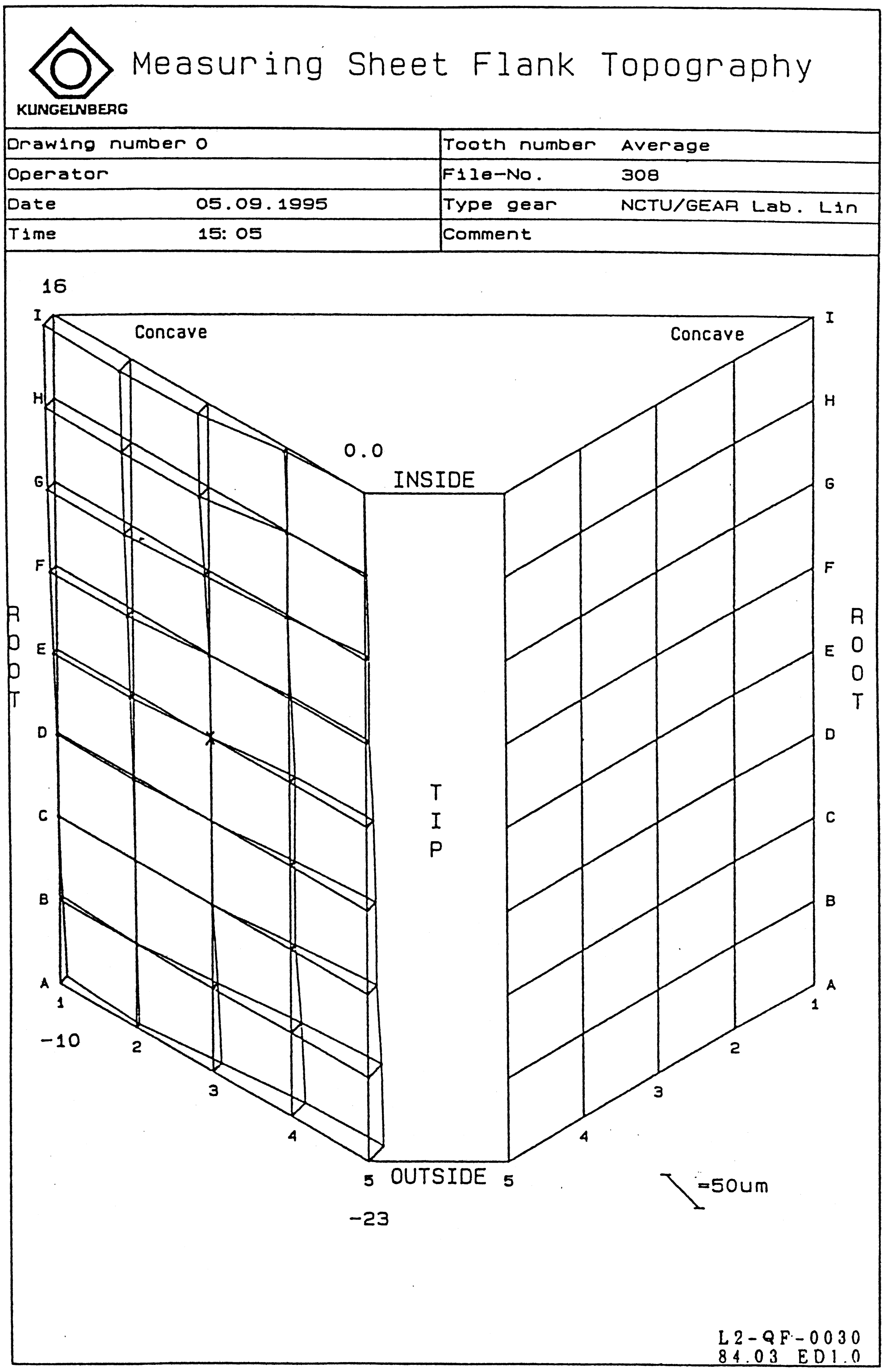

Fig. 15. Pinion surface deviations on concave side cut by using corrective machine-tool settings.

gression method for corrective machine-tool settings calculation that minimize deviations on the real cut gear and pinion tooth-surface within the permissible tolerances has been developed. The results of this paper can be applied to improve the quality and quantity controls of the manufacture of spiral bevel and hypoid gears.

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