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Computer-generated holographic diffuser for color mixing

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Abstract

An iterative algorithm for multiple incoming wavelengths is proposed to design a computer-generated holographic diffuser with the features of high light utilization efficiency and uniform color mixing. The amplitude transmittance of this computer-generated holographic diffuser is constant, while the phase transmittance consists of multiple phase levels. This diffuser is designed according to the Fraunhofer diffraction theory and can uniformly spread the different input light at the same time. The simulation results show that over 88% average far field amplitude transmission efficiency is achieved for each input light and the output uniformity is within 12.4%. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Computer-generated holograms; Holographic diffuser; Diffractive optical element

1. Introduction

The flexibility of optical systems design was enlarged by the invention of computer-generated holograms (CGH) and diffractive optical elements (DOE). They permit modifications of the phase, amplitude, or both of the incoming wavefront in an unlimited manner. Particularly, the phase-only hologram has received much attention due to its high light utilization efficiency.

Many different applications (such as diffuser and beam shaper) for CGH (or DOE) have been reported recently [1–4]. For the case of the diffuser, the phase distribution of a CGH is carefully designed such that the light distribution is uniformly distributed at the far field behind the diffuser. As for the beam shaper, the CGH makes the light distribution at the far field become any prescribed pattern. In addition, several iterative algorithms [5–9] have been reported to design these optical elements.

The current techniques for CGH and DOE design are only suitable for monochromatic light sources. Here we modify the existing iterative algorithm for the computer-generated holographic diffuser (CGHD) design that will be used for color mixing. Our goal is to construct a diffuser by computer-generated holographic technique for mixing different input chromatic lights. Although CGHD is designed here, our idea can be applied to design any optical element when its inputs consist of several different wavelengths. A key feature with our CGHD is its 100% light utilization efficiency such that no energy will be lost when the light passed through a diffuser. Numerical simulation shows that our CGHD achieves uniform color mixing and high far field amplitude transmission efficiency.

2. Model

We consider the system geometry shown in Fig. 1 in our CGHD design. Here the random phase transmittance of the CGHD is directly encoded in the computer-generated hologram, while the amplitude transmittance is constant. The CGHD is illuminated by input chromatic plane waves (with wavelengths $\lambda_1, \lambda_2, \ldots, \lambda_n$) and the Fraunhofer

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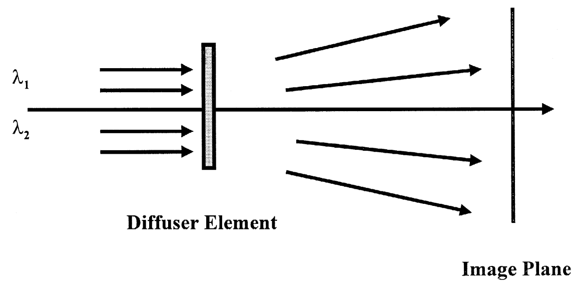


Fig. 1. Schematic of a computer-generated holographic diffuser for color mixing. The diffuser is illuminated by different plane waves with multiple wavelengths.

diffraction theory is used to estimate the diffraction pattern at the far field. To simplify our explanation, only the one-dimensional case is considered.

The transmittance function $f(x)$ of the CGHD is represented solely by the phase variation as

$$f(x) = |f(x)| \exp[j\phi(x)] = \exp[j\phi(x)]. \quad (1)$$

When a coherent plane wave (with wavelength λ_1, λ_2, $\ldots$, λ_n) impinges on this CGHD, the diffraction pattern at a distance z away from the CGHD plane is

$$f_z(x) \propto f(x) \otimes \exp\left(j\frac{\pi x^2}{\lambda z}\right), \quad (2)$$

where $\otimes$ is the convolution operator and $f_z(x)$ is the light distribution at distance z behind the CGHD plane according to the Fresnel-Kirchhoff theorem if z is sufficiently large. Generally speaking when $z > 2L^2/\lambda$, where L is the linear aperture of the CGHD, and the output pattern $f_z(x)$ of the CGHD is equivalent to the Fourier transform of the function $f(x)$. That is,

$$f_z(x) \approx F(\xi) = \int f(x) \exp(j2\pi x\xi)\, \mathrm{d}x$$

$$= |F(\xi)| \exp[j\psi(\xi)]. \quad (3)$$

The proposed iterative algorithm for CGHD design is shown schematically in Fig. 2. Without loss of generality, we assume that the inputs only consist of two different wavelengths. Hereafter, we will explain this algorithm in detail.

The algorithm begins in the frequency domain with the wavelength λ_1 considered. A random number generator [5,9] is used to generate an array of random numbers in the range of π to $-\pi$ that serves as the initial estimate of the phase distribution $\psi_0(\xi)$ at the far field of the CGHD. As for the amplitude transmittance $|F_0(\xi)|$, it is assumed to be constant. With these assumptions, we can obtain the light distribution (with amplitude transmittance and phase distribution, $|f_1(x)|$ and $\phi_1(x)$, respectively) at the CGHD plane through the inverse FFT. Since a uniform amplitude transmittance is desired, the amplitude transmittance $|f_1(x)|$ is then reset to a constant.

The procedure above is applied under the assumption that the wavelength of the input light is λ_1. Since we have

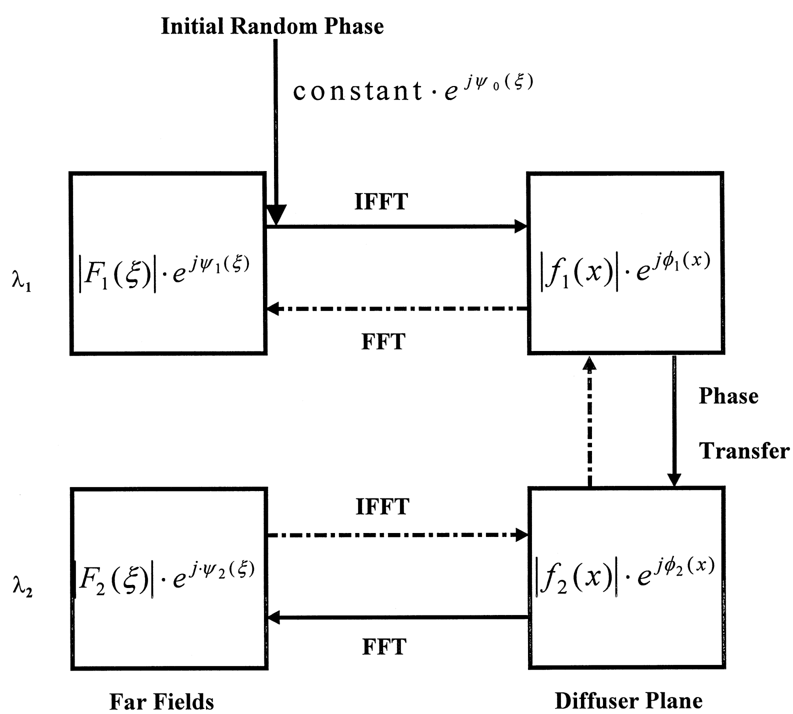


Fig. 2. Flow chart of the iterative algorithms for the computer-generated holographic diffuser with the model of multiple input wavelengths.

two different input wavelengths for CGHD, we must convert the phase distribution $\phi_1(x)$ into phase distribution $\phi_2(x)$ based on the fact that the thickness of the CGHD at any position is fixed. The phase conversion is thus carried out through the following relations:

$$\phi_{j,i}(x) = d_i k_j + (D_0 - d_i) k_j n, \quad k_j = 2\pi/\lambda_j,$$
$$i = 1,2,\dots,l, \; j = 1,2,\dots,m, \tag{4}$$

where $\phi_{j,i}(x)$ is the phase distribution of the CGHD when the input wavelength is λ_j with wave number denoted by k_j. $\phi_1(x)$ and $\phi_2(x)$ mean the set of total pixel numbers in the CGHD for $\phi_{j,i}(x)$, with $j = 1$ for λ_1, $j = 2$ for λ_2, and i for the ith pixel unit, l and m are integer. We then let D_0, d_i and n denote the total etching depth and etching depth of the ith units cell, and the refractive index of the substrate, respectively, because the CGHD is fabricated through the semiconductor etching process in terms of multiple phase levels. For the substrate of quartz and silicon wafers, the index dispersion versus different wavelengths can be neglect because the index difference (between 565 nm and 650 nm) is small than 2% of the substrate index. After phase conversion, we can then obtain the phase distribution ϕ_2 assuming that the thickness of the CGHD is fixed.

Now, we will start the algorithm for wavelength λ_2. With amplitude phase distribution $|f_2(x)|$ and $\phi_2(x)$ at the CGHD plane, we obtain its far field light distribution $|F_2(\xi)|\exp[j\psi_2(\xi)]$ through FFT. Here, we impose another constrain on $|F_2(\xi)|$. That is, the amplitude transmittance should be constant for uniform color mixing. Therefore, we will reset $|F_2(\xi)|$ to a constant. From here, we can reverse the process above to the beginning of the algorithm and then keep repeating.

General speaking, the proposed algorithm is similar to the GS [5] or YG [3] algorithm but with some necessary modifications. Since our problem is to design an optical element for different chromatic light, we must consider the phase conversion shown in Eq. (4). Finally the algorithm stops when the amplitude transmission efficiency at the far field becomes constant or almost constant [9]. A reasonable criterion of comparison is the root-mean-square error ε_{mn} of image amplitude defined by

$$\varepsilon_{mn} = \frac{\sqrt{\left| |A_{mn,\lambda}|^2 - |A_0|^2 \right|}}{|A_0|^2}, \tag{5}$$

where $A_{mn,\lambda}$ is represented by an array of image amplitude at wavelength λ, A_0 is the initial amplitude transmittance at the CGHD plane. The iterative algorithm stops when the root-mean-square amplitude error ε_{mn} for each wavelength falls within an acceptable limit ε_0.

Here, the average amplitude transmission efficiency is the average of the amplitude transmission efficiency over all input wavelengths, i.e. $A = \overline{A_\lambda}$, and the amplitude transmission efficiency is defined as

$$A_\lambda = \sum A_{mn,\lambda}/A_0, \tag{6}$$

where $A_{mn,\lambda}$ is the average amplitude spectrum at the far field over $m \times n$ pixels when the input wavelength is λ and A_0 is the average amplitude spectrum at the CGHD plane. 88% average amplitude transmission efficiency means that most of the light energy at the diffuser plane is also available at the far field.

Now, let us define the amplitude uniformity as

$$U_\lambda = \frac{A_{\max,\lambda} - A_{\min,\lambda}}{A_{\max,\lambda} + A_{\min,\lambda}}, \tag{7}$$

where $A_{\max,\lambda}$ and $A_{\min,\lambda}$ are the maximum and minimum output far field light amplitude for input wavelength λ, respectively. The average amplitude uniformity U thus becomes $U = \overline{U_\lambda}$. According to the simulation, the average amplitude uniformity is 12.4%, which shows that the designed CGHD provides an almost uniform output pattern at the far field.

3. Simulation

To obtain good uniformity and efficiency, the whole simulation is divided into two procedures. (i) Process 1, an initial random generator is searched to solve the optimum converging condition for the iterative algorithm. (ii) Process 2, phase only freedom is explored for this iterative algorithm with multiple input wavelengths.

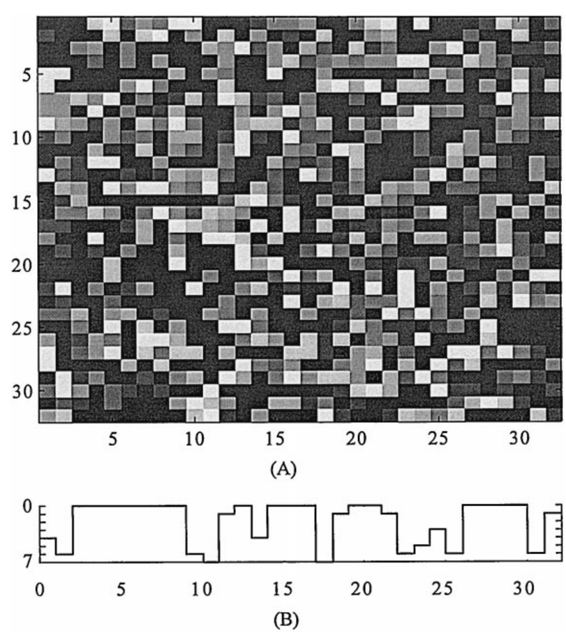
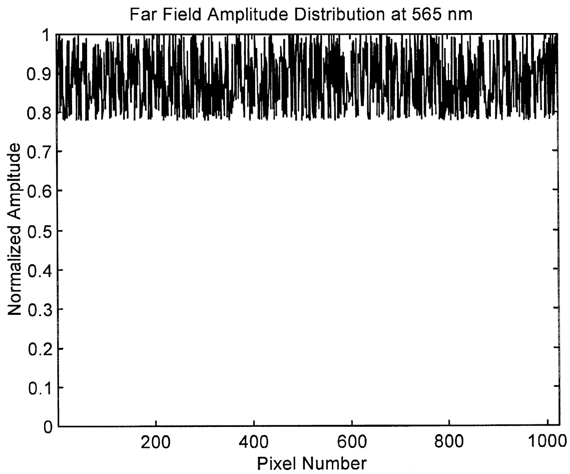
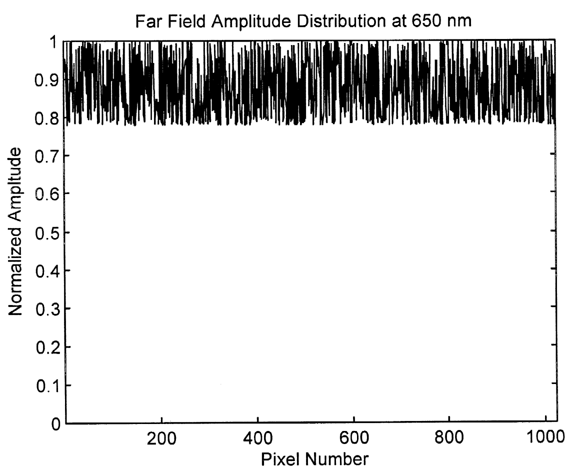


Fig. 3. (A) The image map of the computer-generated holographic diffuser is illustrated eight quantized phase levels. The dark cell means 0-phase level and the bright cells describe other phase levels. (B) Quantization is shown by the one-dimensional spatial profile.



(a)



(b)

Fig. 4. The average far field amplitude transmission efficiency at 565 nm wavelength is 88.32%, and 88.29% for wavelength of 650 nm. The average uniformity of far field amplitude distribution is 12.4%. The abscissa gives the total number of pixel cells.

To simulate the proposed idea, we consider color mixing for two different wavelengths and show its diagram in Fig. 1. The wavelengths considered are 565 and 650 nm, respectively. In total, we have $N \times N$ pixels in the designed CGHD and each pixel consists of p phase levels. Here, N can be 32, 64, 128, 256, or 512, while p can be 2, 4, 8, 16, and 32. The maximum number of p is limited to 32 because it is the maximum phase levels that can be fabricated by the state-of-the-art techniques.

Since the discrete phase is utilized, we show the resulting phase profile of the designed CGHD in Fig. 3 by the gray-scale encoding technique that converts the different phase level values into gray-scale values. For example, if four phase levels (0°, 90°, 180°, and 270°) are encountered, then they are shown via gray-scale values (0, 85, 170, and 255).

Fig. 4 shows the 2-D far field amplitude transmission efficiency (in one dimension) of two different wavelengths of the designed CGHD in 1-D via progressive scan arrangement. We found that the minimum amplitude transmission efficiency is 0.78 and the average of the amplitude transmission efficiency is more than 0.88 while the ideal diffuser should have spectrum 1 at the output. In addition, the designed CGHD achieves 12.4% uniformity. Please note also that the results obtained above are for a diffuser that is specifically designed to diffuse two different input wavelengths at the same time, while the previous results are only designed considering single input wavelength.

4. Conclusion

An iterative algorithm is proposed to design a computer generated holographic diffuser. Our algorithm is especially developed for a diffuser to uniformly spread input lights with different wavelengths. The designed diffuser consists of only phase variation and its amplitude transmittance is constant. Therefore, all the light at the diffuser plane is available at the far field and thus the light utilization efficiency is 100%. In addition, 12.4% uniformity and more than 88% average amplitude transmission efficiency are achieved by our technique.

The proposed algorithm is an extension of the well-known GS algorithm but with some necessary modifications that can solve a more general problem. Although only two-wavelength input is considered here, our algorithm is suitable for many-wavelength cases.

References

[1] R. Brauer, U. Wojak, F. Wyrowski, O. Bryngdahl, Optics Lett. 16 (1991) 1427.
[2] N.C. Robert, Appl. Optics 28 (1989) 31.
[3] Tan, B. Gu, G. Yang, B. Dong, Appl. Optics 34 (1995) 1314.
[4] J.R. Leger, D. Chen, Z. Wang, Optics Lett. 19 (1994) 108.
[5] G. Gerchburg, W.O. Saxton, Optik 35 (1972) 237.
[6] W. Wyrowski, J. Opt. Soc. Am. A 7 (1990) 961.
[7] A. Aleksoff, K.K. Ellis, B.D. Neagle, Opt. Eng. 30 (1991) 537.
[8] Hoßfeld, E. Jager, T. Tschudi, E.G. Churin, SPIE 1574 (1991) 159.
[9] T. Akahori, Appl. Optics 25 (1986) 802.