Electron-phonon scattering times in crystalline disordered titanium alloys between 3 and 15 K

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We have successfully fabricated bulk crystalline disordered titanium alloys ($Ti_{1-x}Sn_x$, $Ti_{1-x}Ge_x$, and $Ti_{0.97-x}Sn_{0.03}Sc_x$) with nominal concentration x of the order of a few at. %. The impurity atoms are gradually introduced into a titanium host to tune the degree of disorder, resulting in residual resistivities ρ_0 varying from $\approx$ 40 to 160 $\mu\Omega$ cm (corresponding to values of $k_F \ell \approx 5-20$, with k_F being the Fermi wave number and ℓ being the electron elastic mean free path). With this wide range of experimentally accessible ρ_0, we are able to perform systematic studies of the disorder ρ_0, or electron elastic mean free path ℓ, behavior of the electron-phonon scattering time τ_{ph}. We have measured the magnetoresistivities of these alloys between 3 and 15 K, and compared our results with weak-localization theoretical predictions to extract $\tau_{ph}(T,\ell)$. Unexpectedly, we obtain a rather diversified temperature dependence and disorder dependence of τ_{ph} as follows. (a) In $Ti_{1-x}Sn_x$ alloys with $\rho_0 \gtrsim 100$ $\mu\Omega$ cm we find $1/\tau_{ph} \sim T^2/\ell$. (b) In $Ti_{1-x}Sn_x$ alloys with $\rho_0 \lesssim 70$ $\mu\Omega$ cm we find $1/\tau_{ph} \sim T^3/\ell$. (c) In $Ti_{1-x}Ge_x$ alloys with $50 \lesssim \rho_0 \lesssim 130$ $\mu\Omega$ cm we find $1/\tau_{ph} \sim T^3$ in some alloys and $1/\tau_{ph} \sim T^4$ in others, while at a given temperature $1/\tau_{ph}$ is essentially independent of disorder. (d) In $Ti_{0.97-x}Sn_{0.03}Sc_x$ alloys with $55 \lesssim \rho_0 \lesssim 75$ $\mu\Omega$ cm we obtain $1/\tau_{ph} \sim T^2$. These experimental observations altogether imply that electron-phonon interactions in the presence of strong impurity scattering are very sensitive to the local material environment (the microscopic quality) of a particular sample system, which might be variously modulated with the doping of different kinds of impurity atoms. Our results are compared with existing theoretical predictions for electron-phonon scattering in disordered conductors. [S0163-1829(98)01118-7]

I. INTRODUCTION

The electron-phonon (e-ph) scattering time τ_{ph} is a physical quantity that determines the most important characteristics of conductors (normal metals and superconductors). For instance, it determines the dephasing scattering time for the electron wave function and the cooling time for electron gas. In the case of pure or clean conductors, the temperature behavior of the e-ph scattering time (namely, $1/\tau_{ph} \sim T^p$ with $p = 3$) is well established both theoretically[1] and experimentally.[2] In the case of disordered conductors, however, both the experimental and theoretical situations still remain to be resolved. Theoretically, the e-ph interaction in disordered metals has been extensively studied by a good number of authors for over two decades, but quite different predictions were reached.[3–9] For example, values for the temperature exponent p ranging from 2 to 4 have been predicted. Experimentally, the temperature dependence of $1/\tau_{ph}$ reported by various measurements on various material systems are not always in agreement with one another.[10,11] This issue becomes even more delicate when one is concerned with the disorder behavior of $1/\tau_{ph}$. On the one hand, the theories have predicted distinctly different disorder dependence of $1/\tau_{ph}$. On the other hand, few experiments in literature have succeeded in reporting a clear dependence of $1/\tau_{ph}$ on disorder.

Over the last 15 years, it has been recognized that weak-localization effects (or coherent backscattering of conduction electrons) result in noticeable magnetoresistivities in disordered metals and superconductors.[10–12] Analysis of those anomalous magnetoresistivities, defined by $\Delta\rho(B)/\rho^2(0) = [\rho(B) - \rho(0)]/\rho^2(0)$, has proved to provide quantitative information of the various electron dephasing scattering times, including inelastic (τ_{in}), spin-orbit (τ_{so}), and magnetic spin-spin scattering times (τ_s). In particular, the temperature dependence of $1/\tau_{in}$ can be extracted by comparison of $\Delta\rho(B)/\rho^2(0)$ obtained at various measuring temperatures with the existing theoretical predictions.

In the weak-localization (WL) studies of the e-ph scattering time τ_{ph} in disordered conductors, however, one frequently encounters various experimental difficulties, depending on the dimensionality of the samples involved. These experimental difficulties, among others, are as follows. (a) In the case of *two* dimensions (e.g., thin metal films), there are usually more than one relevant inelastic processes, i.e., e-ph scattering and electron-electron (e-e) scattering, that are of prime importance in determining the total inelastic electron dephasing scattering time τ_{in}.[10,13] In addition, the appropriate phonon dimensionality in question depends in a complicated way on the film thickness, the phonon wavelength, the acoustic transparency of the film-substrate interface, etc.[10,14–16] These inevitable complications make it (almost) impossible to quantitatively separate τ_{ph} from the total inelastic dephasing scattering time τ_{in} obtained from $\Delta\rho(B)/\rho^2(0)$ measurements. (b) In the case of *three* dimensions (i.e., bulk samples), it is established that e-ph scattering is the *sole* relevant inelastic process while the e-e scattering is comparatively weak and can be totally ignored.[7,10,17] In

TABLE I. Values of relevant parameters for bulk superconducting titanium alloys. The impurity concentration x is *nominal*. The residual resistivity ρ_0 is in $\mu\Omega$ cm, the superconducting transition temperature T_c is in K, and the spin-orbit scattering field B_{so} is in T. For clarity, instead of listing the values of the relevant parameters for all the individual alloys (more than 55) studied in this work, we only list the ranges of values covered in each alloy system. The values of the relevant parameters for Ti-Al alloys are taken from Ref. 19.

Alloys	x	ρ_0	$\rho(300 \text{ K})/\rho_0$	T_c	B_{so}
$Ti_{1-x}Sn_x$	0.025–0.12	43–153	1.14–2.13	1.03–1.63	0.58–0.72
$Ti_{1-x}Ge_x$	0.025–0.07	52–131	1.22–1.88	0.90–1.51	0.32–0.55
$Ti_{0.97-x}Sn_{0.03}Sc_x$	0.01–0.05	56–72	1.65–1.88	<0.4	0.18–0.37
$Ti_{1-x}Al_x$	0.04–0.12	58–143	1.08–1.70	0.60 ± 0.1	0.22–0.27

practice, however, it is very difficult to fabricate microscopically homogeneous bulk samples having a wide range of high impurity resistivities ρ_0. Therefore, an experimental determination of the disorder ρ_0, or electron elastic mean free path ℓ, dependence of $1/\tau_{ph}$ is not readily accessible using bulk samples.

In this work, we have succeeded in fabricating several series of bulk crystalline disordered superconducting titanium alloys ($Ti_{1-x}Sn_x$, $Ti_{1-x}Ge_x$, and $Ti_{0.97-x}Sn_{0.03}Sc_x$) with the nominal impurity concentration x of the order of a few percent. These crystalline disordered alloys are ideal for the studies of WL and e-e interaction effects.[18,19] Compared with other three-dimensional material systems (e.g., thick granular films,[20] bulk samples of solid solutions of insulators in metals,[21] and metallic glasses[11]) usually used in this research area, our bulk superconducting titanium alloys are quite unique and advantageous in several regards: (a) Our various titanium alloys are metal samples essentially *microscopically homogeneous* and having a wide range of high ρ_0, resulting in experimentally measurable three-dimensional (3D) WL contributions to $\Delta\rho(B)/\rho^2(0)$. Particularly, a wide range of ρ_0 makes feasible a reliable experimental determination of the disorder behavior of τ_{ph}. (b) Doping of a titanium host with various impurity atoms (e.g., Al, Sn, Ge, or Sc) is possible. It is expected that different impurity atoms could effect the phonon excitation spectrum of the lattice (and probably also the electronic density of states at the Fermi level) in different ways, and thus could cause different temperature and disorder behavior of τ_{ph}. (c) One more great advantage is that the usual magnetic atoms, such as Fe, Co, and Ni, do *not* form localized moments in a titanium host,[22] and hence result in a *zero* magnetic spin-spin scattering rate in the various titanium alloys.[19,23] (d) The various titanium alloys studied in this work are superconductors with superconducting transition temperatures T_c falling in the range ≈ 0.6–1.6 K. Such values of T_c are very adequate for quantitative studies of τ_{ph}; they are not too high in the sense that there is still a sufficiently wide temperature window left above T_c in which WL contributions to $\Delta\rho(B)/\rho^2(0)$ could be accurately measured.[24] On the other hand, they are not too low so that the upper critical fields B_{c2} for such alloys could be conveniently measured using, say, a ^{3}He cryostat. With B_{c2} being measured, the values of the electron diffusion constant D for the alloy samples can be directly evaluated. Values of D are needed in order to extract values of τ_{ph} in the WL studies. These above-mentioned advantages altogether make feasible quantitative investigations of both the temperature dependence and, particularly, the disorder depen-

dence of τ_{ph}. In short, τ_{ph} can be very reliably extracted from the WL studies using properly tailored bulk titanium alloys, without it being coupled with the e-e scattering time τ_{ee}, and magnetic spin-spin scattering time τ_s. Such reliability is often lacking in other two- or three-dimensional experiments as mentioned above.

This paper is organized as follows. In Sec. II we discuss our experimental method for sample fabrication, magnetoresistivity measurements, and evaluation of values of the electron diffusion constant D. Our experimental results for the measured magnetoresistivities due to WL effects, and the extracted temperature and disorder dependences of τ_{ph} in the various titanium alloys are presented in Sec. III. Our results for $\tau_{ph}(T,\ell)$ are also compared with existing theoretical predictions for e-ph interaction in disordered conductors. Section IV contains our conclusion.

II. EXPERIMENTAL METHOD

We have fabricated $Ti_{1-x}Sn_x$ (Ti-Sn), $Ti_{1-x}Ge_x$ (Ti-Ge), and $Ti_{0.97-x}Sn_{0.03}Sc_x$ (Ti-Sn-Sc) alloys by an arc-melting method as described previously.[18,19] High-purity tin (99.999% pure), germanium (99.999% pure), or scandium (99.99% pure) atoms were gradually introduced into a titanium host (99.995% pure) to serve as compositional disorder. The degree of disorder was carefully controlled by the amount of Sn, Ge, or Sc doped. These kinds of impurity atoms were chosen because, when being present with low concentrations, they could randomly substitute for the titanium host atoms while introducing negligible second (compound) phases.[25] Moreover, it is known that their presence in a titanium host could result in relatively large values of solute resistivity ($\gtrsim 20 \, \mu\Omega$ cm per at. % of Sn or Ge),[26] making comparatively pronounced 3D WL effects. In this work, the *nominal* concentrations x were kept low enough so that all the alloys fabricated were essentially single phased, possessing an hcp structure similar to that of α-Ti, as was evidenced from x-ray measurements performed on a MAC Science M18X diffractometer with Cu $K\alpha$ radiation. Energy dispersive spectroscopy (EDS) studies performed on a Kevex Delta Class 8000 EDS of some of our samples also indicated a uniform distribution of the impurity atoms in the titanium host lattice. In addition, transport measurements demonstrated that, in every alloy system, the impurity resistivity ρ_0 [$=\rho(10 \text{ K})$] increased basically linearly with increasing x for the selected concentration range indicated in Table I, implying, again, a uniform distribution of the impurity atoms. These observations just mentioned suggested that our alloys
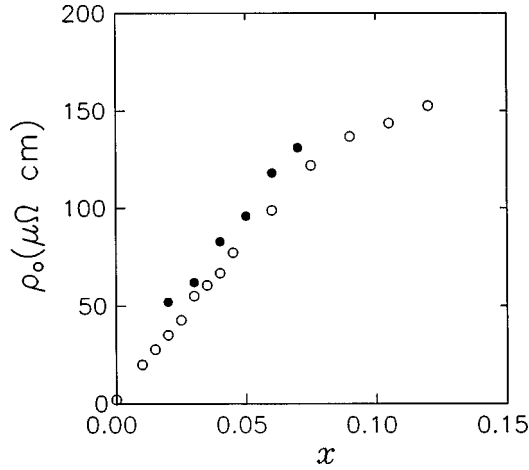
FIG. 1. Impurity resistivities ρ_0 versus the impurity concentration x for $Ti_{1-x}Sn_x$ (open circles) and $Ti_{1-x}Ge_x$ alloys (closed circles), respectively.

did possess homogeneous composition and lattice structure at length scales that were considerably smaller than the relevant length scale in the WL problem (i.e., the electron diffusion length). Table I also lists the corresponding range of the value of ρ_0 in each alloy system. With all alloy systems being taken together, our experimental values of ρ_0 varied from ≈ 40 to ≈ 160 $\mu\Omega$ cm, corresponding to values of $k_F\ell$ in the range $\approx 5-20$, with $k_F \approx 1.9 \times 10^{10}$ m^{-1} being the Fermi wave number.[27,28] Figure 1 shows the representative variations of ρ_0 with x for Ti-Sn (open circles) and Ti-Ge alloys (closed circles), respectively.

Our samples were typically $0.2 \times 0.2 \times 10$ mm^3 in size. Ac magnetoresistivities were measured by a four-probe technique. Great care was taken to make low-noise electrical contacts and wiring. (The electrical contacts for the wiring were made using thermal-free solder, for example.) The measurements were performed in a quiet environment and a signal resolution of a relative resistivity change $\delta\rho/\rho \lesssim 3 \times 10^{-6}$ was achieved. The low-temperature environment was provided with a ^{3}He cryostat and the temperature was monitored with a calibrated carbon-glass and a calibrated RuO$_2$ thermometers. The measuring temperatures were between 3 and 15 K, and magnetic fields below 3 T were used. (Notice, however, that in our comparison of the experimental magnetoresistivities with the 3D WL theoretical predictions, a much lower magnetic-field range was usually used in order to minimize any appreciable contributions from the many-body e-e interaction effects.[19])

Upper critical fields B_{c2} for many of our alloys were measured and the values of the electron diffusion constant were then computed, using the relation[29]

$$D = \frac{4k_B}{\pi e} \frac{1}{|dB_{c2}/dT|},$$

where k_B is the Boltzmann constant, and e is the electronic charge. Our empirical numerical values of D were best interpolated by $D=(885/\rho_0-0.45)\times10^{-5}$ m^2/s for Ti-Sn alloys, $(610/\rho_0+0.30)\times10^{-5}$ m^2/s for Ti-Ge alloys, and $(776/\rho_0)\times10^{-5}$ m^2/s for Ti-Sn-Sc alloys, where ρ_0 is in $\mu\Omega$ cm. In practice, our results indicate that the values of D

for a given value of ρ_0 for the various Ti-Sn, Ti-Ge, and Ti-Sn-Sc alloys [and also for $Ti_{1-x}Al_x$ (Ti-Al) alloys[19]] are essentially the same.

For a few alloys, we had also measured the electronic specific heats, $\gamma T=(\pi^2/3)k_B^2N(0)T$, at liquid-helium temperatures to deduce the e-ph enhanced electronic density of states at the Fermi level $N(0)$. The values of D were then obtained through the Einstein relation $1/\rho_0 = De^2N(0)/(1+\lambda)$, where the e-ph coupling constant $\lambda \approx 0.54$ (Ref. 30) for pure titanium had been used. We observed that the values of D deduced from B_{c2} measurements and those deduced from γT measurements were consistent to within 20%.

III. RESULTS AND DISCUSSION

A. Magnetoresistivities

Before presenting our experimental results, we recall that it is frequently more convenient to deal with the various characteristic fields $B_j = \hbar/4eD\tau_j$ than the scattering times τ_j themselves, where $j=$in, ph, ee, so, and s refer to the inelastic, e-ph, e-e, spin-orbit, and magnetic spin-spin scattering fields (times). This is because the fields B_j are the quantities that explicitly appear in the WL theory. In the most general case, one is concerned with the electron dephasing scattering field given by[10,12]

$$B_\phi(T) = \frac{\hbar}{4eD\tau_\phi(T)} = \frac{\hbar}{4eD}\left[\frac{1}{\tau_{ph}(T)} + \frac{1}{\tau_{ee}(T)} + \frac{1}{\tau_{so}} + \frac{2}{\tau_s}\right], \tag{1}$$

where τ_ϕ denotes the total electron dephasing scattering time. As mentioned above, there is negligible e-e scattering[17] and magnetic spin-spin scattering[23] in the case of bulk superconducting titanium alloys so that Eq. (1) reduces to

$$B_\phi(T) \approx \frac{\hbar}{4eD}\left[\frac{1}{\tau_{ph}(T)} + \frac{1}{\tau_{so}}\right], \tag{2}$$

where only the e-ph scattering time is temperature dependent while the spin-orbit scattering time is not.

The normalized magnetoresistivities $\Delta\rho(B)/\rho^2(0)$ at various temperatures between 3 and 15 K for each of our alloys have been measured and compared with 3D WL theoretical predictions[12] to extract $B_\phi(T)$, and thus $B_{ph}(T)$ and B_{so}. The details of the least-squares fitting procedure had been discussed previously.[19] Here we merely mention that for every alloy studied in this work, the 3D WL predictions (including Maki-Thompson superconducting-fluctuation effects) can well describe our experimental results. Consequently, $B_{ph}(T)$ and B_{so} can be reliably extracted. Our results for the e-e attraction strength $\beta(T)$, defined in the WL theory to describe the Maki-Thompson superconducting-fluctuation contributions, are also consistent with Larkin's theory.[12,19] In this work, we will focus on our results of $B_{ph}(T,\ell)$.

Figure 2 shows representative normalized magnetoresistivities $\Delta\rho(B)/\rho^2(0)$ as a function of the magnetic field B for $Ti_{0.97}Sn_{0.03}$ alloy at several measuring temperatures as indicated in the caption to Fig. 2. The symbols are the experimental results and the solid curves are the 3D WL theo-
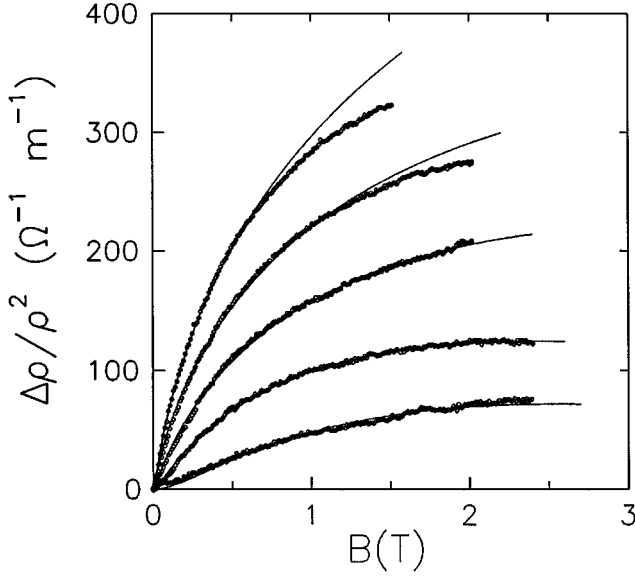
FIG. 2. Normalized magnetoresistivities $\Delta\rho(B)/\rho^2(0)$ versus the magnetic field B for a $Ti_{0.97}Sn_{0.03}$ alloy having $\rho_0 \approx 55 \ \mu\Omega$ cm at (from bottom to top) 12.0, 8.5, 6.0, 4.0, and 3.0 K. The solid lines are the 3D WL theoretical predictions.

retical predictions. This figure reveals that $\Delta\rho(B)/\rho^2(0)$ are monotonically increasing functions of B in all measured magnetic fields. Physically, this is due to the enhanced spin-orbit scattering introduced by the doping of relatively heavy Sn atoms into the Ti host. Our results reveal that, with the addition of 3.0 at. % of Sn, the value of B_{so} increases by a factor ~ 2, e.g., compared with that in Ti-Al alloys. Numerically, we obtain $B_{so} \approx 0.25$ T in Ti-Al alloys (where the Al dopants are light),[19] and $B_{so} \approx 0.58$ T in $Ti_{0.97}Sn_{0.03}$ alloy. (See, also, Table I.)

The magnetoresistivities in Ti-Ge alloys and Ti-Sn-Sc alloys are quite similar to those in $Ti_{0.97}Sn_{0.03}$ alloy shown in Fig. 2, except that the magnitudes are smaller by a factor of approximately 1.5. (The magnitudes are reduced due partly to weaker spin-orbit scattering effects in these two alloy systems.) Therefore, we shall not plot the magnetoresistivities for these alloys. Quantitative analysis indicates moderate-to-strong spin-orbit scattering in these alloys (Table I). This observation is expected, since the atomic weight of Ge atoms is smaller than that of Sn atoms, but is still larger than that of Ti atoms so that the spin-orbit scattering becomes moderate.

B. Temperature dependence of $1/\tau_{ph}(T)$

Figure 3 shows the variations of $B_{ph}(T)$ with temperature between 3 and 15 K for five representative $Ti_{1-x}Sn_x$ alloys with values of x and ρ_0 as indicated in the caption to Fig. 3. The dashed and solid lines (which guide the eye) are drawn proportional to T^2 and T^3, respectively. This figure demonstrates that $B_{ph}(T)$ varies essentially with T^2 for high-ρ_0 samples (open squares and closed circles) while it varies essentially with T^3 for less resistive samples (closed triangles and open circles).[31] More precisely, we find $B_{ph} \sim 1/\tau_{ph} \sim T^2$ in those Ti-Sn alloys with $100 \lesssim \rho_0 \lesssim 160 \ \mu\Omega$ cm, while we find $B_{ph} \sim 1/\tau_{ph} \sim T^3$ in those Ti-Sn alloys with $40 \lesssim \rho_0 \lesssim 70 \ \mu\Omega$ cm. In one alloy with intermediate ρ_0 ($\approx 77 \ \mu\Omega$ cm, open rhombuses), we find the temperature exponent

p between 2 and 3. Furthermore, close inspection of Fig. 3 reveals that the magnitude of B_{ph} at a given temperature increases with increasing ρ_0. This latter issue concerning the disorder behavior of B_{ph} or $1/\tau_{ph}$ will be addressed below in a separate subsection.

Figure 4 shows the variations of $B_{ph}(T)$ with temperature between 3 and 15 K for five representative $Ti_{0.97-x}Sn_{0.03}Sc_x$ alloys with values of x and ρ_0 as indicated in the caption to Fig. 4. The variation of $B_{ph}(T)$ with T for the parent $Ti_{0.97}Sn_{0.03}$ alloy (closed squares) is also plotted for comparison. The dashed and solid lines are drawn proportional to
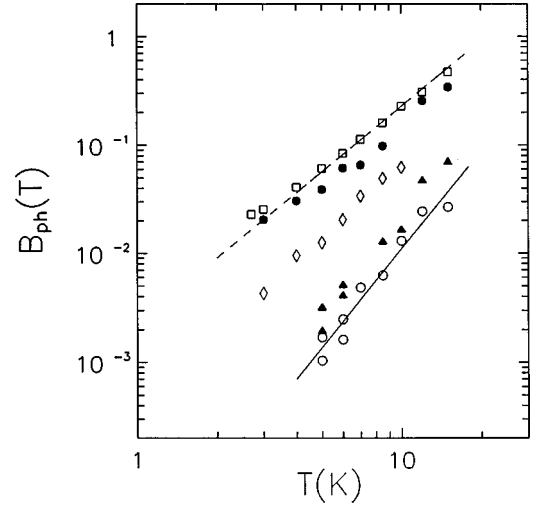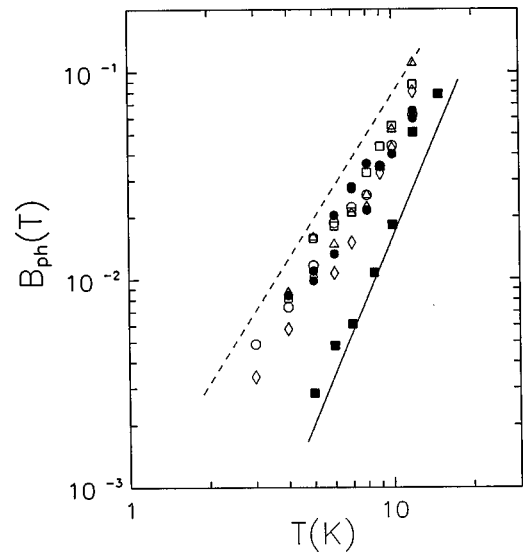

FIG. 3. Electron-phonon scattering fields $B_{ph}(T)$ versus temperature T for five representative $Ti_{1-x}Sn_x$ alloys with $x=0.025$ (open circles), 0.035 (solid triangles), 0.045 (open rhombuses), 0.09 (closed circles), and 0.12 (open squares). The dashed and solid lines are drawn proportional to T^2 and T^3, respectively; they guide the eye.

FIG. 4. Electron-phonon scattering fields $B_{ph}(T)$ versus temperature T for $Ti_{0.97-x}Sn_{0.03}Sc_x$ alloys with $x=0.01$ (open circles), 0.02 (open squares), 0.03 (open triangles), 0.04 (open rhombuses), 0.05 (closed circles), and the parent $Ti_{0.97}Sn_{0.03}$ alloy (closed squares). The dashed and solid lines are drawn proportional to T^2 and T^3, respectively; they guide the eye.
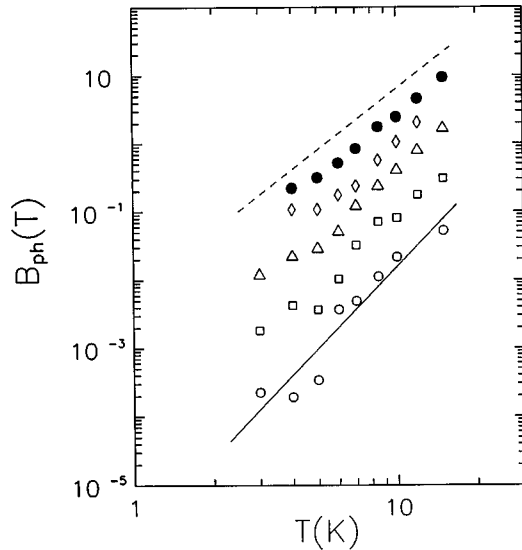
FIG. 5. Electron-phonon scattering fields $B_{ph}(T)$ versus temperature T for five $Ti_{1-x}Ge_x$ alloys with $x = 0.025$ (open circles), 0.03 (open squares), 0.04 (open triangles), 0.06 (open rhombuses), and 0.07 (closed circles). The dashed and solid lines are drawn proportional to T^3 and T^4, respectively; they guide the eye. For clarity, the values of $B_{ph}(T)$ have been shifted by scaling by a factor of 0.4, 2, 5, 10, and 20 for the alloys with $x=0.025$, 0.03, 0.04, 0.06, and 0.07, respectively.

T^2 and T^3, respectively; they are guides to the eye. This figure clearly demonstrates that $B_{ph}(T)$ or $1/\tau_{ph}$ in all the Ti-Sn-Sc alloys varies essentially with T^2, in contrast to that ($\sim T^3$) in the parent $Ti_{0.97}Sn_{0.03}$ alloy. It is intriguing that the value of the exponent p is lowered from ≈ 3 to ≈ 2 by the introduction of only a few at. % of Sc atoms into the parent $Ti_{0.97}Sn_{0.03}$ alloy.

Figure 5 shows the variations of $B_{ph}(T)$ with temperature between 3 and 15 K for five representative $Ti_{1-x}Ge_x$ alloys with values of x and ρ_0 as indicated in the caption to Fig. 5. The dashed and solid lines (which guide the eye) are drawn proportional to T^3 and T^4, respectively. This figure demonstrates that $B_{ph}(T)$ varies approximately with either T^3 or T^4, depending on samples. Our experimental results do not reveal any clear connection between the amount of disorder ρ_0 of the sample and the value of the temperature exponent p for $1/\tau_{ph}$. Nevertheless, unlike in all the other titanium alloys studied (where always $p \lesssim 3$), the values of p in Ti-Ge alloys are always $\gtrsim 3$. [Notice that, for clarity, the values of $B_{ph}(T)$ for every alloy have been shifted by scaling by a certain factor. See the caption to Fig. 5.]

To summarize, the above results indicate that $1/\tau_{ph} \sim T^2$ in both high-resistivity Ti-Sn alloys as well as in Ti-Sn-Sc alloys. Previously, we have already observed a T^2 dependence of $1/\tau_{ph}$ in numerous Ti-Al alloys having $\rho_0 \approx$ 60–150 $\mu\Omega$ cm.[19] Thus, these experimental results indicate that the T^2 behavior of $1/\tau_{ph}$ is quite common in titanium alloys. On the other hand, some of our experimental results for the temperature behavior of $1/\tau_{ph}$ are totally unexpected. For example, it is not understood why the value of p for $Ti_{0.97-x}Sn_{0.03}Sc_x$ alloys is so different from that for the parent $Ti_{0.97}Sn_{0.03}$ alloy (Fig. 4). It is puzzling that even only 1% of Sc doping can already change the T^3 behavior to the T^2 behavior, while additional doping does not cause further change (decrease or

increase) in p. Moreover, it is not understood why the temperature behavior of $1/\tau_{ph}$ in Ti-Ge alloys (Fig. 5) is so much different from that in Ti-Sn, Ti-Sn-Sc, and Ti-Al alloys.

Our experimental results (Figs. 3–5) suggest that there does *not* exist a universal temperature behavior of $1/\tau_{ph}$ in impure conductors. It is conjectured that the phonon excitation spectrum and the frequency dependence of the Eliashberg function $\alpha^2 F(\omega)$, ω being the phonon frequency, could be very sensitive to the local environment of the impure sample, which would likely depend on the types and amounts of the dopants added. A variation in the frequency behavior of the phonon excitation spectrum or in $\alpha^2 F(\omega)$ from one alloy system to another could readily cause a variation in the value of p.[32] On the other hand, it might also be possible that a different temperature dependence of $1/\tau_{ph}$ in the various titanium alloys is due to a different electronic density of states at the Fermi level. How might the density of states at the Fermi level change upon alloying requires further study.

We notice in passing that, at liquid helium temperatures above T_c and in zero magnetic field, the resistivities of our bulk superconducting titanium alloys demonstrate measurable corrections to the constant residual resistivities, i.e., there is a resistivity change $\Delta\rho(T,\ell) = \rho(T,\ell) - \rho_0(\ell)$. Particularly, the temperature dependence of $\Delta\rho(T,\ell)$ for *all* our alloy systems can be *quantitatively* ascribed to e-e interaction effects in the presence of strong impurity scattering.[18,33,34] As a matter of fact, we do not find any appreciable differences among the various titanium alloys, insofar as both the temperature behavior and disorder behavior of $\Delta\rho(T,\ell)$ is concerned. This observation is of fundamental importance: Our experimental results imply that while the e-e interaction effects are only dependent on the value of ρ_0 (and T_c) in titanium alloys, the e-ph interaction not only is dependent on the value of ρ_0 but is also sensitive to the local, microscopic environment characteristic to each sample system.

Finally, we comment briefly on the ''leveling-off'' behavior of $B_{ph}(T)$ in some of the alloys at our lowest measuring temperatures. Close inspection of Figs. 3 and 5 reveals a deviation from the indicated power-law temperature dependences for certain alloys, such as $Ti_{0.91}Sn_{0.09}$ ($T_c \approx 1.63$ K, closed circles in Fig. 3) and $Ti_{0.93}Ge_{0.07}$ ($T_c \approx 1.51$ K, closed circles in Fig. 5), which have *highest* values of T_c among those for the alloys of their own kind. We stress that this leveling-off behavior can be well ascribed to the scattering of the conduction electrons by superconducting fluctuations as T approaches T_c.[19,24] As a matter of fact, a leveling-off behavior is not observed, or much less evident, above 3 K in other alloys with lower values of T_c. (Those samples with lower values of T_c would start to reveal a leveling-off behavior only at some certain temperatures below 3 K.) Therefore, such a deviation behavior from the indicated power-law temperature dependences is *not* a precursor of a saturation of B_{ph} (or, more precisely, B_ϕ) which might have been expected as a result of a finite $1/\tau_s$ (Ref. 23) or $1/\tau_{ee}$. The later scattering rate possesses a temperature exponent of 3/2 in 3D disordered systems,[17] which is appreciably lower than that for 3D e-ph scattering.
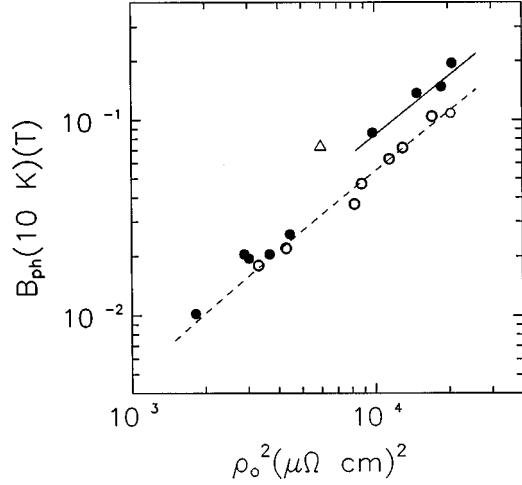
FIG. 6. Variation of the electron-phonon scattering field $B_{ph}(10 \text{ K})$ with the square of impurity resistivity ρ_0 for Ti-Sn (closed circles) and Ti-Al alloys (open circles). The dashed line is the theoretical prediction (4), and the solid line represents a $B_{ph}(10 \text{ K})$ having a magnitude a factor 1.5 times that given by the dashed line. The data for Ti-Al alloys are taken from Ref. 19. The open triangle denotes the $B_{ph}(10 \text{ K})$ of $Ti_{0.955}Sn_{0.045}$ alloy in which p is not equal to 2 or 3.

C. Disorder dependence of $1/\tau_{ph}(\ell)$

In the studies of the e-ph interaction in impure conductors, it is of great importance to focus, apart from the temperature dependence, also on the disorder or elastic mean free path ℓ dependence of $1/\tau_{ph}$. In this work, we have succeeded in performing experimental investigations of the disorder dependence using properly tailored bulk titanium alloys. Our results are summarized in Figs. 6 and 7. Figure 6 shows the variation of our experimental $B_{ph}(10 \text{ K})$ with the square of ρ_0 for Ti-Sn alloys (closed circles). The $B_{ph}(10 \text{ K})$ for Ti-Al alloys (open circles) taken from Ref. 19 are also plotted for comparison. (Previously, we have found $B_{ph} \sim T^2$ and $B_{ph} \sim \rho_0^2$ in numerous Ti-Al alloys.) The dashed and solid lines are drawn proportional to ρ_0^2. Figure 7 shows
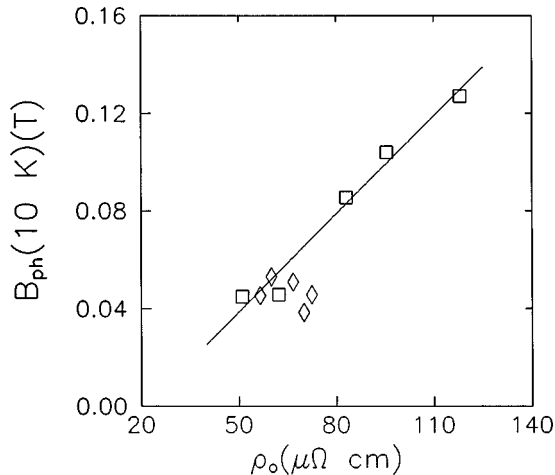

FIG. 7. Variation of the electron-phonon scattering field $B_{ph}(10 \text{ K})$ with impurity resistivity ρ_0 for Ti-Ge (open squares) and Ti-Sn-Sc alloys (open rhombuses). The solid line drawn through the open squares guides the eye.

the variation of our experimental $B_{ph}(10 \text{ K})$ with ρ_0 for Ti-Sn-Sc alloys (open rhombuses) and Ti-Ge alloys (open squares), respectively. The solid line is drawn proportional to ρ_0. Inspection of the data plotted in Figs. 6 and 7 demonstrates the following features. (a) In the case of Ti-Sn alloys, $B_{ph}(10 \text{ K})$ varies essentially *linearly* with ρ_0^2 for both $\rho_0 \gtrsim 100 \ \mu\Omega$ cm (where the temperature behavior is $B_{ph} \sim T^2$, see Fig. 3), and, also, for $\rho_0 \lesssim 70 \ \mu\Omega$ cm (where the temperature behavior is $B_{ph} \sim T^3$, see Fig. 3). A $B_{ph} \sim \rho_0^2$ is equivalent to a $1/\tau_{ph} \sim \rho_0 \sim 1/\ell$. (b) In the case of Ti-Ge alloys, $B_{ph}(10 \text{ K})$ scales approximately with ρ_0 for $\rho_0 \approx 50$–$120 \ \mu\Omega$ cm. This corresponds to a $1/\tau_{ph} \sim \rho_0 D \sim$ constant, implying that $1/\tau_{ph}$ is essentially *independent* of disorder. (c) In the case of Ti-Sn-Sc alloys, in contrast to all the other titanium alloys studied, $B_{ph}(10 \text{ K})$ is approximately independent of ρ_0 for $\rho_0 \approx 55$–$75 \ \mu\Omega$ cm, implying a $1/\tau_{ph} \sim 1/\rho_0 \sim \ell$.[35]

From these observed features, one immediately sees that the disorder behavior of $1/\tau_{ph}$, and thus the nature of the e-ph interaction, in crystalline disordered titanium alloys is rather complicated. It seems that the disorder behavior of $1/\tau_{ph}$ is fairly sensitive to the impurity type (Al, Sc, Sn, or Ge atoms). For instance, this assertion is supported by our observation in Ti-Sn-Sc alloys where the addition of a few percent of Sc atoms totally alters the disorder behavior from that of the parent Ti-Sn alloy. In the former case $1/\tau_{ph}$ increases with increasing disorder, while in the latter case it increases with decreasing disorder. Our experiments strongly indicate that there does *not* exist a universal disorder behavior of $1/\tau_{ph}$ in impure conductors.

D. Comparison with theoretical predictions

Theoretically, Bergmann[3] has calculated the e-ph interaction in a dirty metal. He points out that the impurities participate in the lattice oscillations (i.e., the phonons) and change the e-ph interaction. He finds additional e-ph processes which do not conserve the lattice momentum. These additional e-ph processes contribute particularly to processes with low-energy transfer, so that the Eliashberg function $\alpha^2 F(\omega)$ depends linearly on ω in the low-energy region. Takayama[4] has also studied this problem. The results given by Bergmann and Takayama agree almost exactly. The impurity contribution to the e-ph scattering rate according to Takayama is

$$\frac{1}{\tau_{ph}(T,\ell)} = C \frac{2\pi^2}{k_F \ell} \frac{(k_B T)^2}{\hbar^2 \omega_D} = C \frac{2\pi^2}{k_F \ell} \frac{k_B T^2}{\hbar \theta_D}, \qquad (3)$$

where ω_D (θ_D) is the Debye frequency (temperature) and C is a constant given by $C = n m v_F^2 q_D^2/(6 n_i M v_s^2 k_F^2)$, where n (n_i) is the electron (ion) density, m (M) is the electron (ion) mass, v_F (v_s) is the Fermi (sound) velocity, and $q_D = \omega_D/v_s$. Using $q_D = (2/z)^{1/3} k_F$ and the Bohm-Staver relation for sound velocity $v_s^2 = z m v_F^2/(3M)$ (Ref. 36), we rewrite C in the following form: $C = (2/z)^{2/3}/2$, where z is the nominal valence.

To facilitate a quantitative comparison with experimental results, we rewrite Eq. (3) in the form

$$B_{\mathrm{ph}}(T,\rho_0) = \frac{\hbar}{4eD\tau_{\mathrm{ph}}(T,\rho_0)} = C\frac{ek_Bk_F}{6\hbar D\theta_D}T^2\rho_0. \qquad (4)$$

Since $D\sim 1/\rho_0$, then Eq. (4) predicts a $B_{\mathrm{ph}}\sim T^2\rho_0^2$. Assume that upon dilute alloying, the values of the relevant parameters are barely changed from those of pure Ti. The theoretical prediction of Eq. (4) is then represented by the dashed line shown in Fig. 6, using $k_F\approx 1.9\times 10^{10}$ m^{-1}, $\theta_D=420$ K,[37] $C\approx 0.3$, and the experimentally interpolated value $D=(885/\rho_0-0.45)\times 10^{-5}$ m^2/s for Ti-Sn alloys. Inspection of Fig. 6 clearly reveals that $B_{\mathrm{ph}}(T,\rho_0)$ in those Ti-Sn alloys with $\rho_0\approx 100$–160 $\mu\Omega$ cm (closed circles), and also in those Ti-Al alloys with $\rho_0\approx 60$–150 $\mu\Omega$ cm (open circles) can be fairly well described by Eq. (4). The experimental and theoretical values agree to within a factor of 1.5 or better. [The solid line represents a $B_{\mathrm{ph}}(T,\rho_0)$ having a magnitude a factor 1.5 times that given by the dashed line.] In three dimensions (3D) such close agreement between theory and experiment has rarely been reported in the literature.

Despite the close agreement between experiment and theory as shown in Fig. 6, we must stress that many of our experimental results for $B_{\mathrm{ph}}(T,\rho_0)$ in the various titanium alloys are actually *not* in line with the prediction of Eq. (4). The discrepancies are summarized as follows. (a) In the case of the less resistive Ti-Sn alloys ($\rho_0\lesssim 70$ $\mu\Omega$ cm), the T behavior ($\sim T^3$) is not consistent with the prediction of Eq. (4), although the ρ_0 behavior basically is. (b) In the case of Ti-Sn-Sc alloys, the disorder behavior ($B_{\mathrm{ph}}\sim$ constant) is not consistent with the prediction of Eq. (4), although the T behavior basically is. (c) In the case of Ti-Ge alloys, neither the T behavior nor the ρ_0 behavior is consistent with the prediction of Eq. (4).

Strictly speaking, it is not surprising that a good deal of our experimental results are not consistent with the theoretical prediction of Eq. (4), since, e.g., the Bergmann-Takayama theory was formulated utilizing a simple jellium model with a spherical Fermi surface while real materials often consist of far more complex Fermi surfaces. However, it is then not understood why our experimental data for certain Ti-Sn and many Ti-Al alloys do demonstrate quantitative agreement with the Bergmann-Takayama prediction as discussed above. In fact, recent theoretical works[7–9] on the e-ph interaction in impure conductors have made great advances and reached a consensus that argues *against* the prediction of Eq. (3). Rammer and Schmid have recently reevaluated the problem of e-ph interaction in disordered conductors. They consider impurity atoms that move in phase with the other lattice atoms. In particular, they find that there is a subtle compensation between the coupling of the electrons to the vibrating impurities and the interaction of the electrons with the deformed lattice vibrations. With such a compensation between the two mechanisms properly treated, the resulting e-ph interaction is *weakened* in the *dirty* limit, leading to

$$1/\tau_{\mathrm{ph}}\sim T^4\ell. \qquad (5)$$

Obviously, this prediction is entirely different from the Bergmann-Takayama prediction of Eq. (3). [On the other hand, Rammer and Schmid pointed out that, without such a subtle compensation, each of the two mechanisms would be *enhanced* in the presence of strong disorder and lead to a $1/\tau_{\mathrm{ph}}\sim T^2/\ell$, similar to the prediction of Eq. (3).] The Rammer-Schmid theory has been well accepted by the theoretical community. Furthermore, it has been confirmed independently by the calculations of Reizer and Sergeyev,[8] and Belitz.[9] In spite of such a theoretical consensus, it is rather surprising and puzzling that *none* of our experimental results in any alloy system are in systematic agreement with this presumably more correct theory.

In the case of Ti-Sn alloys, our results do not even agree with the Rammer-Schmid theory in a *qualitative* manner. According to the theory, the value of the temperature exponent p should gradually increase from 3 (the pure limit) to 4 as the sample is made more and more disordered. In other words, the value of p is predicted to increase (up to 4) with increasing ρ_0. This is definitely in disagreement with our observation in Fig. 3, where we see that p *decreases* from 3 down to 2 as ρ_0 is increased from 40 to about 100 $\mu\Omega$ cm.

In the case of Ti-Sn-Sc alloys, we do not observe a T^4 dependence of $1/\tau_{\mathrm{ph}}$ as discussed (see Fig. 4). Nevertheless, it cannot be ruled out that in this alloy system, Sc atoms move more or less in phase with Ti host atoms, and thus $1/\tau_{\mathrm{ph}}\sim 1/\rho_0\sim\ell$ (or, equivalently, $B_{\mathrm{ph}}\sim$ const) as predicted by Eq. (5) is hinted at in Fig. 7 (the open rhombuses). In any case, it is of interest to search for appropriate material systems with impurity atomic weight equal to that of the host atoms and having a wide range of high ρ_0. Such material systems would be useful for performing a critical experimental check of the Rammer-Schmid prediction.

Since it is the e-ph scattering in the dirty limit that most concerns us, we examine whether the disorder criterion $q_T\ell\lesssim 1$ is satisfied in the present work, where $q_T\approx k_BT/\hbar v_s$ is the most frequent phonon-frequency at the temperature T. For our titanium alloys with ρ_0 covering the range 40–160 $\mu\Omega$ cm there is $\ell\approx 2.6$–11 Å. We take $v_s\approx 5000$ m/s for pure Ti.[38] Then, $q_T\ell\approx (0.0068$–$0.029)T$, where T is in K. This indicates that the e-ph processes in our alloys are well within the disordered limit (i.e., $q_T\ell<1$) even at temperatures as high as 30 K or higher. Therefore, our experimental results for $1/\tau_{\mathrm{ph}}$ measured up to 15 K, Figs. 3–7, can be said to fully satisfy the disorder criterion.

E. Further discussion

The nature of the e-ph interaction in disordered conductors (normal metals and superconductors) is still a subject of controversy. Theoretically, quite different predictions have been made for both the temperature and disorder behavior of $1/\tau_{\mathrm{ph}}(T,\ell)$. Experimentally, the temperature dependence of $1/\tau_{\mathrm{ph}}$ reported by various measurements are not always in agreement with one another. For example, a T^3 behavior of $1/\tau_{\mathrm{ph}}$ was found in granular CuO films,[21] and InSb semiconductors.[39] A T^4 behavior of $1/\tau_{\mathrm{ph}}$ was found in Cu-Ti metallic glasses[40] and bismuth films.[14] These and many other results reported in the literature imply that the value of the temperature exponent p for $1/\tau_{\mathrm{ph}}$ might be quite sensitive to the microscopic quality and the intrinsic material properties (electronic structures, phonon excitation spectra, etc.) of the host lattice and impurity atoms. It might not be determined just by the total amount of disorder. In any case, it remains true that a $1/\tau_{\mathrm{ph}}\sim T^2$ is frequently observed in

experiments using both thin films and bulk samples made of various materials.[41–44] A T^2 dependence of $1/\tau_{ph}$ has caused much confusion and debate on the nature of the e-ph interaction in disordered conductors for over the years. On the one hand, the most recent theory, Eq. (5), does *not* support a T^2 dependence as discussed. On the other hand, apart from the experimental results obtained from WL studies, there exist a good number of tunneling measurements in literature that suggest an Eliashberg function $\alpha^2 F(\omega) \sim \omega$.[45] An $\alpha^2 F(\omega)$ linear in ω implies a $1/\tau_{ph} \sim T^2$.[32] This T^2 behavior is one of the key issues which the theory for e-ph interaction in disordered conductors has to properly address. Recently, in a series of carefully designed electron heating experiments using CuCr films of various thicknesses and widths, DiTusa *et al.*[46] have always found a $1/\tau_{ph} \sim T^2$ independent of phonon dimensionality and $q_T \ell$. They argue that there might be heretofore unidentified mechanisms that are responsible for the e-ph scattering in impure conductors.[47,48]

To the authors' knowledge, a linear dependence of $1/\tau_{ph}$ on $1/\ell$ as predicted by Eq. (3) has never been reported in previous experiments. Peters and Bergmann[41] had clearly demonstrated a B_{ph} that scales with (or, quite close to) T^2 in many quenched condensed metal films, including, among others, Mg, Ag, and Au. However, they failed to observe an $B_{ph} \sim \rho_0^2$ as predicted by Eq. (4). Instead, they observed a $B_{ph} \sim \rho_0$ or a disorder independent $1/\tau_{ph}$. Also using heating measurements, Liu and Giordano[16] have recently found in thin Sb films a $1/\tau_{ph}$ which was independent of the sheet resistance $R_\square$ of the films even when the mean free path ℓ increased by more than a factor of 10 in going from the low to high $R_\square$ samples. In these two-dimensional experiments, the extracted values of $1/\tau_{ph}$ might have been more or less obscured by a non-negligible $1/\tau_{ee}$ as well as by some uncertainty in the phonon dimensionality.[13] Such ambiguities do not occur in our case of bulk titanium alloys.

In 3D, $1/\tau_{ph}$ is the sole, dominating inelastic scattering rate. However, microscopically homogeneous 3D samples with high ρ_0 are not easy to make. Thus far, most 3D samples used in the WL studies are thick granular films, bulk samples of solid solutions of insulators in metals, and amorphous metal alloys. In the first two cases, the electronic transport behavior is complicated by the percolating nature of the sample structure, especially when the *macroscopic* disorder of the sample is high. In those systems, a $1/\tau_{ph} \sim T^3$ has often been observed in low-resistance samples, while a disorder *independent* $1/\tau_{ph} \sim T^2$ has been observed in high-resistance samples at $T \lesssim 10$ K.[10,20] It is conjectured that a disorder independent $1/\tau_{ph} \sim T^2$ in high-resistance granular samples might actually come from weakly disordered metal grains. In the case of amorphous metal alloys, although the

residual resistivities are often high, it is usually difficult to tune the value of ρ_0 for over a reasonably wide range.[11] That is, the atomic arrangement in a given amorphous metal alloy is already in the limit of strong randomness, and thus a change in the constituent concentration will hardly change the value of ρ_0. In some metallic glasses, the value of ρ_0 can indeed be adjusted (e.g., by a factor ~ 2) if the concentration is changed by a large amount (e.g., some tens of percent). However, the electronic structures and phonon excitation spectra might be totally altered under such circumstances. Therefore, neither such materials are useful for the experimental investigations of the disorder behavior of $1/\tau_{ph}$. In short, it is not always trivial to obtain and to make material systems that are suitable for systematic experimental studies of $1/\tau_{ph}(T,\ell)$. Therefore, our choice of using bulk titanium alloys deserves close attention.

IV. CONCLUSION

We have experimentally investigated the electron-phonon scattering time $\tau_{ph}(T,\ell)$ in numerous bulk crystalline disordered titanium alloys, including Ti-Sn, Ti-Ge, Ti-Sn-Sc, and Ti-Al alloys. We have studied the WL induced magnetoresistivities in low magnetic fields between 3 and 15 K to extract the values of τ_{ph}. In addition to the temperature behavior, we have determined the disorder behavior of $1/\tau_{ph}$. Our experimental results reveal that the temperature dependence and disorder dependence of $1/\tau_{ph}$ vary among different alloy systems. That is, our experimental results do *not* support the existence of a universal temperature and disorder dependence of $1/\tau_{ph}$, implying that the nature of the electron-phonon interaction in impure conductors could be fairly sensitive to the local material environment of a particular sample system. (The phonon excitation spectrum might be subtly affected by the microscopic quality of the disorder, for example.) From our observations, we have to conclude that much more experimental work is still required before a consensus can be reached on the temperature dependence and disorder dependence of $1/\tau_{ph}$ in impure conductors. Our experimental results strongly suggest that the current theoretical understanding of the electron-phonon interaction in impure conductors is still far from being complete.

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[17] The e-e scattering rate in 3D disordered metals is given by $1/\tau_{ee} = \sqrt{3}(k_B T)^{3/2}/[2\hbar (k_F \ell)^{3/2} \sqrt{E_F}]$, where E_F is the Fermi energy. See, A. Schmid, Z. Phys. B **271**, 251 (1974); B. L. Al'tshuler and A. G. Aronov, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 514 (1979) [JETP Lett. **30**, 482 (1979)]. Using $E_F = \hbar k_F v_F/2$ and the values of k_F quoted in the text and $v_F = 3.2 \times 10^5$ m/s (Ref. 30), we have evaluated the magnitudes of $1/\tau_{ee}$ in the various titanium alloys in our measuring temperature range of 3–15 K. We find that, even in our most disordered Ti-Ge (Ti-Sn and Ti-Al) alloys at the lowest measuring temperatures, the magnitude of $1/\tau_{ee}$ computed from the above expression is still at least a factor of ≈ 10 (≈ 20) smaller than our measured $1/\tau_{in}$. The contribution of $1/\tau_{ee}$ to $1/\tau_{in}$ is even less (by a factor of several tens to two orders of magnitude) in the less disordered alloys and at higher measuring temperatures. Therefore, any appreciable contribution from e-e scattering to the total $1/\tau_{in}$ can be safely ignored in this work. In fact, our focus on measuring temperatures *above*, but not below, 3 K also minimizes the role played by e-e scattering.

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[23] Our assertion of the absence of magnetic spin-spin scattering is supported by the fact that all of our alloys (except those containing Sc) superconduct at values of T_c appreciably higher than that (≈ 0.40 K) for pure Ti, see Table I. This property of enhanced, but not depressed, superconductivity is strongly suggestive of there being no spin-flip scattering in our samples due to alloying or any specific defect structures. Our use of high-purity starting materials as well as careful precautions for sample fabrication also minimize any appreciable sample contamination (see Sec. II). In addition, our best least-squares fits of the measured magnetoresistivities with the WL theoretical predictions are obtained with a zero, but not any finite, spin-spin scattering rate.

[24] In practice, at temperatures above but sufficiently close to T_c, in addition to the e-ph scattering, scattering of the conduction electrons by superconducting fluctuations might also contribute to the total $1/\tau_{in}$. This additional scattering could cause a leveling-

off and, finally, a divergence of the electron dephasing scattering rate as T approaches T_c. [See, e.g., W. Brenig, M. C. Chang, E. Abrahams, and P. Wolfle, Phys. Rev. B **31**, 7001 (1985); Ref. 19; and W. B. Jian, M.S. thesis, National Taiwan University, 1996.] This situation makes it less desirable for a quantitative separation of τ_{ph} from the measured $1/\tau_{in}$. We find that, for all our titanium alloys having T_c below 1.6 K, at T above 3 K or so the electron-superconducting-fluctuation scattering is negligibly weak compared with the e-ph scattering. On the other hand, the WL contributions to $\triangle \rho(B)/\rho^2(0)$ in our *bulk* samples are too small to be correctly detected above about 15 K. (Notice that WL effects are generally smaller in 3D than in two dimensions.) These considerations taken together lead to an experimental temperature window of $\approx$ 3–15 K.

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[27] This value of k_F for Ti is evaluated by using the free-electron model result: $k_F = 3.63/(r_s/a_0)$ Å^{-1}, where a_0 is the Bohr radius, and r_s is the radius of a sphere whose volume is equal to the volume per conduction electron. The electron elastic mean free path is then obtained through the relation $\ell = 3\pi^2 \hbar/(k_F^2 e^2 \rho_0)$.

[28] In practice, titanium alloys with $\rho_0 < 40$ $\mu\Omega$ cm can be readily produced. (See, e.g., Fig. 1.) However, experimentally it is extremely difficult to detect the 3D WL contributions to $\Delta\rho(B)/\rho^2(0)$ in such less disordered samples. Therefore, we have to restrict our work to those alloys having $\rho_0 \gtrsim 40$ $\mu\Omega$ cm or, equivalently, $k_F \ell \lesssim 20$. On the other hand, we cannot keep increasing the doping level while maintaining the alloy samples single phased. This latter constraint compels us to focus on alloy samples with $\rho_0 \lesssim 160$ $\mu\Omega$ cm or, equivalently, $k_F \ell \gtrsim 5$.

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[31] However, notice that the values of ρ_0 in these what we call less resistive samples are already much higher than that (≈ 2 $\mu\Omega$ cm) in pure Ti. That is, these so-called less resistive samples are already rather disordered.

[32] The temperature dependence of $1/\tau_{ph}$ is determined by the frequency behavior of $\alpha^2 F(\omega)$ through the relation $1/\tau_{ph} = (4\pi/\hbar) \int d\omega [\alpha^2 F(\omega)/\sinh(\hbar\omega/k_B T)]$. In general, an $\alpha^2 F(\omega) \sim \omega^s$ would result in a $1/\tau_{ph} \sim T^{s+1}$, where s is a positive integer. See, e.g., Ref. 15.

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[35] We have found in this work that ρ_0 cannot be tuned over a wide range by Sc doping. Nevertheless, although the accessible ρ_0 range for this alloy system is limited, our data do indicate a somewhat constant, instead of an increasing $B_{ph}(10 \text{ K})$, with in-

creasing disorder. Notice that our experimental uncertainty would already allow a detection of an increase or a decrease in $B_{ph}(10 \text{ K})$ over a ρ_0 range of $\approx 20 \ \mu\Omega$ cm.

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