Letters to the Editor

An Optimal Variable Structure Control with Integral Compensation for Electrohydraulic Position Servo Control Systems

Tzuen-Lih Chern and Yung-Chun Wu

Abstract—**An approach employing variable structure control with integral compensation is presented for an electrohydraulic position servo control system to achieve accurate servo tracking in the presence of load disturbance and plant parameter variation. Simulations show that the proposed approach may give a rather accurate servo-tracking result and is fairly robust to plant parameter variation and load disturbance.**

I. Introduction

Processes requiring large driving forces or torques are often actuated by hydraulic servo systems. The dynamic characteristics of such systems are usually very complex and highly nonlinear due to the flow–pressure relationship of the hydraulic components. For a practical control system, it is usually desired to have a fast accurate response with small overshoot. To achieve this result, an approach using variable structure control (VSC) with integral compensation for an electrohydraulic position servo control system is presented.

II. Variable Structure Control with Integral Compensation

The system using VSC with integral compensation is described as

$$\dot{X}_i = X_{i+1} \qquad i = 1,\cdots,n-1 \tag{1a}$$

$$\dot{X}_n = -\sum_{i=1}^{n} a_i X_i + bU - f \tag{1b}$$

$$\dot{Z} = r - X_1 \tag{1c}$$

where X_1 is the output, r is the input, a_i and b are the plant parameters, f is the disturbance, and U is a piecewise linear control function of the form

$$U = \begin{cases} U^+(x,t) & \text{if } \sigma > 0 \\ U^-(x,t) & \text{if } \sigma < 0 \end{cases} \tag{2}$$

where σ is the switching function given by

$$\sigma = c_1(X_1 - K_I Z) + \sum_{i=2}^{n} c_i X_i \qquad c_n = 1 \tag{3}$$

in which K_I is the integral control gain and c_i are constants.

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Design of such a system involves 1) and the choice of the control function U to guarantee the existence of a sliding mode, 2) the determination of the switching function σ and the integral control gain K_I such that the system has an optimal motion with respect to a quadratic performance index, and 3) the elimination of chattering of the control input.

A. Choice of Control Function

From (1) and (3), one obtains

$$\dot{\sigma} = c_1[X_2 - K_I(r - X_1)] + \sum_{i=2}^{n-1} c_i X_{i+1} + \left(-\sum_{i=1}^{n} a_i X_i + bU - f\right) \tag{4}$$

Let

$$a_i = a_i^0 + \Delta a_i \qquad i = 1 \cdots n$$

$$b = b^0 + \Delta b$$

where a_i^0 and b^0 are nominal values of a_i and b, and Δa_i and Δb are the deviations, respectively. Let the control function U be decomposed into

$$U = U_{eq} + \Delta U \tag{5a}$$

where U_{eq}, called the equivalent control, is defined as the solution of the equation $\dot{\sigma} = 0$ under $f = 0$, $a_i = a_i^0$ and $b = b^0$, that is,

$$U_{eq} = \left[c_1 K_I(r - X_1) - \sum_{i=2}^{n} c_{i+1} X_i + \sum_{i=1}^{n} a_i^0 X_i\right] / b^0. \tag{5b}$$

The function ΔU is used to eliminate the influence due to the presence of Δa_i, Δb, and f so as to guarantee the existence of a sliding mode. This function is constructed as

$$\Delta U = \Psi_1(X_1 - K_I Z) + \sum_{i=2}^{n} \Psi_i X_i + \Phi \tag{5c}$$

where

$$\Psi_1 = \begin{cases} \alpha_1 & \text{if } (X_1 - K_I Z)\sigma > 0 \\ \beta_1 & \text{if } (X_1 - K_I Z)\sigma < 0 \end{cases} \tag{5d}$$

$$\Psi_i = \begin{cases} \alpha_i & \text{if } X_i\sigma > 0 \\ \beta_i & \text{if } X_i\sigma < 0 \end{cases} \qquad i = 2,\cdots,n \tag{5e}$$

and

$$\Phi = \begin{cases} \gamma & \text{if } \sigma > 0 \\ \delta & \text{if } \sigma < 0. \end{cases} \tag{5f}$$

It is known that the condition for the existence and reachability of a sliding motion is [1], [2].

$$\sigma\dot{\sigma} < 0 \tag{6}$$

Substitution of (5) into (4) yields

$$\dot{\sigma}\sigma = (-\Delta a_1 + a_1^0 \Delta b/b^0 + b\Psi_1)(X_1 - K_I X)\sigma + \sum_{i=2}^{n} [(-\Delta a_i + a_i^0 \Delta b/b^0 - c_{i-1}\Delta b/b^0 + b\Psi_i)X_i\sigma] + [b\Phi + N(t)]\sigma \tag{7}$$

where

$$N(t) = \{-K_I Z(\Delta a_1 - a_1^0 \Delta b/b^0) + [c_1 K_I (r - X_1)]\Delta b/b^0 - f\}$$

Thus, the conditions for satisfying the inequality (6) are

$$\Psi_i = \begin{cases} \alpha_i < (\Delta a_i - a_i^0 \Delta b/b^0 + c_{i-1}\Delta b/b^0)/b \\ \beta_i > (\Delta a_i - a_i^0 \Delta b/b^0 + c_{i-1}\Delta b/b^0)/b \end{cases} \quad i = 1,\cdots,n \quad c_0 = 0 \tag{8a}$$

and

$$\Phi = \begin{cases} \gamma < -N(t)/b \\ \delta > N(t)/b. \end{cases} \tag{8b}$$

B. Determination of Switching Plane and Integral Control Gain

Under sliding motion, the system described by (1) can be reduced to [1], [2]

$$\dot{X}_i = X_{i+1} \quad i = 1,\cdots,n-2 \tag{9a}$$

$$\dot{X}_{n-1} = -\sum_{i=1}^{n-1} c_i X_i + c_1 K_I Z \tag{9b}$$

$$\dot{Z} = r - X_1 \tag{9c}$$

or, in the matrix form,

$$\dot{X} = AX + BV + Er \tag{9d}$$

$$V = GX \tag{9e}$$

where

$$X = \begin{bmatrix} Z \\ \hline X_1 \\ \vdots \\ X_{n-1} \end{bmatrix}_{n\times 1} \quad A = \begin{bmatrix} 0 & -1 & 0 & \cdots & 0 \\ \hline 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & & 0 \end{bmatrix}_{n\times n}$$

$$B = \begin{bmatrix} 0 \\ \hline 0 \\ \vdots \\ 1 \end{bmatrix}_{n\times 1} \quad E = \begin{bmatrix} 1 \\ \hline 0 \\ \vdots \\ 0 \end{bmatrix}_{n\times 1}$$

and

$$G = [c_1 K_I \quad -c_1 \quad -c_2 \quad \cdots \quad -c_{n-1}]_{1\times n}$$

In order to find the optimal gain matrix G by means of the optimal linear regulator technique, the quadratic index I as shown in the following equation must be minimized [3]:

$$I = \frac{1}{2}\int_{t_s}^{\infty} (X^T Q^T X + V^T R V) \tag{10}$$

where $Q = Q^T > 0$ and $R = R^T > 0$ are weighting matrices and t_s is the time from which the sliding mode begins. The weighting matrix Q can be chosen as

$$Q = D^T D \tag{11}$$

where D is a $1 \times n$ vector and the pair (A, D) is observable. Then the optimal gain matrix G is given by

$$G = -R^{-1}B^T P \tag{12}$$

where P is the solution of the matrix Riccati equation

$$PA + A^T P - PBR^{-1}B^T P + Q = 0. \tag{13}$$

C. Chattering Considerations

For the control law given by (5), if Φ and Ψ_i are chosen as

$$\Phi = \gamma = -\delta \qquad \Psi_i = \alpha_i = -\beta_i \qquad i = 1,\cdots,n$$

then the control function U can be represented as

$$U = \left[c_1 K_I (r - X_1) - \sum_{i=2}^{n} c_{i-1} X_i + \sum_{i=1}^{n} a_i^0 X_i\right] \Big/ b^0 + \left(\Psi_1 |X_1 - K_I Z| + \sum_{i=2}^{n} \Psi_i |X_i| + \Phi\right) \operatorname{sign}(\sigma) \tag{14}$$

Since the control U contains the sign function $\operatorname{sign}(\sigma)$, direct application of such a control signal to the plant may give rise to chatterings. To obtain a continuous control signal, the discontinuous function $\operatorname{sign}(\sigma)$ in (14) can be replaced by a proper continuous function [4] as

$$S_\delta(\sigma) = \frac{\sigma}{|\sigma| + \delta} \tag{15}$$

where δ is a positive number. If this number is too small, the chattering phenomenon may not be effectively suppressed, and if it is too large, the sliding action may be slow so that the advantage of robustness of VSC is lost. For improving the result, the value of δ is therefore chosen as a function of $|X_1 - r|$ as

$$\delta = \delta_0 + \delta_1 |X_1 - r|$$

where δ_0 and δ_1 are positive constants, and the proper continuous function is modified as

$$M_\delta(\sigma) = \frac{\sigma}{|\sigma| + \delta_0 + \delta_1 |X_1 - r|}. \tag{16}$$

III. An Electrohydraulic Position Control Servo Problem

The block diagram of the electrohydraulic position servo control system to be studied is shown in Fig. 1. The relation between the valve displacement X_v and the load flow rate Q_L is described as [5], [6]

$$Q_L = X_v K_j \sqrt{P_s - \operatorname{sign}(X_v) P_L} = X_v K_s \tag{17}$$

where K_j is a constant for a specific hydraulic motor, P_s is the supply pressure, P_L is the load pressure, and K_s is the valve flow gain that varies under different operating points. The flow continuity property of the motor chamber yields

$$Q_L = D_m \omega_c + K_{ce} P_L + (V_t/4\beta)\dot{P}_L \tag{18}$$

where D_m is the volumetric displacement, K_{ce} is the total leakage coefficient, V_t is the total volume of the oil, β is the bulk modulus of the oil, and ω_c is the velocity of the motor shaft. The torque balance equation for the motor is given by

$$D_m P_L = J\dot{\omega}_c + B_m \omega_c + T_L \tag{19}$$

where B_m is the viscous damping coefficient, J is the inertia of motor and T_L is the load disturbance.

Based on the block diagram as shown in Fig. 1, by combining (17)–(19), the servo valve gain K_v, and the VSC with integral

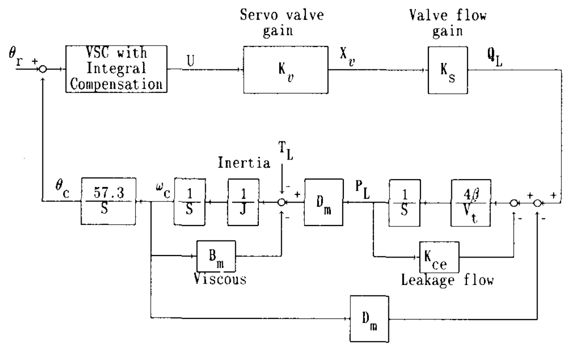

Fig. 1. The electrohydraulic position servo system using VSC with integral compensation.

compensation the following set of state equations can be obtained:

$$\dot{X}_1 = X_2 \tag{20a}$$

$$\dot{X}_2 = X_3 \tag{20b}$$

$$\dot{X}_3 = -a_2 X_2 - a_3 X_3 + bU - f \tag{20c}$$

$$\dot{Z} = r - X_1 \tag{20d}$$

where

$$a_2 = \frac{4\beta}{V_t}\frac{D_m^2}{J} + \frac{4\beta}{V_t}\frac{B_m}{J}K_{ce} \qquad a_3 = \frac{B_m}{J} + \frac{4\beta}{V_t}K_{ce}$$

$$b = 57.3 K_v K_s \frac{4\beta}{V_t}\frac{D_m}{J} \qquad f = 57.3\frac{4\beta}{V_t}\frac{K_{ce}}{J}T_L + 57.3\frac{1}{J}\dot{T}_L$$

$X_1 = \theta_c$

is the position of the motor shaft and $r = \theta_r$ is the reference input.

Following the design procedure given in Section II, one obtains the control function

$$U = [c_1 K_I (r - X_1) - c_1 X_2 - c_2 X_3 + a_2^0 X_2 + a_3^0 X_3]/b^0 + (\Psi_1 |X_1 - K_I Z| + \Psi_2 |X_2| + \Psi_3 |X_3| + \Phi) M_\delta(\sigma) \tag{21}$$

with

$$\Psi_i < -|\Delta a_i - a_i^0 \Delta b/b^0 + c_{i-1}\Delta b/b^0|/b \qquad i = 1,\cdots,3 \qquad c_0 = 0 \tag{22a}$$

and

$$\Phi < -|N(t)|/b \tag{22b}$$

where

$$N(t) = \{-K_I Z(\Delta a_1 - a_1^0 \Delta b/b^0) + [c_1 K_I (r - X_1)]\Delta b/b^0 - f\}.$$

The σ function is obtained from (3) as

$$\sigma = c_1 (X_1 - K_I Z) + c_2 X_2 + X_3 \tag{23}$$

and, by suitably choosing Q and R, one can obtain the optimal gains of c_1, c_2, and K_I.

IV. Simulation Results and Discussions

The robustness of the proposed approach against large plant parameter variations and external load disturbance has been simulated for demonstration. The nominal values of the hydraulic system parameters are listed in Table I. The weighting matrices are chosen as

TABLE I
System Parameters for Simulation

Parameter	Value	Dimension
K_s	$0.03 \times \sqrt{P_s - \text{sign}(X_v) P_L}$	in^2/s
P_s	2000	psi
β	50000	psi
V_t	2.0	in^3
K_{ce}	0.001	$\text{in}^3/\text{s}/\text{psi}$
D_m	1.0	in^3/rad
J	0.5	$\text{in-lb-s}^2/\text{rad}$
B_m	75	in-lb.s/rad
K_v	20.	in/V

$$Q = \begin{bmatrix} 10^5 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \qquad \text{and} \qquad R = 10^{-5}.$$

Then, from (12), the optimal gain matrix can be obtained as

$$G = [-10^5 \quad -5873.1 \quad -147.3]$$

so that $K_I = 17$, $c_1 = 5873.1$, and $c_2 = 147.3$.

Gains Ψ_1, Ψ_2, Ψ_3, and Φ must be chosen to satisfy (22) and, based on simulations, one possible choice is

$$\Psi_1 = -1 \qquad \Psi_2 = -0.01 \qquad \Psi_3 = -0.00002 \qquad \Phi = -0.001.$$

This VSC with integral compensation approach gives a control function

$$U = [c_1 K_I (r - X_1) - c_1 X_2 - c_2 X_3 + a_2^0 X_2 + a_3^0 X_3]/b^0 + (\Psi_1 |X_1 - K_I Z| + \Psi_2 |X_2| + \Psi_3 |X_3| + \Phi) M_\delta(\sigma)$$

where $\sigma = 5873.1(X_1 - K_I Z) + 147.3 X_2 + X_3$ and $\delta = 20000|X_1 - r| + 500$.

The following approaches are presented for performance comparison.

1) Conventional VSC approach: Let the control function U be

$$U = (-1|X_1 - r| - 0.01|X_2| - 0.00002|X_3|) M_\delta(\sigma)$$

where $\sigma = 800(X_1 - r) + 40X_2 + X_3$ and $\delta = 20000|X_1 - r| + 500$.

2) Linear PI approach: Let the transfer function of the controller be

$$K_p + K_I/S$$

where $K_p = 0.0095$ and $K_I = 0.0158$.

Fig. 2 shows the dynamic responses of the three approaches. It is seen that, in the presence of a shaft-angle-dependent external load disturbance T_L and the variations of plant parameters K_v and J, the responses of the proposed approach can be maintained almost identically but vary significantly for other approaches. Fig. 3 shows the waveform of the control function U. It is clear that by using a modified proper continuous function the chattering phenomena can be effectively suppressed. Thus, the proposed approach seems amenable for practical implementation.

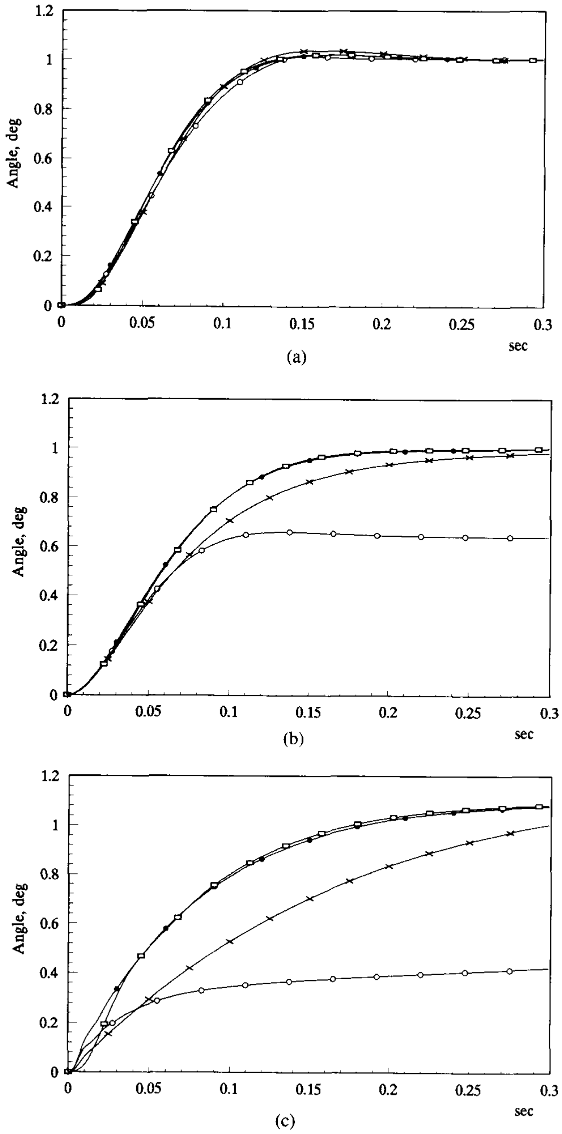

Fig. 2. Angular responses in the presence of load disturbance T_L and variations of plant parameters K_v and J. (a) The proposed VSC with integral compensation approach. (b) The conventional VSC approach. (c) The linear PI approach.

—●—: normal ($T_L = 0$, $K_v = 20$ in/V, $J = 0.5$ in-lb-s^2/rad)
—○—: $T_L = 500|\theta_c|$
—×—: -50% change in K_v
—□—: 1000% change in J

V. Conclusions

A VSC with integral compensation for an electrohydraulic position servo control system is presented. It has been shown that the proposed approach is theoretically robust to the plant parameter variations. It can achieve a zero steady-state error for step input and has an optimal motion with respect to a quadratic performance index. Simulations show that the proposed approach can give a quite accurate servo-tracking response in the face of large plant parameter variations and external load disturbance.

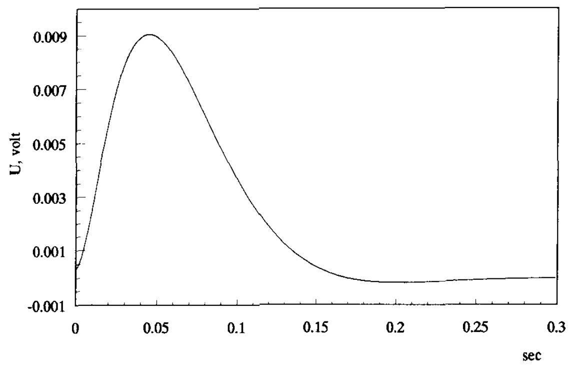

Fig. 3. Control signal of the proposed approach.

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A Rotor Time Constant Evaluation for Vector-Controlled Induction Motor Drives

Piotr J. Chrzan and Piotr Kurzyński

***Abstract*—The on-line identification method of the rotor time constant of an induction machine is derived from the steady-state analysis of the machine space vectors. Simulation of the indirect field orientation system is performed to verify the method convergence in quasi-steady-state operation, independently of the initial controller parameters.**

I. Introduction

Digital vector control techniques incorporating PWM inverters have made possible the development of high-performance induction motor drives. However, in these solutions the control gains depend heavily on the motor parameters, particularly on the rotor resistance or the rotor time constant, which change widely with temperature, frequency, and current amplitude.

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