Static and dynamical anisotropy effects in the mixed state of d-wave superconductors

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We describe the effects of anisotropy caused by the crystal lattice in d-wave superconductors using an effective free-energy approach in which only one order parameter, the d-wave order-parameter field, is used. The Abrikosov parameter β_A is calculated analytically for both static and moving vortex lattices. The analytic expression provides an unambiguous determination of the vortex-lattice structure. We also calculate both direct and Hall I-V curves as functions of the angle between the current and the crystal-lattice orientation. In particular, we show that near H_{c2} the fourfold symmetry of the crystal lattice causes asymmetric motions of particles and holes, resulting in a nonvanishing Hall current. [S0163-1829(98)07909-0]

I. INTRODUCTION

It is widely believed that the major pairing channel in layered high-T_c cuprates is the $d_{(x^2-y^2)}$ pairing.[1] The $d_{(x^2-y^2)}$ pairing comes with fourfold symmetry, which has been observed to change the vortex-lattice structure.[2–4] There are also indications that even though the major bulk pairing mechanism is of d-wave nature, there is a small admixture of the s-wave pairs in the condensate. Several derivations have been given, aiming to derive Ginzburg-Landau (GL) theory for the mixing of d and s waves from microscopic models that respect the D_{4h} symmetry of the Cu-O plane.[5–8] The free energy constructed thus has D_{4h} symmetry and contains two fields d and s,[9,10]

$$f=\alpha_s|s|^2-\alpha_d|d|^2+\beta_1|s|^4+\beta_2|d|^4+\beta_3|s|^2|d|^2+\beta_4(s^{*2}d^2+d^{*2}s^2)+\gamma_s|\mathbf{\Pi}s|^2+\gamma_d|\mathbf{\Pi}d|^2+\gamma_v[s^*(\Pi_y^2-\Pi_x^2)d+\text{c.c.}], \tag{1}$$

where $\mathbf{\Pi}\equiv-i\nabla-e^*\mathbf{A}$ is the covariant derivative and e^* is the charge of the Cooper pair (throughout this paper we use the convention $c=\hbar=1$). Within a particular microscopic model there might be some relations between these coefficients, but since the ultimate microscopic theory is not known as yet, all of them should be considered as phenomenologically fixed parameters.

Using equations following from this free energy or more fundamental equations (see the recent quasiclassical Eilenberger equation treatment in Ref. 7), one obtains a characteristic four-lobe structure with four zeros for the s wave inside a single vortex.[9,10] Therefore the vortex core loses full rotational symmetry and only fourfold D_{4h} symmetry remains.

It was later pointed out by Affleck, Franz, and Amin[11] (AFA) that because s is induced by gradients of d [with the approximated relation $s\approx(-\gamma_v/\alpha_s)(\Pi_y^2-\Pi_x^2)d$], a single component ($d$-wave) effective free energy that incorporates the corrections due to the s wave is sufficient,

$$f_{\text{eff}}[d]=\frac{1}{2m_d}|\mathbf{\Pi}d|^2-\alpha_d|d|^2+\beta|d|^4-\eta d^*(\Pi_y^2-\Pi_x^2)^2d. \tag{2}$$

The corresponding GL equation is

$$\left(\frac{1}{2m_d}\Pi^2-\alpha_d\right)d-\eta(\Pi_y^2-\Pi_x^2)^2d+2\beta|d|^2d=0, \tag{3}$$

where we have replaced γ_d by a more conventional notation $1/2m_d$. The parameter $\eta\equiv\gamma_v^2/\alpha$ is due to the nonvanishing of the s wave, representing D_{4h} symmetry. The contributions to it might come not only from s-d mixing, which always gives a positive η, but also from other sources. In $YBa_2Cu_3O_7$ (YBCO), twinning might be an important contribution to it. By simple dimensional analysis, one can easily show that up to dimension five, $d^*(\Pi_y^2-\Pi_x^2)^2d$ is the only term that breaks rotational symmetry down to D_{4h}. Because fourfold symmetry is primarily located inside the core, the η term will be important near H_{c2}. In the one-component approach, since only fourfold symmetry is retained, in principle it applies to conventional superconductors with fourfold symmetry as well.[12] In this case, the origin of the η term is not s-d mixing, and its effect on the vortex lattice has been recently observed.[13] The one-field formulation greatly reduces the number of parameters (instead of nine as in the two-field approach, it has only four parameters), and thus is more analytically tractable.

In the work by AFA, the one-field approach was used to investigate the structure of the static vortex lattice in the London approximation.[11] Here we apply this formalism to study two properties that depend the most on the core structure, that is, the static and moving vortex-lattice structure and the I-V curve for the flux flow near H_{c2}. The simplicity of the formulation allows us to obtain an analytic expression of the Abrikosov parameter β_A to leading order in η. Unlike previous approaches, the numerical values of β_A can be evaluated to a very high precision and thus provide an unambiguous way to determine the vortex-lattice structure. The degrees of freedom we include in the analysis contain (1) an arbitrary rotation angle φ between the crystal lattice and the vortex lattice and (2) all the possible lattices, not only the rectangular ones considered before.[6,9] The lattice is demonstrated to be body-centered rectangular (CR) with the most general lattice included in the analysis. Moreover, the treatment can be easily extended to moving flux lattices, which, as is well known[14] are more demanding, as far as calculational complexity is concerned. Our results indicate that moving lattices are still the CR type. More importantly, a nonvanishing Hall current arises near H_{c2}. The Hall current is due to the asymmetric motions of particles and holes caused by the anisotropy of the crystal lattice. It is nonlinear and can become large enough to be observed if the field **E** is large. This is very different from the simple s-wave case in which one has to introduce a complex relaxation time in order to break the particle-hole symmetry.[15]

This paper is organized as follows. In Sec. II, we show that the single-vortex solution that is obtained in the one-field approach is almost identical to the solutions obtained earlier within the two-field approach. One still can define the ''effective s-wave field by $s=(-\gamma_v/\alpha_s)(\Pi_y^2-\Pi_x^2)d$ and observe the four-lobe structure; see Figs. 2 and 3 for d and s components, respectively. Relations to earlier work (discrepancies or common points) are summarized in Appendix A.

The vortex lattice near H_{c2} is studied in Sec. III. The simplicity of the formulation allows for an analytic study of all the possibilities, not considered before or considered using uncontrollable approximations. We tabulate the lattice characteristics for different η in Fig. 7. At a certain value of η there is a phase transition from rectangular to a more symmetric square lattice first noticed.[9] The existence of a phase transition becomes obvious in our formulation in which the effective strength of the fourfold symmetry is proportional to the magnetic field, characterized by a dimensionless parameter $\eta'\equiv\eta m_d e^* H$. In low fields, the fourfold symmetry is subdominant, and so the lattice is closer to a triangle lattice. In high fields, the fourfold symmetry dominates, and so the lattice becomes square. We find that the transition occurs at $\eta_c'=0.0235$.

The moving lattice solutions are derived in Sec. IV from an appropriately generalized time-dependent Ginzburg-Landau equation (TDGL), which in the one-field formalism, only one additional parameter is introduced: the relaxation time for the d field. They are not only needed for the nonlinear conductivity calculation, but are also interesting in their own right, since they are, in principle, observable. Unlike in the case of the pure s-wave superconductor, the moving (with arbitrary, not infinitesimal, velocity) lattice solution in this case cannot be obtained from the static one by a simple Galilean boost.[14] It is a nontrivial problem and we were able to solve it perturbatively in η. While the s-wave moving lattices are triangular,[14] the orientation is determined by the direction of the crystal lattice as well as by the current direction. The Abrikosov β_A now depends on the angle Θ between the electric field **E** and an axis of the underlying atomic lattice axis. The dynamical phase transition line as a function of current and its orientation with respect to the atomic lattice are quantitatively discussed in Sec. IV A.

In Sec. V, we derive the nonlinear conductivity. The result is remarkably simple. In addition to the isotropic linear parts, there is an anisotropic direct current, cubic in the electric field **E**,

$$\Delta\mathbf{J}_{\rm dir}=-\eta'\frac{\alpha_d m_d^2\gamma^3E^2}{\beta\beta_A^0H^4}(1+\cos 4\Theta)\mathbf{E}, \tag{4}$$

and the Hall current is

$$\Delta\mathbf{J}_{\rm Hall}=-\eta'\frac{\alpha_d m_d^2\gamma^3E^2}{\beta\beta_A^0H^4}\sin 4\Theta(\mathbf{E}\times\hat{z}). \tag{5}$$

The presence of the Hall current is argued to be entirely due to D_{4h} symmetry. In these expressions γ is the order-parameter relaxation rate. Both direct and Hall I-V curves depend on the angle between the current and the crystal-lattice orientation via the fourth harmonic only. The result contains only a cubic dependence of the currents on the electric field, higher orders being canceled.

Finally in Sec. VI we conclude by briefly discussing possible experiments to observe the various above mentioned effects, as well as some generalizations.

II. SINGLE-VORTEX SOLUTION

In this section we shall find an isolated vortex solution of the one-component equation, Eq. (3), near H_{c1}. The opposite case in which the magnetic field is close to H_{c2} will be considered in the next section. We measure the order-parameter field in units of the vacuum expectation value $\Psi_0=\sqrt{\alpha_d/2\beta_2}$ and length in units of the coherence length $\xi_d=1/\sqrt{2m_d\alpha_d}$. In strongly type-II materials (when the Ginzburg-Landau parameter κ is very large), as is the case in high-T_c superconductors, we can safely ignore the magnetic field and the dimensionless GL equation becomes

$$(-\nabla^2-1)d-\lambda(\nabla_y^2-\nabla_x^2)^2d+|d|^2d=0, \tag{6}$$

where $\lambda\equiv4\eta m_d^2\alpha_d$ is the dimensionless small perturbative parameter characterizing the anisotropy near H_{c1}. Equation (6) can be solved perturbatively in λ by setting $d=d_0+\lambda d_1+\cdots$, where $d_0=f_0(r)e^{i\phi}$ is the solution of the standard unperturbed GL equation. Then the first-order equation in λ is

$$(-\nabla^2-1)d_1+(2|d_0|^2d_1+d_0^2d_1^*)=(\nabla_y^2-\nabla_x^2)^2d_0. \tag{7}$$

The angular dependence of d_1 is easily observed to contain only three harmonics $e^{-3i\phi}$, $e^{+i\phi}$, and $e^{5i\phi}$. This is consistent with fourfold symmetry which is built into the theory. We therefore decompose d_1 into a combination of these three harmonics:

$$d_1(r,\phi)=f_{-3}(r)e^{-3i\phi}+f_1(r)e^{i\phi}+f_5(r)e^{5i\phi}. \tag{8}$$

The equation becomes

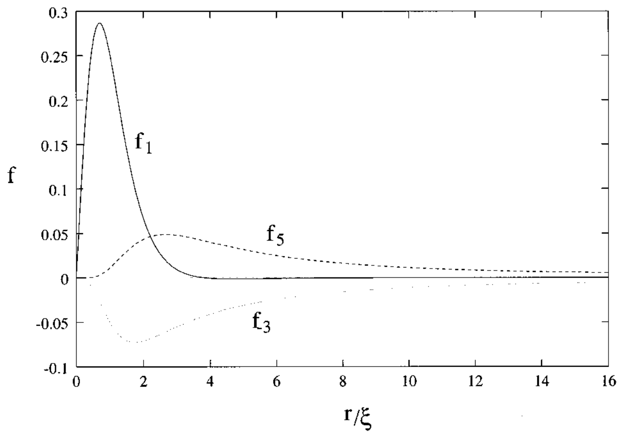

FIG. 1. A single-vortex solution of the one-component GL equation. The coefficient functions f_1, f_{-3}, f_5 are for the harmonics $e^{i\phi}, e^{-3i\phi}, e^{5i\phi}$, respectively. f_i's are given in units of $\Psi_0=\alpha_d/2\beta$, and r is given in units of ξ_d. See Eqs. (9), (10), and (11).

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{9}{r^2}+1\right)f_{-3}-f_0^2(2f_{-3}+f_5)=-J_{-3}(r), \tag{9}$$

$$\left(\frac{d^2}{dr}+\frac{1}{r}\frac{d}{dr}-\frac{25}{r^2}+1\right)f_5-f_0^2(2f_5+f_{-3})=-J_5(r), \tag{10}$$

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}+1\right)f_1-3f_0^2f_1=-J_1(r), \tag{11}$$

with J_i's are defined by

$$(\nabla_y^2-\nabla_x^2)^2[f_0(r)e^{i\phi}]=e^{i\phi}J_1(r)+e^{-3i\phi}J_{-3}(r)+e^{5i\phi}J_5(r).$$

As is well known, an analytic expression for f_0 does not exist; however, there are a number of known good approximations. Using one of them,[16] $f_0(r)=r/\sqrt{r^2+\xi_v^2}$, the set of linear equations is then solved numerically (the third equation decouples from the first two). The results are shown in Fig. 1. The d-wave configuration is basically indistinguishable from that of the two-field formalism; we show the solution for $\lambda=0.15$ in Fig. 2.

Note also that within the same approximation and normalization, the s component is $s=\lambda'(\nabla_y^2-\nabla_x^2)d_0$ with $\lambda'=2\gamma_v m_d(\alpha_d/\alpha_s)=\lambda/2\gamma_v m_d$ being another dimensionless small parameter. It is easy to see that s has the asymptotic behaviors

$$s\sim re^{-i\phi}, \quad r\to 0, \quad s\sim\frac{1}{r^2}e^{+3i\phi}, \quad r\to\infty. \tag{12}$$

The s field is plotted in Fig. 3. The different winding numbers in the near and far asymptotic regions give rise to four additional poles in the s component in the intermediate region. This confirms calculations in Ref. 9 even though some asymptotic analytic expressions used there to obtain the numerical results disagree with ours. A comparison with Refs. 9 and 30 is presented in Appendix A.

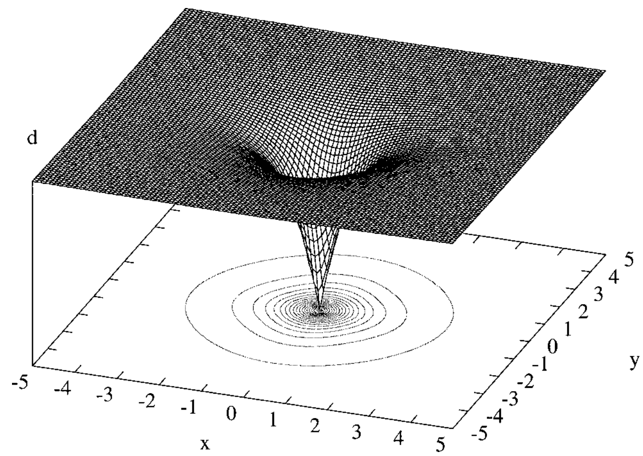

FIG. 2. The d field of a single vortex for $\eta=0.15$. Only the absolute value of the d field in units of Ψ_0 is shown. (a) Contour plot. (b) Three-dimensional plot.

III. VORTEX LATTICE NEAR H_{c2}

In this section we follow a generalization of Abrikosov's procedure[17,18] to investigate the structure of the vortex lattice near H_{c2}. One first ignores the nonlinear terms in the GL equation and finds a set of the lowest-energy solutions $\Psi_{k_n}(x,y)$ of the linearized equation. The vortex-lattice solution is constructed as a linear superposition

$$d(x,y)=\sum_n C_n\Psi_{k_n}(x,y) \tag{13}$$

in such a way that it is invariant under the corresponding symmetry group of a given lattice structure. It is well known that the free energy near H_{c2} is monotonic in Abrikosov's parameter β_A, defined by $\beta_A=\langle|d|^4\rangle/\langle|d|^2\rangle^2$, so that minimizing β_A equivalently minimizes the free energy.

A general lattice in two dimensions (2D) can be specified by three parameters a, b, and α, where a and b are the two lattice constants, while α is the angle between the two primitive lattice vectors (see Fig. 4). Flux quantization gives a constraint $Hab\sin\alpha=\Phi_0$, so that there are two free parameters. In the d-wave superconductors rotational symmetry is broken; therefore the relative orientation of the vortex lattice

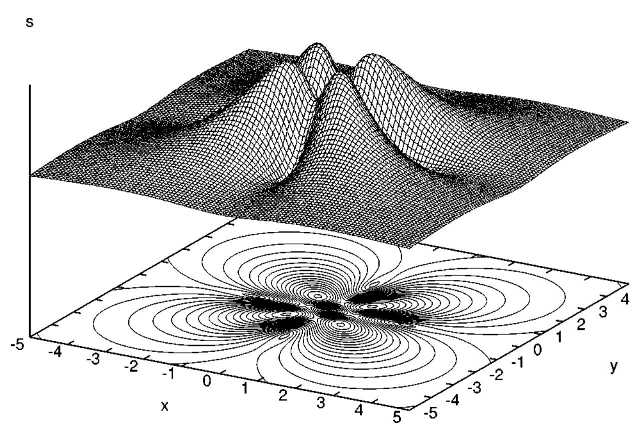

FIG. 3. The s field of a single vortex for $\eta=0.15$. (a) Contour plot. (b) Three-dimensional plot. Note that there are four singularities on which the s field vanishes.

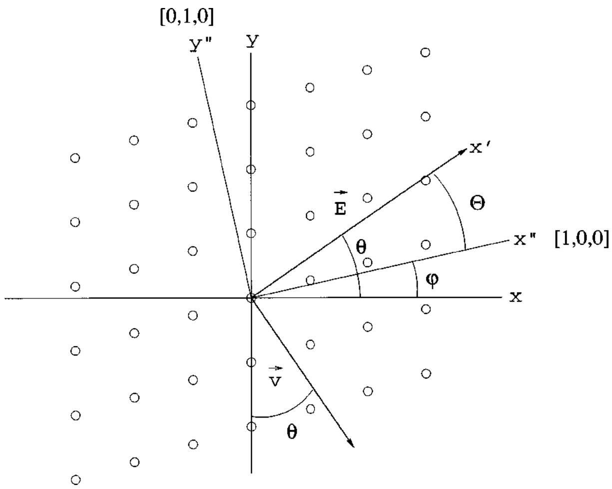

FIG. 4. The coordinate system used in our calculations; this defines the angles θ, φ, and Θ.

to the underlying lattice becomes important. Later we will denote φ to be the angle between **a** and one of the axes of the underlying lattice. In Abrikosov's original paper[17] he had assumed $C_n = C_{n+1}$ and obtained the square lattice; later Kleiner, Roth, and Autler[19] generalized the procedure to the case where $C_n = C_{n+2}$. In this way all the rectangular body-centered lattices can be included in the analysis. In previous work on d-wave superconductivity,[9] the same formalism was used; however, it did not include the most general lattice. In this section we follow a more generalized formulation of Ref. 18 to cover all possible lattice types.

A. Perturbative solution to the linearized GL equations

We start from the one-component linearized GL equation to find Ψ_{k_n},

$$\frac{1}{2m_d}\Pi^2 d - \eta(\Pi_y^2 - \Pi_x^2)^2 d = \alpha_d d, \tag{14}$$

where for later convenience we have moved $\alpha_d d$ to the right-hand side. It is important to note that in Eq. (14) we have assumed that the coordinate system and the underlying microscopic lattice coincide. Later it will be convenient to orient the coordinate system (x,y) with the Abrikosov vortex lattice rather the atomic crystal. In general, if the crystal is rotated by an angle φ clockwise with respect to the coordinate system, Eq. (14) becomes

$$\frac{1}{2m_d}\Pi^2 d - \eta[\cos 2\varphi(\Pi_x^2 - \Pi_y^2) + \sin 2\varphi(\Pi_x\Pi_y + \Pi_y\Pi_x)]^2 d = \alpha_d d. \tag{15}$$

It is convenient to introduce dimensionless creation and annihilation operators, $\hat{a} = i\Pi_+ l_H/\sqrt{2}$ and $\hat{a}^\dagger = -i\Pi_- l_H/\sqrt{2}$, where $\Pi_\pm \equiv \Pi_x \pm i\Pi_y$ and the scaling parameter $l_H = 1/\sqrt{|e^*|H}$ is the magnetic length. In terms of $\hat{a}$ and $\hat{a}^\dagger$, Eq. (14) becomes

$$\left[\hat{a}^\dagger\hat{a} + \frac{1}{2} - \eta'(e^{+2i\varphi}\hat{a}^{\dagger 2} + e^{-2i\varphi}\hat{a}^2)^2\right] d(x,y) = \frac{H^0}{2H} d(x,y). \tag{16}$$

Here the dimensionless parameter η' is given by $\eta' = \eta m_d |e^*| H$. For later convenience, we have defined an unperturbed (conventional) upper critical fields $H^0 \equiv \Phi_0/(2\pi\xi_d^2) = 2m_d\alpha_d/|e^*|$.

In the Landau gauge $\mathbf{A} = Hx\hat{y}$, the y dependence is trivially separated and we can write $d(x,y)$ as $\exp(iky)\psi_k(x)$. The operators $\hat{a}$ and $\hat{a}^\dagger$ then become

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\frac{d}{d\tilde{x}} + \tilde{x}\right), \tag{17}$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{2}}\left(-\frac{d}{d\tilde{x}} + \tilde{x}\right), \tag{18}$$

where $\tilde{x} \equiv (x - x_0)/l_H$ is dimensionless with $x_0 \equiv k l_H^2$. Using standard perturbation theory, we found the lowest eigenvalue to be

$$\frac{H^0}{2H} = \frac{1}{2} - 2\eta' + O(\eta^2). \tag{19}$$

This determines the upper critical field

$$H_{c2}(T) = \frac{2m_d\alpha'}{|e^*|}[(T_c - T) + 8\eta m_d^2\alpha'(T_c - T)^2], \tag{20}$$

where we have written α_d as $\alpha'(T_c - T)$. Note that the relative angle φ does not affect H in the lowest order. We observe that around T_c for a positive η the $H(T)$ curve bends upwards, in agreement with the two-field results.[9,20] This effect has been reported in some experiments. However, since the coefficient $\alpha_d = \alpha'(T_c - T)$ is only accurate to first order in $T_c - T$, one should be cautious about taking this too seriously.

Finally, the corresponding eigenfunction $\psi(x)$ is

$$\psi(\tilde{x}) = \left(\frac{1}{\pi l_H^2}\right)^{1/4}\left[1 + \eta'\frac{e^{+4i\varphi}}{16}H_4(\tilde{x})\right]\exp\left(-\frac{\tilde{x}^2}{2}\right), \tag{21}$$

where $H_4(\tilde{x})$ is the fourth Hermit polynomial.

B. Abrikosov parameter and optimal vortex-lattice structure

Now we proceed to calculate Abrikosov's $\beta_A \equiv \langle |d|^4\rangle / \langle |d|^2\rangle^2$. Here the angular brackets are defined as $\langle f\rangle \equiv (1/A)\int^2 \mathbf{r} f(\mathbf{r})$ with A being the total area of the system. If the function $f(\mathbf{r})$ is periodic, it is sufficient to calculate the average over one unit cell.

In the Landau gauge, a generic solution of the linearized equation takes the form $\Psi_k(x,y) = \exp(iky)\psi(x - kl_H^2)$. Periodicity in the y direction (our lattice vector **a** by assumption is aligned with this axis; see Fig. 4) allows the following linear combinations:

$$d(x,y) = \sum_{n=-\infty}^{\infty} C_n \Psi_{k_n}(x,y) = \sum_{n=-\infty}^{\infty} C_n \exp\left(i\frac{2\pi n}{a}y\right)\psi\left(x - n\frac{2\pi l_H^2}{a}\right). \tag{22}$$

If the second lattice constant is b and it makes an angle α relative to the y axis, the periodicity in the $\hat{b}$ direction requires that $d(x-b\sin\alpha, y+b\cos\alpha)=d(x,y)$ (up to a phase). One can achieve it by setting $b\sin\alpha=p(2\pi l_H^2/a)$ and $C_{n+p}=C_n\exp(i2\pi nb\cos\alpha/a)$, where p is an integer. For simple Bravais lattices, there is only one vortex in each unit cell. Therefore, we can take $p=1$. The area of the unit cell is then $ab\sin\alpha=\Phi_0/H=2\pi l_H^2$. As a result, all C_n can be fixed up to an overall constant, to be fixed later,

$$C_n=\exp\left[2\pi i\frac{b}{a}\cos\alpha\frac{n(n-1)}{2}\right]. \tag{23}$$

It is convenient to use new rectilinear coordinates whose axes coincide with the vortex-lattice directions (Fig. 4). We shall denote them as X and Y. Their relations to the old x-y coordinates are $y=Y+X\cos\alpha$ and $x=-X\sin\alpha$.

$\langle|d|^2\rangle$ and $\langle|d|^4\rangle$ are found by integrating $|d|^2$ and $|d|^4$ over $0<X<b$ and $0<Y<a$. The details are relegated to Appendix B. It will be convenient to introduce the complex variable $\zeta\equiv(b/a)\exp(i\alpha)\equiv\rho+i\sigma$. The unperturbed β_A has been shown to be[18]

$$\beta_A^0=\sqrt{\sigma}\left\{\left|\sum_{n=-\infty}^{\infty}\exp(2\pi i\zeta n^2)\right|^2+\left|\sum_{n=-\infty}^{\infty}\exp\left[2\pi i\zeta\left(n+\frac{1}{2}\right)^2\right]\right|^2\right\}. \tag{24}$$

The above calculation can be straightforwardly extended to include the perturbation of η. The relevant integral is

$$\int_{-b\sin\alpha}^{0}dx\ \psi_1(x-nb\sin\alpha)\psi_0(x-m\sin\alpha)\times\psi_0(x-n'\sin\alpha)\psi_0(x-m'\sin\alpha), \tag{25}$$

where ψ_1 is now given by Eq. (21). The correction of β_A in the first order of η', after some algebra, is

$$\beta_A^1=\frac{\eta'}{4}\sqrt{\sigma}\mathrm{Re}\left\{\exp(4i\varphi)\left[\sum_{n'}\exp(-2\pi i\zeta^* n'^2)\right]\times\left[\sum_n\exp(2\pi i\zeta n^2)(64\pi^2\sigma^2n^4-48\pi\sigma n^2+3)\right]+\left(n\to n+\frac{1}{2},n'\to n'+\frac{1}{2}\right)\right\}. \tag{26}$$

From the calculated Abrikosov parameter β_A, one finds the vortex structure by minimizing it with respect to φ, ρ, and σ. The minimization with respect to the angle φ between the vortex lattice and the crystal axes is easily done analytically. The general form of β_A is

$$\beta_A(\varphi,\rho,\sigma)=\beta_A^0(\rho,\sigma)+\eta'[e^{4i\varphi}\delta(\rho,\sigma)+e^{-4i\varphi}\delta^*(\rho,\sigma)]. \tag{27}$$

Obviously the minimum of β_A is achieved when $\varphi=-\arg[\delta(\rho,\sigma)]/4\pm\pi/4$. The minimum of β_A is $\beta_A^{\min}(\rho,\sigma)=\beta_A^0(\rho,\sigma)-|\eta'\delta(\rho,\sigma)|$.

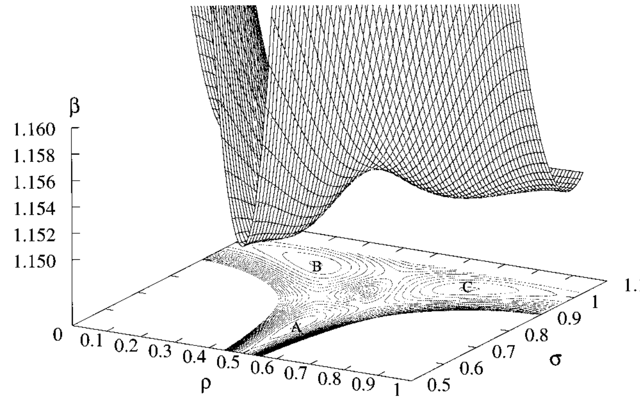

FIG. 5. The Abrikosov parameter β_A as a function of the lattice parameters (ρ,σ). There are three degenerate local minima. Oblique lattices are on the lines $\rho=1/2$ and $\rho^2+\sigma^2=1$. The two points A and B are related by $\rho\to1/\rho$ and therefore represent the same lattice. Point C represents the same rectangular lattice rotated by 90°.

The further minimization of $\beta_A^{\min}(\rho,\sigma)$ is done numerically. In Fig. 5, we show a plot of $\beta_A^{\min}(\rho,\sigma)$ for $\eta'=0.0193$. Due to the fact that the same vortex lattice might be represented by several sets of (ρ,σ), it is enough to consider the region $0<\rho<1/2$; see discussions in Ref. 18. For every η', there are two degenerate minima. One is at $\rho=0.5,\sigma=0.663$, and is clearly a rectangular body-centered lattice with $\alpha=53°$. The corresponding φ is zero. Therefore the vortex lattice coincides with the crystal axes, which was also claimed in Ref. 9. The other minimum is at $\rho=0.275$ and $\sigma=0.961$ and $\alpha=74°$, but with φ equal to 37°. This corresponds to the previous lattice rotated by $\pi/2$. To conclude, we observed rectangular body-centered lattices only. The lowest-energy state is doubly degenerate. It is interesting to note that D_{4h} symmetry is not completely broken in the static vortex lattice: Rotations of π and reflections are retained. The η' dependence of α and $\beta_A^{\min}$ is plotted in Fig. 6 and Fig. 7. A phase transition occurs at $\eta'=0.0235$ where the lattice goes continuously from rectangular to square.

Despite the fact that general oblique lattices were considered for d waves, our numerical analysis shows that they have higher energy than the rectangular body centered ones.

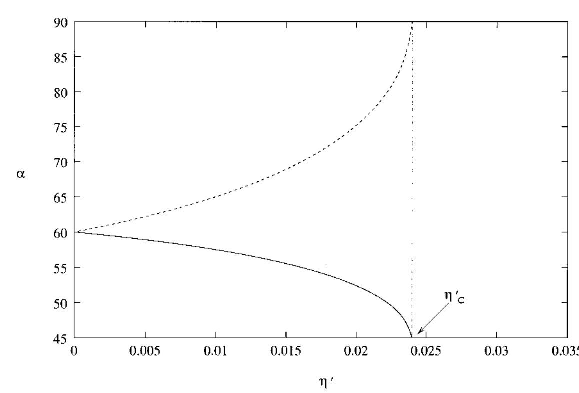

FIG. 6. The angle α as a function of η'; the two branches correspond to lattices related by a rotation of 90°. A continuous transition from the rectangular lattice to the square lattice happens at $\eta_c'=0.235$.

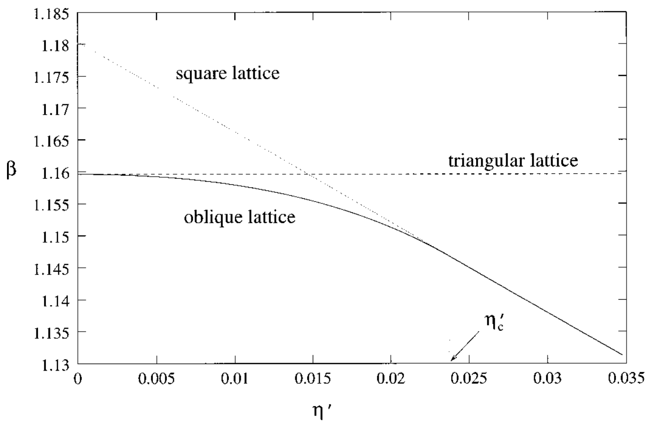

FIG. 7. The Abrikosov parameter β_A as a function of η' for triangular, square, and optimal rectangular body-centered lattices, respectively. At the transition point η_c', the rectangular lattice is taken over by the square lattice. Note that η' is proportional to the magnetic field H.

Intuitively in the symmetric case this is understandable because the rectangular lattices are more symmetric. Although for the s-wave superconductors this fact has been established,[21] for rotationally nonsymmetric superconductors this ''argument'' is not invalid. We are not aware of any mathematical investigations of this question. Moreover, when the vortex lattice starts moving, rotational symmetry is further explicitly broken. As we will see in the next section, general oblique lattices nevertheless are not formed.

IV. MOVING LATTICE SOLUTIONS

In this section we generalize the above procedure to find the structure for a moving vortex lattice near H_{c2}. For this purpose, we need a time-dependent equation for $d(\mathbf{r},t)$. The simplest scenario for the vortex lattice to move is that it moves as a whole. In this case, the s wave simply comoves with the cores of the d wave so that its dynamics is completely determined by that of the d wave. Therefore, we shall assume that a time-dependent GL equation for the d field is sufficient,[22]

$$\gamma\left(\frac{\partial}{\partial t}+ie^*\Phi\right)d=-\left(\frac{1}{2m_d}\Pi^2-\alpha_d\right)d+\eta(\Pi_y^2-\Pi_x^2)^2 d -2\beta|d|^2 d, \tag{28}$$

where Φ is the electric potential and γ is the order-parameter relaxation rate.

In principle γ can be complex; its imaginary part represents so-called particle-hole asymmetry in conventional superconductors, resulting in nonvanishing Hall currents.[15,23] Such Hall currents do not depend on the orientation of the electric field $\mathbf{E}$ to the crystal lattice. In the d-wave superconductors, however, because of the anisotropy due to the crystal lattice, particles and holes can move asymmetrically. As a result, orientionally dependent Hall currents can arise. This is captured by the η term. To see this oriontional dependence, we shall only consider real γ and concentrate on the Hall current induced by the η term.

The vortex-lattice velocity is perpendicular to both electric and magnetic fields (which is assumed not to be tilted for simplicity and taken to be in the $+z$ direction): $\mathbf{E}=-\mathbf{v}\times\mathbf{B}$. For a general direction of the electric field the fourfold symmetry of the system is completely (explicitly) broken, except for several special directions, along the crystal axes $[1,\underline{0},0]$, $[0,1,0]$ or along the diagonal lines $[1,1,0]$ or $[1,1,0]$. Even for the simple s-wave time-dependent GL equations the problem of finding the moving lattice solution is nontrivial. However, there exists the ''Galilean boost'' trick[14] to solve the linearized (and sometimes even a nonlinear problem for linear response[23]) problem. As we will see shortly, for the d-wave equations, even the linearized equation does not seem to possess a boosted static solution.

Technically the steps follow those of the static case. First we find a complete set of solutions of the linearized equation using perturbation theory in η. Then we impose periodicity conditions to construct the vortex-lattice wave functions. It is more convenient to perform the first step in the gauge aligned in the direction of the electric field, while for the second step it is preferable to use a gauge aligned in the direction of the vortex lattice. We will combine the two steps using the gauge transformation. After the wave function is found, it is straightforward to apply the procedure described in the previous section to minimize Abrikosov's β_A and find the lattice structure.

A. Linearized TDGL equation and the dependence of H_{c2} on the electric field

To simplify the presentation, we first assume that the direction of the electric field is special: along the crystalline x (or $[1,0,0]$) direction. In this case the vortices are moving in the negative y direction of the coordinate system. We will return to the general case afterwards. The perturbative solution to the linearized TDGL equation can be most easily constructed in the Landau gauge. In this case, if we choose a time- and y-independent electric potential $\Phi=-vHx$, the variables t and y trivially separate from x: $d(x,y,t)=\exp(iky)\exp(-\omega t/\gamma)\psi(x)$. As shown in Appendix C, after substituting $d(x,y,t)$ into the linearized TDGL equation, the equation reduces to a one-dimensional Schrödinger-type equation with an anti-Hermitian dissipation term which comes from $ie^*\Phi$ in Eq. (28). To compensate this term, we allow ω to have an imaginary part: $\omega=\omega_R+i\omega_I$ with $\omega_I=-ik\gamma v$. Another effect of this anti-Hermitian dissipation term is to shift the argument x to $x-igl_H$ with $g=\gamma m_d v l_H$. The final solution is simply the shifted $\psi(x)$ together with other factors

$$d(x,y,t)=\exp[ik(y+vt)]\exp\left[-\frac{1}{2l_H^2}(x-kl_H^2-igl_H)^2\right]\times\frac{1}{\sqrt{L}}\left(\frac{1}{\pi l_H^2}\right)^{1/4}\left[1+\eta\sum_{n=1}^{4}c_n\frac{1}{\sqrt{2^n n!}}\times H_n\left(\frac{x}{l_H}-kl_H-ig\right)\right], \tag{29}$$

where c_n are given by Eq. (C12) in Appendix C.

This solution is restricted to the case when the direction of the electric field is along the crystalline x direction. When the electric field is in arbitrary direction, the solution is still given by Eq. (29) but with different c_n. In this case, c_n depend on the angle between the crystal [1,0,0] axis and the electric field $\Theta = \theta - \varphi$ (see Fig. 4); their explicit expressions are given by Eq. (C13).

From Eq. (C10), one deduces that in the simpler case when the electric field is parallel to one of the crystal axes, the new phase boundary equation follows,

$$H_{c2}=H_{c2}^0+\eta H_{c2}^1=\frac{m_d}{e^*}(2\alpha_d-\gamma^2 m_d v^2)+\frac{2\eta m_d^3}{e^*}(5m_d^2\gamma^4 v^4 -12m_d\gamma^2 v^2\alpha_d+8\alpha_d^2), \tag{30}$$

where the second term is a perturbation. The entire temperature dependence is contained inside $\alpha_d=\alpha'(T_c-T)$. The phase transition line is therefore still quadratic in T,

$$H_{c2}(T)=h_0+h_1(T_c-T)+h_2(T_c-T)^2, \tag{31}$$

but the coefficients have a nontrivial dependance on velocity v,

$$h_0=\frac{m_d^2}{e^*}(-1+10\eta m_d^3\gamma^2 v^2)\gamma^2 v^2,$$

$$h_1=2\alpha'\frac{m_d}{e^*}(1-12\eta m_d^3\gamma^2 v^2), \tag{32}$$

$$h_2=16\alpha'^2\eta\frac{m_d^3}{e^*}.$$

Note that the curvature has not changed compared to the static case, but we have two new effects. First of all, the electric field (or, equivalently, electric current) reduces H_{c2}; this is expected. Second, although the curvature h_2 does not change compared with the static case, the slope h_1 acquires a negative contribution proportional to E^2.

In the general case of an arbitrary orientation of the electric field, only the coefficient h_0 gets modified:

$$h_0=\frac{m_d^2}{e^*}\{-1+[9+\cos 4\Theta]\eta m_d^3\gamma^2 v^2\}\gamma^2 v^2. \tag{33}$$

One sees that the shift in H_{c2} due to the electric field actually depends on the direction of the electric field relative to the crystal lattice. This result should be checked experimentally.

In the s-wave case (or $\eta=0$) the boundary was found and discussed in Ref. 14. There are a couple of peculiarities associated with it like the existence of a metastable normal state and the unstable superconducting state. The same applies to the present case. As far as we know, these peculiarities have not been directly observed in low-T_c materials. It would be interesting to reconsider this question for high-T_c materials. Note also that the phase transition is not the usual one (second order which probably turns to weakly first order due to fluctuations). In the presence of flux flow the two phases are stationary states rather than states in thermal equilibrium. There exists therefore a phase diagram in the space containing the current as an external parameter (both magnitude and direction).

B. Construction of the moving vortex lattice

We would like to follow a procedure similar to that described in Sec. III B for the static case to construct a moving vortex-lattice solution. It turns out not to be a straightforward generalization. In earlier sections, we used the gauge freedom to make both the scalar and the vector potentials independent of y and t. This allows for separation of variables. The fact that y variable factored into the form $\exp(iky)$ helped us implement the periodicity in the y direction (with discrete values of k). However, in general, the vortex lattice will not be periodic along this special direction. To construct this general periodic solution, one has to solve a very complicated periodicity constraint equation for the coefficients C_k, where k is now a continuous index.

In the static vortex-lattice case, we used the gauge freedom to align the vector potential to the vortex lattice. This choice allows us to solve the constraint equation on C_k easily since we already had periodicity along the y axis which is built in. This reduced the problem to a discrete one. Furthermore, only a few k_n's were coupled, and it turned out to be solvable, at least for $p=1$. This is not the case for the moving vortex lattice. In Sec. IV A, for the problem with electric field and time dependence, we used the gauge freedom to align the vector potential with respect to the electric field in order to find the general solution of the perturbed Hamiltonian. Now, when we have to use this general solution to construct the periodic solution we encounter the problem that we cannot use the gauge freedom to simultaneously simplify *both* problems. Fortunately, in the unperturbed (s-wave) case a simple ansatz for the construction of moving vortex-lattice solution exists. This works for the linearized TDGL equation with an arbitrary direction of the electric field.[14] We shall use this observation to guide us in obtaining the periodic solution for the moving vortex lattice in the presence of perturbation. The solution can be explicitly checked to satisfy the TDGL equation and the periodicity constraints.

As mentioned earlier, in the previous subsection, we adopted a Landau gauge (which will be referred to later as the gauge I):

$$\mathbf{A}^{\mathrm{I}}(x',y',t)=Hx'\hat{y}', \quad \Phi^{\mathrm{I}}(x',y',t)=v\cdot\mathbf{A}^{\mathrm{I}}=-vHx'. \tag{34}$$

Here, for later convenience we use x',y' to represent the coordinate in which the electric field is along the x' direction, while x,y is the coordinate in which the vortex lattice is aligned to the y direction (see Fig. 4). The relation is $x' = x\cos\theta+y\sin\theta$ and $y'=-x\sin\theta+y\cos\theta$. In this gauge, it is very difficult to impose periodicity. Fortunately we can transform our solution to a gauge in which the periodicity is manifest and the standard procedure works (referred as gauge II):[14]

$$\mathbf{A}^{\mathrm{II}}(x,y,t)=H(x-v_x t)\hat{y}+\gamma\frac{m_d}{e^*}v\times\hat{z},$$

$$\Phi^{\mathrm{II}}(x,y,t)=v\cdot\mathbf{A}^{\mathrm{II}}=v_y H(x-v_x t), \tag{35}$$

where $v_x=v\sin\theta, v_y=-v\cos\theta$. The gauge transformation between the two is determined by a phase $\chi(x,y,t)$ satisfying $\nabla\chi=\mathbf{A}^{\mathrm{I}}-\mathbf{A}^{\mathrm{II}}$ and $-\partial_t\chi=\Phi^{\mathrm{I}}-\Phi^{\mathrm{II}}$. One of the solutions is

$$\chi=\frac{\gamma m_d v}{e^*}x'+\frac{H}{2}\sin\theta\cos\theta[(y'+vt)^2-x'^2] + H\sin^2\theta[x'(y'+vt)]. \tag{36}$$

In this gauge, the *unperturbed* vortex lattice can be easily formed using ''boosted'' solutions,

$$\Psi_n^{\mathrm{II}}(x,y,t)=\frac{1}{\sqrt{L}}\left(\frac{1}{\pi l_H^2}\right)^{1/4}\exp[ik_n(y-v_y t)] \times\exp\left[-\frac{1}{2l_H^2}(x-v_x t-k_n l_H^2)^2\right], \tag{37}$$

with standard coefficients C_n: $\Psi^{\mathrm{II}}=\Sigma_n C_n\Psi_n^{\mathrm{II}}$. After the gauge transformation is performed, these elementary solutions are linearly related to the unperturbed normalized eigenfunctions in the gauge I found in the previous subsection by

$$e^{ie^*\chi}\Psi_n^{\mathrm{II}}=\frac{L}{2\pi}\int_{-\infty}^{\infty}dk\, B_{nk}\Psi_k^{\mathrm{I}}, \tag{38}$$

where Ψ_k^{I} is given by

$$\Psi_k^{\mathrm{I}}(x',y',t)=\frac{1}{\sqrt{L}}\left(\frac{1}{\pi l_H^2}\right)^{1/4}\exp\left(-\frac{g^2}{2}\right)\exp[ik(y'+vt)] \times\exp\left[-\frac{1}{2l_H^2}(x'-igl_H-kl_H^2)^2\right], \tag{39}$$

where $\exp(-g^2/2)$ is a normalization factor. Note that the gauge transformation and hence the quantities B_{nk} are in general time dependent. However, since we are looking for boosted solutions so that the Abrikosov parameter β_A (which is also gauge invariant) is time independent, we do not need to keep track of the time dependence here, and to simplify the calculation, we can set $t=0$.

The coefficients B_{nk} can be found by performing the overlap integrals

$$B_{nk}=\int dx'dy'[\Psi_k^{\mathrm{I}*}(x',y',0)e^{ie^*\chi(x',y',0)}\Psi_n^{\mathrm{II}}(x,y,0)]=\frac{\sqrt{\pi}l_H}{L}\frac{1}{\sqrt{ie^{i\theta}\sin\theta}}\exp\left[-\frac{1}{2}\frac{\cos\theta}{\sin\theta}(k^2+k_n^2)l_H^2 +ikl_H^2\left(\frac{k_n}{\sin\theta}+\gamma m_d v\right)\right]. \tag{40}$$

The first-order corrections $O(\eta)$ to the wave function in two different gauges are also related by $e^{ie^*\chi}\delta\Psi_n^{\mathrm{II}}=(L/2\pi)\int_{-\infty}^{\infty}dk\, B_{nk}\delta\Psi_k^{\mathrm{I}}$. $\delta\Psi_k^{\mathrm{I}}$ can be read off from Eq. (29); it turns out that after a lengthy calculation, the correction to the wave function in gauge II is amazingly simple:

$$\delta\Psi_n^{\mathrm{II}}(x,y)=e^{-ie^*\chi}\frac{L}{2\pi}\int_{-\infty}^{\infty}dk\, B_{nk}\delta\Psi_k^{\mathrm{I}}=\frac{1}{\sqrt{L}}\left(\frac{1}{\pi l_H^2}\right)^{1/4}\exp(ik_n y)\exp\left[-\frac{1}{2l_H^2}(x-k_n l_H^2)^2\right]\eta' \times\sum_{m=1}^{4}c_m\frac{e^{im\theta}}{\sqrt{2^m m!}}H_m\left(\frac{x}{l_H}-k_n l_H-ige^{-i\theta}\right). \tag{41}$$

An important observation is that the corrected moving lattice solution at $t=0$ is

$$\Psi^{\mathrm{II}}(x,y)=\sum_{n=-\infty}^{\infty}C_n[\Psi_n^{\mathrm{II}}+\delta\Psi_n^{\mathrm{II}}]=\sum_{n=-\infty}^{\infty}C_n\frac{1}{\sqrt{L}}\left(\frac{H}{\pi}\right)^{1/4}\exp(ik_n y)\exp\left[-\frac{1}{2l_H^2}(x-k_n l_H^2)^2\right]\left[1+\eta'\sum_{m=1}^{4}c_m\frac{e^{im\theta}}{\sqrt{2^m m!}} \times H_m\left(\frac{x}{l_H}-k_n l_H-ige^{-i\theta}\right)\right], \tag{42}$$

where $k_n=2\pi n/a$, and C_n, being again given by Eq. (23), are still invariant under vortex-lattice symmetries.

C. Structure and magnetization of the moving lattice

The standard Abrikosov procedure to develop an approximation for a small order parameter around H_{c2} can be applied also in the flux flow case (see Ref. 14). This time, however, the minimization of the Abrikosov parameter β_A does not correspond to the minimization of energy, but rather to the smallest deviation from the exact solution of the TDGL equation. The ''derivation'' closely follows the static one. Using the expression for the vortex-lattice solution found in the previous subsection, the correction term in an expansion of the Abrikosov parameter β_A in η', $\beta_A=\beta_A^0+\eta'\beta_A^1$, is

$$\beta_A^1=\frac{\sqrt{\sigma}}{4}\mathrm{Re}\left\{\left[\sum_{n'}\exp(-2\pi i\zeta^* n'^2)\right]\times\left[\sum_n \exp(2\pi i\zeta n^2)G(n)\right]+\left(n\to n+\frac{1}{2},n'\to n'+\frac{1}{2}\right)\right\}. \tag{43}$$

Here the function $G(n)$ is defined by

$$G(n)=e^{4i\varphi}(64\pi^2\sigma^2 n^4-48\pi\sigma n^2+3)-8e^{2i\varphi}g^2\cos 2\Theta(8\pi\sigma n^2-1), \tag{44}$$

where $\Theta\equiv\theta-\varphi$ is the angle between the electric field and the crystal lattice. One immediately observes a surprising fact—the dependence on the angle Θ and velocity v is only via the combination $g^2\cos 2\Theta$ where $g\equiv\gamma m_d v l_H$. It factors out as

$$\beta_A(\varphi,\rho,\sigma)\equiv\beta_A^0(\rho,\sigma)+\eta'\mathrm{Re}[e^{4i\varphi}\delta(\rho,\sigma)+g^2\cos 2\Theta e^{2i\varphi}\delta'(\rho,\sigma)]. \tag{45}$$

For example, the resulting lattice for $\Theta=\pi/4$ and arbitrary g will be the same as without an electric field at all. Also apparent complete breaking of rotational symmetry by the general direction of the electric field is not felt by β_A. Indeed, the lattice for some arbitrary Θ and g^2 is the same as for $\Theta=0$ and $g^{2\prime}=g^2\cos 2\Theta$. Fourfold symmetry has been reduced, however. These results are nontrivial and can be checked experimentally. This degeneracy in velocity and Θ in the determination of the vortex lattice may be a reflection of some dynamical symmetries which we have so far failed to see yet.

We get the $e^{\pm 2i\varphi}$ harmonics in Eq. (45) in addition to the fourth harmonic that appeared in the static case. The minimization with respect to φ still can be done analytically, although the algebraic equation in this case is quartic. For fixed η' and $g^2\cos 2\Theta$, the minimization with respect to ρ and σ was performed numerically and we again obtain only rectangular body-centered lattices aligned to either crystalline axis. The angle α turns out to be only weakly dependent on the combination $g^2\cos 2\Theta$. For example, for positive $\eta'=0.015$, we found $\alpha=\alpha|_{g=0}+1.0g^2|\cos 2\Theta|$ (in degrees) where $\alpha|_{g=0}=69.3°$. The Abrikosov β_A is simply related to the slope of the magnetization curve,

$$4\pi\frac{dM}{dH}=\frac{1}{(2\kappa^2-1)\beta_A}, \tag{46}$$

as well as to other thermodynamic quantities. All of them therefore exhibit a very simple dependence on the velocity $\mathbf{v}$ or, equivalently, on the current $\mathbf{J}$.

The fact that the optimal lattice is rectangular body centered is a bit mysterious. Rotational symmetry is completely broken by both the electric field and by the underlying crystal lattice. It is not easy to attribute the advantage of this lattice structure to some simple physical origin. It might be that it is a consequence of using the Abrikosov approximation, and therefore beyond this approximation the lattices might not be rectangular. Note also that it was also surprising that β_A was independent of the orientation of the electric field even in the s-wave calculation.[14] As far as we know, the orientation dependence of the electric field for the moving lattice has not been observed in either low-T_c or high-T_c type-II superconductors.

V. NONLINEAR CONDUCTIVITY NEAR H_{c2}

In this section we consider the dissipation in vortex cores due to flux flow. As is well known, fourfold symmetry forces the conductivity tensor σ_{ij} to be rotationally symmetric, namely, $\sigma_{ij}=\sigma\delta_{ij}+\sigma^H\varepsilon_{ij}$. Here σ is the usual (Ohmic) conductivity, σ^H is the Hall conductivity, and ε_{ij} is the antisymmetric tensor. Both σ and σ^H can be calculated via the Kubo formula.[24] When γ is real, σ^H is identically zero,[15] and the correction to σ is of order η (which is a small effect). So, to see nonzero Hall current, we need to go beyond a linear response. Even in the simple s-wave case, isotropy implies that there is no particular direction perpendicular to the $\mathbf{E}$ field so that there is no nonlinear Hall current. This was confirmed in Ref. 14. The situation changes when the crystal lattice is included. In this case, directions perpendicular to the $\mathbf{E}$ field are not all the same; the anisotropy of the crystal lattice causes a nonvanishing nonlinear Hall current (however, the linear Hall current is still zero.). This simple picture implies that the leading Hall current $\mathbf{J}_{\mathrm{Hall}}$ must have the form $\eta\sin 4\Theta$ or $\eta\cos 4\Theta$.

A. Condensate for the moving lattice

To calculate the transport properties due to flux flow, we have to determine the normalization of the order parameter and the expression for the electric current to first order in η'. Note that in previous sections we do not need the normalization because Abrikosov's β_A is normalization independent. One can expand the order-parameter field as follows

$$d=N\sum C_n(\Psi_n+\eta'\delta\Psi_n)\equiv N(\Psi+\eta'\delta\Psi)\cong(N_0+\eta' N_1)(\Psi+\eta'\delta\Psi). \tag{47}$$

Here C_n,Ψ_n, and $\delta\Psi_n$ have been calculated in the previous section, N is the normalization and N_0 is the unperturbed normalization of Ψ. The calculation is standard. We again expand it to first order in η'. The normalization is determined from the minimization of the free energy as $\langle d^*d\rangle=\alpha_d/2\beta\beta_A$, where $\langle\cdots\rangle$ denotes the space average, and α_d and β are coefficients of the GL equation. The Abrikosov parameter β_A has its own η' expansion calculated in Sec. VI C. Combining the two one obtains

$$N^2\cong N_0^2\left(1+\eta'\frac{2N_1}{N_0}\right)=\frac{\alpha_d}{2\beta}\frac{1}{\beta_A^0\langle\Psi^*\Psi\rangle}\times\left[1-\eta'\left(\frac{\beta_A^1}{\beta_A^0}+\frac{2\mathrm{Re}\langle\Psi^*\delta\Psi\rangle}{\langle\Psi^*\Psi\rangle}\right)\right], \tag{48}$$

which will be used to calculate the current.

B. Direct and Hall currents

We will neglect pinning and consider the motion of a very large bundle. While there is a normal component of the conductivity, here we will concentrate on the contribution of the supercurrent only. For a discussion of the relative contribution of the two see Ref. 14. As usual, the supercurrent is given by $\mathbf{J}=-\delta F_{\mathrm{eff}}/\delta\mathbf{A}$ with F_{eff} ($\equiv\int d\mathbf{r}\, f_{\mathrm{eff}}$) being the total free energy. Since the η term in the free energy, Eq. (2), contains four covariant derivatives, consequently the electric current, in addition to the usual expression, contains additional terms. The leading order current is given by $\mathbf{J}^a=(e^*/2m_d)\langle d^*(\mathbf{\Pi}d)+(\mathbf{\Pi}d)^*d\rangle$, while the correction from the η perturbation is

$$\mathbf{J}^b(d)=e^*\eta\hat{\mathbf{x}}''\langle[(\Pi_y''^2-\Pi_x''^2)d]^*(\Pi_x''d)+[\Pi_x''(\Pi_y''^2-\Pi_x''^2)d]^*d+\text{c.c.}\rangle-e^*\eta\hat{\mathbf{y}}''\langle[(\Pi_y''^2-\Pi_x''^2)d]^*(\Pi_y''d)+[\Pi_y''(\Pi_y''^2-\Pi_x''^2)d]^*d+\text{c.c.}\rangle, \tag{49}$$

where $\Pi_x''=\cos\varphi\Pi_x+\sin\varphi\Pi_y$, $\Pi_y''=\cos\varphi\Pi_y-\sin\varphi\Pi_x$, and $\hat{\mathbf{x}}''=\cos\varphi\hat{\mathbf{x}}+\sin\varphi\hat{y}$, $\hat{\mathbf{y}}''=\cos\varphi\hat{\mathbf{y}}-\sin\varphi\hat{x}$ (see Fig. 4).

Substituting the condensate $d=N(\Psi+\eta'\delta\Psi)$ with Ψ and $\delta\Psi$ determined in the previous subsection, one obtains the expansion of $\mathbf{J}^a$ to the first order in η',

$$\mathbf{J}^a\cong\mathbf{j}_0+\eta'\delta\mathbf{j}_1, \tag{50}$$

where

$$\mathbf{j}_0=N_0^2\left(\frac{e^*}{2m_d}\right)[\langle\Psi^*\mathbf{\Pi}\Psi\rangle+\langle\Psi\mathbf{\Pi}\Psi^*\rangle]=\left(\frac{e^*\alpha_d}{4\beta m_d}\frac{1}{\beta_A^0}\right)\frac{2\mathrm{Re}\langle\Psi^*\mathbf{\Pi}\Psi\rangle}{\langle\Psi^*\Psi\rangle}, \tag{51}$$

and $\delta\mathbf{j}_1$ contains both the correction to $N^2$and the correction to the wave function $\delta\Psi$,

$$\delta\mathbf{j}_1=-\left(\frac{\beta_A^1}{\beta_A^0}+\frac{2\mathrm{Re}\langle\Psi^*\delta\Psi\rangle}{\langle\Psi^*\Psi\rangle}\right)\mathbf{j}+\left(\frac{e^*\alpha_d}{4\beta m_d}\frac{1}{\beta_A^0}\right)\frac{4\mathrm{Re}\langle\delta\Psi^*\Pi\Psi\rangle}{\langle\Psi^*\Psi\rangle}. \tag{52}$$

The expansion to first order in η' of $\mathbf{J}^b$ is $\mathbf{J}^b(N\Psi)\simeq N_0^2\mathbf{J}^b(\Psi)\equiv\eta'\delta\mathbf{j}_2$ with $\delta\mathbf{j}_2$ being given by

$$\delta\mathbf{j}_2=\left(\frac{e^*\alpha_d}{2\beta m_d}\frac{1}{\beta_A^0}\right)l_H^2\left\{\hat{\mathbf{x}}''\frac{\langle[(\Pi_y''^2-\Pi_x''^2)\Psi]^*(\Pi_x''\Psi)+[\Pi_x''(\Pi_y''^2-\Pi_x''^2)\Psi]^*\Psi+\text{c.c.}\rangle}{\langle\Psi^*\Psi\rangle}-\hat{\mathbf{y}}''\frac{\langle[(\Pi_y''^2-\Pi_x''^2)d]^*(\Pi_y''d)+[\Pi_y''(\Pi_y''^2-\Pi_x''^2)d]^*d+\text{c.c.}\rangle}{\langle\Psi^*\Psi\rangle}\right\}. \tag{53}$$

The total current is then given by $\mathbf{J}=\mathbf{J}^a+\mathbf{J}^b=\mathbf{j}_0+\eta'(\delta\mathbf{j}_1+\delta\mathbf{j}_2)$. Note that although the total wave function is a linear combination of $\Psi_n(x,y)$, after averaging over 2D space all the components decouple due to the $\exp(ik_ny)$ factors and the fact that the current is quadratic in Ψ_n. We find that

$$\frac{-2\mathrm{Re}\langle\Psi^*\delta\Psi\rangle}{\langle\Psi^*\Psi\rangle}\mathbf{J}=\left(\frac{|e^*|\alpha_d}{\beta}\frac{1}{\beta_A^0}\right)\left\{-g^4\left[1+\frac{2}{3}\cos 4\Theta\right]+4g^2\right\}(\gamma v\times\hat{\mathbf{z}}), \tag{54}$$

$$\left(\frac{e^*\alpha_d}{4\beta m_d}\frac{1}{\beta_A^0}\right)\frac{4\mathrm{Re}\langle\delta\Psi^*\Pi\Psi\rangle}{\langle\Psi^*\Psi\rangle}=\left(\frac{|e^*|\alpha_d}{\beta}\frac{1}{\beta_A^0}\right)\left\{g^4\left[1+\frac{2}{3}\cos 4\Theta\right]-4g^2-2\right\}(\gamma v\times\hat{\mathbf{z}}), \tag{55}$$

$$\left(\frac{e^*\alpha_d}{2\beta m_d}\frac{1}{\beta_A^0}\right)l_H^2\left\{\hat{\mathbf{x}}''\frac{2\mathrm{Re}\langle\Psi^*\{\Pi_y''^2-\Pi_x''^2,\Pi_x''\}\Psi\rangle}{\langle\Psi^*\Psi\rangle}-\hat{\mathbf{y}}''\frac{2\mathrm{Re}\langle\Psi^*\{\Pi_y''^2-\Pi_x''^2,\Pi_y''\}\Psi\rangle}{\langle\Psi^*\Psi\rangle}\right\}=\left(\frac{|e^*|\alpha_d}{\beta}\frac{1}{\beta_A^0}\right)\{[g^2(1+\cos 4\Theta)+2](\gamma v\times\hat{\mathbf{z}})-g^2\sin 4\Theta(\gamma v)\}, \tag{56}$$

where $\Theta\equiv\theta-\varphi$ as before. Here we have used the fact that Π is Hermitian, and the curly brackets denote anticommutation. Note that since g carries the sign of e^*, combining with the e^* in front of the expression for current gives a factor $|e^*|$, which ensures that the I-V relation is independent of the sign of charge. Summing them up we got our final expression

$$\delta\mathbf{j}=-\frac{\beta_A^1}{\beta_A^0}\mathbf{j}+\frac{|e^*|\alpha_d}{\beta}\frac{1}{\beta_A^0}g^2(1+\cos 4\Theta)(\gamma v\times\hat{\mathbf{z}})-\frac{|e^*|\alpha_d}{\beta}\frac{1}{\beta_A^0}g^2\sin 4\Theta(\gamma v). \tag{57}$$

From this, one obtains the simple results in Eqs. (4) and (5) as advertised earlier. Note that all the g^4 terms are canceled. The Hall current is proportional to $\eta \sin 4\Theta$. In fact, only the fourth harmonics of the angle between **E** and the crystal lattice orientation are retained. Furthermore, only the cubic power of **E** contributes; all the higher orders terms are canceled. The Hall current $\mathbf{J}_{\mathrm{Hall}}$ does not depend on the sign of charge e^*, and depending on the value of Θ, it can be chargelike or holelike. This reflects its anisotropy origin.

VI. CONCLUDING REMARKS

Instead of summarizing the results (which has been done in Sec. I), we briefly comment on the possibility of the observation of various phenomena quantitatively discussed in this paper.

(1) Internal structure of a single anisotropic vortex. Although direct observation of the order parameter using scanning tunneling microscopy[26] (STM) or the magnetic field distribution [10] using electron holography[28] or other techniques is possible, the detailed effects hotly debated by theoreticians (where the zeros of the s field are located, small distance asymptotics) probably do not have a significant impact on such experiments. One also should note that the Ginzburg-Landau framework adopted here might not be applicable close to the vortex center where microscopic excitation spectrum becomes important. An approach using elements of the microscopic theory (via Bogoliubov–de Gennes equations along the lines of work in Refs. 27 and 7) will be necessary.

(2) Structure of the static and moving anisotropic vortex lattice. The static vortex lattice has been observed by using small angle neutron scattering[3] and tunneling spectroscopy.[4] Although moving vortices have been directly observed using electron tomography,[25] to our knowledge the shape and orientation of moving large bundles has not been observed as yet. The moving vortex lattice is much more sensitive to pinning effects than the static lattice. In the static case pinning can just slightly distort or cause breakup of the crystal to smaller pieces. For the moving flux lattice pinning is expected to be much more significant. The orientation effect that we predict is very small, but the asymmetry in magnetization might be quite significant.

We found that the transition point for the parameter $\eta' = \eta m_d |e^*| H$ is at $\eta'_c = 0.0235$. This transition between the rectangular and the square lattices might be seen in neutron scattering experiments, since the square lattice has higher symmetry (number of spots is reduced to four at the transition). Note that by increasing the magnetic field the critical η'_c can be exceeded without changing the sample (η is independent of the magnetic field).

(3) Static transition to the normal phase. As is well known, in the presence of fluctuations, the second-order phase transition from superconducting to normal state becomes a weakly first-order melting line into the vortex liquid. This is the reason that the present study of the diagram will be useless for $Bi_2Sr_2CaCu_2O_{8+\delta}$ which has a relatively large Ginzburg number. For YBCO and the low-temperature superconductors, the curvature of the phase transition line can in principle provide an estimate of η [see Eq. (20)], with reservations mentioned in the end of Sec. III A.

(4) Dynamic phase diagram. The dynamical phase diagram, namely, the transition from the moving lattice to normal (or moving liquid), should be complicated by pinning effects. However, provided these could be overcome a number of interesting effects could be observed. First, even neglecting rotational symmetry breaking effects, there are a number of peculiarities associated with the normal-superconducting boundary noted by Thompson and Hu like the existence of a metastable normal state and the unstable superconductive state. As far as we know, these peculiarities have not been convincingly observed[29] in low-T_c materials. It would be interesting to reconsider this question for the high-T_c materials.

In the s-wave case, however, the phase diagram cannot depend on the orientation of the current. We calculated this orientation dependence on the angle between the atomic lattice and the direction of current or electric field to first order in η [see Eqs. (31), (32), and (33)]. New effects include the change in slope of H_{c2} as a function of temperature, not only in curvature.

Nonlinear *I-V* curves and magnetization

One should be able to measure currents in the same sample oriented differently with respect to the atomic crystal. Note that the effect can be seen in low-temperature anisotropic superconductors, not necessarily in YBCO. The simplicity of the expressions for both direct and Hall currents, Eqs. (4) and (5), calls for some special ways to verify it experimentally. The angular dependence of the magnetization near the transition, given by Eqs. (43) and (46), might be large enough to be measurable.

There are number of limitations of our approach which can be lifted by possible extensions. One of them is the assumption of exact fourfold symmetry. Deviations from it in a form of different coefficients of the gradient terms in x and y directions have already been studied recently[30] using the two-field formalism. If they happen to be small, they can be easily added perturbatively. These effects of explicit breaking are clearly quite different from those of the spontaneous breaking of fourfold symmetry studied here. Our results for the lattices are limited to fields close to H_{c2} only. It is possible, although more difficult, to extend them to lower magnetic fields. Another interesting direction is the influence of anisotropy on vortex fluctuations in the lattice. We hope to address these issues in the future.

In addition, the effective one-component approach allows us to consider possibilities not apparent within the two-field one. For example, the coefficient η, in principle, can be negative despite the fact that within the two-field formalism it should be positive. Twinning is expected to reduce the value of the parameter.

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APPENDIX A: COMPARISON OF THE TWO-FIELD AND THE ONE-FIELD RESULTS FOR A SINGLE VORTEX

In the two-field formulation, the small r asymptotics of the solution for the d-wave component of the isolated vortex is given by

$$d(r,\phi)\simeq(d_1 r+d_3 r^3)e^{i\phi}, \tag{A1}$$

where the subleading term coefficient is

$$d_3=-\frac{d_1}{8\xi_d^2}\left[1+\frac{h_0}{H_{c2}(0)}\right]\simeq-\frac{d_1}{8\xi_d^2},$$

neglecting terms proportional to $h_0/H_{c2}(0)$.[9] The s-component asymptotics is

$$s(r,\phi)\simeq-\frac{\gamma_v}{\alpha_s}(\Pi_y^2-\Pi_x^2)d(r)\simeq-\frac{\gamma_v}{\alpha_s}(4d_3 r e^{-i\phi})$$
$$=+\frac{1}{2}\left(\frac{\gamma_v}{\alpha_s\xi_d^2}\right)d_1 r e^{-i\phi}. \tag{A2}$$

The expression, Eq. (A2), is different from what was obtained in Ref. 9, Eq. (22), but qualitatively the behavior is not affected. We also found that it does not follow from their Eq. (19), because to the same order of approximation Ref. 9 had a nonvanishing term proportional to $e^{3i\phi}$. Nevertheless, the concluding statement in Ref. 9 is basically correct. Following the same argument leading to an estimate of the maximal amplitude of $s(r)$ as in Ref. 9 we obtained

$$\frac{s_{\max}}{d_0}\simeq\frac{1}{4}\left(\frac{\gamma_v}{\alpha_s\xi_d^2}\right).$$

Apparently this correction accounts for the 20% error cited in Ref. 9.

The asymptotic form of the wave function was used to make a topological argument about poles in the s wave. Due to the different winding number of small r and large r asymptotics of $s(r,\phi)$, there must exist four poles in the intermediate region. This was shown numerically in Ref. 9. Xu *et al.*,[31] however, performed a similar calculation, but did not get the poles. Our calculation, which is much simpler than the two-field one, confirms the former and shows clearly four poles on the x and y axes, independent of what kind of approximate d-component wave function one chooses. We suspect that the numerical simulation in Ref. 31 was not sensitive enough to resolve these poles.[32]

APPENDIX B: EVALUATION OF $\langle|d|^2\rangle$ AND $\langle|d|^4\rangle$

In this appendix, we shall find $\langle|d|^2\rangle$ and $\langle|d|^4\rangle$. The average of $|d|^2$ is found by integrating $|d|^2$ over $0<X<b$ and $0<Y<a$. The integration over Y enforces a δ function and simplifies the double summation to

$$\langle|d|^2\rangle=\frac{1}{ab\sin\alpha}\sum_{n=-\infty}^{\infty}a\sin\alpha\times\int_0^b dX|\psi[(-X-nb)\sin\alpha]|^2, \tag{B1}$$

where $a\sin\alpha$ is the Jacobian. The summation over n converts the integration domain into $(-\infty,\infty)$. We thus obtain

$$\langle|d|^2\rangle=\frac{|C_0|^2}{b\sin\alpha}\int_{-\infty}^{\infty}dx|\psi(x)|^2. \tag{B2}$$

A similar manipulation on $\langle|d|^4\rangle$ leads to

$$\langle|d|^4\rangle=\frac{1}{ab\sin\alpha}\sum_{n,m,n',m'}(a\sin\alpha)\delta_{m+m',n+n'}\exp\left[2\pi i\frac{b}{a}\cos\alpha\frac{-m^2-m'^2+n^2+n'^2}{2}\right]$$
$$\times\int_0^b dX\psi^*[(-X-mb)\sin\alpha]\psi^*[(-X-m'b)\sin\alpha]\psi[(-X-nb)\sin\alpha]\psi[(-X-n'b)\sin\alpha]$$
$$=\frac{1}{b\sin\alpha}\sum_{n,m,n',m'}\delta_{m+m',n+n'}\exp\left[2\pi i\frac{b}{a}\cos\alpha\frac{-m^2-m'^2+n^2+n'^2}{2}\right]$$
$$\times\int_{-b\sin\alpha}^{0}dx\,\psi^*(x-mb\sin\alpha)\psi^*(x-m'b\sin\alpha)\psi(x-nb\sin\alpha)\psi(x-n'b\sin\alpha). \tag{B3}$$

Equation (B2) and (B3) are general expressions for $\langle|d|^2\rangle$ and $\langle|d|^4\rangle$. We can specialize them to our perturbed d field solution, Eq. (21). It is easy to see that the correction to $\langle|d|^2\rangle$ starts from η'^2. We found that

$$\langle|d|^2\rangle=\frac{1}{b\sin\alpha}\left(1+\frac{3}{2}\eta'^2\right). \tag{B4}$$

The first-order term vanishes, because according to Eq. (B2), it is proportional to the inner product of ψ_0 and ψ_1. Since we shall be interested only in $O(\eta')$ corrections, we will drop this second-order term.

The calculation of $\langle|d|^4\rangle$ is more involved even in zeroth order.[18] Because of the presence of the Kronecker δ, there are only three independent summations in Eq. (B3). We choose the summation variables to be $Z=n+n'=m+m'$, $N=n-n'$, and $M=m-m'$. Note that the new discrete variables Z, M, and N are not completely independent since they have to be either all even or all odd simultaneously. The summation in Eq. (B3) then becomes

$$\sum_{m,m',n,n'}\delta_{m+m',n+n'}=\sum_{\text{even}Z}\sum_{\text{even}M}\sum_{\text{even}N}+\sum_{\text{odd}Z}\sum_{\text{odd}M}\sum_{\text{odd }N}. \tag{B5}$$

To zeroth order in η, the integrand in Eq. (B3), after appropriate rearrangement, has a simple Gaussian form

$$\exp\left[-\frac{2\sin^2\alpha}{l_H^2}\left(X+\frac{Z}{2}b\right)^2\right]\exp\left\{-\frac{b^2\sin^2\alpha}{l_H^2}\left[\left(\frac{M}{2}\right)^2+\left(\frac{N}{2}\right)^2\right]\right\}. \tag{B6}$$

As before, the summation over Z in Eq. (B5) extends the range of the integral over X to $(-\infty,\infty)$, so that the Gaussian integral becomes a common factor

$$\int_{-\infty}^{\infty}dX\exp\left(-\frac{2\sin^2\alpha}{l_H^2}X^2\right)=\sqrt{\frac{\pi}{2}}\frac{l_H}{\sin\alpha}. \tag{B7}$$

Pulling out this factor, we obtain

$$\langle|d|^4\rangle_0=\sqrt{\frac{\pi}{2}}\frac{l_H}{b\sin\alpha}\left(\sum_{\text{even }M,N}\exp\left\{2\pi i\frac{b}{a}\cos\alpha\left[-\left(\frac{M}{2}\right)^2+\left(\frac{N}{2}\right)^2\right]-\frac{b^2\sin^2\alpha}{l_H^2}\left[\left(\frac{M}{2}\right)^2+\left(\frac{N}{2}\right)^2\right]\right\}\right. \tag{B8}$$

$$\left.+\sum_{\text{odd }M,N}\exp\left\{2\pi i\frac{b}{a}\cos\alpha\left[-\left(\frac{M}{2}\right)^2+\left(\frac{N}{2}\right)^2\right]-\frac{b^2\sin^2\alpha}{l_H^2}\left[\left(\frac{M}{2}\right)^2+\left(\frac{N}{2}\right)^2\right]\right\}\right). \tag{B9}$$

Using the variable $\zeta=(b/a)\,e^{i\alpha}=\rho+i\sigma$ introduced in Sec. III B, the expression reduces to Eq. (24). Similar calculations apply to the correction term as well.

APPENDIX C: THE PERTURBATIVE SOLUTION TO THE LINEARIZED TDGL EQUATION

The linearized TDGL equation is simply Eq. (28) without the β term,

$$\gamma\left(\frac{\partial}{\partial t}+ie^*\Phi\right)d=-\left(\frac{1}{2m_d}\Pi^2-\alpha_d\right)d+\eta(\Pi_y^2-\Pi_x^2)^2d. \tag{C1}$$

We shall work in the Landau gauge and choose the electric potential to be time and y independent: $\Phi=-vHx$. In this gauge, the variables t and y trivially separate from x: $d(x,y,t)=\exp(iky)\exp(-\omega t/\gamma)\psi(x)$, where ω can have an imaginary part, $\omega=\omega_R+i\omega_I$. The equation then reduces to a one-dimensional "Schrödinger-type" equation

$$\left\{\frac{1}{2m_d}\left[\hat{p}_x^2+\left(k-\frac{x}{l_H^2}\right)^2\right]-i\gamma ve^*Hx-\alpha_d-\eta(\Pi_y^2-\Pi_x^2)^2\right\}\psi(x)=\omega\psi(x).$$

Note that there is an anti-Hermitian dissipation term $-i\gamma ve^*Hx$. Completing the square, rearranging the equation, and choosing $\omega_I=-ik\gamma v$, one obtains

$$\left\{\frac{1}{2m_d}\left[-\frac{d^2}{dx^2}+\frac{1}{l_H^4}(x-x_0-igl_H)^2\right]-\eta(\Pi_y^2-\Pi_x^2)^2-\alpha_d+\frac{1}{2}\gamma^2m_dv^2\right\}\psi(x)\equiv\left[(\hat{K}-\eta\hat{V})-\alpha_d+\frac{1}{2}\gamma^2mv^2\right]\psi(x)=\omega_R\psi(x), \tag{C2}$$

where the dimensionless quantity g is defined by $g\equiv\text{sgn}(e^*)\gamma m_d v l_H$ and $x_0=kl_H^2$; for later convenience we have absorbed the sign of e^* into the definition of g. The parameter α_d should be adjusted (i.e., changing the temperature) such that the lowest eigenvalue ω_R becomes zero; otherwise one gets runaway solutions. This is nothing but the H_{c2} condition generalized to include an arbitrary electric field. The operator $\hat{K}$ defined in Eq. (C2) is simply $\widetilde{K}\equiv\Pi^2/2m_d$ with $\widetilde{x}\equiv(x-x_0)/l_H$ shifted by an imaginary amount $-ig$, and so we can write it as

$$\hat{K}=\exp(g\ l_H\hat{p}_x)\widetilde{K}\exp(-gl_H\hat{p}_x). \tag{C3}$$

The perturbation theory to Eq. (C2) is most conveniently performed on the shifted ψ field defined by

$$\psi(\widetilde{x})\equiv\exp(gl_H\hat{p}_x)\widetilde{\psi}(\widetilde{x})=\widetilde{\psi}(\widetilde{x}-ig). \tag{C4}$$

The transformed Hamiltonian is

$$\widetilde{H}\equiv\exp(-gl_H\hat{p}_x)(\hat{K}-\eta\hat{V})\exp(gl_H\hat{p}_x).$$

Going to the creation and annihilation operator $\hat{a}^\dagger$ and $\hat{a}$ representation, Eq. (C2) becomes

$$\left[\hat{a}^\dagger\hat{a}+\frac{1}{2}-\eta'\widetilde{V}(\hat{a},\hat{a}^\dagger)\right]\widetilde{\psi}(x)=\frac{m_d}{e^*H}\left(\omega_R+\alpha_d-\frac{1}{2}\gamma^2m_dv^2\right)\widetilde{\psi}(x)\equiv\xi\widetilde{\psi}(x). \tag{C5}$$

Here

$$\widetilde{V}(\hat{a},\hat{a}^\dagger)=\exp\left[-i\frac{g}{\sqrt{2}}(\hat{a}^\dagger-\hat{a})\right](\hat{a}^{\dagger2}+\hat{a}^2)^2\times\exp\left[i\frac{g}{\sqrt{2}}(\hat{a}^\dagger-\hat{a})\right]. \tag{C6}$$

The "potential energy" can be further simplified using the identities

$$\exp\left[-i\frac{g}{\sqrt{2}}(\hat{a}^{\dagger}-\hat{a})\right]\hat{a}\exp\left[i\frac{g}{\sqrt{2}}(\hat{a}^{\dagger}-\hat{a})\right]=\hat{a}+i\frac{g}{\sqrt{2}}, \tag{C7}$$

$$\exp\left[-i\frac{g}{\sqrt{2}}(\hat{a}^{\dagger}-\hat{a})\right]\hat{a}^{\dagger}\exp\left[i\frac{g}{\sqrt{2}}(\hat{a}^{\dagger}-\hat{a})\right]=\hat{a}^{\dagger}+i\frac{g}{\sqrt{2}}. \tag{C8}$$

It is helpful to note that the state resulting from the action of the shifting operator on $|0\rangle$ is a coherent state,

$$\left|-i\frac{g}{\sqrt{2}}\right\rangle \equiv \exp\left[-i\frac{g}{\sqrt{2}}(\hat{a}^{\dagger}-\hat{a})\right]|0\rangle. \tag{C9}$$

The correction to the eigenvalue ξ (used later to find the phase transition boundary) to the first order η is then easily found:

$$\xi=\frac{1}{2}-\eta'(g^4-2g^2+2). \tag{C10}$$

To the first order in η, the perturbed ground state is given by

$$\tilde{\psi}=|0\rangle+\sum_{n=1}^{4}\frac{\eta'}{n}|n\rangle\langle n|\left[\left(\hat{a}^{\dagger}+i\frac{g}{\sqrt{2}}\right)^2+\left(\hat{a}+i\frac{g}{\sqrt{2}}\right)^2\right]^2|0\rangle$$
$$\equiv|0\rangle+\eta'\sum_{n=1}^{4}c_n|n\rangle, \tag{C11}$$

where

$$c_1=-2\sqrt{2}ig(g^2-1),\quad c_2=-2\sqrt{2}g^2,$$
$$c_3=\frac{4\sqrt{3}}{3}ig,\quad c_4=\frac{\sqrt{6}}{2}. \tag{C12}$$

The solution in the x representation is the shifted $\psi(x)$ together with other factors; see Eq. (29).

The above solution is restricted to the case when the direction of the electric field is along the crystalline x direction. Now we generalize the calculation to an arbitrary direction of the electric field. The calculation is just a little bit more complicated. It still will be convenient to choose a coordinate system in which the direction of the electric field and that of the x axis coincide. The perturbed Hamiltonian then becomes

$$\tilde{V}(\hat{a},\hat{a}^{\dagger})=\exp\left[-i\frac{g}{\sqrt{2}}(\hat{a}^{\dagger}-\hat{a})\right](e^{-2i\Theta}\hat{a}^{\dagger 2}$$
$$+e^{2i\Theta}\hat{a}^2)^2\exp\left[i\frac{g}{\sqrt{2}}(\hat{a}^{\dagger}-\hat{a})\right].$$

The corrected solution has the same form as Eq. (C11), with the coefficients c_n which now depend on the angle $\Theta=\theta-\varphi$ (see Fig. 4), as follows:

$$c_1=-\sqrt{2}ig[(1+e^{-4i\Theta})g^2-2],$$
$$c_2=-\frac{\sqrt{2}}{2}(1+3e^{-4i\Theta})g^2,$$
$$c_3=\frac{4\sqrt{3}}{3}ige^{-4i\Theta},\quad c_4=\frac{\sqrt{6}}{2}e^{-4i\Theta}. \tag{C13}$$

The corresponding eigenvalue becomes

$$\xi=\frac{1}{2}-\eta'\left[\frac{1}{2}(1+\cos 4\Theta)g^4-2g^2+2\right]. \tag{C14}$$

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