Throughput Performance of a Class of Continuous ARQ Strategies for Burst-Error Channels

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***Abstract*—This paper studies the throughput performance of a class of continuous automatic repeat request (ARQ) strategies for burst-error channels modeled by two-state Markov chains. The operation of the investigated ARQ strategies can be described as follows. A chunk of m or fewer copies of each data block is transmitted contiguously to the receiver. Whenever a repeat request is received, the transmitter goes back to and retransmits that data block and all successive ones. However, the maximal number of copies transmitted is equal to n in each retransmission. It is proved that the optimal repetition sequence which maximizes the throughput efficiency among a more general set of ARQ schemes falls in the class of investigated strategies. Numerical results reveal that throughput efficiency is very likely to be maximized at $m = n$.**

I. Introduction

Automatic repeat request (ARQ) is a common technique adopted to handle transmission errors which occur inevitably because of the existence of channel noise. Many researchers have devoted much effort to the design and analysis of various ARQ strategies (see, for example [1]–[5]). However, most of previous results regarding the performance of ARQ strategies were based on the independent error model. This model becomes unrealistic when transmission errors occur in bursts. Instead, Markov models [8], [9] are usually used to describe the behavior of such burst-error channels.

It is the purpose of this paper to analyze the throughput efficiency of a class of continuous ARQ strategies for burst-error channels modeled by two-state Markov chains. We assume all the data blocks are of fixed length, and time is divided into slots so that the duration of each slot is equal to the transmission time of a data block. The round-trip delay r, which is defined as the time interval between the end of a transmission and the receipt of its response, is assumed to be fixed and is equal to an integral number of slots. For simplicity, the feedback channel is assumed to be noiseless.

The Markov channel model studied here consists of one quiet state and one noisy state. In each slot, the channel is either in the quiet state or the noisy state. The transmission is a success (failure) with probability one if the channel is in the quiet (noisy) state. Let X_i denote the channel state in the ith slot so that $X_i = 0$ or 1 means the channel is in the quiet state or the noisy state, respectively. Then the channel model can be described as $\Pr[X_{i+1} = 0|X_i = 0] = p$ and $\Pr[X_{i+1} = 1|X_i = 1] = q$. The same model was studied in [9] to evaluate the throughput performance of the classic go-back-N (GBN) ARQ scheme. It was found that the classic GBN ARQ scheme is more efficient for a Markov system than a system under the independent error model if and only if $p + q \geq 1$. Similar work was done by Towsley [8].

The operation of the investigated ARQ strategies is described in Section II. Analysis of the limiting throughput efficiency of the investigated ARQ strategies is presented in Section III. Illustrative examples are studied and the results are discussed in Section IV. Finally, some conclusions are drawn in Section V.

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II. Investigated ARQ Strategies

The operation of the investigated ARQ strategies can be described as follows. A chunk of m ($m \geq 1$) of fewer copies of each data block are transmitted contiguously to the receiver. An error detection procedure is performed at the receiver on each received copy. A positive (ACK) or a negative (NAK) acknowledgment is sent to the transmitter according to whether the copy is received successfully or erroneously. The data block is considered to be successfully delivered as long as at least one of the copies is correctly received. If all the m copies of a data block are negatively acknowledged, then, just as in the classic GBN ARQ strategy, the transmitter goes back to and retransmits that data block and all successive ones. However, the maximal number of copies transmitted is equal to n in each retransmission. It is noted that successive data blocks are considered as new ones, and hence at most m copies are transmitted after retransmission of the negatively acknowledged data block. In general, n could be different from m.

Obviously, if $m > r+1$, then an ACK for a data block may arrive at the transmitter before it exhausts all the m copies. Whenever this occurs, the transmitter will start transmitting the next data block rather than continuing with the transmission of the remaining copies. Therefore, the phrase "or fewer" is used in the description of the operation of the strategies. Fig. 1 illustrates an example for $m = 4$, $n = 2$, and $r = 2$. A similar situation may occur during retransmissions if $n > r + 1$.

We now explain why we investigate the class of ARQ strategies with the same maximal number of copies in each retransmission. Consider the more general set of continuous ARQ strategies studied in [5]. A chunk of m_0 or fewer copies is transmitted contiguously to the receiver for each data block. If a repeat request is received, then at most m_i copies are transmitted for the ith retransmission. Let $f(m_0, m_1, m_2, m_3, \cdots)$

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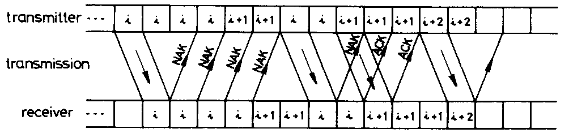


Fig. 1. An example of transmission for $m = 4$, $n = 2$, and $r = 2$.

denote the average number of slots spent to successfully deliver a data block. Consider the transmission of a particular data block, and let x denote the probability of failure for the first copy of the data block. It can be shown that

$$f(m_0, m_1, m_2, m_3, \cdots) = d(m_0; x) + xq^{m_0-1} \cdot \left[r + h(m_1, m_2, m_3, \cdots)\right] \quad (1)$$

where (2) is true (shown at the bottom of the page), and $h(m_1, m_2, m_3, \cdots)$ represents the average number of slots spent in retransmissions. It is not hard to see that $h(m_1, m_2, m_3, \cdots)$ can be expressed as

$$h(m_1, m_2, m_3, \cdots) = d(m_1; y) + yq^{m_1-1}\left[r + h(m_2, m_3, \cdots)\right] \quad (3)$$

where y denotes the probability of failure for the first copy in each retransmission. Notice that this probability is the same for each retransmission under the given Markov channel model. Let $(m_0^*, m_1^*, m_2^*, m_3^*, \cdots)$ denote the optimal repetition sequence which minimizes $f(m_0, m_1, m_2, m_3, \cdots)$, i.e.,

$$f(m_0^*, m_1^*, m_2^*, m_3^*, \cdots) \le f(m_0, m_1, m_2, m_3, \cdots) \quad \text{for all choices of } (m_0, m_1, m_2, m_3, \cdots).$$

In particular, we have $f(m_0^*, m_1^*, m_2^*, m_3^*, \cdots) \le f(m_0^*, m_1, m_2, m_3, \cdots)$ which implies $h(m_1^*, m_2^*, m_3^*, \cdots) \le h(m_1, m_2, m_3, \cdots)$ for all choices of $(m_1, m_2, m_3, \cdots)$. If $m_0 = m_0^*$ and $m_1 = m_1^*$, then, by (2) and (3), we get $h(m_2^*, m_3^*, \cdots) \le h(m_2, m_3, \cdots)$ for all choices of $(m_2, m_3, \cdots)$. Consequently, one can select $m_1^* = m_2^* = m_3^* = \cdots$ to minimize $f(m_0, m_1, m_2, m_3, \cdots)$. In other words, the optimal repetition sequence that maximizes the throughput efficiency among the more general set of strategies falls in the class of strategies we investigate here.

III. Throughput Efficiency

Let $T = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$ denote the transition matrix of the channel state. It can be shown [12] that the k-step transition matrix is given by

$$T^k = \frac{1}{2-(p+q)}\begin{bmatrix} 1-q & 1-p \\ 1-q & 1-p \end{bmatrix} + \frac{(p+q-1)^k}{2-(p+q)}\begin{bmatrix} 1-p & p-1 \\ q-1 & 1-q \end{bmatrix}.$$

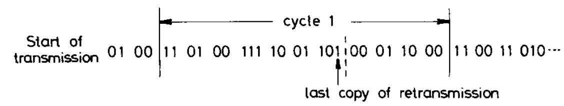


Fig. 2. A typical transmission sequence for $m = 2$, $n = 3$, and $r = 4$.

For the remainder of this paper, we will use $t_{ij}^k (i, j = 0, 1)$ to denote the (i, j)th entry of T^k.

Notice that, under the investigated model, the channel state variable X_i can also be used as the outcome of the ith transmission so that $X_i = 0$ or 1 means the transmission is a success or a failure, respectively. As a result, $\{X_i\}_{i=1}^{\infty}$ represents the transmission sequence. Let us now consider a particular pair of (m, n). The transmission sequence can be divided into cycles so that each cycle starts with m consecutive 1's, i.e., an unsuccessful transmission, of the first outstanding data block (FODB). A data block becomes the FODB if at the time of its transmission, all data blocks transmitted before it will never be retransmitted. Fig. 2 shows a typical transmission sequence from $m = 2$, $n = 3$, and $r = 4$. For clarity, spaces are inserted in Fig. 2. The limiting throughput efficiency of the investigated strategy, denoted by $\eta(m, n)$, is defined as

$$\eta(m, n) = \lim_{k\to\infty} \frac{\alpha_k(m, n)}{k}$$

where $\alpha_k(m, n)$ denotes the average number of data blocks successfully delivered in slots 1–k. By regenerative theorems [12], $\eta(m, n)$ can be computed by $\eta(m, n) = M(m, n)/L(m, n)$, where $L(m, n)$ and $M(m, n)$ represent, respectively, the average length of a cycle and the average number of data blocks successfully transmitted in a cycle.

Now consider a particular cycle. For convenience, we will use X_i to denote the outcome of the transmission in the ith slot of the cycle. Furthermore, let X_R denote the outcome of the last copy of retransmission (in Fig. 2, $R = 16$ and $X_R = 1$). To evaluate $\eta(m, n)$, we need to define the following variables.

- $L_S(m, n)$ and $L_f(m, n)$: the average length of a cycle minus $m + r$ conditioning on $X_{m+r} = 0$ or 1, respectively;
- $L_{sn}(m)$ and $L_{fn}(m)$: the average length in a cycle contributed by data blocks other than the retransmitted one conditioning on $X_R = 0$ or 1, respectively;
- $M_S(m, n)$ and $M_f(m, n)$: the average number of data blocks successfully transmitted in a cycle conditioning on $X_{m+r} = 0$ or 1, respectively;
- $M_{sn}(m)$ and $M_{fn}(m)$: the average number of data blocks successfully delivered in a cycle, excluding the retransmitted one, conditioning on $X_R = 0$ or 1, respectively.

$$d(m_0; x) = \begin{cases} m_0, & m_0 \le r+1 \\ (1-x)(1+r) + x(1-q)\sum\limits_{i=0}^{m_0-r-2} q^i(r+2+i) + m_0 x q^{m_0-r-1}, & m_0 > r+1 \end{cases} \quad (2)$$

$$a_S: \ \Pr(X_R = 0 | X_{m+r} = 0)$$
$$b_S: \ \Pr(X_R = 0 | X_{m+r} = 1)$$

It is not hard to see that $L(m,n)$ and $M(m,n)$ can be evaluated by

$$L(m,n) = m + r + t_{10}^r L_S(m,n) + t_{11}^r L_f(m,n) \quad (4)$$
$$M(m,n) = t_{10}^r M_S(m,n) + t_{11}^r M_f(m,n). \quad (5)$$

Moreover, $M_S(m,n)$ and $M_f(m,n)$ are given by

$$M_s(m,n) = 1 + a_s M_{sn}(m) + (1 - a_S) M_{fn}(m) \quad (6)$$
$$M_f(m,n) = 1 + b_s M_{sn}(m) + (1 - b_s) M_{fn}(m). \quad (7)$$

Therefore, the remaining work is to compute the values of $L_S(m,n)$, $L_f(m,n)$, $M_{sn}(m)$, $M_{fn}(m)$, a_s, and b_s. These values have to be computed using different equations according to the relationship among m, n, and r. For example, consider the case when $m \le r+1$ and $n \le r+1$. For this case, all the m copies in each transmission (or n copies in each retransmission) have to be sent before the transmitter receives any response. Therefore, a recursive formula for $L_S(m,n)$ can be obtained as

$$\begin{aligned} L_S(m,n) &= p\left[n + t_{00}^{n-1} L_{sn}(m) + t_{01}^{n-1} L_{fn}(m)\right] \\ &\quad + (1-p)(1-q)\left[n + t_{00}^{n-2} L_{sn}(m) + t_{01}^{n-2} L_{fn}(m)\right] \\ &\quad + (1-p)q(1-q)\left[n + t_{00}^{n-3} L_{sn}(m) + t_{01}^{n-3} L_{fn}(m)\right] \\ &\quad + \cdots \\ &\quad + (1-p)q^{n-3}(1-q)[n + t_{00} L_{sn}(m) + t_{01} L_{fn}(m)] \\ &\quad + (1-p)q^{n-2}(1-q)[n + L_{sn}(m)] \\ &\quad + (1-p)q^{n-1}[n + r + t_{10}^r L_s(m,n) + t_{11}^r L_f(m,n)]. \end{aligned} \quad (8)$$

Similarly, a recursive formula for $L_f(m,n)$ can be obtained as

$$\begin{aligned} L_f(m,n) &= (1-q)\left[n + t_{00}^{n-1} L_{sn}(m) + t_{01}^{n-1} L_{fn}(m)\right] \\ &\quad + q(1-q)\left[n + t_{00}^{n-2} L_{sn}(m) + t_{01}^{n-2} L_{fn}(m)\right] \\ &\quad + q^2(1-q)\left[n + t_{00}^{n-3} L_{sn}(m) + t_{01}^{n-3} L_{fn}(m)\right] \\ &\quad + \cdots \\ &\quad + q^{n-2}(1-q)\left[n + t_{00} L_{sn}(m) + t_{01} L_{fn}(m)\right] \\ &\quad + q^{n-1}(1-q)[n + L_{sn}(m)] \\ &\quad + q^n[n + r + t_{10}^r L_s(m,n) + t_{11} L_f(m,n)]. \end{aligned} \quad (9)$$

After some algebraic manipulations, we get

$$\begin{aligned} &a_{11} L_s(m,n) + a_{12} L_f(m,n) \\ &= \left[p t_{00}^{n-1} + (1-p)(1-q) \sum_{i=0}^{n-2} q^i t_{00}^{n-2-i}\right] L_{sn}(m) \\ &\quad + \left[p t_{01}^{n-1} + (1-p)(1-q) \sum_{i=0}^{n-3} q^i t_{01}^{n-2-i}\right] L_{fn}(m) \\ &\quad + n + (1-p)q^{n-1} r \end{aligned} \quad (10)$$

and

$$\begin{aligned} &a_{11} L_s(m,n) + a_{12} L_f(m,n) \\ &= \left[(1-q) t_{00}^{n-1} + q(1-q) \sum_{i=0}^{n-2} q^i t_{00}^{n-2-i}\right] L_{sn}(m) \\ &\quad + \left[(1-q) t_{01}^{n-1} + q(1-q) \sum_{i=0}^{n-3} q^i t_{01}^{n-2-i}\right] L_{fn}(m) \\ &\quad + n + q^n r \end{aligned} \quad (11)$$

where

$$\begin{aligned} a_{11} &= 1 - (1-p) q^{n-1} t_{10}^r, \\ a_{12} &= -(1-p) q^{n-1} t_{11}^r \end{aligned} \quad (12)$$

and

$$\begin{aligned} a_{21} &= -q^n t_{10}^r, \\ a_{22} &= 1 - q^n t_{11}^r. \end{aligned} \quad (13)$$

As for the computation of $L_{sn}(m)$ and $L_{fn}(m)$, two recursive expressions can be similarly derived and the results after simplifications are

$$b_{11} L_{sn}(m) + b_{12} L_{fn}(m) = m\left[1 - (1-p) q^{m-1}\right] \quad (14)$$

and

$$b_{21} L_{sn}(m) + b_{22} L_{fn}(m) = m(1 - q^m), \quad (15)$$

where

$$\begin{aligned} b_{11} &= 1 - p t_{00}^{m-1} - (1-p)(1-q) \sum_{i=0}^{m-2} q^i t_{00}^{m-2-i}, \\ b_{12} &= -\left[p t_{01}^{m-1} + (1-p)(1-q) \sum_{i=0}^{m-3} q^i t_{01}^{m-2-i}\right] \end{aligned} \quad (16)$$

and

$$\begin{aligned} b_{21} &= -\left[(1-q) t_{00}^{m-1} + q(1-q) \sum_{i=0}^{m-2} q^i t_{00}^{m-2-i}\right], \\ b_{22} &= 1 - (1-q) t_{01}^{m-1} - q(1-q) \sum_{i=0}^{m-3} q^i t_{01}^{m-2-i}. \end{aligned} \quad (17)$$

Following similar derivations, one can compute the values of $M_s(m,n)$ and $M_f(m,n)$ as follows:

$$M_s(m,n) = 1 + a_s M_{sn}(m) + (1 - a_s) M_{fn}(m) \quad (18)$$

and

$$M_f(m,n) = 1 + b_s M_{sn}(m) + (1 - b_s) M_{fn}(m) \quad (19)$$

where a_s, b_s, $M_{sn}(m)$, and $M_{fn}(m)$ satisfy

$$a_{11} a_s + a_{12} b_s = p t_{00}^{n-1} + (1-p)(1-q) \sum_{i=0}^{n-2} q^i t_{00}^{n-2-i} \quad (20)$$

$$a_{21} a_s + a_{22} b_s = (1-q) t_{00}^{n-1} + q(1-q) \sum_{i=0}^{n-2} q^i t_{00}^{n-2-i} \quad (21)$$

$$b_{11} M_{sn}(m) + b_{12} M_{fn}(m) = 1 - (1-p) q^{m-1} \quad (22)$$

$$b_{21} M_{sn}(m) + b_{22} M_{fn}(m) = 1 - q^m \quad (23)$$

where a_{11}, a_{12}, a_{21}, a_{22} are given in (12) and (13) and b_{11}, b_{12}, b_{21}, b_{22} are given in (16) and (17).

TABLE I
EQUATION REFERENCES FOR DIFFERENT CASES

m and n / Equations / Variables	$m \le r+1$, $n \le r+1$	$m \le r+1$, $n > r+1$	$m > r+1$, $n \le r+1$	$m > r+1$, $n > r+1$
$L_s(m,n), L_f(m,n)$	(10), (11)	(24), (25)	(10), (11)	(24), (25)
$L_{sn}(m), L_{fn}(m)$	(14), (15)	(14), (15)	(26), (27)	(26), (27)
a_s, b_s	(20), (21)	(28), (29)	(20), (21)	(28), (29)
$M_{sn}(m), M_{fn}(m)$	(22), (23)	(22), (23)	(30), (31)	(30), (31)

Results for the other cases are presented in Table I. The equations used in Table I are listed below.

$$\begin{aligned} a_{11}L_s(m,n) + a_{12}L_f(m,n) &= \left\{\left[p + (1-p)(1-q)\sum_{i=0}^{n-r-2} q^i\right]t_{00}^r + \left[(1-p)q^{n-r-2}(1-q)\sum_{i=1}^{r} q^i t_{00}^{r-i}\right]\right\}L_{sn}(m) \\ &+ \left\{\left[p + (1-p)(1-q)\sum_{i=0}^{n-r-2} q^i\right]t_{01}^r + \left[(1-p)\,q^{n-r-2}(1-q)\sum_{i=1}^{r-1} q^i t_{01}^{r-i}\right]\right\}L_{fn}(m) \\ &+ p(r+1) + (1-p)(1-q)\sum_{i=0}^{n-r-2} q^i(r+2+i) \\ &+ n(1-p)q^{n-r-1}(1-q)\sum_{i=0}^{r-1} q^i \\ &+ (1-p)q^{n-1}(n+r) \end{aligned} \tag{24}$$

$$\begin{aligned} a_{21}L_s(m,n) + a_{22}L_f(m,n) &= \left\{\left[(1-q) + q(1-q)\sum_{i=0}^{n-r-2} q^i\right]t_{00}^r + \left[q^{n-r-1}(1-q)\sum_{i=1}^{r} q^i t_{00}^{r-i}\right]\right\}L_{sn}(m) \\ &+ \left\{\left[(1-q) + q(1-q)\sum_{i=0}^{n-r-2} q^i\right]t_{01}^r + \left[q^{n-r-1}(1-q)\sum_{i=1}^{r-1} q^i t_{01}^{r-i}\right]\right\}L_{fn}(m) \\ &+ (1-q)(r+1) + q(1-q)\sum_{i=0}^{n-r-2} q^i(r+2+i) \\ &+ nq^{n-r}(1-q)\sum_{i=0}^{r-1} q^i + q^n(n+r) \end{aligned} \tag{25}$$

$$b'_{11}L_{sn}(m) + b'_{12}L_{fn}(m) = p(r+1) + (1-p)(1-q)\sum_{i=0}^{m-r-2} q^i(r+2+i) + m(1-p)q^{m-r-1}(1-q)\sum_{i=0}^{r-1} q^i, \tag{26}$$

$$\begin{aligned} b'_{21}L_{sn}(m) + b'_{22}L_{fn}(m) &= (1-q)(r+1) \\ &+ q(1-q)\sum_{i=0}^{m-r-2} q^i(r+2+i) \\ &+ mq^{m-r}(1-q)\sum_{i=0}^{r-1} q^i \end{aligned} \tag{27}$$

$$a_{11}a_s + a_{12}b_s = \left[p + (1-p)(1-q)\sum_{i=0}^{n-r-2} q^i\right]t_{00}^r + (1-p)q^{n-r-2}(1-q)\sum_{i=1}^{r} q^i t_{00}^{r-i} \tag{28}$$

$$a_{21}a_s + a_{22}b_s = \left[(1-q) + q(1-q)\sum_{i=0}^{n-r-2} q^i\right]t_{00}^r + q^{n-r-1}(1-q)\sum_{i=1}^{r} q^i t_{00}^{r-i} \tag{29}$$

$$b'_{11}M_{sn}(m) + b'_{12}M_{fn}(m) = 1 - (1-p)q^{m-1} \tag{30}$$

$$b'_{21}L_{sn}(m) + b'_{22}L_{fn}(m) = 1 - q^m. \tag{31}$$

In the above equations, $b'_{11}, b'_{12}, b'_{21},$ and b'_{22} are given by

$$b'_{11} = 1 - \left[p + (1-p)(1-q)\sum_{i=0}^{m-r-2} q^i\right]t_{00}^r - (1-p)q^{m-r-2}(1-q)\sum_{i=1}^{r} q^i t_{00}^{r-i} \tag{32}$$

$$b'_{12} = -\left[p + (1-p)(1-q)\sum_{i=0}^{m-r-2} q^i\right]t_{01}^r - (1-p)q^{m-r-2}(1-q)\sum_{i=1}^{r-1} q^i t_{01}^{r-i} \tag{33}$$

$$b'_{21} = -\left[(1-q) + q(1-q)\sum_{i=0}^{m-r-2} q^i\right] t_{00}^r - q^{m-r-1}(1-q)\sum_{i=1}^{r} q^i t_{00}^{r-i} \tag{34}$$

and

$$b'_{22} = 1 - \left[(1-q) + q(1-q)\sum_{i=0}^{m-r-2} q^i\right] t_{01}^r - q^{m-r-1}(1-q)\sum_{i=1}^{r-1} q^i t_{01}^{r-i}. \tag{35}$$

One can easily verify that, for the classic GBN ARQ scheme, we have

$$\eta(1,1) = \frac{(1-q)\left[1-(p+q-1)^{r+1}\right]}{(r+1)\left[2-(p+q)\right](1-p) + (1-q)\left[1-(p+q-1)^{r+1}\right]}$$

the same as [9, Eq. (2.10)]. Futhermore, when $p + q = 1$, which corresponds to the independent error model, we get, after some computations,

$$\eta(m,n) = \frac{1-q^n}{m(1-q^n) + (n+r)q^m}$$

which can also be obtained using the computation algorithm presented in [5].

It should be pointed out that the above derivations can not be applied to compute the throughput efficiency for cases when $m = \infty$ or $n = \infty$. The reason is that when $m = \infty$ or $n = \infty$, we will get $L(m,n) = M(m,n) = \infty$. The author derived in [10] the recursive formulas and the resulting set of equations for these cases.

IV. Numerical Results

The performance of the investigated ARQ schemes depends on the round-trip delay r and the values of the parameters $m, n, p,$ and q. The values of p and q are determined by the transmission channel. The classic GBN scheme may have the best throughput performance among the investigated class of ARQ schemes. This is usually the case for small round-trip delays and/or quiet channels, i.e., channels having large p and small q. However, the performance of the classic GBN scheme could be far worse than that of the optimal investigated scheme when round-trip delay is large and/or the channel is noisy. In other words, a significantly better performance than the classic GBN scheme offers can be achieved if one select appropriately the values of m and n. An interesting result of our experiments is that the throughput efficiency is maximized at $m = n$. Since the optimal values of m and n are identical in our examples, we will restrict our study in such strategies for simplicity.

Figs. 3(a)–(c) illustrate the throughput efficiency versus number of repetitions m (and n) for $r = 10$ under different channel conditions. The case $m = 1$ corresponds to the classic GBN scheme. One can see from these figures that the throughput performance of the classic GBN scheme can be significantly improved for quite noisy channels (see the curves for $p = 0.1$) if the number of repetitions is appropriately selected. Moreover, the throughput efficiency quickly converges to a constant, namely $\eta(\infty, \infty)$, as m increases. In Fig. 3(c), the curve for $p = 0.1$ can be shown [5] is monotonic increasing with $\eta(\infty, \infty) = 0.05$. According

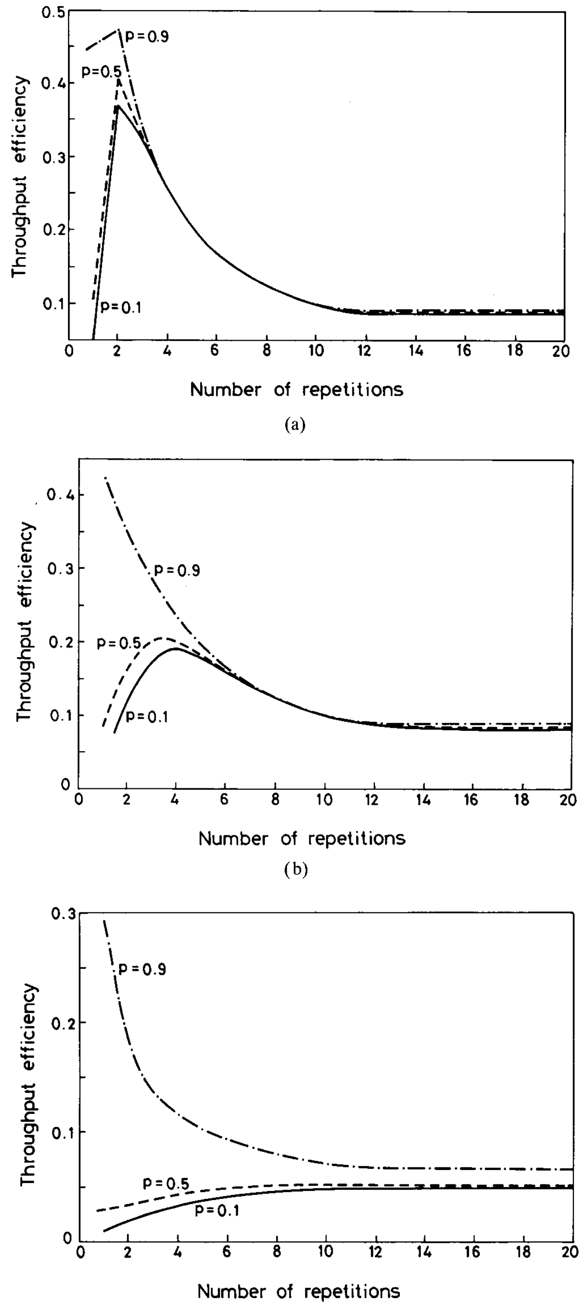
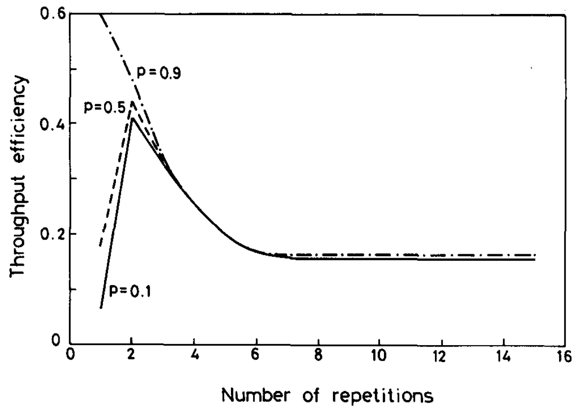
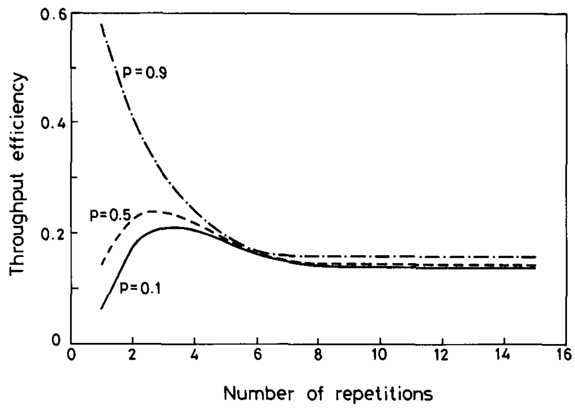


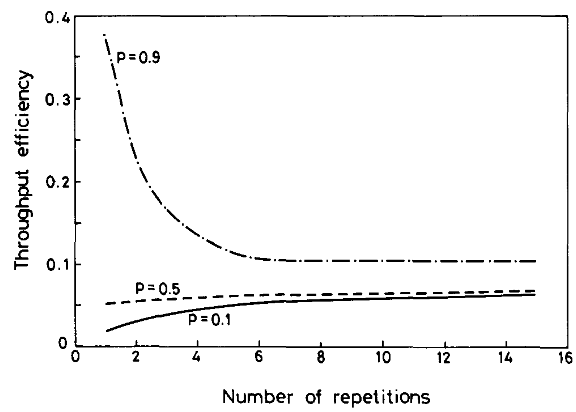
Fig. 3. Throughput efficiency versus number of repetitions for $r = 10$. (a) $q = 0.1$. (b) $q = 0.5$. (c) $q = 0.9$.



(a)



(b)



(c)

Fig. 4. Throughput efficiency versus number of repetitions for $r = 5$. (a) $q = 0.1$. (b) $q = 0.5$. (c) $q = 0.9$.

to our numerical results, the curve for $p = 0.5$ is also monotonic increasing with $\eta(\infty, \infty) \cong 0.0517$. In fact, from the curves we plotted in Figs. 3(a)–(c), one can see that $\eta(m, m)$ seems to be a monotonic function of m when $m \geq r + 1$. However, it is difficult to prove this formally unless for the independent error model (see [5]). The curves for $r = 5$ are very similar to those for $r = 10$ and are presented in Figs. 4(a)–(c).

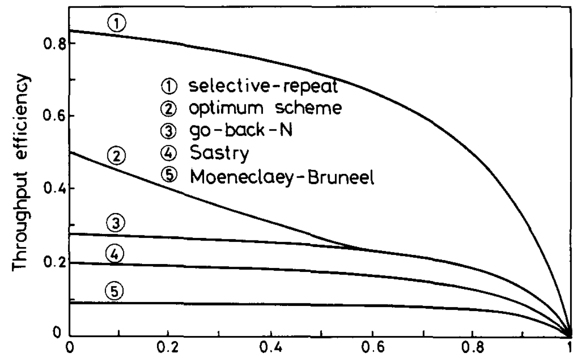


Fig. 5. Comparison of throughput efficiencies of various ARQ schemes for $r = 10$ and $p = 0.08$.

Fig. 5 illustrates the comparison among the selective-repeat, the classic GBN $(m = n = 1)$, The Sastry's $(m = 1, n = \infty)$ [2], the Moeneclaey–Bruneel's $(m = n = \infty)$ [4], and the optimal one selected from the investigated schemes for $r = 10$ and $p = 0.8$. The limiting throughput efficiency for the selective-repeat ARQ strategy is equal to $(1 - q)/(2 - p - q)$. One can see that the optimal investigated scheme can provide a considerable improvement over other related schemes.

V. Conclusion

We have analyzed in this paper the limiting throughput efficiency of a class of continuous ARQ strategies with repeated transmissions for burst-error channels modeled by two-state Markov chains. It was shown that the throughput efficiency can be significantly improved by transmitting multiple copies contiguously to the receiver, especially under high error rate conditions. Numerical results show that the throughput efficiency is very likely to be maximized at $m = n$, a confirmation of the results for the independent error model [5]. This is important because a strategy with $m \neq n$ requires more logic than the one with $m = n$.

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