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Publisher: Taylor & Francis

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Journal of the Chinese Institute of Engineers

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/tcie20>

Comparison of time domain BEM for 2d elastodynamic

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Published online: 03 Mar 2011.

To cite this article: Chung-Cheng Wang & Gin-Show Liou (1998) Comparison of time domain BEM for 2d elastodynamic, Journal of the Chinese Institute of Engineers, 21:4, 479-484, DOI: [10.1080/02533839.1998.9670410](https://doi.org/10.1080/02533839.1998.9670410)

To link to this article: <http://dx.doi.org/10.1080/02533839.1998.9670410>

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COMPARISON OF TIME DOMAIN BEM FOR 2D ELASTODYNAMIC

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Key Words: time domain, BEM, elastodynamic.

ABSTRACT

The three (LC, QC and QL) new time domain BEM methods are compared with each other and with an analytical method, the Laplace domain BEM method and another time domain boundary element method by solving two example problems of a circular cylindrical cavity subjected to a suddenly applied internal pressure, and a finite bar subjected to a ramp-step loading. Numerical study reveals that the three (LC, QC and QL) new methods are more accurate and stable than the other numerical methods used for comparison. Demonstration of the numerical computational time versus time steps of the three BEM methods is also given for a simple two-dimensional elastodynamics problem of a bar with 12-quadratic-element mesh due to an end traction. The difference in computational times by QC and LC methods is small.

I. INTRODUCTION

Time domain BEM methods have attracted much attention, since they have the advantage of treating transient and nonlinear problems. However, the temporal variation of field variables is assumed to be either zeroth or first order (constant or linear) and piece-wise continuous only in one time step for most research. Therefore, the paper uses two examples to present two quadratic temporal solution procedures (the second order, piece-wise continuous in two time steps) developed by Wang *et al.* [7, 8] in order to demonstrate the advantage of the problems. Among these two procedures, one is called QC method in which quadratic temporal variation for displacement and

constant temporal variation for traction are adopted and spatial fields variations are assumed to be quadratic. The other is called QL method in which quadratic temporal variation for displacement and linear instead of constant temporal variation for traction are chosen, and spatial fields variations are also quadratic. As expected, the QC and QL methods give better result than the methods with constant or linear temporal variation do.

In these methods, the temporal integrations can be obtained analytically and the spatial integration is evaluated by using Gaussian quadrature at each time step. One numerical example of cylindrical cavity subjected to transient compressive wave is used to demonstrate the efficiency, effectiveness and

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numerical stability of the presented two advanced time domain boundary element methods and LC (linear approximation in displacement and constant approximation in traction) method. Some comparative studies for the QL method with the QC and LC methods are first time made in the paper. As another illustrative example, a 2D bar subjected to a ramp-step end traction is solved by using several different time intervals. Comparative study for the calculation cost of the three (LC, QC and QL) advanced time BEM methods is also focused in the paper.

II. GOVERNING EQUATION OF ELASTODYNAMICS

Considering a domain V bounded by a surface S , the displacement $u_j(\bar{\xi}, t)$ at a point $\bar{\xi}$ and at time t can be obtained by using the dynamic reciprocal work theorem in an integral form as follows:

$$\begin{aligned} C_{ij}(\bar{\xi})u_j(\bar{\xi}, t) = & \int_S \{G_{ij}(\bar{\xi}, \tau; \bar{x}, t) * t_j(\bar{x}, \tau) \\ & - F_{ij}(\bar{\xi}, \tau; \bar{x}, t) * u_j(\bar{x}, \tau)\} dS(\bar{x}) \\ & + \rho \int_V G_{ij}(\bar{\xi}, \tau; \bar{x}, t) * b_j(\bar{x}, \tau) dV(\bar{x}) \\ & + \rho \int_V G_{ij}(\bar{\xi}, 0; \bar{x}, t) * \dot{u}_j(\bar{x}, 0) \\ & + \dot{G}_{ij}(\bar{\xi}, 0; \bar{x}, t)u_j(\bar{x}, 0) dV(\bar{x}). \quad (1) \end{aligned}$$

In the above equation, $u_i(\bar{x}, 0)$ and $\dot{u}_i(\bar{x}, 0)$ are the initial displacement and velocity, respectively. $C_{ij}(\bar{\xi})$ is the well known discontinuity term which is dependent on local geometry. The symbol * stands for Reimann time convolution integral, *i.e.*, for example,

$$\begin{aligned} f(\bar{x}, t) * g(\bar{x}, t) & \equiv \int_0^t (\bar{x}, t - \tau)g(\bar{x}, \tau) d\tau \\ & = \int_0^t f(\bar{x}, \tau)g(\bar{x}, t - \tau) d\tau \quad (2) \end{aligned}$$

for $t \geq 0$ and for two functions f and g . Or,

$$F_{ij}(\bar{\xi}, \tau; \bar{x}, t) * u_j(\bar{x}, \tau) \equiv \int_0^t F_{ij}(\bar{\xi}, \tau; \bar{x}, t) * u_j(\bar{x}, \tau) d\tau. \quad (3)$$

The terms $G_{ij}(\bar{\xi}, \tau; \bar{x}, t)$ and $F_{ij}(\bar{\xi}, \tau; \bar{x}, t)$ are the fundamental solutions and represent respectively the displacement and traction at the field point \bar{x} and time t due to a unit point force applied at the source point

$\bar{\xi}$ and a preceding time τ . Eq. (1) is valid for both bounded and unbounded regions.

For the numerical implementation of Eq. (1), both time and space domains have to be discretized. The time convoluted integrations can be performed analytically in the sense of piece-wise continuity while the spatial integrations are treated numerically. In order to integrate the time convolution analytically, the time span of interest, from zero to t , is divided into N equal increments with duration Δt for each time step. Therefore $t_n = n\Delta t$ for $n=1, 2, \dots, N$. As for spatial discretization, one can choose suitable boundary elements, for example using a quadratic element, to model the geometry of the problems. It is of interest to note that Eq. (1) is an implicit time-domain formulation, since the displacements at time t are being calculated by taking account of the histories of surface tractions and displacements up to time t . As mentioned before, the temporal variations of field variables (displacements and tractions) can be constant, linear or quadratic. Eq. (1) with zero initial conditions and in the absence of the body force can be rewritten by using the discretization of time and surface boundary. For the time-marching process for different temporal interpolation functions (quadratic, linear and constant), one can refer to Reference [7].

III. NUMERICAL EXAMPLES

The following examples are presented to demonstrate the capability of the developed time-convoluted BEM algorithm. The geometry for each solution procedure is modeled with the same continuous isoparametric quadratic elements in order to maintain accuracy and apply boundary conditions easily. For two elements attached to the same node at a corner, different traction components can be assigned in the computing program.

In general, the spatial variation of the condensed convoluted kernels is a Heaviside logarithmic or square root decay function for two dimensional problems and hence uniform subsegmentation techniques are used for obtaining accurate evaluation of the spatial integrals. Twelve subsegments and 32 Gaussian points per each subsegment are used for evaluating the integrals of singular time steps, while 8 subsegments and 10 Gaussian points per each segment are used for evaluating the integrals of nonsingular time steps.

The nondimensional time step β is defined as

$$\beta \equiv \frac{c_1 \Delta t}{l} \quad (4)$$

where l is the quadratic element length (not length between two nodes), Δt is the time step used and c_1 is the compressive wave velocity. Only $0.5 \leq \beta \leq 2.5$ are

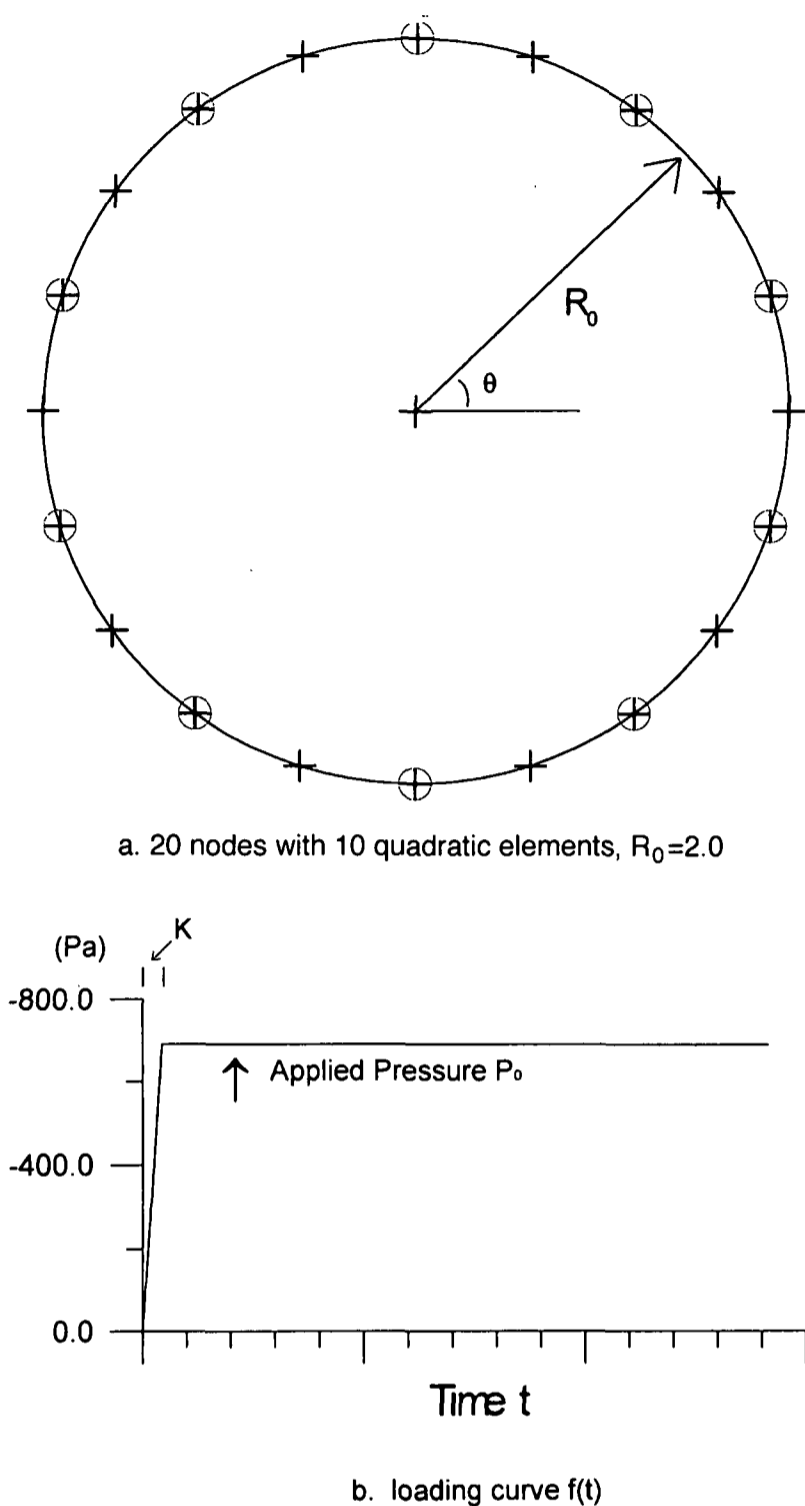


Fig. 1. Discretization of the circular cylindrical cavity, (a) 20 nodes and 10 quadratic elements, (b) loading curve.

selected in the numerical study. The boundary stresses are calculated directly from the computed displacements and tractions by simple differentiation of the shape functions.

1. Transient Compressive Wave from a Cylindrical Cavity

Consider a circular cylindrical cavity with radius R_0 in an infinite elastic medium subjected to a suddenly applied internal uniform pressure of intensity P_0 . As shown in Fig. 1b the applied Heaviside loading can be approximated by a ramp-step loading. This problem has been solved by many authors and it is also frequently used as a reference to validate their methods. It is found that it is appropriate to discretize the circumference of the cavity into 20 segments of

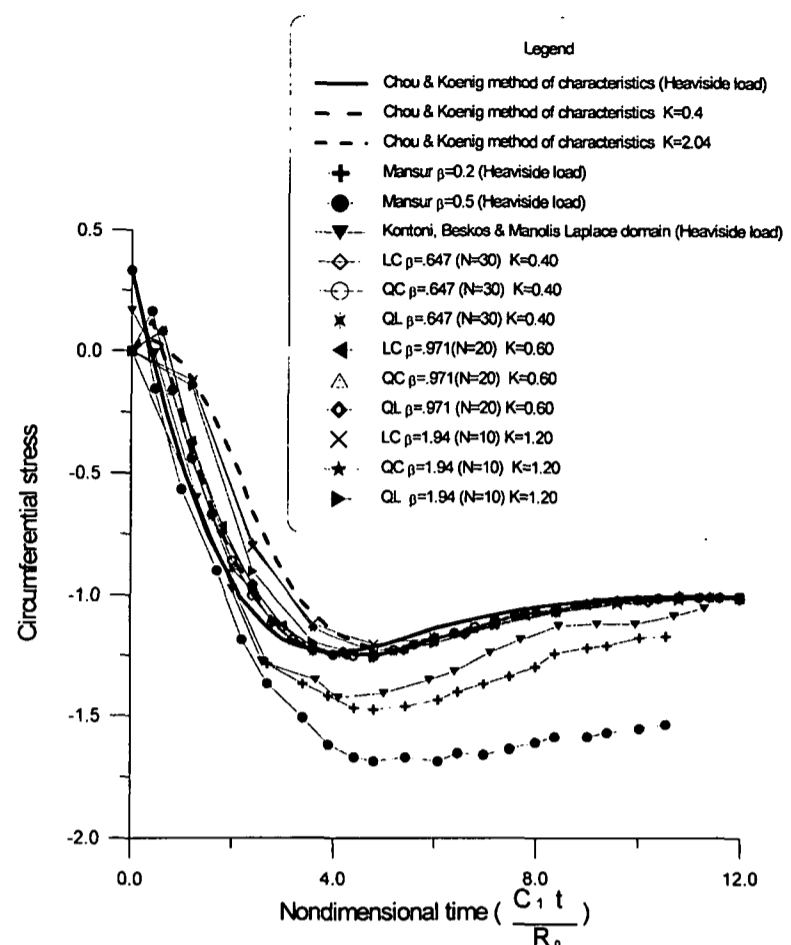


Fig. 2. Circumferential stresses at cavity surface due to a ramp-step transient compressive wave ($R_0=2$, $\nu=1/4$, plane strain) for 20 nodes mesh.

equal length and 10 quadratic elements as shown in Fig. 1a. The numerical data are $R_0=2$ m, $E=62000$ MPa, $\rho=2670.0$ kg/m³, $\nu=1/4$, $\lambda=\mu=24800$ MPa, which correspond physically to granite (hard rock) and $P_0=-690$ Pa and time span $T=0.00454653$ sec. Three numbers of time steps ($N=10, 20$ and 30) were selected for the example. This problem has been solved by Chou and Koenig [1] using the method of characteristics under plane stress condition, by Fu [2] using the FEM under plane strain condition, by Kontoni *et al.* [3] using the Laplace domain and by Mansur and Brebbia [4] using the first generation time domain BEM under plane stress condition. All the above methods are numerical methods. The same problem has also been solved analytically by Selberg [5, 6] under plane strain condition. Within the accuracy of curve plotting, the results by Selberg [6] agree exactly with those by Chou and Koenig [1].

Figure 2 depicts the time history of the circumferential stress $\sigma_{\theta\theta}$ at R_0 by the three (LC, QC and QL) methods. In the figure, K is defined as the rise time of the ramp-step loading. The results are in good agreement with the analytical solution, except for the first time step. This is due to the difference between Heaviside loading and ramp-step loading. Moreover, these results from the QC method are almost the same as those from the LC and QL methods.

Within plotting accuracy, the results of the three (QL, QC and LC) methods with $\beta=0.971$ ($N=20$ steps)

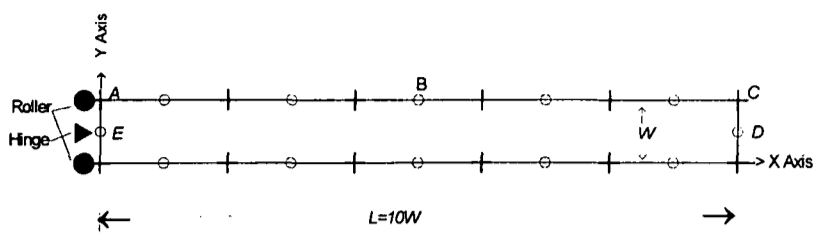


Fig. 3. Boundary element discretization of a rectangular bar, $L/W=10$, 24 nodes, 12 quadratic elements.

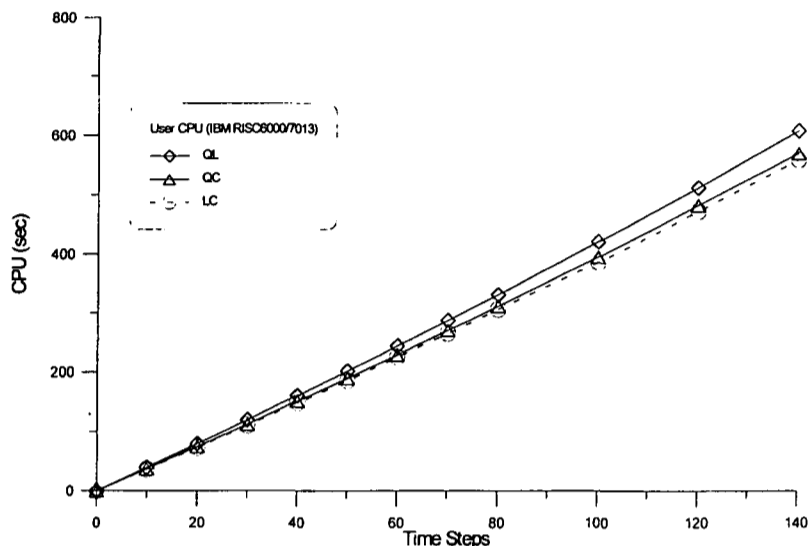


Fig. 4. Comparison of computing time for a bar with 12 quadratic elements.

and $\beta=0.647$ ($N=30$ steps) agree exactly with those of Chou and Koenig [1] using ramp input, and especially match up to those curves between $K=0$ (Heaviside) and $K=0.4$. Fig. 2 also reveals that the results from QL, QC and LC are better than the results from the Laplace domain BEM [3] and the first generation time domain BEM [4] methods.

2. A Bar Subjected to a Uniform Ramp-Step Load

A rectangular bar with length-to-width ratio $L/W=10$ is fixed at its left end with traction free on its top and bottom sides as shown in Fig. 3 and subjected to a ramp-step load. This rectangular bar is used in the investigation of efficiency of the aforementioned three advanced (LC, QC and QL) methods. The material constants for the rectangular bar are $E=7.8 \text{ Pa}$, $\nu=0$ and $c_1=100 \text{ m/sec}$. The boundary is discretized into twelve quadratic boundary elements as shown in Fig. 3.

Under the ramp-step load, the magnitude of the applied load starts linearly increasing from zero and then is kept constant after time $t=T_r=L/c_1$. This means that when $t=T_r$, the P-wave front has just arrived at the point of the fixed end.

Figure 4 shows comparison of the computing time versus time steps. From the figure, the computing time (CPU time on IBM RISC 6000 model 7013) increases almost linearly with the time steps for all

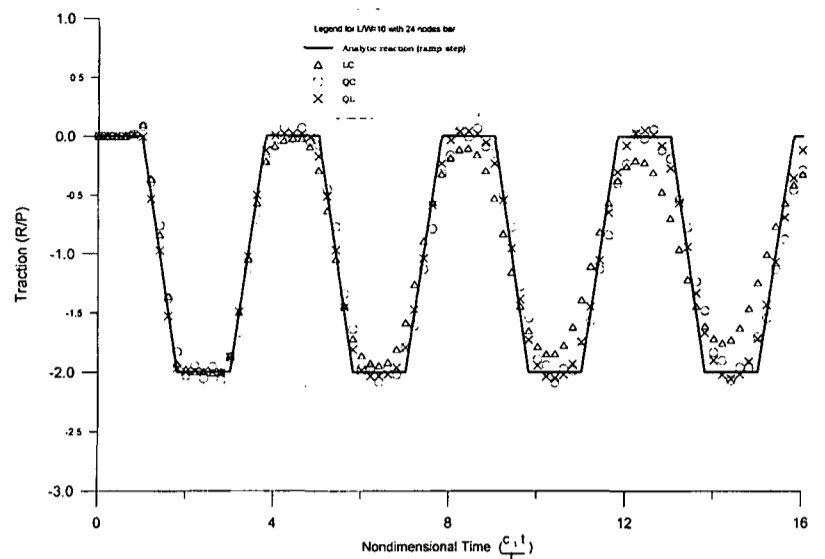


Fig. 5. Traction at point A for $\beta=1.0$ subjected to a ramp-step loading.

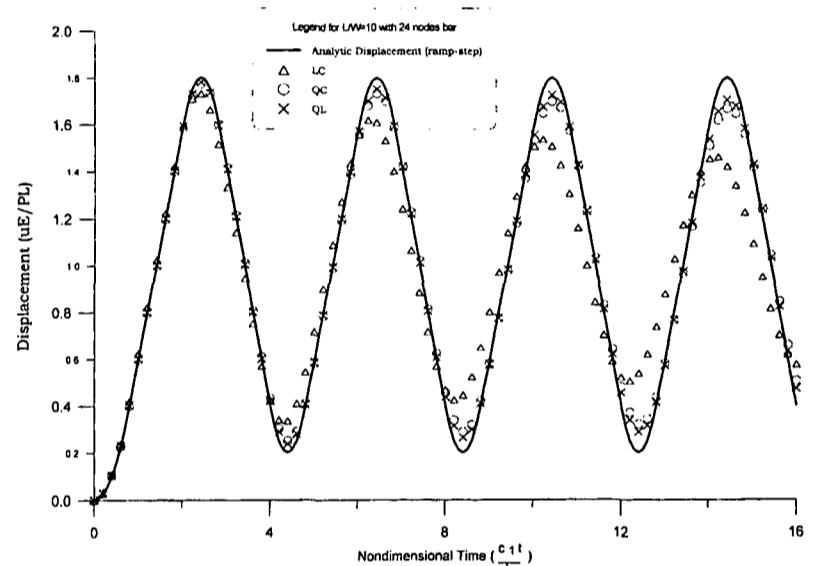


Fig. 6. Displacement at point C for $\beta=1.0$ due to a ramp-step loading.

three advanced time domain BEM methods. Although the condensed convoluted kernel of the QL method is more complicated than those of the QC and LC methods, the difference in computing time among the QL, QC and LC methods is small, and the QL method needs only a little more computing time than the others.

The displacement at point C and the reaction at point A (see Fig. 3) is investigated for $\beta=1.0$ and $\beta=2.5$. The numerical solutions from the QL, QC and LC methods are plotted together and compared to the analytical solution as shown in Figs. 5-8. The time history of traction at point A is depicted in Fig. 5 and Fig. 7 for $\beta=1.0$ and $\beta=2.5$, respectively. The time history of displacement at point C is depicted in Fig. 6 and Fig. 8 for $\beta=1.0$ and $\beta=2.5$, respectively. Notably, numerical damping is observed in the figures. From these figures, the QL and QC methods are much better and more accurate than the LC method as shown in Figs. 5-6. From Figs. 5-8, it is also revealed that

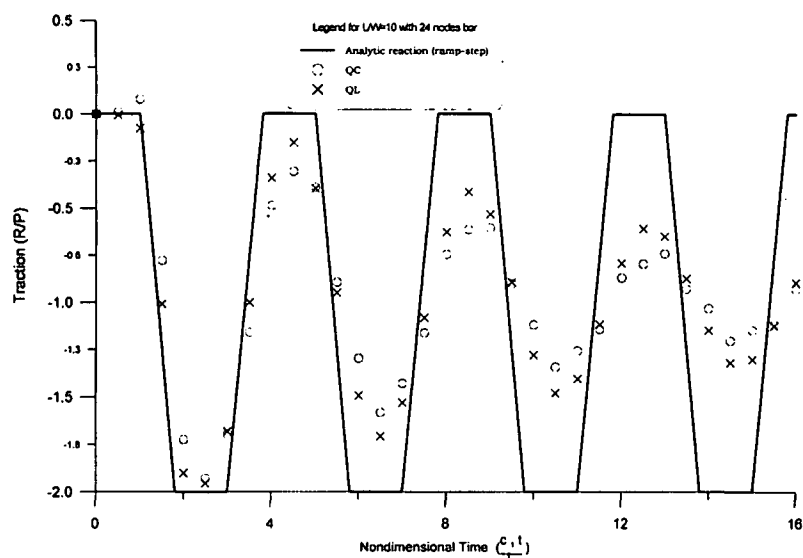


Fig. 7. Traction at point A for $\beta=2.5$ subjected to a ramp-step loading.

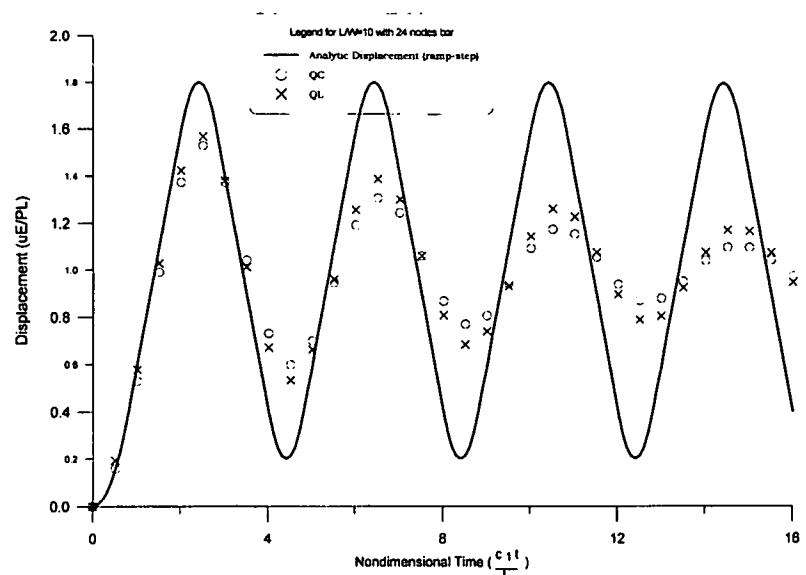


Fig. 8. Displacement at point C for $\beta=2.5$ due to a ramp-step loading.

the QL method is better and more accurate than the QC method when using a larger β . A larger β means less computational time.

IV. CONCLUSION

After some extensive numerical study of the presented BEM scheme, the following conclusions can be drawn:

- (1) Comparing with the three advanced time domain BEMs with each other for $0.5 \leq \beta \leq 2.0$ in solving the problem of the infinite domain of a cavity under the same integration scheme, numerical study revealed that all three methods are accurate and stable for the infinite domain. But, for the finite domain, for example a finite bar, the QC and QL methods are more accurate and stable than the LC method for $\beta \geq 1.0$. In addition, QL is more accurate and stable than QC for a large β as revealed in Figs. 7 and 8. This implicitly reveals that using BEM results for elastodynamic problems with an infinite domain are more easily obtained than that for elastodynamic problems with finite domain.
- (2) The relationship between computing time and the number of time steps is almost linear as shown in Fig. 4. The difference in computing time among the three advanced time domain BEM methods is small.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to the Chung Shan Institute of Science and Technology (CSIST) of Taiwan for their financial support which made this research possible, and to Prof. H.C. Wang for his valuable advice. In addition, the authors also thank the National Science Council of

Taiwan for partially sponsoring the research presented in the paper.

NOMENCLATURES

c_1	compressive wave velocity
E	Young's modulus
$F_{ij}(\bar{\xi}, \tau; \bar{x}, t)$	traction fundamental singular solution
$G_{ij}(\bar{\xi}, \tau; \bar{x}, t)$	displacement fundamental singular solution
l	quadratic element length
N	No. of time steps
P_0	applied pressure of intensity
R_0	radius of a circular cylindrical cavity
S	surface of boundary
t, T, τ	time
$t_j(\bar{\xi}, t)$	traction vector
$u_j(\bar{\xi}, t)$	displacement vector
V	domain of interest
\bar{x}	field point
*	Reimann time convolution integral
β	nondimensional time step
Δt	time step
λ, μ	elastic constants
ν	Poisson ratio
$\bar{\xi}$	source point
ρ	density of material

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Discussions of this paper may appear in the discussion section of a future issue. All discussions should be submitted to the Editor-in-Chief.

Manuscript Received: Nov. 06, 1997

Revision Received: Feb. 28, 1998

and Accepted: Mar. 06, 1998

時域邊界元素法在二維應力波傳之比較

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摘 要

本文主要以兩個例題：一圓柱體狀空洞承受爆炸內壓力與一短柱承受斜階段負荷，來說明作者推導之 LC、QC 與 QL 時域邊界元素法在與理論解析、Laplace 域邊界元素法及其他時域邊界方法作比較，顯示此等方法比其餘數值方法具有優越精確度與數值穩定度。此外，比較 LC、QC 與 QL 三者所需計算時間與 time steps 關係，結果顯示 LC 與 QC 方法所需計算時間相差很少。

關鍵詞：時間域、邊界元素法、應力波傳。