Analysis on the depth of focus in flying optics

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Abstract. Based on a simplified model of Gaussian beam focusing, in which nontruncating, thin-lens, aberration-free, and on-axis optical systems are assumed and the thermal-lensing effect is neglected, a general and concise expression is derived to describe the variation of $1/e^2$ radius of the focused beam as a function of input beam waist to lens spacing, lens-to-target plane spacing, input beam Rayleigh range, and focal length of lens. From this expression, we get the depth of focus (DOF) of flying optics with a fixed flying range, which is used as a merit function to determine the optimal solutions for system parameters. The results are very useful in the design and analysis of flying optics in laser material processing. Finally, a practical example describing the optimization of the output coupler for CO_2 laser resonator is given. [S0091-3286(98)02505-7]

Subject terms: Gaussian beam; beam focusing; flying optics; depth of focus; laser material processing.

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1 Introduction

Due to the beam feature, the behavior of a focused Gaussian beam can not be precisely approached to the practical level by geometrical optics. Based on a simplified model in which nontruncating, thin-lens, aberration-free, and on-axis optical systems are assumed and the thermal-lensing effect is neglected, the expressions of output beam parameters are derived with ray transfer matrix and mode matching in the review paper of Kogelnik and Li.[1] From diffraction theory, Dickson[2] derived a somewhat more general expression including aperture truncation effect. With the safe truncation in which the radius of the physical aperture is twice larger than the $1/e^2$ radius of Gaussian beam at the aperture, this more general expression reduces to the Kogelnik and Li's expression. To be analogous to geometrical optics, Self[3] simplified this derivation and got concise expressions describing the variations of beam waist radius, Rayleigh range, and beam waist to lens spacing for a focused Gaussian beam as a function of those of input beam and focal length of lens. Luxon et al.[4] verified Self's equations to be valid for all higher order modes and useful in practice.

In laser material processing, the power density (irradiance) of working beam has a tolerance range for a particular application. If the range is exceeded, it results in poor quality or even cessation of the process. As a fixed laser power is applied during processing, this tolerance makes a constraint on the acceptable variation in the value of working beam radius. The depth of focus (DOF) is defined as a distance from the focusing point over which the working beam radius varies within the required region. In flying optics, beam guiding over long distance during processing changes the input beam waist to lens spacing and results in the variations of beam waist radius, Rayleigh range, and beam waist to lens spacing of working beam. Much work has been done to reduce these variations. Luxon[5] optimized the surface curvature of output coupler in Spectra-Physics 825 configuration CO_2 laser resonator. Zoske and Giesen[6] optimized the interspacing of beam expander. Haferkamp et al.[7] used adaptive optics (deformable mirror) to compensate the variation of input beam waist to lens spacing.

In this paper, a general expression is derived to describe the variation of $1/e^2$ radius of focused Gaussian beam as a function of input beam waist to lens spacing, lens-to-target plane spacing, input beam Rayleigh range, and focal length of lens. From this expression, we get the DOF of flying optics with a fixed flying range, which is used as a merit function to find out the optimal solutions for system parameters. The results are very useful for the design and analysis of flying optics in laser material processing.

Finally, we provide an analysis example of the optimum design of output coupler in laser-gantry-robot systems to show how to use our derived equations to find the optimal parameters and the DOF value.

2 The $1/e^2$ Radius Equation of the Focused Gaussian Beam

Based on the mentioned simplified model of Gaussian beam focusing, the geometry and parameters of focused beam, as shown in Fig. 1, can be well described by Self's equations. We summarize his equations as

$$\begin{cases} w_0' = m w_0 \\ Zr' = m^2 Zr \\ s' = f + m^2 (s - f) \end{cases}, \tag{1a}$$

where

$$\begin{cases} m = \left[\dfrac{f^2}{(s-f)^2 + Zr^2} \right]^{1/2} \\ w_0 = \left(\dfrac{\lambda Zr}{\pi} \right)^{1/2}. \end{cases} \tag{1b}$$

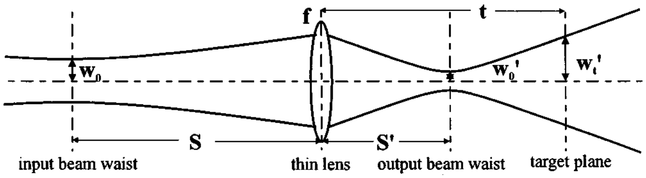

Fig. 1 Geometry of Gaussian beam focusing.

With Self's convention w_0, Zr, s, w_0', Zr', and s' are waist radii, Rayleigh ranges, and beam waists to lens spacings of input and output beams, respectively. The s is positive if the input beam waist locates in front of lens, but the s' is positive if the output beam waist locates behind lens. Both m and f are magnification and focal length of lens, respectively.

The equation for the $1/e^2$ radius of a focused Gaussian beam at the target plane after lens (see Fig. 1) can be derived with Eqs. (1a) and (1b) and expressed as

$$w'(s,t;Zr,f)=w_0'\left[1+\left(\frac{t-s'}{Zr'}\right)^2\right]^{1/2}$$
$$=\left(\frac{\lambda Zr}{\pi f^2}\left\{(t-f)^2+\left[\frac{(t-f)(s-f)-f^2}{Zr}\right]^2\right\}\right)^{1/2}, \tag{2}$$

and

$$w'(t\equiv f;s,Zr)=\left(\frac{\lambda f^2}{\pi Zr}\right)^{1/2}. \tag{3}$$

Equation (2) describes the output beam radius w', which varies as a function of the parameters s, t, Zr, and f. From Eq. (3) we can see that the output beam radius w' at back focal plane is independent of the s value. This feature was also noted by Dickson.[2]

Now, we can express the output beam radius in a normalized form as

$$w'_{\mathrm{nor}}(x,y)=\frac{w'(s,t;Zr,f)}{w'(t\equiv f;s,Zr)}$$
$$=[y^2+(xy-1)^2]^{1/2} \tag{4a}$$
$$=\left[(1+x^2)\left(y-\frac{x}{1+x^2}\right)^2+\frac{1}{1+x^2}\right]^{1/2}, \tag{4b}$$

where

$$\begin{cases} x=(s-f)/Zr \\ y=(t-f)Zr/(f^2). \end{cases} \tag{4c}$$

From Eq. (4b), we can see that the output beam waist locates at the position $y=x/(1+x^2)$ and the normalized radius at waist reaches the minimum value as $1/(1+x^2)^{1/2}$.

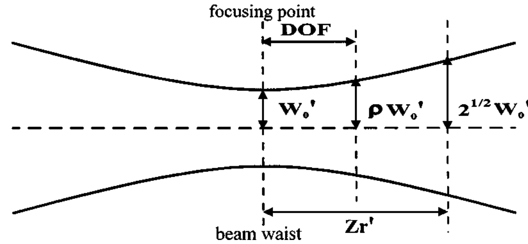

Fig. 2 Convention of Rayleigh range (Zr') and DOF.

3 DOF Features in Nonflying Optics

With a fixed input beam-waist-to-lens spacing during processing, s remains constant and the system is called nonflying optics here. From Eq. (4b), it can be seen that the focusing point can be clearly chosen as the position of output beam waist for the minimum value of focused beam radius. As with Eq. (1a), the focusing point is usually different from the focal point of lens except that $s=f$. The DOF, which is defined as a distance from the focusing point over which the working beam radius varies within the required region, as shown in Fig. 2, can be expressed as

$$\mathrm{DOF}=(\rho^2-1)^{1/2}Zr'=(\rho^2-1)^{1/2}\,\frac{f^2Zr}{Zr^2+(s-f)^2}, \tag{5a}$$

where

$$\rho\equiv\frac{w'(t=s'\pm\mathrm{DOF})}{w_0'(t=s')}. \tag{5b}$$

From Eq. (5a) and curve B in Fig. 3 given by Eqs. (1a) and (1b), we can find the optimal conditions for the DOF of nonflying optics as follows:

1. The input beam waist should locate at front focal points ($s\rightarrow f$).
2. Input beam Rayleigh range Zr should be small when $s\rightarrow f$.
3. The lens of longer focal length should be used when $s\rightarrow f$.

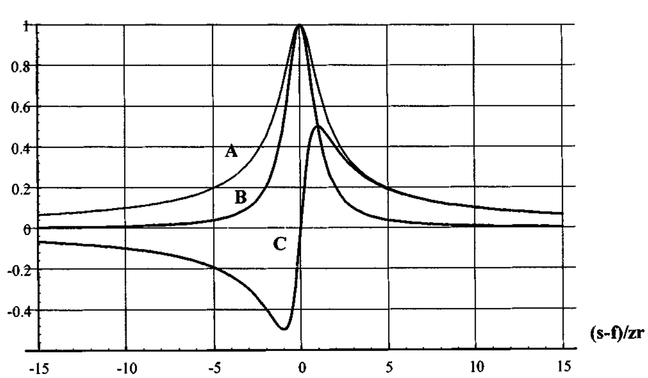

Fig. 3 Relation of focused beam parameters w_0' and Zr' and s' with s. The abscissa is a normalized variable $(s-f)/Zr$ and the ordinates for curves A, B, and C are these normalized functions $[(\pi/\lambda)(Zr/f^2)]^{1/2}w_0'$, $(Zr/f^2)Zr'$, and $(Zr/f^2)(s'-f)$, respectively.

In the limiting case where $s=f$ and $Zr=0$, coming into the case of geometrical optics, the DOF becomes infinite.

4 DOF in Flying Optics

As the input beam waist to lens spacing s varies within a fixed flying region during processing, the system is called flying optics; for example, laser-gantry-robot systems and multiple workstations in series in laser material processing. In flying optics, the output beam radius in any target plane except the back focal plane varies during processing. Figure 3 shows all the variations of Zr', w_0', and s' as functions of s. It can be seen that no region of s has less variations for all the parameters and at the same time larger Zr' values so as to make the DOF optimum.

The DOF calculation is very complicated as a function of two parameters (x and y). Within the flying region $s_0-\Delta s\leqslant s\leqslant s_0+\Delta s$, i.e., in the normalized form the region $x_0-\Delta x\leqslant x\leqslant x_0+\Delta x$, there exists a ratio ρ of the maximum to the minimum output beam radii within the region $y_0-\Delta y\leqslant y\leqslant y_0+\Delta y$ in the output space of the focusing lens. For any x_0 value, there always exists an optimum y_0 value that makes the beam radius ratio ρ minimum. On the contrary, if an optimum y_0 is chosen, the Δy should be maximum for a fixed ρ value. The maximum Δy is the DOF in the normalized form. Figure 4, given by Eq. (4a), shows that the maximum Δy is optimum on the condition $x_0=0$ (i.e., $s_0=f$) and $y_0=0$ (i.e., $t_0=f$). The maximum Δy value decreases as x_0 changes. The optimum y_0 value increases with increased x_0 for $x_0>0$, but decreases with increased $|x_0|$ for $x_0<0$. Figure 5, given by Eq. (4a) with $\Delta x=0.2$, shows the tendency that the less $|x_0|$ has the larger DOF. This means that the DOF value decreases as the $|x_0|$ value increases. Therefore, the following analysis of the limited region $|x_0|\leqslant\Delta x$ is actually an optimum analysis of the DOF in flying optics.

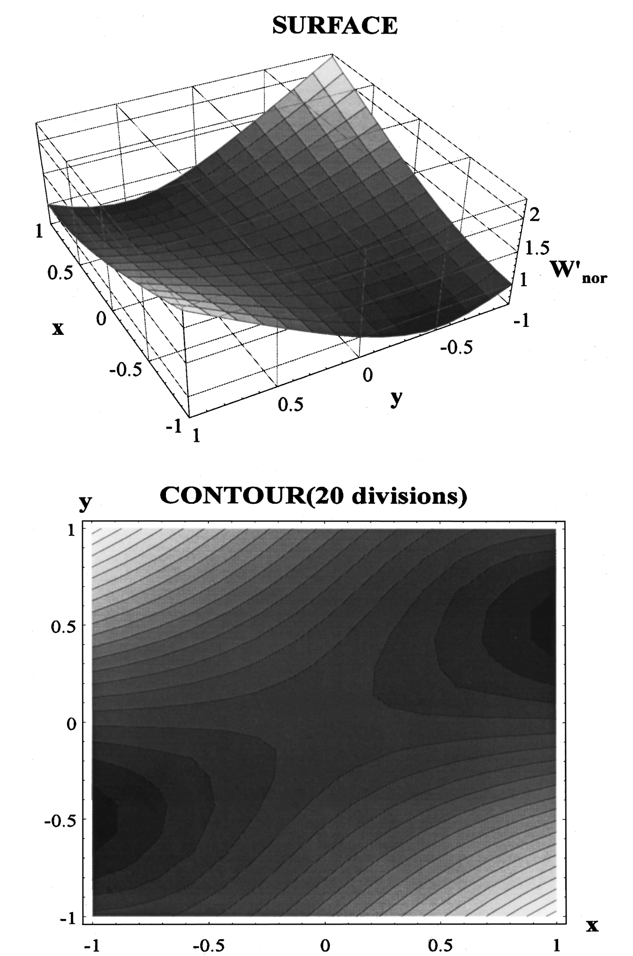

Fig. 4 Surface and contour of focused beam radius $W_{\text{nor}}'=[y^2+(xy-1)^2]^{1/2}$.

5 DOF Determination in Flying Optics

Now, we explore the acceptable region of both the x and y variations in Eq. (4a). To reduce the variation of beam radius at lens, we limit the flying of s in the collimated region $-Zr\leqslant(s-f)\leqslant Zr$, i.e., $-1\leqslant x\leqslant 1$. For y, we consider that if the maximum variation ratio of beam radius after lens is taken to be ρ, from Eq. (5a) we get $s'-(\rho^2-1)^{1/2}Zr'\leqslant t\leqslant s'+(\rho^2-1)^{1/2}Zr'$ for every s value and have

$$\frac{-1-(\rho^2-1)^{1/2}}{2}\leqslant\frac{x-(\rho^2-1)^{1/2}}{1+x^2}\leqslant y\leqslant\frac{x+(\rho^2-1)^{1/2}}{1+x^2}\leqslant\frac{1+(\rho^2-1)^{1/2}}{2} \tag{6a}$$

and

$$-1\leqslant y\leqslant 1,\quad \text{if } 1\leqslant\rho\leqslant 2^{1/2}. \tag{6b}$$

The focused beam radii at any plane in the region $-1\leqslant y\leqslant 1$, during flying region $-1\leqslant x\leqslant 1$, are always inside the region constrained by curves of $x_{\min}=x_0-\Delta x$ and $x_{\max}=x_0+\Delta x$. The reason can be expressed as follows. According to Eq. (4a), the minimum w_{nor}' occurs at $x=1/y$ if the y remains constant and a larger deviation from this x value causes larger w_{nor}'. Then, the absolute value of optimum x is always not less than one in the region $-1\leqslant y\leqslant 1$ and the largest absolute x value always has the minimum w_{nor}' during flying region $-1\leqslant x\leqslant 1$.

Figure 6, given by Eq. (4b), shows the variation of the focused beam radius as a function of y for a given positive value of x. The maximum Δy, i.e., the DOF, occurs as the focusing point locates at the beam waist where $y_0=x/(1+x^2)$. Once the focusing point shifts from beam waist, the maximum ρ value within the region $y_0-\Delta y\leqslant y\leqslant y_0+\Delta y$ occurs at

$$y_0|_{\rho_{\max}}=\frac{x}{1+x^2}\pm\left[\Delta y^2+\frac{1}{(1+x^2)^2}\right]^{1/2}. \tag{7}$$

Here if $0\leqslant x\leqslant 1$, we can find that $y_0|_{\rho_{\max}}$(minus root)<0 or $y_0|_{\rho_{\max}}$(plus root)$>2x/(1+x^2)$. This means that within the region $y_0|_{\rho_{\max}}$(minus root)$\leqslant y_0\leqslant y_0|_{\rho_{\max}}$(plus root) marked as region I in Fig. 6, if the focusing point comes closer to the beam waist, the Δy value becomes larger. This

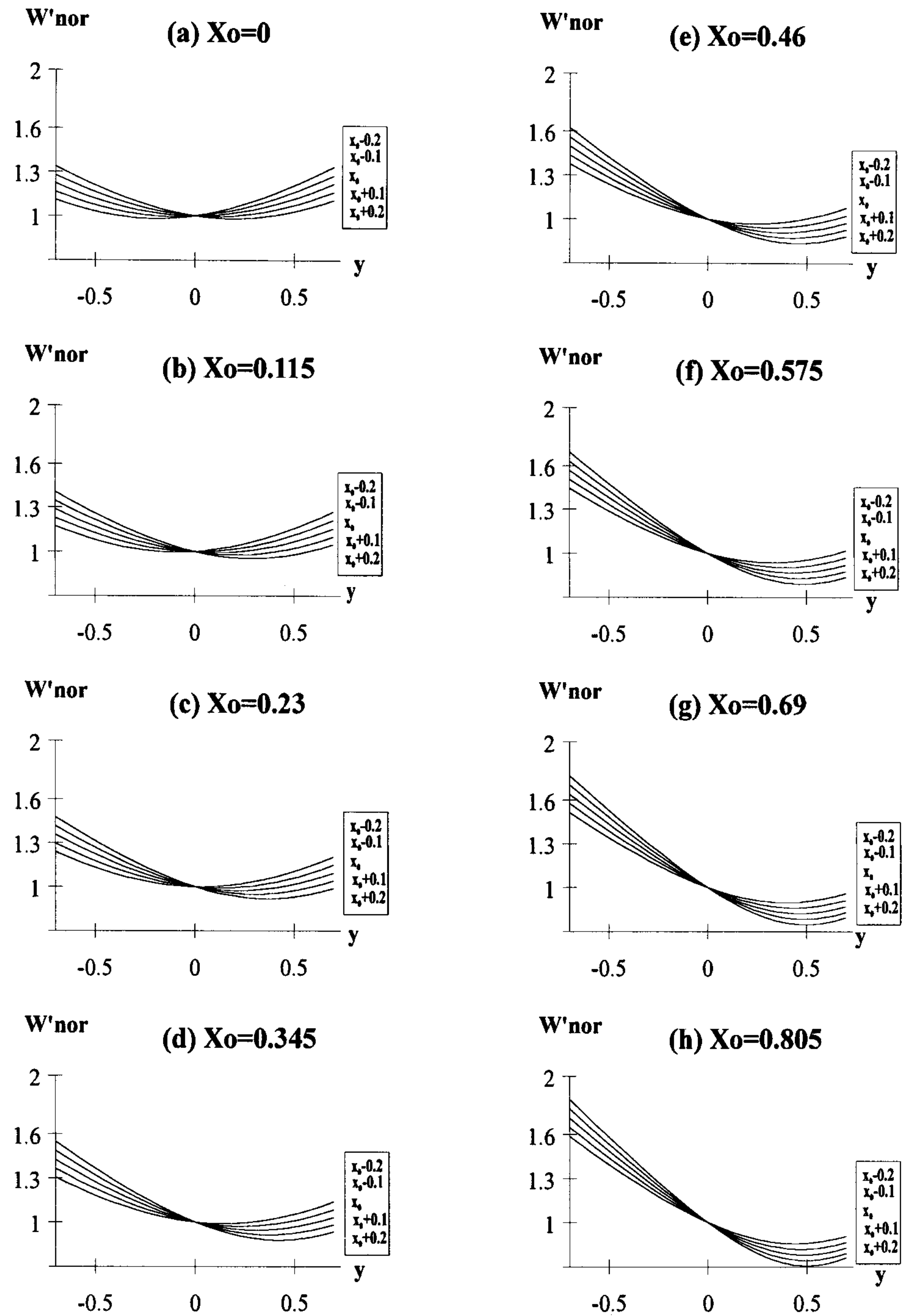

Fig. 5 Variation of W'_{nor} in the case $x_0-\Delta x\leqslant x\leqslant x_0+\Delta x$, where $\Delta x=0.2$.

conclusion can also be available in the region $y_0|_{\rho_{max}}$(plus root)$\leqslant y_0\leqslant y_0|_{\rho_{max}}$(minus root) for $-1\leqslant x\leqslant 0$.

The DOF of flying optics for any flying region $x_{min}\leqslant x\leqslant x_{max}$, where $|x_0|\leqslant\Delta x$, $x_{min}\geqslant -1$, $x_{max}\leqslant 1$, can be determined by the condition $w'_{nor}(x_{max},y_{min})\equiv w'_{nor}(x_{min},y_{max})$ under which the beam radius ratio ρ within the region $y_{min}\leqslant y\leqslant y_{max}$ is minimum. The reason is expressed as follows. In the case where y_{max} does not exceed the beam waist position of x_{max} curve, i.e., $y_{max}\leqslant y_R$, as shown in Fig. 7(a), an increasing y_0 value causes the maximum and minimum radii in the $2\Delta y$ region to move from A (or Aa) and B to A_R and B_R, respectively, and the beam radius ratio ρ increases. Otherwise, decreasing y_0 value causes the maximum and minimum radii in the $2\Delta y$ region to move from A (or Aa) and B to A_L and B_L, respectively, and according to the discussion on Fig. 6, the beam radius ratio ρ still increases. In the case where y_{max} does exceed the beam waist position of x_{max} curve, i.e., $y_{max}>y_R$, as shown in Fig. 7(b), the minimum radius is at B, increasing or

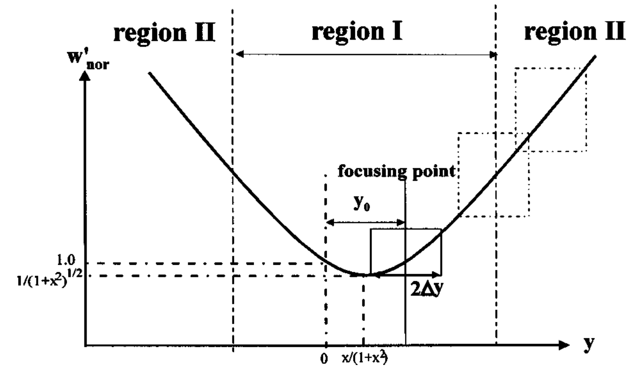

Fig. 6 Variation of the focused beam radii ($W'_{\rm nor}$) as a function of y for a given positive value of x.

decreasing the y_0 value causes the maximum radius A to increase to A_R or A_L, respectively, and the beam radius ratio ρ increases.

6 Derivative of the DOF in Flying Optics

With the flying range $2L$, s varies within the region $s_0-L\leqslant s\leqslant s_0+L$, or $-L+(f+d)\leqslant s\leqslant L+(f+d)$, where $d\equiv s_0-f$ and $0\leqslant d\leqslant L$, and then we have

$$-1\leqslant x_0-\Delta x\leqslant x\leqslant x_0+\Delta x\leqslant 1,$$

and

$$0\leqslant x_0\leqslant \Delta x,$$

where

$$\begin{cases} x=(s-f)/Zr \\ x_0=d/Zr \\ \Delta x=L/Zr \end{cases}. \tag{8}$$

The required region of y is

$$-1\leqslant y_0-\Delta y\leqslant y\leqslant y_0+\Delta y\leqslant 1,$$

where

$$\begin{cases} y=(t-f)Zr/(f^2) \\ y_0=(t_0-f)Zr/(f^2) \\ \Delta y=(\mathrm{DOF})Zr/(f^2). \end{cases} \tag{9}$$

On the $x_{\rm min}$ curve, the normalized waist radius is

$$w'_{\rm nor}(L)=\left[\frac{1}{1+(x_{\rm min})^2}\right]^{1/2} \tag{10}$$

at the location

$$y_L=\frac{x_{\rm min}}{1+(x_{\rm min})^2}. \tag{11}$$

On the $x_{\rm max}$ curve, the normalized waist radius is

$$w'_{\rm nor}(R)=\left[\frac{1}{1+(x_{\rm max})^2}\right]^{1/2} \tag{12}$$

at the location

$$y_R=\frac{x_{\rm max}}{1+(x_{\rm max})^2}. \tag{13}$$

The DOF is evaluated from point $y=0$ where the $w'_{\rm nor}$ remains constant when x varies as shown in Fig. 7.

In the region $y\geqslant 0$ ($t\geqslant f$), the maximum $w'_{\rm nor}$ locates on the $x_{\rm min}$ curve and the value increases with the y value. The minimum $w'_{\rm nor}$ locates on the $x_{\rm max}$ curve, and the value decreases with the y value in the region $0\leqslant y\leqslant y_R$ but increases with the y value in the region $y>y_R$. Therefore, we get

$$\max\, w'_{\rm nor}=\{(y_{\rm max})^2+[(y_{\rm max})(x_{\rm min})-1]^2\}^{1/2} \tag{14}$$

$$\min\, w'_{\rm nor}=\begin{cases} \{(y_{\rm max})^2+[(y_{\rm max})(x_{\rm max})-1]^2\}^{1/2} \\ \quad \text{if } 0\leqslant y_{\rm max}\leqslant y_R \\ \left[\dfrac{1}{1+(x_{\rm max})^2}\right]^{1/2} \\ \quad \text{if } y_{\rm max}>y_R. \end{cases} \tag{15}$$

According to the requirement $\max w'_{\rm nor}/\min w'_{\rm nor}=\rho$, we can calculate the $y_{\rm max}$ from the preceding equations and have

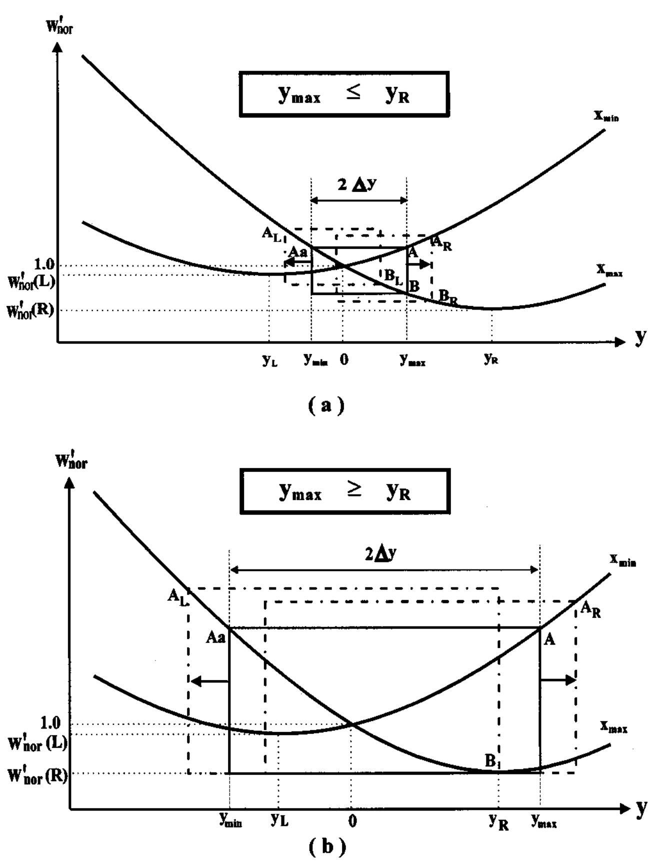

Fig. 7 Determination of the DOF in flying optics ($0\leqslant x_0\leqslant \Delta x$).

$$y_{t>f}\equiv(y_{\max})(\Delta x)=\begin{cases}\dfrac{(\rho^2+1)+(\rho^2-1)a-[4\rho^2-(\rho^2-1)^2(1/\Delta x)^2]^{1/2}}{(\rho^2-1)[(1+a)^2+(1/\Delta x)^2]+4a} & \text{if } \Delta x\geqslant\Delta x_{\text{bp}}\\ \dfrac{-(1-a)[(1+a)^2+(1/\Delta x)^2]+((1/\Delta x)^2[(1+a)^2+(1/\Delta x)^2]\{(\rho^2-1)[(1+a)^2+(1/\Delta x)^2]-4\rho^2 a\})^{1/2}}{[(1+a)^2+(1/\Delta x)^2][(1/\Delta x)^2+(1-a)^2]} & \text{if } \Delta x\leqslant\Delta x_{\text{bp}},\end{cases} \tag{16a}$$

where

$$\begin{cases}a=d/L\\ \Delta x=L/Zr\end{cases} \tag{16b}$$

Using the relationship $y_{t>f}(\Delta x=\Delta x_{\text{bp}})|_{\Delta x\geqslant\Delta x(\text{bp})}\equiv y_{t>f}(\Delta x=\Delta x_{\text{bp}})|_{\Delta x\leqslant\Delta x(\text{bp})}\equiv(y_R)(\Delta x)$ and Eq. (16a) in which $\Delta x=\Delta x_{\text{bp}}$, we can find Δx_{bp} as

$$\Delta x_{\text{bp}}=\left(\frac{[(\rho^2-1)a^2+2(\rho^2-3)a+(\rho^2-5)]+\{[(\rho^2-1)a^2+2(\rho^2-3)a+(\rho^2-5)]^2+16(\rho^2-1)(1+a)^2\}^{1/2}}{8(1+a)^2}\right)^{1/2}. \tag{17}$$

In the region $y<0$ $(t<f)$, we can calculate the $y_{\min}$ with the condition $w'_{\text{nor}}(x_{\max},y_{\min})\equiv w'_{\text{nor}}(x_{\min},y_{\max})$ and have

$$y_{t<f}\equiv(y_{\min})(\Delta x)=\frac{(1+a)-([(1/\Delta x)^2+(1+a)^2]\{[(1/\Delta x)^2+(1-a)^2]y_{t>f}{}^2+2(1-a)y_{t>f}\}+(1+a)^2)^{1/2}}{(1/\Delta x)^2+(1+a)^2} \tag{18}$$

Therefore, the normalized DOF can be expressed as

$$\text{DOF}_{\text{nor}}\equiv\frac{(\text{DOF})L}{f^2}=\frac{y_{t>t}-y_{t<f}}{2}. \tag{19}$$

The nominal focusing point, i.e., the center of the allowable region, has a position shift from the back focal point of lens. This shift can be normalized and expressed as

$$\Delta f_{\text{nor}}\equiv\frac{(t_0-f\,)L}{f^2}=\frac{y_{t>f}+y_{t<f}}{2}. \tag{20}$$

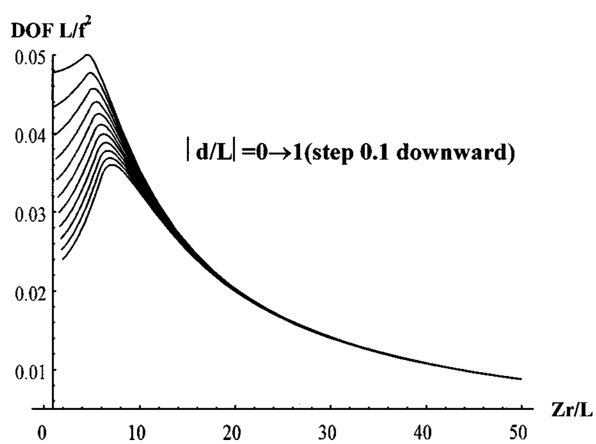

Fig. 8 System DOF of flying optics as a function of *Zr* and *d*.

In the region $-L\leqslant d\leqslant 0$, i.e., $-\Delta x\leqslant x_0\leqslant 0$, with a similar procedure the same result can be obtained except that $|d|$ takes place of d and Δf_{nor} changes its sign.

Figures 8 and 9, given by the Eqs. (19) and (20), show the variations of DOF_{nor} and Δf_{nor}. From these figures, it can be seen that the left-side start point of each curve with different $|d|/L$ value is different. The reason is that present analysis has the limited region $-1\leqslant x\leqslant 1$, i.e., $1/\Delta x\geqslant 1+|x_0|/\Delta x$. Because $1/\Delta x\equiv Zr/L$ and $|x_0|/\Delta x\equiv|d|/L$, we

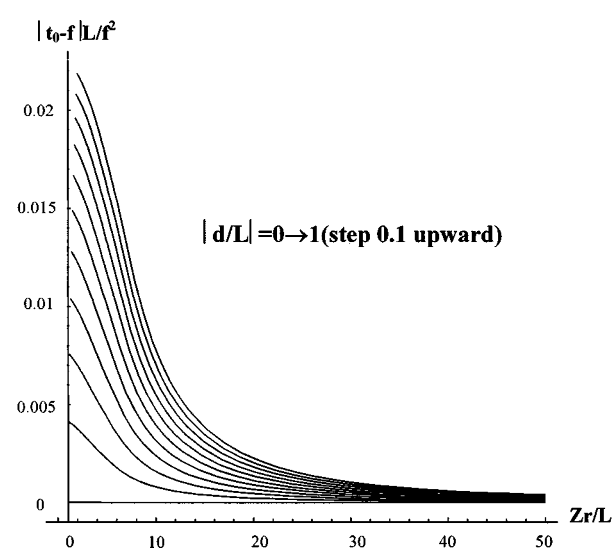

Fig. 9 System focusing point shift of flying optics as a function of *Zr* and *d*.

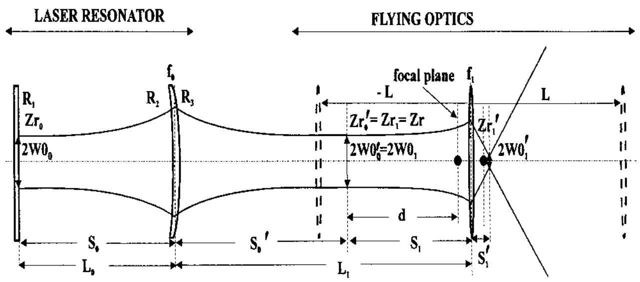

Fig. 10 Geometry of the flying optics.

find the condition $Zr \geqslant L + |d|$. We can find the conditions for the optimal DOF of flying optics as follows:

1. An optimum Zr should be taken.
2. The center of flying region should locate at front focal point ($d \to 0$).
3. The lens of longer focal length should be used.
4. Less value in flying range ($2L$) should be taken.

7 Flying Optics with $d=0$

In case of $d=0$, i.e., $a=0$, the $1/e^2$ radius curves are symmetrical with respect to the $t=f$ plane [$x_0=0$, see Fig. 5(a)]. From Eqs. (16a) and (18), we have

$$y_{t>f} = -y_{t<f} = \begin{cases} \dfrac{(\rho^2+1) - [4\rho^2 - (\rho^2-1)^2 (1/\Delta x)^2]^{1/2}}{(\rho^2-1)[1+(1/\Delta x)^2]} & \text{if } \Delta x \geqslant \Delta x_{\mathrm{bp}} \\ \dfrac{-1+(\rho^2-1)^{1/2}(1/\Delta x)}{1+(1/\Delta x)^2} & \text{if } \Delta x \leqslant \Delta x_{\mathrm{bp}}. \end{cases} \tag{21}$$

From Eq. (17), we have

$$\Delta x_{\mathrm{bp}} = \frac{(\rho^2-1)^{1/2}}{2}. \tag{22}$$

From Eq. (21), we have the optimal results as

$$\begin{cases} \Delta x|_{\mathrm{opt}} = \left(\dfrac{\rho-1}{\rho+1}\right)^{1/2} \\ \mathrm{DOF}_{\mathrm{nor}}|_{\mathrm{opt}} = \dfrac{\rho-1}{2}. \end{cases} \tag{23}$$

In laser material processing, if we take $\rho=1.1$ and $d=0$, from Eq. (23), the optimal condition of input beam is $Zr=\sqrt{21}L$ and the optimum DOF is $0.05 f^2/L$.

8 Analysis Example

To demonstrate the use of our derived equations, an example is presented of the optimum design of output coupler in laser-gantry-robot systems. The system geometry is shown in Fig. 10, which consists of a stable resonator with $R_1=\infty$, $R_2=30$ m, $L_0=8.05$ m, a focusing lens with $f_1=0.254$ m, and flying optics with $L=6$ m. The curvature radius R_3 of the outside surface in this output coupler is considered as a variable to find the optimum DOF value for $\rho=1.1$.

Following Luxon's analytical approach,[5] the focal length of output coupler was modified for the thermal-lensing effect with the equation $1/f_T = 1/f_0 + 1/f_{\mathrm{ther}}$, in which $f_{\mathrm{ther}}=83.9$ m, see Figs. 11(a) and 11(b). The output beam parameters (s_0', Zr_0') of the laser resonator as the functions of the radius R_3, are shown in Figs. 11(c) and 11(d). In this system where the flying region is from 4 to 16 m after the output coupler, the center of the flying region is at the position $L_1=10$ m. The output beam parameters of

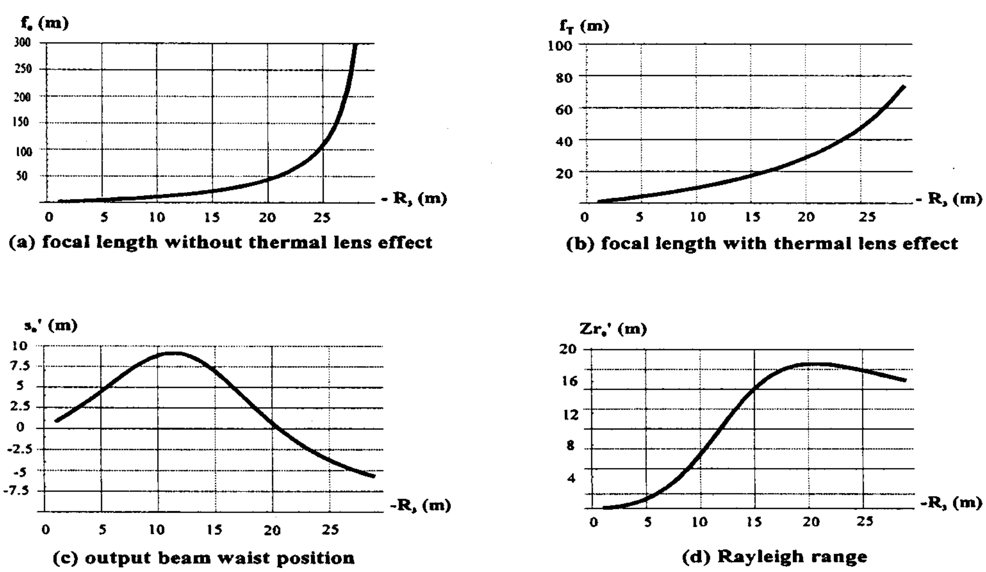

Fig. 11 Optimizing the output coupler ($R_1=\infty$, $R_2=30$ m, $L_0=8.05$ m, $f_{\mathrm{ther}}=83.9$ m, and $n_{\mathrm{ZnSe}}(\lambda_{\mathrm{co_2}})=2.4$) and output beam parameters (s_0', Zr_0') of laser resonator as the functions of R_3, respectively.

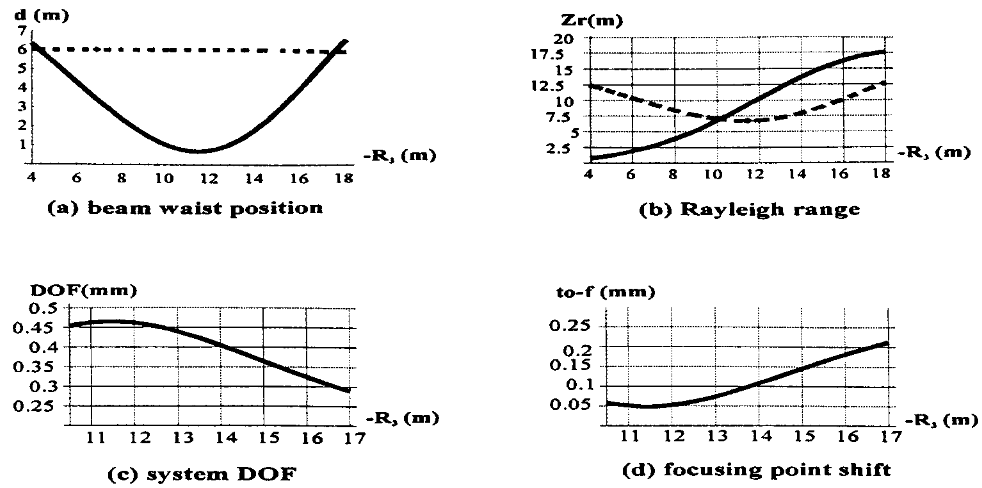

Fig. 12 Optimizing the output coupler (flying region 4, 16 m; 10-in. focusing lens, $\rho=1.1$) and focusing beam parameters (system DOF, focusing point shift) as functions of R_3, respectively. The dashed lines in (a) and (b) are drawn by the equations $|d|=L$ and $Zr=L+|d|$, respectively.

the laser resonator should be transferred to the input beam parameters (d,Zr) of flying optics with $d=(L_1-s_0')-f_1$ and $Zr=Zr_1=Zr_0'$. Figures 12(a) and 12(b) show d and Zr as functions of R_3, respectively. According to our assumptions $-1\leqslant x\leqslant 1$ and $-\Delta x\leqslant x_0\leqslant \Delta x$, we have the constraints $Zr\geqslant L+|d|$ and $|d|\leqslant L$, as the dashed curves show in these figures, which limit the considered region of R_3. Then using Eqs. (19) and (20), we calculate the DOF and focusing point shift shown in Figs. 12(c) and 12(d) in which the optimum DOF is about 0.47 mm when $R_3=-11.5$ m. If $R_3=-17$ m is chosen, we get DOF≅0.29 mm, which is shorter. Figure 13 shows that $R_3=-11.5$ m has larger variations in output beam parameters (s_1',Zr_1') of focusing lens during flying, but with a larger Zr_1' value, which results in a larger DOF.

This example shows that we can not get the optimum DOF of flying optics only by minimizing the variations of w_0', Zr', and s'.

9 Discussion and Conclusions

In laser material processing the power density (irradiance) of working beam has a tolerance range for a particular application. As a fixed laser power is applied during processing, this tolerance becomes a constraint on the acceptable variation in the working beam radius. Because the machined materials do have a certain thickness, the DOF of

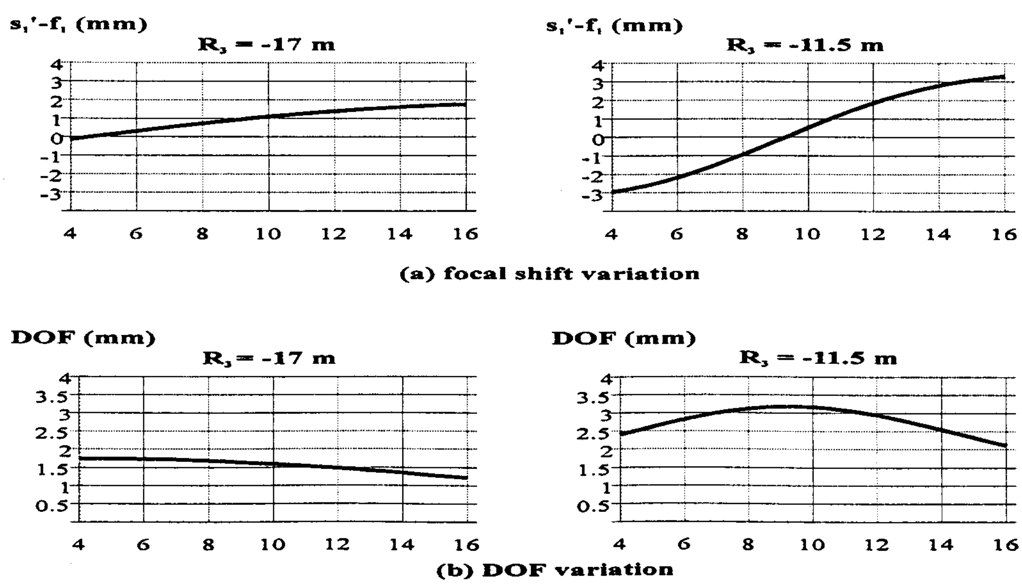

Fig. 13 Optimizing the output coupler: the variations of focus and DOF for $R_3=-17$ m and $R_3=-11.5$ m during the flying region (4 to 16 m).

laser beam must be considered as a significant factor in system design to keep the spot size within the tolerance in the processed thickness.

In flying optics, due to the s variation in processing, the complication of the variation in the afterlens beam radii, as shown in Figs. 3, 4, and 5, makes the DOF analysis difficult. The maximum value of the DOF requires both the fewer variations of w_0', Zr', and s' and a larger Zr' value at the same time. From Fig. 3, fewer variations of w_0' and Zr' requires a larger Zr/f value and $s_0 \rightarrow f$, but less variation of s' requires a larger Zr/f value and $s_0 \rightarrow f \pm Zr$. However, the larger Zr' requires a smaller Zr/f value. In earlier works, these inconsistent requirements made it difficult to find the optimal DOF value and associated parameters in system design. In this paper, the total effect of these parameters on the flying optics are derived and explicitly shown in Fig. 8. The DOF of flying optics can be evaluated with Eq. (19) as a merit function to give the optimal system parameters (d, Zr) and the optimal DOF itself. In the nonflying optics, as discussed in the preceding section, a minimum Zr value is required when optimum condition $s = f$ are taken.

In the laser material processing system design, both the working beam radius and the DOF must be considered. In this paper, the DOF analysis in flying optics can be used to optimize the DOF during the system design.

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