Short Papers

Distribution of Current Induced on Metal-Strip Gratings by Plane Wave

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Abstract—In this paper, we present a rigorous analysis of current distribution induced on a metal-strip grating by an incident plane wave. The metal strips of the grating are characterized by a complex permittivity, with a large imaginary part to account for their finite conductivity. Such a scattering problem is formulated by the mode-matching method to determine the scattered fields everywhere, so that the volume distribution of current within a metal strip can be explicitly obtained. Numerical results are given to illustrate the effects of the dielectric constant of the surrounding media, as well as the incident angle and polarization on the current distribution induced by an incident plane wave. The air and metal modes form the basis for physical explanations of the numerical results obtained.

Index Terms—Current density, electromagnetic scatterings, gratings, periodic structures.

I. INTRODUCTION

In this paper, we present a new approach to the determination of current distribution on the metal strips of a grating, as induced by an incident plane wave. The metal strips are realistically characterized by a finite thickness and a complex dielectric constant with a large imaginary part to account for their large, but finite, conductivity. Thus, the metal grating can be treated as a dielectric one, in which the electromagnetic fields can be obtained everywhere, and this permits the evaluation of the volume distribution of current within the metal strips.

In the ideal case of perfectly conducting strips, the grating has been often assumed to have a vanishing thickness, and the induced current distributes over the conducting surface. It is well known that the current density is finite for the component perpendicular to the edges of the strips and exhibits singular behavior for the component parallel to the edges [1], [2]. Such a singular behavior can be related to the edge condition of a conducting wedge, which was first studied by Meixner [3]; for the strip-type transmission lines with perfectly conducting strips of zero thickness, the singular behavior had been included as a built-in factor in the assumed current distribution. For example, in the spectral-domain method, the unknown current distribution on a microstrip line or a coplanar waveguide had been expressed as a summation of basis functions which exhibit singular or smooth behavior near the edges, depending on the polarization of the guided wave. In doing so, the resultant system of linear equations derived from the electric-field integral equation can be solved with relative ease.

In the scattering of a plane wave by a metal grating, it is expected that the singular behavior of the field component parallel to the edge should still manifest itself in the current distribution on the metal strips, but should depend on other factors, such as the thickness of the grating. In fact, when the thickness is larger than the skin depth, as is usually the case in practice, we have to deal with the volume
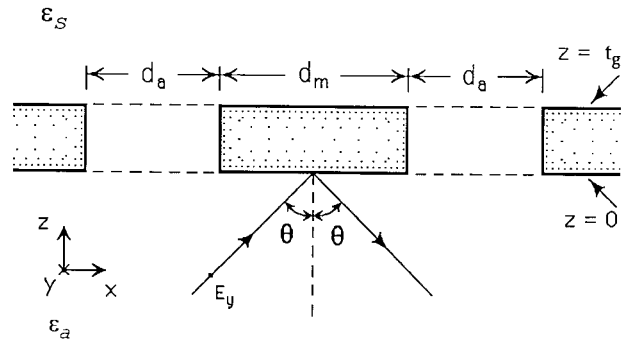
Fig. 1. Plane-wave scattering by a metal grating in a uniform medium.

distribution of current, which has not been studied so far, and this is what we set forth to do in this paper. Specifically, with a finite conductivity of metal considered, the electromagnetic fields in the grating region may be expressed as a superposition of Floquet-mode functions, so that we may investigate the volume current distribution on a metal-strip grating, as induced by an incident plane wave. Thus, the singular or smooth behavior of the current distribution may be interpreted in terms of the simple field variations of the Floquet-mode functions. This provides a transparent physical insight into the problem on hand.

II. STATEMENT OF PROBLEM AND THEORETICAL BACKGROUND

A metal-strip grating is sandwiched between two uniform media, and a plane wave is incident at an angle θ, as depicted in Fig. 1. For simplicity, the incident medium will be referred to as the air region, with the dielectric constants ε_a, and the transmitted medium will be referred to as the substrate region, with the dielectric constants ε_s. The metal strips are characterized by two parameters: the conductivity σ and the thickness t_g. In this paper, σ will take a very large, but finite, value, while t_g must take a nonzero value in order for the grating to be meaningful. The other grating parameters are: the width of metal strip d_m and the width of air space d_a. The period of grating is denoted by $d = d_m + d_a$. With a finite conductivity, the metal regions are characterized by a complex permittivity $\varepsilon_m = 1 - j60\sigma\lambda$, where λ is the incident wavelength. With such a characterization, the metal grating may be treated as a dielectric one, which had been extensively studied in the literature [4].

In this paper, our effort is to evaluate the current distribution that is related directly to the local fields on the surfaces of the grating; therefore, the results to be presented in this paper will be a critical test of the accuracy obtainable from the numerical analysis. In the case of periodic array of metal layers, the dispersion relation may be rewritten for either of the two polarizations in the following form [4]:

$$\sin \kappa_a d_a \, \sin \kappa_m d_m$$
$$= \frac{2Z_a Z_m}{Z_a^2 + Z_m^2} [\cos \kappa_a d_a \, \cos \kappa_m d_m - \cos(k_0 d \, \sin \theta_{\text{inc}})] \quad (1)$$

where κ_a and κ_m are the propagation constant, and Z_a and Z_m are the wave impedance in the air and metal region, respectively. The dispersion relation in (1) may be taken as a transcendental equation to solve for the roots of κ_a or κ_m, as will be further explained.

When the finite conductivity of the metal is taken into consideration, we have $\sigma < \infty$ and $Z_m \neq 0$, and the dispersion relation (1) can be manipulated into the following two equations:

$$\sin \kappa_a d_a = \Delta_a \qquad (2)$$

$$\sin \kappa_m d_m = \Delta_m \qquad (3)$$

where Δ_a and Δ_m are small, but nonvanishing, quantities for good conductors. It then follows that the two subsets of eigenvalues can be obtained as

$$\kappa_a d_a = n\pi + \sin^{-1} \Delta_a, \qquad \text{for } n = 0, 1, 2, \cdots \qquad (4)$$

$$\kappa_m d_m = n\pi + \sin^{-1} \Delta_m, \qquad \text{for } n = 1, 2, \cdots \qquad (5)$$

which reduce to the ideal sets in the limit of infinite conductivity. We observe that the dispersion roots in the case of a metal grating with a finite conductivity differ only slightly from those of the ideal case of perfectly conducting grating. The modes determined from (4) have their fields distributed mostly inside the air regions, with a tail penetrating laterally into the adjacent metal region due to the skin effect (for simplicity, they will be referred to as the air modes). On the other hand, those mode determined from (5) have their fields distributed almost sinusoidally across the metal strips, and they have their energy residing mostly inside the metal strips (for simplicity, they will be referred to as the metal modes). Mathematically, these two subsets of modes constitute a complete set to form a basis for a judicial representation of the electromagnetic fields inside the grating layer, as done in this paper.

With the dispersion roots explicitly determined as described above, the Floquet-mode functions can be obtained in a closed form, as is the current distribution associated with each mode. Referring to Fig. 1, we have the coordinate system with the origin at the center of the bottom surface of a metal strip. From the Floquet-mode functions of the grating region, the current distribution inside a metal strip comes from the contributions of both air and metal modes. For TE-polarized air modes, we have

$$J_y^{(a)}(x, z) = J_a(z) e^{-(\alpha_m + j\beta_m)(d_m/2 - x)}, \qquad \text{for } 0 < x < \frac{d_m}{2} \qquad (6)$$

where $J_a(z)$ represents the vertical variation of the current density to be determined by the boundary-value problem, and α_m and β_m are the lateral decay and propagation constants of the air mode, respectively. Such a current distribution is exponentially laterally decaying away from the edges of the strips; therefore, when the strips are very thin, the currents appear only near the edges, and are called the edge currents. From each mode function, it can be shown that $J_a(z)$ is proportional to $\sqrt{\sigma}$, the square root of the conductivity of the metal. As expected, when the conductivity is increased to infinity, the edge current will become singular in its distribution. On the other hand, for a TE-polarized metal mode, we have

$$J_y^{(m)}(x, z) = J_m(z) \cos \kappa_m x, \qquad \text{for } 0 < x < \frac{d_m}{2} \qquad (7)$$

where $J_m(z)$ represents the vertical variations of the current density to be determined by the boundary-value problem, and κ_m is the propagation constant of the metal mode in the metal region. Such a sinusoidal current distribution will be responsible for the surface current on the broadside of the metal strips.

III. NUMERICAL RESULTS

The current distribution associated with an individual mode can be obtained simply, as described above, and it will form a basis for our analysis. For the total current on a strip, it is necessary to analyze the
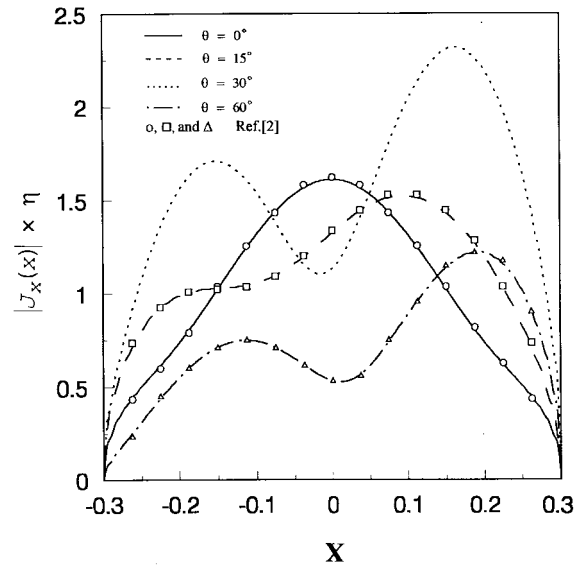

Fig. 2.   The induced sheet current distribution in the lateral direction for TM plane-wave incidence with different incident angles where $\eta = 120\pi$. The structure parameters are $\sigma = 5.8 \times 10^4$ S/mm, $d_a = d_m = 0.6$ mm, and $t_g = 10\delta$.

scattering problem as a rigorous boundary-value problem from which all the modal amplitudes in the grating region have to be determined. In this paper, it is noted that we consider only the special case of principal plane incidence, e.g., an incidence with the azimuth angle $\phi = 0°$, so that the polarization is conserved in the scattering process. Specifically, in such a special case, the electric field has only a single component (E_y) for the case of TE incidence, while it has two components (E_x and E_z) for the case of TM incidence. Thus, the induced current will flow along the metal strips for the case of TE incidence and will flow in the cross-sectional plane in the case of TM incidence.

In general, the current density for metal strips of finite thickness is a volume distribution, i.e., a function of the two cross-sectional coordinates. In order to compare with the published results by other methods, such as the spectral-domain approach [2], we first consider a grating of very small thickness ($t_g = 10\delta$), where δ is the skin depth of the metal. For the case of TM incidence, the induced current density has two components (J_x and J_z), as does the electric field. Since the strips are so thin that the J_z component of the current can be ignored, we present here numerical results only for the current component J_x. In order for a comparison with published results, we integrate the volume current density in the vertical direction to obtain a sheet-current density distributed in the lateral direction across a metal strip. Fig. 2 shows the results for four different angles of incidence: $\theta = 0°$, $\theta = 15°$, $\theta = 30°$, and $\theta = 60°$; they agree extremely well with those published in the literature [2]. It is noted that the case of $\theta = 30°$ was not shown in [2]; nevertheless, it is included here to stress the peculiar behavior of the induced current which is more pronounced than those at other larger or smaller incident angles. Such a phenomenon may be explained as follows. For the width of the metal strips chosen, we have the eigenvalue of the fundamental mode obtained from (5) as $\kappa_m = k_0/2$. For the incident angle of $\theta = 30°$, the propagation constants of the fundamental and $n = -2$ space harmonics are given by $k_{x, 0} = -k_{x, -2} = k_0/2$. This means physically that the metal modes happen to be in resonance with the space harmonics at the incident angle of $\theta = 30°$, leading to the pronounced effect on the current distribution.

In this paper, our analysis allows for the general case of a grating sandwiched between two half-spaces of different dielectric constants.
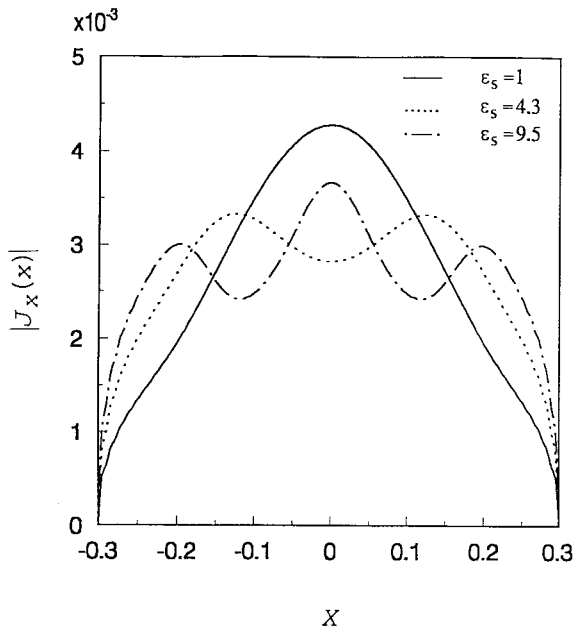
Fig. 3. The induced sheet current distribution in the lateral direction for TM plane-wave normal incidence with different upper half-space medium. The parameters are the same as in Fig. 2.
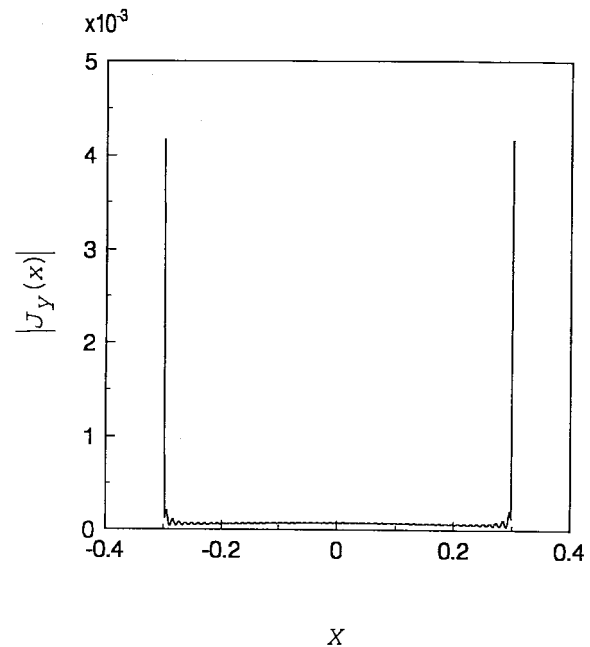

Fig. 4. The induced sheet current distribution in the lateral direction for TE plane-wave normal incidence. The parameters are the same as in Fig. 2.

Fig. 3 compares the current distributions induced by a plane wave at the normal incidence for three different values of ε_s, while keeping $\varepsilon_a = 1$ fixed. We observe that the current distributions are all symmetrical with respect to the center line of the strips. The reason for such a behavior is that the antisymmetrical metal modes cannot be excited because both the structure and excitation are symmetrical. From the shape of the current–distribution curve, the fundamental metal mode is the dominate one for the case of $\varepsilon_s = 1$, although the existence of the higher order symmetrical modes are evident. When the dielectric constant of the substrate is increased (say, $\varepsilon_s = 4.3$), the current distribution changes considerably; here, the second higher order symmetrical metal mode becomes important and it interferes with the fundamental one to produce the resonant behavior of the curve. As the dielectric constant of the substrate is further increased (say, $\varepsilon_s = 9.5$), the results show that the third higher order symmetrical metal mode comes into play. Evidently, the higher the dielectric constant, the larger the number of metal modes excited. Since the characteristic impedance of a plane wave in the substrate is proportional to $1/\sqrt{\varepsilon_s}$, the substrate becomes a medium of vanishing impedance in the limit of $\varepsilon_s \rightarrow \infty$. In this case, the presence of the thin metal strips on the interface between the air and substrate half-spaces is immaterial, because the whole interface has a uniformly vanishing impedance. When a plane wave is scattered by such an interface, only the specular reflection will occur, without the presence of any higher space harmonics. This means physically that the current distribution should appear piecewise uniform across the metal strips, which will require a large number of the metal modes to synthesize. In this paper, our results exhibit such a trend on the physical basis.

Consider now, the case of TE incidence onto the same structure as before. Fig. 4 shows the current distribution, exhibiting a very high concentration of near the edges of the strips. This is a case where

the electric field has only a single component parallel to the strips and must exhibit the singular behavior in the extreme case of infinite conductivity, as proven by Meixner [3]. With this paper's method, the edge current comes from the contribution of the air modes, as explained in Section II, and it should have a finite amplitude, as shown here. In comparison, the current distribution obtained from our calculation is more concentrated than those previously published [2].

IV. CONCLUSION

The scattering of a plane wave by a metal grating is analyzed by the mode-matching method, and the distribution of current induced on the metal strips are determined in terms of the Floquet modes of the grating layer. The effects of the incident angle and dielectric constant on the current distribution are investigated for incident plane waves of both polarizations. This approach yields transparent insight into the physical processes involved and provides a better understanding of the scattering of a plane wave by a metal grating.

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