

Transportation investment project selection using fuzzy multiobjective programming

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Received October 1993; revised September 1996

Abstract

This paper proposes the fuzzy multiobjective programming for the problem of transportation investment project selection (TIPS). The programming then uses the fuzzy spatial algorithm, which calculates the performance of objective achievement and the requirement of resource utilization as of fuzziness. Under a complex and uncertain decision-making environment, there exists a certain degree of interdependence among these transportation investment projects. This paper uses expert evaluation, and conducts, respectively, with the consensus of most of the experts, the decision on interdependence type (complementary and substitutive) and on the degree of fuzzy interdependence. In every iteration of the fuzzy spatial algorithm, the method of ranking fuzzy numbers must be used so as to obtain the ranking for selecting investment projects. This paper has modified the method provided by Kim and Park in order that the preference of most of the decision makers or experts can be overlooked, since the degree of optimism or pessimism can be demonstrated in the profitability of every investment project. In the end, this paper will employ a numerical example to illustrate the method forwarded. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Transportation investment project selection; Fuzzy multiobjective programming; Interdependence; Fuzzy spatial algorithm; Majority rule; Ranking of fuzzy numbers

1. Introduction

Transportation investment decision-making is under a rather fuzzy environment as the process will find itself within a society of much uncertainty and complexity to the future, and such decision-making issues are considered as ill-structured [29, 24]. Therefore, when transportation investment planning is performed, not only the nature of multiple objectives but also the fuzzy characteristics have to be considered. Since it is difficult to measure the achieved objectives of transportation investment projects, they will be managed with the fuzzy set theory.

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The transportation investment project selection (TIPS) is a significant part of transportation investment planning. The application of fuzzy set theory on the selection and evaluation of multiobjective transportation investment planning can be categorized as fuzzy multiobjective mathematical programming (FMOMP) and fuzzy multicriteria decision-making (FMCMD) [5, 29]. Upon the application of FMOMP, objectives achievement and resource utilization are both considered; then nondominated solutions can be located through mathematical optimization or approximate solutions, through heuristic algorithms. As to FMCMD method, it reckons resource utilization to be an objective mode, and it goes into the comprehensive evaluation of transportation investment projects so as to decide its priority; Siskos [20], Roy and Hugonnard [18], and Roy et al. [19] had once used fuzzy outranking methods for the evaluation and selection of transportation investment planning.

This paper has employed FMOMP method to select transportation investment projects. The application of FMOMP in TIPS problems generally belongs to a 0–1 multiobjective fuzzy linear programming (0–1 MFLP) problem. Wiedey and Zimmermann [26] tried to find out the solution for the problem, still using non-0–1 MFLP method for the solution [30]. Initially, 0–1 MFLP problems were changed into the LP model and solved. Dias Jr. [9] put forward the nondominated solutions method to a 0–1 MFLP problem and had it utilized on an R&D project selection problem. Teng and Tzeng [21] have already developed a heuristic algorithm to the 0–1 MFLP problem and applied it to the TIPS problem. Yet they limited it to dealing with fuzziness of objectives. Resources will be considered as nonfuzziness, and the fuzzy achievement value will be transformed into crisp value before a mathematical model is to be used for finding solutions. Strictly speaking, the techniques of fuzzy mathematical programming have not been applied to look for a solution. This paper will abide by the research of Tzeng and Teng [23], and a proposed solution method to the 0–1 MFLP problem, aside from deliberating on the interdependence among transportation investment projects.

In the application of fuzzy set theory to the selection of a nonindependent transportation investment project, both the performance of objective achievement and requirement of resource utilization are considered as fuzziness, while the relative weights of objectives and resources are to be dealt with by crisp value. In Section 2, there is an explanation of fuzzy multiobjective transportation investment selection (FMOTIPS) problems; Section 3 will present the decision-making method to decide the degree of interdependence of transportation investment projects; Section 4 will explain the structuring of fuzzy multiobjective programming models and fuzzy spatial algorithm for non-independent transportation investment projects; Section 5 will use an approximate illustrated example to expound the method put forward in this paper; finally, some conclusions and a future research approach will then be reached.

2. The fuzzy multiobjective transportation investment project selection problem

Transportation investment project selection (TIPS) is a typical example of a multiobjective decision problem. A multiobjective decision problem is defined as a problem in which there is more than one objective and the objectives cannot be combined in any way. In most cases TIPS is a discrete multiobjective problem, and transportation investment projects are supposedly known. Thus, a multiobjective transportation investment decision problem can be defined as: a given finite set of n potential transportation investment projects $\{x_1, x_2, \dots, x_n\}$ which is evaluated with respect to m objectives $\{O_1, O_2, \dots, O_m\}$ and q resource constraints $\{B_1, B_2, \dots, B_q\}$ and the best subset of projects (n_1) is chosen from the given finite set of potential projects. Mathematically, it can be denoted as a multiobjective transportation investment project selection (MOTIPS) problem:

$$\text{MOTIPS: } \text{maximize } Z = \mathbf{w}^T \mathbf{G} \mathbf{x} \quad (1a)$$

$$\text{subject to } \mathbf{x} \in X, \quad (1b)$$

where \mathbf{G} is the $m \times n$ matrix whose generic element G_{ij} is the value of j project according to objective i ($i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$). \mathbf{w} is the m -dimensional decision vector $\{w_i\}$ of the relative importance (weights) of the objectives; $w_i > 0$, $\forall i$. \mathbf{x} is the n -dimensional decision vector $\{x_i\}$; if project j is selected, then $x_j = 1$; otherwise, then $x_j = 0$. X is a feasible set, that is

$$X = \{\mathbf{x} \mid \mathbf{Ax} \leq \mathbf{B}; x_j = 0, 1; j = 1, 2, \dots, n\}, \quad (2)$$

where \mathbf{A} is $q \times n$ matrix $\{A_{kj}\}$ of the coefficients of the linear constraints; \mathbf{B} is the q -dimensional vector $\{B_k\}$ of resource constraints.

Among MOTIPS problems, if coefficients G_{ij} ($i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$), A_{kj} ($k = 1, 2, \dots, q$; $j = 1, 2, \dots, n$), and resource requirements B_k are denoted by fuzzy numbers \tilde{G}_{ij} , \tilde{A}_{kj} , and \tilde{B}_k , then MOTIPS problem becomes FMOTIPS(1) problem:

$$\text{FMOTIPS(1):} \quad \text{maximize} \quad \tilde{Z} = \mathbf{w}^T \tilde{\mathbf{G}} \mathbf{x} \quad (3a)$$

$$\text{subject to} \quad \mathbf{x} \in X, \quad (3b)$$

$$X = \{\mathbf{x} \mid \tilde{\mathbf{A}}\mathbf{x} \leq \tilde{\mathbf{B}}; x_j = 0, 1; j = 1, 2, \dots, n\},$$

where \mathbf{w} , \mathbf{x} and X were already defined in Eqs. (1) and (2); $\tilde{\mathbf{G}}$ and $\tilde{\mathbf{A}}$ are the fuzzy matrix whose generic element are the fuzzy sets \tilde{G}_{ij} and \tilde{A}_{kj} , respectively. $\tilde{\mathbf{B}}$ is the fuzzy vector whose generic element is the fuzzy set \tilde{B}_k .

In addition to achieving the maximization of m objectives, the TIPS problem expects its resources can be fully used without being idled. In other words, the gap between the utilization quantity and affordable quantity of every resource of selected investment projects should be at its minimization. Also, since the relative importance among q resource varies, different weights are attained. Thus, FMOTIPS (1) can be rewritten as FMOTIPS (2) problem:

$$\text{FMOTIPS(2):} \quad \text{maximize}_{i=1,2,\dots,m} \quad \tilde{Z}_i(\mathbf{x}) = \sum_{j=1}^n w_i \tilde{G}_{ij} x_j \quad (4a)$$

$$\text{minimize}_{k=1,2,\dots,q} \quad \tilde{H}_k(\mathbf{x}) = \lambda_k \left(\tilde{B}_k \ominus \sum_{j=1}^n \tilde{A}_{kj} x_j \right) \quad (4b)$$

$$\text{subject to} \quad \mathbf{x} \in X, \quad (4c)$$

where $\tilde{Z}_i(\mathbf{x})$ indicates the achieved value of objective i , $\tilde{H}_k(\mathbf{x})$ indicates the quantity of resource k laid idle, λ_k indicates the weight of resource k , and \ominus denotes fuzzy subtraction, \sum indicates the summation of fuzzy number:

$$\sum_{j=1}^n \tilde{A}_j = \tilde{A}_1 \oplus \tilde{A}_2 \oplus \dots \oplus \tilde{A}_n, \quad (5)$$

where \oplus indicates fuzzy addition.

When the interdependence of transportation investment projects is taken into consideration, the achieved value of m objectives will be affected; in other words, the attained value $Z_i(\mathbf{x})$ towards objective i of n_1 selected investment projects has to be added to $\tilde{Q}_i(\mathbf{x})$ of the increased achieved value due to complementarity and the lessened achieved value due to substitution, aside from reaching the sum of the achievement value of

individual project. Thus, FMOTIPS (2) can be rewritten as FMOTIPS (3) problem:

$$\text{FMOTIPS(3):} \quad \underset{i=1,2,\dots,m}{\text{maximize}} \quad \tilde{Z}_i(\mathbf{x}) = w_i \left(\sum_{j=1}^n \tilde{G}_{ij} x_j \oplus \tilde{Q}_i(\mathbf{x}) \right) \quad (6a)$$

$$\underset{k=1,2,\dots,q}{\text{minimize}} \quad \tilde{H}_k(\mathbf{x}) = \lambda_k \left(\tilde{B}_k \ominus \sum_{j=1}^n \tilde{A}_{kj} x_j \right) \quad (6b)$$

$$\text{subject to} \quad \mathbf{x} \in X. \quad (6c)$$

3. Project types and degree of interdependence

3.1. Types of transportation investment projects

There is a certain degree of interdependence among transportation investment projects, and such degree of interdependence varies under different objectives. Under every objective, various factors influencing investment projects vary in degree and some of them cannot be measured with objective materials. As a result, this paper will bring in a decision group consisting of specialists from relevant expertise to provide professional cognition and judgment, hoping to produce phenomenal effect as a group and brain tank and cut down bias of personal judgment.

This paper has categorized four types of transportation investment projects based on their relevant characteristics; they are independent projects, complementary projects, substitutive projects and common complementary substitutive projects (see Fig. 1).

3.1.1. Independent investment projects

Independent investment projects are those whose performance of all m objectives has been attained and will not be affected by other projects and vice versa. The set of independent investment projects is denoted

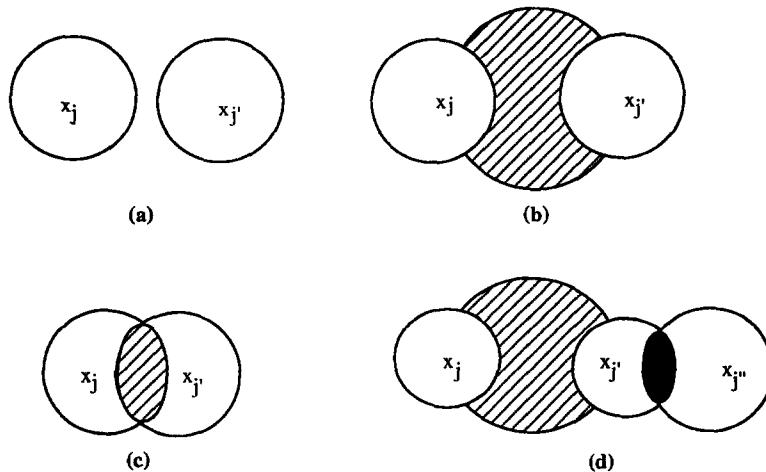


Fig. 1. Effect of different types of investment projects: (a) independent projects; (b) complementary projects; (c) substitutive projects; (d) common complementary substitutive projects.

in A^I . If x_j and $x_{j'}$ are two independent investment projects, then x_j will not affect $x_{j'}$ and $x_{j'}$ will not affect x_j , either. It can be denoted either in $x_jIx_{j'}$ or $x_{j'}Ix_j$ and its influences of investment results are shown in Fig. 1(a).

3.1.2. Complementary investment projects

Complementary investment projects, whose performance was once attained from objective i , will be influenced by other projects and vice versa. Thus, when two investment projects are implemented at the same time, investment results can also be increased to the designated objective. Yet, complementary investment projects do not necessarily have to affect complementarity to all m objectives so as to be thus dubbed. As long as one or more than one objective is complementary, they are complementary investment projects. The set of complementary investment projects is denoted by A^C . If x_j and $x_{j'}$ are two complementary investment projects, then $x_jCx_{j'}$ is used to indicate the complementarity of x_j to $x_{j'}$ and $x_{j'}Cx_j$ is used to indicate the complementarity of $x_{j'}$ to x_j . Complementary projects are shown in Fig. 1(b), and the slant part shows the added performance of $x_jCx_{j'}$ and $x_{j'}Cx_j$.

3.1.3. Substitutive investment projects

Substitutive investment projects are those which can partly or wholly be substituted for another after performance of objective i is achieved. In other words, another project can replace the original one to achieve partial or integral performance of objective i . Substitutive investment projects do not have to be substitutive to all m objectives so as to be thus named. Being substitutive to only one or more than one objective, they can be categorized as substitutive investment projects. Substitutive investment projects will not increase the achieved value of performance. But they can only use one project to substitute for the achieved value of another. The set of substitutive investment projects is demonstrated with A^S . If x_j and $x_{j'}$ are two substitutive investment projects, then $x_jSx_{j'}$ indicates x_j is substitutive to $x_{j'}$ while $x_{j'}Sx_j$ indicates $x_{j'}$ is substitutive to x_j . Substitutive projects are shown in Fig. 1(c); the slant line indicates the performance $x_{j'}$ is replaced by x_j (i.e. $x_jSx_{j'}$).

3.1.4. Common complementary substitutive projects

Transportation investment project x_j could be complementary to project $x_{j'}$ and substitutive to project $x_{j''}$ (as indicated in Fig. 1(d)). This form of projects is demonstrated with A^{CS} set, which is

$$A^{CS} = A^C \cap A^S. \quad (7)$$

3.2. Classification of transportation investment projects

Among these four types of transportation investment projects, once complementary projects (A^C) and substitutive projects (A^S) are decided, independent projects (A^I) and common complementary substitutive projects (A^{CS}) can be found through the following formula:

$$A^I = A^T \setminus (A^C \cup A^S \cup A^{CS}), \quad (8)$$

where $A^{CS} = A^C \cap A^S$, $A^T = \{x_1, x_2, \dots, x_n\}$ indicates the constructed set of n transportation investment projects.

Following their professional knowledge and experiences to proceed to their subjective judgment and employing pairwise comparison to decide if every two investment projects are related to each other, experts can then see the objective achievement of n transportation investment projects under separate m objectives and show if it is of A^C or A^S category. Let $d_{jj'}^{hi}$ and $e_{jj'}^{hi}$ for instances, they indicate discretely under i objective ($i = 1, 2, \dots, m$), and h expert ($h = 1, 2, \dots, R$) judges investment projects of x_j and $x_{j'}$ ($j, j' = 1, 2, \dots, n$; $j \neq j'$). They belong to the degree of membership of complementary projects and substitutive projects and this degree of membership, lies in $[0, 1]$. If $d_{jj'}^{hi} = 0$ or $e_{jj'}^{hi} = 0$, then it shows that the investment projects

enjoy no complementarity or substitution if $d_{jj'}^{hi} > 0$ or $e_{jj'}^{hi} > 0$, it shows that the investment projects might be complementary or substitutive to each other. Thus, disregarding complementarity or substitution of x_j to $x_{j'}$ or $x_{j'}$ to x_j , the investment projects of x_j and $x_{j'}$ investment projects can be categorized either as complementary or substitutive projects. In other words, $d_{jj'}^{hi} = d_{j'j}^{hi}$ and $e_{jj'}^{hi} = e_{j'j}^{hi}$, so it would be sufficient only to take expert h one judgment. Thus, under every objective the fuzzy matrix of some R experts' judgment can be obtained:

$$D^{hi} = \{d_{jj'}^{hi}\}, \quad \forall h \quad (9)$$

$$E^{hi} = \{e_{jj'}^{hi}\}, \quad \forall h. \quad (10)$$

Whether or not these two transportation investment projects are of complementary or substitutive fuzzy judgment matrix remains dubious; obviously, they are symmetric matrixes.

R experts view differently as to if every two investment projects x_j and $x_{j'}$ are complementary or substitutive. It has become an important issue how the consensus of their views can be reached, and this paper will abide by the majority rule as the criteria of judgment. Let $\hat{D}_{jj'}^{Ti}$ and $\hat{E}_{jj'}^{Ti}$ denote some R complementary or substitutive degree of membership under objective i , the fuzzy set achieved ranking from the greatest to the smallest will be

$$\hat{D}_{jj'}^{Ti} = \{\hat{d}_{jj'}^{h'i} \mid \hat{d}_{jj'}^{1i} \geq \hat{d}_{jj'}^{2i} \geq \dots \geq \hat{d}_{jj'}^{R'i}\}, \quad \forall i, \quad (11)$$

$$\hat{E}_{jj'}^{Ti} = \{\hat{e}_{jj'}^{h'i} \mid \hat{e}_{jj'}^{1i} \geq \hat{e}_{jj'}^{2i} \geq \dots \geq \hat{e}_{jj'}^{R'i}\}, \quad \forall i, \quad (12)$$

where

$$\hat{d}_{jj'}^{1i} = \max_{h=1,2,\dots,R} \{\hat{d}_{jj'}^{hi}\}, \quad (13)$$

$$\hat{d}_{jj'}^{R'i} = \min_{h=1,2,\dots,R} \{\hat{d}_{jj'}^{hi}\}, \quad (14)$$

$$\hat{e}_{jj'}^{1i} = \max_{h=1,2,\dots,R} \{\hat{e}_{jj'}^{hi}\}, \quad (15)$$

$$\hat{e}_{jj'}^{R'i} = \min_{h=1,2,\dots,R} \{\hat{e}_{jj'}^{hi}\}. \quad (16)$$

To decide if there is complementarity or substitution among n transportation investment projects, a consensus of some R experts can be reached based on majority rule and can help obtain complementary and substitutive comprehensive matrixes D^{Ti} and E^{Ti} under every objective:

$$D^{Ti} = \{\hat{d}_{jj'}^{Mi}\}, \quad \forall i, \quad (17)$$

$$E^{Ti} = \{\hat{e}_{jj'}^{Mi}\}, \quad \forall i, \quad (18)$$

where

$$M = \begin{cases} (R/2) + 1 & \text{if } R \text{ is even,} \\ [(R/2 - 1)/2] + 1 & \text{if } R \text{ is odd.} \end{cases} \quad (19)$$

The principle of over half for M value is employed in Eq.(19). Should experts of the decision group have to be more consensual, the principle of $\frac{2}{3}$ or $\frac{3}{4}$ can be used to decide. According to the consensual judgment values of \bar{d} and \bar{e} of complementary and substitutive investment projects, matrix D^{Ti} and E^{Ti} , R experts can decide whether or not investment projects x_j and $x_{j'}$ are the fuzzy threshold values \bar{d} and \bar{e} of complementary and substitutive investment projects. If $\hat{d}_{jj'}^{Mi} \geq \bar{d}$ and $\hat{e}_{jj'}^{Mi} \geq \bar{e}$, it can be said that x_j and $x_{j'}$ are complementary

and substitutive projects:

$$\text{if } \hat{d}_{jj'}^{Mi} \geq \bar{d} \Rightarrow x_j, x_{j'} \in A^C, \quad (20)$$

$$\text{if } \hat{e}_{jj'}^{Mi} \geq \bar{e} \Rightarrow x_j, x_{j'} \in A^S. \quad (21)$$

This paper will take $\bar{d} = 0.5$ and $\bar{e} = 0.5$; values of d and e can be lifted up if stricter sense of recognition is required.

3.3. Determination of interdependent degree

When transportation investment projects are many (n is great), the above-mentioned methods should be used initially for categorization. Then thoughts can be placed on the fuzzy degree of complementarity and fuzzy degree of substitution in complementary projects and substitutive projects. As a result, the level of sophistication of judgment and decision-making of experts can be lowered. When n is smaller, experts can judge directly on the relevance of n projects:

Let $\widetilde{CC}_{jj'}^{hi}(j, j' \in A^C)$ and $\widetilde{SC}_{jj'}^{hi}(j, j' \in A^S)$ indicate individually the fuzzy degree of complementarity of expert h towards investment project x_j to $x_{j'}$ under objective i ; then $\widetilde{CC}_{jj'}^{hi}$ and $\widetilde{SC}_{jj'}^{hi}$ can be demonstrated in triangular fuzzy numbers (TFN) as

$$\widetilde{CC}_{jj'}^{hi} = (LC_{jj'}^{hi}, MC_{jj'}^{hi}, RC_{jj'}^{hi}), \quad \forall h, i, \quad (22)$$

$$\widetilde{SC}_{jj'}^{hi} = (LS_{jj'}^{hi}, MS_{jj'}^{hi}, RS_{jj'}^{hi}), \quad \forall h, i, \quad (23)$$

where $LC_{jj'}^{hi} \leq MC_{jj'}^{hi} \leq RC_{jj'}^{hi}$ and $LS_{jj'}^{hi} \leq MS_{jj'}^{hi} \leq RS_{jj'}^{hi}$, its value will be $[0, 1]$. The degree of membership on the left of $LC_{jj'}^{hi}$ is 0; the value between $[LC_{jj'}^{hi}, 0]$ and $[MC_{jj'}^{hi}, 1]$ is continuous and strictly increasing. The value between $[MC_{jj'}^{hi}, 1]$ and $[RC_{jj'}^{hi}, 0]$ will be continuous and strictly decreasing. The degree of membership on the right of $RC_{jj'}^{hi}$ is 0. Likewise, the membership function of $\widetilde{SC}_{jj'}^{hi}$ enjoys similar form as $\widetilde{CC}_{jj'}^{hi}$. Values of $\widetilde{CC}_{jj'}^{hi}$ and $\widetilde{SC}_{jj'}^{hi}$ are not exactly the same; nor are $\widetilde{SC}_{jj'}^{hi}$ and $\widetilde{SC}_{j'j}^{hi}$.

R experts share diverse views of the fuzzy degree of complementarity on complementary investment projects, and at this point majority rule is applied to decide the fuzzy degree of complementarity $\widetilde{CC}_{jj'}^{hi}$ among R experts of consensus,

$$\widetilde{CC}_{jj'}^{hi} = (LC_{jj'}^{Mi}, MC_{jj'}^{Mi}, RC_{jj'}^{Mi}), \quad \forall i, \quad (24)$$

where $LC_{jj'}^{Mi}$, $MC_{jj'}^{Mi}$, and $RC_{jj'}^{Mi}$ indicate the consensual part of their judgment of degree of complementarity of M experts towards x_j and $x_{j'}$ investment projects. In terms of $LC_{jj'}^{Mi}$, its method of decision will be like the classification of investment projects. In the beginning, $LC_{jj'}^{hi}$ ($h = 1, 2, \dots, R$) values will be ranked according to each value from the greatest to the smallest, then the value of M will be decided by Eq. (19). At last the M th $LC_{jj'}^{Mi}$ value will be found. As for $MC_{jj'}^{Mi}$ and $RC_{jj'}^{Mi}$ values, they can be decided similarly. Also, the fuzzy degree of substitution $\widetilde{SC}_{jj'}^{Mi}$ shared by most of the experts of consensus can also be spotted:

$$\widetilde{SC}_{jj'}^i = (LS_{jj'}^{Mi}, MS_{jj'}^{Mi}, RS_{jj'}^{Mi}), \quad \forall i. \quad (25)$$

The degree of correlation will have direct impact on the degree of objective achievement of investment projects and on whether they can be selected. If the average value of R experts is taken, overestimation may occur as there are some greater outliers. Yet if $\widetilde{CC}_{jj'}^i$ and $\widetilde{SC}_{jj'}^i$ are obtained, as outlined in this paper, they point out that the consensual part of most of the experts will be the intersection of these experts. Thus overestimation is avoided.

4. Fuzzy multiobjective programming model and its algorithm

4.1. Fuzzy measurement and aggregation

Methods such as quantitative and qualitative are available for the measurement of objective achievement under m objectives, necessary workable amount of q resource and the affordable amount of q resources of n transportation investment projects. The former can use statistical method or experiential rule to estimate, such as the achievement situation of quantifiable objective, necessary workable amount of q resource, and affordable amount of q resource. The latter will call for the judgment of experts, such as the achievement situation of unquantifiable objective. Under future uncertainty, both quantifiable and unquantifiable items cannot be demonstrated in exact numbers and their measurement values will be a fuzzy number form. This paper will simplify measurement work and demonstrate every item of measurement value in TFN.

4.1.1. Measurement of quantifiable item

If m_1 ($m_1 \leq m$) quantifiable objectives are displayed as set C , then the fuzzy achievement value $\tilde{T}_{i'j}$ ($i' = 1, 2, \dots, m_1$; $j = 1, 2, \dots, n$) of n investment projects towards m_1 quantifiable objectives can be shown in TFN as follows:

$$\tilde{T}_{i'j} = (LT_{i'j}, MT_{i'j}, RT_{i'j}), \quad i' \in C, \quad (26)$$

where $LT_{i'j}$, $MT_{i'j}$ and $RT_{i'j}$ are real numbers, and $LT_{i'j} \leq MT_{i'j} \leq RT_{i'j}$.

In terms of the fuzzy amount \tilde{A}_{kj} ($k = 1, 2, \dots, q$; $j = 1, 2, \dots, n$) of the necessary utilizable q resources and affordable amount \tilde{B}_k of q resources of n investment projects, they can be measured according to the statistical method and experiential judgment by the investment planning group and be shown in TFN as follows:

$$\tilde{A}_{kj} = (LA_{kj}, MA_{kj}, RA_{kj}), \quad \forall k, \quad (27)$$

$$\tilde{B}_k = (LB_k, MB_k, RB_k), \quad \forall k, \quad (28)$$

where $LA_{kj} \leq MA_{kj} \leq RA_{kj}$ and $LB_k \leq MB_k \leq RB_k$.

4.1.2. Measurement of unquantifiable items

The major unquantifiable items are the unquantifiable m_2 objectives ($m_2 \leq m, m_1 + m_2 = m$) and are indicated as set O . The achievement value of m_2 unquantifiable objectives of n investment projects relies on R experts from relevant speciality for their professional experiences to proceed to subjective judgment. Experts can use linguistic variables as “High”, “Medium”, and “Low” to group, and each linguistic variable can be demonstrated in TFN as well as in integer scale $\{0, 1, 2, \dots, L\}$ for classification; for instance, an expert considered the range for “High” is 60–80 (L will be 100), and the more exact value will be 70. If $\tilde{T}_{i''j}$ ($i'' = 1, 2, \dots, m_2$; $j = 1, 2, \dots, n$) indicates the fuzzy achievement value of x_j investment projects towards unquantifiable objectives, it can be displayed in TFN as follows:

$$\tilde{T}_{i''j} = (LT_{i''j}, MT_{i''j}, RT_{i''j}), \quad i'' \in O \quad (29)$$

where $LT_{i''j} \leq MT_{i''j} \leq RT_{i''j}$.

R experts will attain several judgments towards achieving m_2 unquantifiable objectives degree of n investment projects. This paper will integrate preference of R experts in average, where $\tilde{\tilde{T}}_{i''j}$ indicates the balanced view of R experts towards x_j investment project achieving i'' objective ($i'' \in O$); then

$$\tilde{\tilde{T}}_{i''j} = \left(\frac{1}{R} \right) \ominus [\tilde{T}_{i''j}^1 \oplus \tilde{T}_{i''j}^2 \oplus \dots \oplus \tilde{T}_{i''j}^R], \quad i'' \in O. \quad (30)$$

Where \ominus symbolizes the fuzzy multiplication, $\tilde{T}_{i''j}$ will be shown in TFN as follows:

$$\tilde{T}_{i''j} = (\overline{LT}_{i''j}, \overline{MT}_{i''j}, \overline{RT}_{i''j}), \quad i'' \in O, \quad (31)$$

where

$$\overline{LT}_{i''j} = \left(\sum_{h=1}^R LT_{i''j} \right) / R. \quad (32)$$

The rest of $\overline{MT}_{i''j}$ and $\overline{RT}_{i''j}$ can be spotted likewise.

According to the measurement results of quantifiable and unquantifiable objectives, the achievement degree (\tilde{G}_{ij}) of m objectives can be expressed in following general equations:

$$\tilde{G}_{ij} = (LG_{ij}, MG_{ij}, RG_{ij}), \quad \forall i, j, \quad (33)$$

$$LG_{ij} = \begin{cases} LT_{i'j}, & i = i' \in C, \\ \overline{LT}_{i''j}, & i = i'' \in O, \end{cases} \quad (34)$$

$$MG_{ij} = \begin{cases} MT_{i'j}, & i = i' \in C, \\ \overline{MT}_{i''j}, & i = i'' \in O, \end{cases} \quad (35)$$

$$RG_{ij} = \begin{cases} RT_{i'j}, & i = i' \in C, \\ \overline{RT}_{i''j}, & i = i'' \in O. \end{cases} \quad (36)$$

4.2. Model formulation

The necessary total amount of q resources for n transportation investment projects will be signified as (\tilde{U}_k) and displayed as follows:

$$\tilde{U}_k = \sum_{j=1}^n \tilde{A}_{kj} = (LU, MU, RU) = \left(\sum_{j=1}^n LA_{kj}, \sum_{j=1}^n MA_{kj}, \sum_{j=1}^n RA_{kj} \right). \quad (37)$$

If $\tilde{B}_k = \tilde{U}_k$ ($k = 1, 2, \dots, q$), n transportation investment projects can be selected for investment in terms of all k ; under such circumstances, its total fuzzy performance value (\tilde{Z}^*) of the achieved m objectives will be the greatest, that is

$$\tilde{Z}^*(x) = (\tilde{Z}_1^*(x), \dots, \tilde{Z}_i^*(x), \dots, \tilde{Z}_m^*(x)), \quad (38)$$

where

$$\tilde{Z}_i^*(x) = (LZ_i, MZ_i, RZ_i), \quad \forall i \quad (39)$$

and

$$\begin{aligned} \tilde{Z}_i^*(x) &= \sum_{j \in A^T} \tilde{G}_{ij} \oplus \sum_{j, j' \in A^C} [\widetilde{CC}_{jj'}^i \tilde{G}_{ij'} \oplus \widetilde{CC}_{j'j}^i \tilde{G}_{ij}] \\ &\oplus \sum_{j \in A^S} \left\{ \widetilde{\max} \left(\sum_{j' \in A^S} \widetilde{\min}[(\widetilde{SC}_{jj'}^i \tilde{G}_{ij'} \ominus \widetilde{SC}_{j'j}^i \tilde{G}_{ij}), 0] \right), -\tilde{G}_{ij} \right\}, \quad \forall i. \end{aligned} \quad (40)$$

The first item is the total objective achievement value of n investment projects towards objective i ; the second item is the increased performance value due to complementarity arisen from complementary projects;

and the third item is the decreased performance value due to substitution occurring from substitutive projects. There cannot be any increment of objective performance value among every two substitutive investment projects; also, the transition of substitution to other investment projects does not exist. Thus, $\widetilde{\min}$, denoting the invalidated part of substitution and substituted, cannot be a positive value, while $\widetilde{\max}$ denotes the maximal achievable part (\tilde{G}_{ij}) by itself of x_j investment projects and that part can be wholly absorbed by other $x_{j'}$ ($j' \in A^S$), yet not exceeding the value of \tilde{G}_{ij} . In addition, $\widetilde{\max}(\cdot)$ and $\widetilde{\min}(\cdot)$ denote the maximum and minimum operation of two fuzzy numbers. If $\tilde{A} = (LA, MA, RA)$, $\tilde{B} = (LB, MB, RB)$ and $\tilde{0} = (0, 0, 0)$, then $\widetilde{\max}(\tilde{A}, \tilde{B})$ and $\widetilde{\min}(\tilde{A}, \tilde{0})$ can be thus defined:

$$\widetilde{\max}(\tilde{A}, \tilde{B}) = [(LA \vee LB), (MA \vee MB), (RA \vee RB)], \quad (41)$$

$$\widetilde{\min}(\tilde{A}, \tilde{0}) = [(LA \wedge O), (MA \wedge O), (RA \wedge O)]. \quad (42)$$

Since fuzzy multiplication is rather complex, calculation has been simplified so as to improve computation efficiency in practice, and the finding of approximate fuzzy numbers would be quite enough (Kaufmann and Gupta, 1988). If the two TFNs are, respectively, $\tilde{A}_1 = (a_1, b_1, c_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2)$, the approximate multiplication computation is as follows:

$$\tilde{A} = \tilde{A}_1 \oplus \tilde{A}_2 = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2) = (a_1 a_2, b_1 b_2, c_1 c_2). \quad (43)$$

The constructed solution $\tilde{Z}^*(x)$ of m , $\tilde{Z}_i^*(x)$ is called an ideal solution; in fact, this ideal solution is hardly attainable because the affordable resources are scarce, thus

$$\tilde{U}_k > \tilde{B}_k \Rightarrow (LU_k \geq LB_k, MU_k \geq MB_k, RU_k > RB_k). \quad (44)$$

Therefore, the issue of investment project selection occurs under such situation.

During the selection of investment projects, the process of normalization has to be put forward first, as the measurement scale of m objectives and q resources is different. This paper will abide by the following equation to proceed with the normalization:

$$\tilde{f}_{ij} = \tilde{G}_{ij} \textcircled{R} RZ_i, \quad \forall i, j, \quad (45)$$

$$\tilde{h}_{ij} = \tilde{A}_{in} \textcircled{R} RB_i, \quad \forall i, j, \quad (46)$$

where \textcircled{R} denotes fuzzy division.

FMOTIPS (3) problem can be transformed into scaled value optimization problem after normalization:

$$\text{FMOTIPS(4):} \quad \text{maximize} \quad \tilde{\phi}(x) = \sum_{i=1}^m \sum_{j=1}^n w_i \tilde{f}_{ij} x_j \quad (47a)$$

$$\text{minimize} \quad \tilde{I}(x) = \sum_{k=1}^q \lambda_k \left[\tilde{I}_k \ominus \sum_{j=1}^n \tilde{h}_{kj} x_j \right] \quad (47b)$$

$$\text{subject to} \quad \sum_{j=1}^n \tilde{h}_{kj} x_j \leq \tilde{I}_k, \quad \forall k, \quad (47c)$$

$$x_j = 0 \text{ or } 1, \quad \forall j, \quad (47d)$$

where (47a) equation is to transform m objective into a scalar objective $\tilde{\phi}(x)$, (47b) equation is to transform the equation of q resource working efficiency objective equation into a scaled objective equation $\tilde{I}(x)$, (47c)

equation is a resource limit equation. In Eqs. (47b) and (47c), \tilde{l}_k indicates the normalization of affordable amount B_k of k resource in TFN, that is

$$\tilde{l}_k = (LB_k/RB_k, MB_k/RB_k, 1). \quad (48)$$

Weights of m objectives and q resources can be found through the eigenvector method proposed by Saaty (1977), then the geometric mean of R experts can be obtained as the weights $W = (w_1, w_2, \dots, w_m)$ for m objectives and the weight $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_q)$ for q resources.

4.3. Algorithm

Limited by q resource and wishing to achieve m objectives by TIPS issues, the known n transportation investment projects are considered an 0–1 MFLP problem under such fuzzy environment. This paper will follow the spirit of spatial algorithm [21] and have it expanded to the management of fuzzy data. Before further elaboration of algorithm, symbols have to be defined first:

I_t : set of selected investment projects

A_t^T : set of all investment projects

\tilde{G}_i^t : the fuzzy achievement value on i objective by all investment projects within the set I_t :

$$\begin{aligned} \tilde{G}_i^t = & \sum_{j \in I_t} \tilde{f}_{ij} \oplus \sum_{j, j' \in (I_t \cap A^C)} [\tilde{CC}_{jj'}^i \tilde{f}_{ij'} \oplus \tilde{CC}_{j'j}^i \tilde{f}_{ij}] \\ & \oplus \sum_{j \in (I_t \cap A^S)} \left\{ \max \left(\sum_{j' \in (I_t \cap A^S)} \min \left[\left(\tilde{SC}_{jj'}^i \tilde{G}_{ij'} \ominus \tilde{SC}_{j'j}^i \tilde{G}_{ij} \right), 0 \right] \right), -\tilde{f}_{ij} \right\}, \quad \forall i. \end{aligned} \quad (49)$$

\tilde{G}^t : the fuzzy vector of m objectives achievement, respectively, by all investment projects within the set I_t :

$$\tilde{G}^t = (\tilde{G}_1^t, \dots, \tilde{G}_i^t, \dots, \tilde{G}_m^t). \quad (50)$$

\tilde{R}_k^t : k 's fuzzy resource requirement by all investment projects within the set I_t :

$$\tilde{R}_k^t = \sum_{j \in I_t} \tilde{h}_{kj}, \quad \forall k. \quad (51)$$

\tilde{R}^t : fuzzy vector of q resource utilization, respectively, required by all investment projects within the set I_t :

$$\tilde{R}^t = (\tilde{R}_1^t, \dots, \tilde{R}_k^t, \dots, \tilde{R}_q^t). \quad (52)$$

F_t : set of candidate investment projects, i.e.,

$$F_t = A_t^T \setminus \{x_j \in A_t^T \mid \tilde{R}_k^t \oplus \tilde{h}_{kj} > \tilde{l}_k, \exists k\}. \quad (53)$$

Next, the efficiency concept of fuzzy spatial solution will be briefed. The attained fuzzy performance index of x_j investment projects in objective space is denoted by \tilde{FB}_j^t . Since x_j could be of A^I, A^C , and A^S situations, \tilde{FB}_j^t can be solved, respectively, by the following equations:

$$\tilde{FB}_j^t = \left[\prod_{i=1}^m (\tilde{G}_i^t \oplus \tilde{f}_{ij}) \right]^{1/m}, \quad j \in (F_t \cap A^I), \quad (54)$$

$$\widetilde{FB}_j^t = \left[\overline{\prod}_{i=1}^m \left\{ \tilde{G}_i^t \oplus \tilde{f}_{ij} \oplus \sum_{j' \in I_t} (\widetilde{CC}_{jj'}^i \tilde{f}_{ij'} \oplus \widetilde{CC}_{j'j}^i \tilde{f}_{ij}) \right\} \right]^{1/m}, \quad j \in (F_t \cap A^C), \quad (55)$$

$$\widetilde{FB}_j^t = \left[\overline{\prod}_{i=1}^m \left\{ \tilde{G}_i^t \oplus \widetilde{\max} \left\{ \sum_{j' \in I_t} \widetilde{\min}[(\widetilde{SC}_{jj'}^i \tilde{f}_{ij'} \ominus \widetilde{SC}_{j'j}^i \tilde{f}_{ij}), 0], -\tilde{f}_{ij} \right\} \right\} \right]^{1/m}, \quad j \in (F_t \cap A^S) \quad (56)$$

$$\begin{aligned} \widetilde{FB}_j^t = & \left[\overline{\prod}_{i=1}^m \left\{ \tilde{G}_i^t \oplus \left[\sum_{j' \in I_t} (\widetilde{CC}_{jj'}^i \tilde{f}_{ij'} \oplus \widetilde{CC}_{j'j}^i \tilde{f}_{ij}) \right] \right. \right. \\ & \left. \left. \oplus \widetilde{\max} \left\{ \sum_{j' \in I_t} \widetilde{\min}[(\widetilde{SC}_{jj'}^i \tilde{f}_{ij} \ominus \widetilde{SC}_{j'j}^i \tilde{f}_{ij}), 0], -\tilde{f}_{ij} \right\} \right\} \right]^{1/m}, \quad j \in (F_t \cap A^{CS}) \end{aligned} \quad (57)$$

where $\widetilde{\prod}$ denotes the cumulative fuzzy product of fuzzy numbers, i.e.,

$$\overline{\prod}_{i=1}^m \tilde{G}_i = \tilde{G}_1 \ominus \tilde{G}_2 \ominus \cdots \ominus \tilde{G}_m. \quad (58)$$

The purpose of m root is to make the scale enlarged to [0,1]. For practical application it is of interest to consider simplification and obtain an approximate fuzzy number in terms of a TFN. If a TFN is $\tilde{A} = (a, b, c)$, then approximate $(\tilde{A})^{1/m}$ by \tilde{B} as

$$\tilde{B} = (\tilde{A})^{1/m} = (a, b, c)^{1/m} = (a^{1/m}, b^{1/m}, c^{1/m}). \quad (59)$$

As to the resource space, the fuzzy resource requirement index of x_j investment projects can be denoted by \widetilde{FB}_j^t and solved by the following equation:

$$\widetilde{FR}_j^t = \left[\overline{\prod}_{k=1}^q (\tilde{R}_k^t \oplus \tilde{h}_{kj}) \right]^{1/q}, \quad j \in F_t. \quad (60)$$

If $m = 2$ and $q = 2$, \widetilde{FB}_j^t and \widetilde{FR}_j^t are for finding out the fuzzy size of the objective space and resource space. \widetilde{FB}_j^t is the constructed fuzzy size (as in Fig. 2(a), the range from ABCD to AEFG) of the cumulated value of the two achieved objectives. If the interdependence of investment projects are considered, then the constructed fuzzy size of the cumulated value of the two achieved objectives will increase because of complementarity or decrease due to substitution (as shown in Fig. 2(a). Due to the existence of interdependent projects, the range of objective achievement value will increase from AB'C'D' to AE'F'G') \widetilde{FR}_j^t is the constructed fuzzy size of the two resources' cumulated working amount (as shown in Fig. 2(b) with its range from QRST to QUVW).

Based upon the two fuzzy efficiency index of \widetilde{FB}_j^t and \widetilde{FR}_j^t , the fuzzy index of the profitability \widetilde{PR}_j^t of x_j investment project can be thus defined:

$$\widetilde{PR}_j^t = \widetilde{FB}_j^t \circledR \widetilde{FR}_j^t, \quad j \in F_t. \quad (61)$$

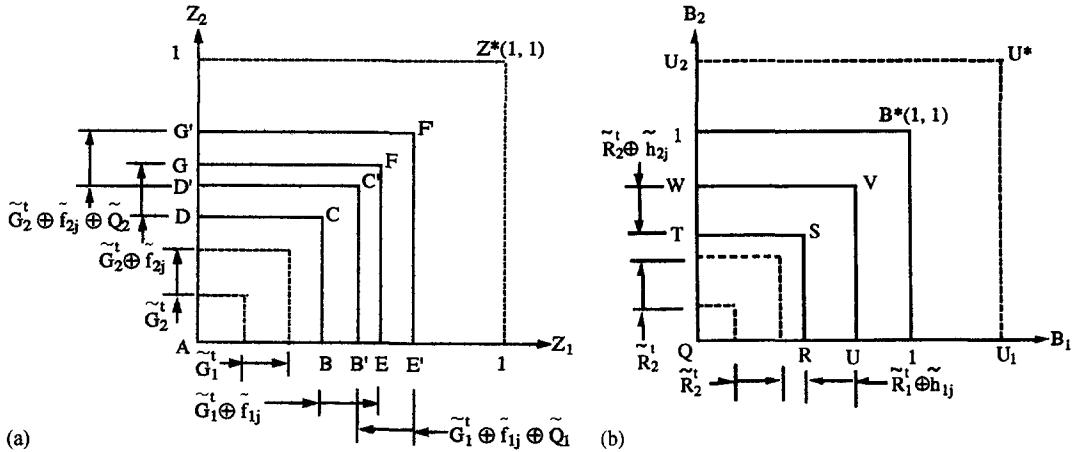


Fig. 2. The fuzzy efficiency index of objective achievement and resource use: (a) fuzzy performance of objective achievement; (b) fuzzy requirement of resource use.

From Eqs. (54)–(61), approximate TFNs of \widetilde{FB}_j^t and \widetilde{FR}_j^t can be solved, and in practice approximate fuzzy numbers [13] would be good enough since fuzzy division is rather complicated. If the two TFNs are, respectively, $\tilde{A}_1 = (a_1, b_1, c_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2)$, then

$$\tilde{A} = \tilde{A}_1 \circledast \tilde{A}_2 = (a_1, b_1, c_1) \circledast (a_2, b_2, c_2) = (a_1/c_2, b_1/b_2, c_1/a_2). \quad (62)$$

In conclusion, the greatest fuzzy index of the profitability \widetilde{PR}_j^t will be used as the selection criteria for x_j investment projects in t iteration, that is

$$\widetilde{PR}_{j*}^t = \widetilde{\max}_{j \in F_t} \{\widetilde{PR}_j^t\}, \quad x_{j*}^t = 1 \text{ and } x_{j*}^t \in I_t, \quad (63)$$

where $\max(*)$ denotes the extended maximum operation of two fuzzy sets [11]. This paper will explain the ranking of fuzzy numbers in Section 4.4 so as to find out the best fuzzy number of profitability in t iteration.

Founded on previous definitions and summary of fuzzy spatial algorithm, we can spot the approximate solution to an FMOTIPS (4) problem. Solutions disclosing steps of fuzzy spatial algorithm can be summarized as follows:

Step 1: Set the initial value

$$t = 1, \quad I_1 = \emptyset, \quad A_1^T = \{x_1, x_2, \dots, x_n\}, \quad x = 0, \quad \forall j.$$

Step 2: Based on Eqs. (49) and (51), objective value and resource working amount of the investment projects within set I_t should be found separately.

Step 3: Eq. (53) is used to decide the investment projects within set F_t . If $F_t = \emptyset$ we will move directly to step 10.

Step 4: Find the value of \widetilde{FB}_j^t :

- (1) If $j \in (F_t \cap A^I)$, find out the value according to Eq. (54);
- (2) If $j \in (F_t \cap A^C)$, find out the value according to Eq. (55);
- (3) If $j \in (F_t \cap A^S)$, find out the value according to Eq. (56);
- (4) If $j \in (F_t \cap A^{CS})$, find out the value according to Eq. (57).

Step 5: Find out the value of \widetilde{FR}_j^t according to Eq. (60).

Step 6: Find out the \widetilde{PR}_j^t value of all the investment projects within set F_t according to Eq. (61).

Step 7: Rank all \widetilde{PR}_j^t TFNs and find out throughout t th iteration the selected investment project $x_{j^*}^t$, that is $x_{j^*} = 1$.

Step 8: $I_{t+1} = I_t \cup \{x_{j^*}\}$; $A_{t+1}^T = A_t^T \setminus \{x_{j^*}\}$.

Step 9: $t = t + 1$, back to step 2.

Step 10: The investment project within set I_t is the approximate solution to FMOTIPS (4).

4.4. Ranking of fuzzy numbers

Since 1976, many methods for ranking two or more fuzzy numbers have been suggested in the literature as surveyed in Bortolan and Degani [4]. All ranking methods are based on some types of indices. How to generate a proper index becomes the issue in devising ranking methods. Generally, they are classified into two approaches:

(1) *Methods using ranking function*, $f: F(\mathbf{R}) \rightarrow \mathbf{R}$, where $F(\mathbf{R})$ is the set of fuzzy numbers. Adamo [1], Yager [27], Chang [6], de Campos Ibanez and Gonzalez Munoz [8], and McCahon and Lee [16] have followed this approach.

(2) *Methods using comparison function*, $\tilde{D} = \{j / \mu_{\tilde{D}}(j)\}$, where $\mu_{\tilde{D}}(j)$ is the degree to which the j th project (alternative, plan, action, etc.) may be considered the best project. Jain [12], Baas and Kwakernaak [2], Baldwin and Guild [3], Watson et al. [24], Dubois and Prade [10], Chen [7], Ovchinnikov et al. [17], Tseng and Klein [22], and Kim and Park [14] have followed this approach.

For past developments of ranking methods, some were counter-intuitive and mainly placed thoughts on a specific discriminating index to determine the ups and downs of fuzzy numbers. Consequently, alternating ranking phenomena occurred as various ranking methods were employed for the same issue. Due to such a situation, Kim and Park [14] extended the method presented by Jain [12] and Chen [7] and brought optimistic index and pessimistic index into the comparison function. Meanwhile, they integrated optimistic and pessimistic indices according to the classical decision theory by Hurwicz (1957) and convex combination of fuzzy sets by Zadeh [28] and reached the comprehensive judgment index of participating decision maker's preference. Let

$$\widetilde{PR}_j^t = (LP_j^t, MP_j^t, RP_j^t), \quad x_j \in F_t. \quad (64)$$

Then the Kim–Park method should be used for \widetilde{PR}_j^t TFNs ranking to all the investment projects within set F_t so as to locate the fuzzy decision set $\tilde{D} = \{j / \mu_{\tilde{D}}(j)\}$ and find the greatest degree of membership $\mu_{\tilde{D}}(j)$ of the investment project x_j . Sometimes $\mu_{\tilde{D}}(j)$ can be found according to the following equation:

$$\mu_{\tilde{D}}(j) = k\mu_{\tilde{D}_o}(j) + (1 - k)\mu_{\tilde{D}_p}(j), \quad (65)$$

where k varies in $[0, 1]$ and indicates the attitude of the decision maker. If DM is an optimist, the value of k will be near 1 and its decision set will be demonstrated as $\tilde{D}_o = \{j / \mu_{\tilde{D}_o}(j)\}$; if DM is a pessimist, the value of k will be near 0 and its decision set will be demonstrated as $\tilde{D}_p = \{j / \mu_{\tilde{D}_p}(j)\}$. As for the membership grade $\mu_{\tilde{D}_o}(j)$ of every \widetilde{PR}_j^t which has the possibility of being maximum as well as the membership grade $\mu_{\tilde{D}_p}(j)$ which has the possibility of being minimum, they can be resolved according to the following equations:

$$\mu_{\tilde{D}_o}(j) = hgt(\widetilde{PR}_j^t \cap \tilde{G}_{\max}), \quad x_j \in F_t, \quad (66)$$

$$\mu_{\tilde{D}_p}(j) = 1 - hgt(\widetilde{PR}_j^t \cap \tilde{G}_{\min}), \quad x_j \in F_t, \quad (67)$$

$$\tilde{G}_{\max} = \{s / \mu_{\tilde{G}_{\max}}(s)\}, \quad s \in S, \quad (68)$$

$$\tilde{G}_{\min} = \{s / \mu_{\tilde{G}_{\min}}(s)\}, \quad s \in S, \quad (69)$$

$$\mu_{\tilde{G}_{\max}}(s) = (s - s_{\min})/(s_{\max} - s_{\min}), \quad (70)$$

$$\mu_{\tilde{G}_{\min}}(s) = (s_{\max} - s)/(s_{\max} - s_{\min}), \quad (71)$$

$$s_{\max} = \sup S, \quad (72)$$

$$s_{\min} = \inf S, \quad (73)$$

$$S = \bigcup_{j \in F_t} PR_j^t, \quad (74)$$

where $hgt \tilde{A}$ denotes the maximum value of membership function of \tilde{A} . S is a crisp support set (of universe of discourse) and s is an element of set S .

In terms of TIPS problems, experts from the decision group are to decide the weights of objectives and resources and the achieved degree of unquantifiable items; therefore, their work is to evaluate before selecting investment projects. Once investment projects reach the final stage, DMs or experts will not bring in any preference information. For this reason the Kim–Park method should be rectified. Besides, it is dubious that the Kim–Park method should hypothesize identical preferences for the optimistic and pessimistic degrees of every fuzzy number, because to the \tilde{PR}_j^t of every investment project, its degree of optimism and pessimism will have to depend on the profit ranges likely to be achieved, more or less. In other words, the profit range more likely to be achieved will be at $[MP_j^t, RP_j^t]$ as based on the TFN of \tilde{PR}_j^t , while the profit range likely to be cut down will be at $[LP_j^t, MP_j^t]$. The greater range of $[MP_j^t, RP_j^t]$ indicates the results of prior comprehensive evaluation by the decision group and reveals higher degree of optimism of profitability towards x_j investment project, and vice versa. As a matter of course, this paper has revised the Kim–Park method to become the following:

$$\mu_{\tilde{D}}(j) = k_j \mu_{\tilde{D}_o}(j) + (1 - k_j) \mu_{\tilde{D}_p}(j), \quad x_j \in F_t \quad (75)$$

and

$$k_j = (RP_j^t - MP_j^t) / (RP_j^t - LP_j^t), \quad (76)$$

where k_j indicates the optimistic degree of possible achievable profit of x_j investment project, and $k_j \in [0, 1]$. When $LP_j^t = MP_j^t$, and $k_j = 1$ denotes x_j investment project enjoys a complete optimistic degree. When $MP_j^t = RP_j^t$ and $k_j = 0$ symbolizes x_j investment project radiates a complete pessimistic degree.

Preference of DMs and experts will then be out of the scene during the selection of transportation investment projects after this rectification. In the meantime, the degree of optimism and pessimism of profitability of every investment project can be well manifested.

5. Illustrated example

5.1. Problem description

From the point of an entire regional transportation investment planning, it is hoped that the more profitable investment project can be selected to achieve the greatest designated objective from ten feasible transportation investment projects ($n = 10$) under three resource constraints. Four objectives ($m = 4$) desired to be achieved in this package of transportation investment planning are Z_1, Z_2, Z_3, Z_4 , while Z_1 and Z_2 are quantifiable objectives, Z_3 and Z_4 are not. Since these ten transportation investment projects are related in degree to the achievement of these four objectives, investment projects have to be classified in advance and have their degree of interdependence discovered.

Table 1
 $d_{jj''}^{hi}$ and $d_{j'j}^{hi}$

Projects	x_1	x_2	x_3	x_4	x_5
x_1	—	(0.1, 0.2, 0.6)	(0.2, 0.1, 0.3)	(0.8, 0.7, 0.6)	(0.1, 0.5, 0.2)
x_2	(0.1, 0.5, 0.6)	—	(0.1, 0.0, 0.3)	(0.8, 0.6, 0.9)	(0.1, 0.1, 0.3)
x_3	(0.2, 0.1, 0.3)	(0.1, 0.0, 0.3)	—	(0.1, 0.1, 0.3)	(0.3, 0.2, 0.2)
x_4	(0.8, 0.7, 0.6)	(0.8, 0.6, 0.9)	(0.1, 0.1, 0.3)	—	(0.5, 0.1, 0.3)
x_5	(0.1, 0.5, 0.2)	(0.1, 0.1, 0.3)	(0.3, 0.2, 0.2)	(0.5, 0.1, 0.3)	—
x_6	(0.8, 0.6, 0.3)	(0.3, 0.7, 0.6)	(0.3, 0.3, 0.2)	(0.9, 0.8, 0.4)	(0.4, 0.2, 0.1)
x_7	(0.4, 0.2, 0.3)	(0.1, 0.2, 0.2)	(0.1, 0.2, 0.4)	(0.2, 0.1, 0.1)	(0.3, 0.7, 0.1)
x_8	(0.7, 0.4, 0.7)	(0.4, 0.6, 0.5)	(0.3, 0.1, 0.1)	(0.3, 0.5, 0.7)	(0.1, 0.1, 0.0)
x_9	(0.0, 0.2, 0.1)	(0.2, 0.5, 0.2)	(0.6, 0.2, 0.1)	(0.2, 0.2, 0.1)	(0.1, 0.4, 0.1)
x_{10}	(0.1, 0.1, 0.2)	(0.1, 0.4, 0.6)	(0.1, 0.1, 0.0)	(0.1, 0.1, 0.5)	(0.2, 0.1, 0.3)
Projects	x_6	x_7	x_8	x_9	x_{10}
x_1	(0.8, 0.6, 0.3)	(0.4, 0.2, 0.3)	(0.7, 0.4, 0.7)	(0.0, 0.2, 0.1)	(0.1, 0.1, 0.2)
x_2	(0.3, 0.7, 0.6)	(0.1, 0.2, 0.2)	(0.4, 0.6, 0.5)	(0.2, 0.5, 0.2)	(0.1, 0.4, 0.6)
x_3	(0.3, 0.3, 0.2)	(0.1, 0.2, 0.4)	(0.3, 0.1, 0.1)	(0.6, 0.2, 0.1)	(0.1, 0.1, 0.0)
x_4	(0.9, 0.8, 0.4)	(0.2, 0.1, 0.1)	(0.3, 0.5, 0.7)	(0.2, 0.2, 0.1)	(0.1, 0.1, 0.5)
x_5	(0.4, 0.2, 0.1)	(0.3, 0.7, 0.1)	(0.1, 0.1, 0.0)	(0.1, 0.4, 0.1)	(0.2, 0.1, 0.3)
x_6	—	(0.2, 0.3, 0.6)	(0.2, 0.1, 0.1)	(0.1, 0.2, 0.5)	(0.3, 0.0, 0.1)
x_7	(0.2, 0.3, 0.6)	—	(0.1, 0.2, 0.1)	(0.3, 0.3, 0.1)	(0.2, 0.4, 0.5)
x_8	(0.2, 0.1, 0.1)	(0.1, 0.2, 0.1)	—	(0.2, 0.0, 0.1)	(0.1, 0.0, 0.1)
x_9	(0.1, 0.2, 0.5)	(0.3, 0.3, 0.1)	(0.2, 0.0, 0.1)	—	(0.0, 0.0, 0.0)
x_{10}	(0.3, 0.0, 0.0)	(0.2, 0.4, 0.5)	(0.1, 0.0, 0.1)	(0.0, 0.0, 0.0)	—

^a (a, b, c) represents a , b , and c expert's judgment value.

Let us say the decision group is composed of three experts ($R = 3$), its main responsibilities are to determine the type of investment projects, degree of interdependence, weights of objectives, and the achieved degree of unquantifiable objectives; it will also provide professional counseling as a planning team to the achieved degree of quantifiable objectives and resource working amount.

5.2. Classification of transportation investment projects

For these ten feasible transportation investment projects, some degree of interdependence can be traced among these investment projects under every objective, which includes independence, complementarity, substitution, and complementary substitution. To simplify the issue, common interdependence is assumed under these four objectives in this paper, and complementary and substitutive integrated judgment materials are obtained individually from the judgment results of the three experts, which are shown in Tables 1 and 2.

Based on the judgment results of Tables 1 and 2, complementary and substitutive investment projects can be spotted through the application of majority rule (this example takes $M = 2$) and postulation of fuzzy threshold values ($\bar{d} = 0.5$ and $\bar{e} = 0.5$) by the three experts, they are as follows:

$$A^C = \{x_1, x_2, x_4, x_6, x_8\}, \quad A^S = \{x_5, x_6, x_8, x_9\}.$$

According to Eqs. (7) and (8), common complementary substitutive and independent investment projects can be obtained as follows:

$$A^{CS} = \{x_6, x_8\}, \quad A^I = \{x_3, x_7, x_{10}\}.$$

Table 2
 $e_{jj'}^{hi}$ and $e_{j'j}^{hi}$

Projects	x_1	x_2	x_3	x_4	x_5
x_1	—	(0.0, 0.0, 0.1)	(0.5, 0.4, 0.3)	(0.2, 0.2, 0.1)	(0.2, 0.1, 0.6)
x_2	(0.0, 0.0, 0.1)	—	(0.3, 0.2, 0.2)	(0.2, 0.1, 0.1)	(0.4, 0.6, 0.2)
x_3	(0.5, 0.4, 0.3)	(0.3, 0.2, 0.2)	—	(0.2, 0.4, 0.2)	(0.3, 0.4, 0.6)
x_4	(0.2, 0.2, 0.1)	(0.2, 0.1, 0.1)	(0.2, 0.4, 0.2)	—	(0.6, 0.4, 0.3)
x_5	(0.2, 0.1, 0.6)	(0.4, 0.6, 0.2)	(0.3, 0.4, 0.6)	(0.6, 0.4, 0.2)	—
x_6	(0.0, 0.2, 0.0)	(0.1, 0.1, 0.3)	(0.7, 0.3, 0.3)	(0.6, 0.3, 0.1)	(0.7, 0.5, 0.9)
x_7	(0.4, 0.6, 0.2)	(0.3, 0.3, 0.6)	(0.2, 0.4, 0.1)	(0.2, 0.0, 0.0)	(0.3, 0.5, 0.1)
x_8	(0.1, 0.1, 0.1)	(0.2, 0.2, 0.2)	(0.1, 0.5, 0.1)	(0.1, 0.2, 0.5)	(0.5, 0.7, 0.8)
x_9	(0.2, 0.2, 0.8)	(0.1, 0.0, 0.2)	(0.2, 0.4, 0.5)	(0.1, 0.0, 0.3)	(0.5, 0.8, 0.6)
x_{10}	(0.6, 0.1, 0.4)	(0.0, 0.2, 0.1)	(0.3, 0.2, 0.3)	(0.2, 0.2, 0.0)	(0.1, 0.3, 0.0)
Projects	x_6	x_7	x_8	x_9	x_{10}
x_1	(0.0, 0.2, 0.0)	(0.4, 0.6, 0.2)	(0.1, 0.1, 0.1)	(0.2, 0.2, 0.8)	(0.6, 0.1, 0.4)
x_2	(0.1, 0.1, 0.3)	(0.3, 0.3, 0.6)	(0.2, 0.2, 0.2)	(0.1, 0.0, 0.2)	(0.0, 0.2, 0.1)
x_3	(0.7, 0.3, 0.3)	(0.2, 0.4, 0.1)	(0.1, 0.5, 0.1)	(0.2, 0.4, 0.5)	(0.3, 0.2, 0.3)
x_4	(0.6, 0.3, 0.1)	(0.2, 0.0, 0.0)	(0.1, 0.2, 0.5)	(0.1, 0.0, 0.3)	(0.2, 0.2, 0.0)
x_5	(0.7, 0.5, 0.9)	(0.3, 0.5, 0.1)	(0.5, 0.7, 0.8)	(0.5, 0.8, 0.6)	(0.1, 0.3, 0.0)
x_6	—	(0.2, 0.1, 0.1)	(0.6, 0.6, 0.5)	(0.8, 0.6, 0.6)	(0.2, 0.1, 0.3)
x_7	(0.2, 0.1, 0.1)	—	(0.4, 0.4, 0.5)	(0.1, 0.1, 0.0)	(0.3, 0.5, 0.3)
x_8	(0.6, 0.6, 0.5)	(0.4, 0.4, 0.5)	—	(0.8, 0.9, 0.4)	(0.1, 0.1, 0.1)
x_9	(0.8, 0.6, 0.6)	(0.1, 0.1, 0.0)	(0.8, 0.9, 0.4)	—	(0.0, 0.2, 0.3)
x_{10}	(0.2, 0.1, 0.3)	(0.3, 0.5, 0.3)	(0.1, 0.1, 0.1)	(0.0, 0.2, 0.3)	—

^a (a, b, c) represents a , b , and c expert's judgment value.

5.3. Determination of interdependent degree

The three experts will determine the fuzzy degree of complementarity and fuzzy degree of substitution of complementary investment projects and substitutive investment projects, and their results are shown in Tables 3 and 4.

From the information in Tables 3 and 4 and the majority rule (this instance takes $M = 2$) of Eqs. (24) and (25), consensual fuzzy degree of complementarity and fuzzy degree of substitution can be attained as shown in Tables 5 and 6.

5.4. Fuzzy measurement of objective achievement and resource utilization

In terms of objective fuzzy achievement value, measurement manner varies to whether it is quantifiable. Z_1 and Z_2 are quantifiable objectives whose fuzzy achievement values have located their possible domain through statistical estimation, and their measurement unit of objectives are, respectively, one billion per year and 1000 h per year. As Z_3 and Z_4 are unquantifiable objectives, their fuzzy achievement values will be liable to the subjective judgment of the three experts of the decision group according to their professional experiences, and the numerical value of their measurement scale will be from 0 to 100.

In terms of the resource fuzzy requirement, the planning team will first judge from their past experiences and with their statistical techniques, and then place it to be discussed and decided by experts of the decision group. The measurement units of B_1, B_2, B_3 are, respectively, billions, manpower-month, and vehicles; as for

Table 3
 $\widetilde{CC}_{jj'}^{hi}$ and $\widetilde{CC}_{j'j}^{hi}$

Projects	x_1	x_2	x_4	x_6	x_8
x_1	—	(0.2, 0.4, 0.5)	(0.1, 0.2, 0.3)	(0.2, 0.3, 0.4)	(0.0, 0.1, 0.2)
		(0.1, 0.1, 0.2)	(0.1, 0.3, 0.4)	(0.0, 0.1, 0.2)	(0.1, 0.2, 0.3)
		(0.0, 0.1, 0.2)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)
x_2	(0.1, 0.2, 0.5) (0.2, 0.2, 0.4) (0.3, 0.3, 0.4)	— (0.2, 0.3, 0.4) (0.1, 0.2, 0.3)	(0.3, 0.5, 0.7) (0.2, 0.3, 0.4) (0.0, 0.1, 0.2)	(0.2, 0.3, 0.4) (0.1, 0.2, 0.3) (0.0, 0.1, 0.2)	(0.0, 0.1, 0.1) (0.0, 0.0, 0.0) (0.0, 0.0, 0.0)
x_4	(0.1, 0.2, 0.3) (0.0, 0.1, 0.2) (0.0, 0.1, 0.1)	(0.0, 0.0, 0.0) (0.0, 0.1, 0.1) (0.0, 0.0, 0.0)	— (0.1, 0.2, 0.3) (0.0, 0.0, 0.1)	(0.1, 0.2, 0.3) (0.0, 0.1, 0.1) (0.0, 0.0, 0.1)	(0.2, 0.3, 0.4) (0.1, 0.2, 0.3) (0.1, 0.2, 0.4)
x_6	(0.0, 0.0, 0.0) (0.0, 0.0, 0.0) (0.0, 0.0, 0.1)	(0.0, 0.0, 0.0) (0.0, 0.0, 0.0) (0.0, 0.0, 0.0)	(0.2, 0.3, 0.4) (0.1, 0.1, 0.3) (0.2, 0.3, 0.5)	— (0.0, 0.0, 0.1) (0.0, 0.0, 0.0)	(0.1, 0.1, 0.2) (0.0, 0.0, 0.0) (0.0, 0.0, 0.0)
x_8	(0.2, 0.3, 0.4) (0.2, 0.3, 0.5) (0.3, 0.4, 0.6)	(0.0, 0.1, 0.3) (0.1, 0.2, 0.33) (0.1, 0.2, 0.4)	(0.0, 0.0, 0.0) (0.1, 0.2, 0.3) (0.0, 0.0, 0.0)	(0.0, 0.0, 0.0) (0.0, 0.0, 0.1) (0.0, 0.0, 0.0)	— (0.0, 0.0, 0.1) (0.0, 0.0, 0.0)

Table 4
 $\widetilde{SC}_{jj'}^{hi}$ and $\widetilde{SC}_{j'j}^{hi}$

Projects	x_5	x_6	x_8	x_9
x_5	—	(0.4, 0.6, 0.2)	(0.1, 0.1, 0.1)	(0.2, 0.2, 0.8)
		(0.3, 0.3, 0.6)	(0.2, 0.2, 0.2)	(0.1, 0.0, 0.2)
		(0.2, 0.4, 0.1)	(0.1, 0.5, 0.1)	(0.2, 0.4, 0.5)
x_6	(0.6, 0.3, 0.1) (0.7, 0.5, 0.9) (0.2, 0.1, 0.1)	— (0.2, 0.1, 0.1)	— (0.3, 0.5, 0.3)	(0.2, 0.2, 0.0) (0.5, 0.8, 0.6) (0.2, 0.1, 0.3)
x_8	(0.2, 0.1, 0.1) (0.6, 0.6, 0.5) (0.8, 0.6, 0.6)	(0.2, 0.1, 0.1) (0.4, 0.4, 0.5) (0.1, 0.1, 0.0)	— (0.1, 0.1, 0.1)	(0.3, 0.5, 0.3) (0.8, 0.9, 0.4) (0.2, 0.1, 0.1)
x_9	(0.2, 0.1, 0.3) (0.2, 0.1, 0.1) (0.2, 0.1, 0.1)	(0.3, 0.5, 0.3) (0.2, 0.1, 0.1) (0.2, 0.1, 0.1)	(0.1, 0.1, 0.1) (0.6, 0.6, 0.5) (0.4, 0.4, 0.5)	(0.2, 0.1, 0.1) (0.3, 0.5, 0.3)

the weights of the four objectives and three types of resources, the three experts will proceed to pairwise comparison and eigenvector method to find their weights. Then the weights of the three experts will be averaged out as $\mathbf{W} = (0.395, 0.130, 0.300, 0.175)$ and $\lambda = (0.444, 0.387, 0.169)$.

Fuzzy achievement values of the four objectives and fuzzy necessity amount of the three types of resources about these ten investment projects are specifically shown in Table 7. The greatest affordable amount of the three types of resources are, respectively, $\tilde{B}_1 = (250, 300, 400)$, $\tilde{B}_2 = (20, 30, 40)$, and $\tilde{B}_3 = (80, 100, 1200)$.

Table 5
 $\widetilde{CC}_{jj'}^i$ and $\widetilde{CC}_{j'j}^i$

Projects	x_1	x_2	x_4	x_6	x_8
x_1	—	(0.1, 0.1, 0.2)	(0.1, 0.2, 0.4)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)
x_2	(0.2, 0.2, 0.4)	—	(0.2, 0.3, 0.4)	(0.1, 0.2, 0.3)	(0.0, 0.0, 0.0)
x_4	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.0)	—	(0.0, 0.0, 0.0)	(0.1, 0.2, 0.4)
x_6	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.2, 0.3, 0.4)	—	(0.0, 0.0, 0.0)
x_8	(0.2, 0.3, 0.5)	(0.1, 0.2, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	—

Table 6
 $\widetilde{SC}_{jj'}^i$ and $\widetilde{SC}_{j'j}^i$

Projects	x_5	x_6	x_8	x_9
x_5	—	(0.0, 0.1, 0.2)	(0.1, 0.2, 0.2)	(0.0, 0.0, 0.0)
x_6	(0.1, 0.2, 0.4)	—	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.2)
x_8	(0.0, 0.0, 0.0)	(0.2, 0.4, 0.4)	—	(0.1, 0.2, 0.4)
x_9	(0.2, 0.3, 0.4)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	—

Table 7
The fuzzy numbers of objective achievement and resource requirement

Projects (weights)	Z_1 (0.395)	Z_2 (0.130)	Z_3 (0.300)	Z_4 (0.175)	B_1 (0.444)	B_2 (0.387)	B_3 (0.169)
x_1	(30, 35, 40)	(36, 40, 48)	(24, 32, 40)	(60, 75, 82)	(40, 50, 70)	(3, 5, 6)	(12, 16, 18)
x_2	(48, 55, 66)	(28, 30, 36)	(28, 40, 60)	(42, 45, 54)	(50, 60, 70)	(3, 5, 7)	(15, 20, 22)
x_3	(18, 25, 33)	(8, 10, 13)	(12, 20, 32)	(23, 30, 38)	(30, 35, 40)	(2, 3, 5)	(10, 12, 15)
x_4	(26, 35, 42)	(50, 60, 68)	(20, 28, 40)	(48, 75, 90)	(60, 65, 75)	(4, 6, 8)	(10, 14, 15)
x_5	(15, 20, 24)	(18, 20, 25)	(8, 20, 28)	(24, 30, 36)	(20, 25, 35)	(1, 2, 4)	(5, 8, 10)
x_6	(26, 30, 38)	(25, 30, 36)	(28, 40, 60)	(45, 52, 63)	(30, 40, 50)	(3, 4, 5)	(5, 10, 12)
x_7	(30, 40, 50)	(14, 20, 30)	(40, 48, 60)	(30, 38, 45)	(40, 45, 55)	(3, 4, 6)	(10, 13, 15)
x_8	(47, 55, 63)	(20, 30, 34)	(40, 60, 80)	(42, 52, 60)	(60, 70, 85)	(3, 6, 8)	(12, 18, 20)
x_9	(50, 60, 70)	(35, 40, 50)	(48, 64, 88)	(67, 83, 80)	(40, 50, 65)	(6, 8, 10)	(16, 22, 24)
x_{10}	(40, 45, 54)	(16, 20, 30)	(32, 48, 72)	(30, 45, 57)	(50, 60, 75)	(3, 5, 6)	(15, 17, 19)
Total	(330, 400, 480)	(250, 300, 370)	(280, 400, 560)	(411, 525, 615)	(420, 500, 620)	(31, 48, 65)	(110, 150, 170)
Total resources provided					(250, 300, 40)	(20, 30, 40)	(80, 100, 120)

5.5. Investment project selection

Based upon types and degree of interdependence of these ten transportation investment projects, the ideal achievement value achieved under every single objective and without resource constraints will be:

$$\begin{aligned}\tilde{Z}_1^* &= (276.8, 444.0, 646.3); & \tilde{Z}_2^* &= (222.9, 363.0, 541.7); \\ \tilde{Z}_3^* &= (185.6, 429.8, 744.0); & \tilde{Z}_4^* &= (351.9, 602.6, 883.0).\end{aligned}$$

Table 8
 $\mu_{\tilde{D}}(j)$ index

Projects	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	Selected projects
x_1	0.2560	0.3772	0.4856	S	S	S	x_1
x_2	0.2151	0.3709	0.4401	0.4877	S	S	x_2
x_3	0.0619	0.1764	0.2291	0.2578	0.3825	0.4860	x_3
x_4	0.2496	0.4856	S	S	S	S	x_4
x_5	0.3283	0.3039	0.2983	0.3085	0.4476	0.4145	—
x_6	0.4917	S	S	S	S	S	x_6
x_7	0.2497	0.2993	0.2975	0.2233	0.3597	*	—
x_8	0.2742	0.2065	0.3134	0.4261	0.4873	S	x_8
x_9	0.3420	0.2124	0.2307	0.2233	0.3597	*	—
x_{10}	0.1905	0.2349	0.2569	0.2698	0.3854	*	—

Note: S denotes selected investment alternative at t iteration.

* denotes that the iteration of the amount of resource requirement is over constraint.

In terms of resource requirements, the amount of the three types of resources these ten investment projects will need is: $U_1 = (420, 500, 620)$; $U_2 = (31, 48, 65)$; $U_3 = (110, 150, 170)$.

Since affordable resources are limited, the issue of selection has thus occurred. Initially, we will proceed with normalization according to Eqs. (45) and (46), so as to even out effects of various measurement units and then resort to the fuzzy spatial algorithm recommended for solution in this paper. After seven iterations we will calculate the approximate solution as $\{x_1, x_2, x_3, x_4, x_6, x_8\}$. In every iteration, the index of every investment project $\mu_{\tilde{D}}(j)$ ($j \in F_t$) will be detailed as shown in Table 8.

Among these six selected investment projects, $\{x_3\}$ is an independent project, $\{x_1, x_2, x_4, x_6, x_8\}$ are complementary projects, and $\{x_6, x_8\}$ are common complementary substitutive projects. Of these six investment projects, their total achievement value towards the four objectives and the immobile amount of the three resources are, respectively, as follows:

$$\tilde{Z}_1 = (195, 235, 282); \quad \tilde{Z}_2 = (167, 200, 235); \quad \tilde{Z}_3 = (152, 220, 312); \quad \tilde{Z}_4 = (260, 329, 387);$$

$$\tilde{H}_1 = (-140, -20, 130); \quad \tilde{H}_2 = (-19, 1, 22); \quad \tilde{H}_3 = (-22, 10, 56).$$

6. Conclusions

The evaluation and selection of transportation investment projects is one of the major public investment decisions made by government. Under an ever complex social infrastructure and international environment, criterion for one single objective (promotion of economic development for instance) is no longer valid. Thus, varying objectives and utilizable resources must be considered. Thus, the decision obviously belongs to the dominion of multiobjective investment decision making.

Since the effects of objective achieved will only be disclosed in steps after transportation investment projects have been implemented for some time, it would then be very difficult to thoroughly evaluate the objective achievement value and resource working amount of every investment project at the incipient planning stage of transportation investment projects. For convenience sake, experts take advantage of statistical estimations and experiential rules so as to reach such plausible domain for objective achievement value and resource working amount, as befits practices. Because this paper will indicate such plausible domain in TFN; therefore, the selection of transportation of investment projects will be a fuzzy multiobjective investment decision problem.

Inherent interdependence among transportation investment projects exists, and this paper classifies four groups: independent, complementary, substitutive, and common complementary substitutive. Once complementary and substitutive modes are decided, independent and common complementary substitutive ones will be known. Employment of objective quantifiable materials to resolve the degree of complementarity and degree of substitution for complementary and substitutive modes is rather demanding, yet it is hoped that expert judgment can help classify types and determine measurement of interdependence. Majority rule is adopted so as to uncover consensual views of most of the experts.

The selection issue of transportation investment projects under the fuzzy multiobjective is a 0–1 MFLP problem. A greater amount of time and manpower will have to be expended for precisely locating the non-dominated solution. In practice, strict calculation for the optimal solution would not be necessary; on the contrary, simple computation is enough for a rather exact approximate solution. This paper uses the ideas of efficiency from objective space and resource space for spotting the fuzzy efficiency index of every investment project as well as the yardstick for selection. Having revised the Kim–Park [14] method, this paper wishes to consider degree of optimism and pessimism even without a decision maker's participation so as to compare the ups and downs of a fuzzy efficiency index bearing TFN.

As this paper manages the selection of transportation investment projects, weights of objective and resource will be handled in a nonfuzzy manner. Though in fact it is likely to be dealt within fuzzy weights, management in fuzzy operation will bring about greater bias. Moreover, for now only interdependence among objectives is taken care of in this paper; however, interdependence prevails also among resources, which poses another approach for future research. Finally, since transportation investment projects are a long-term planning, research direction should as well move toward fuzzy multistage investment planning so as to lessen elements of uncertainty.

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