Fuzzy Pattern Recognition Model for Diagnosing Cracks in RC Structures

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Abstract: This paper examines a diagnostic model based on the concept of cause-and-effect diagramming and fuzzy pattern recognition, which contributes a new methodology for diagnosing engineering problems. Three examples are presented to demonstrate the feasibility of the model in diagnosing crack formations in reinforced concrete structures. Two levels of parameters representing the causes of cracks in concrete are used to form fuzzy sets. The parameters represent the materials used, fabrication of structural elements, loading, and environmental conditions. An expert system that links the parameters by means of fuzzy set theory is constructed using finite universal sets consisting of membership functions and fuzzy vectors. Pattern recognition is used to identify a fuzzy vector that represents the most likely causes of the crack.

INTRODUCTION

Concrete is a versatile building material that has been used widely in construction since the invention of cement in 1824. Major concrete structures include dams, bridges, highways, buildings, and pipe systems. Because concrete is strong in compression but weak in tension, concrete structures often develop cracks that ultimately affect the performance of the structure. The assessment of cracks in concrete members of existing buildings is a complex process that requires information on the aggregate used, mixing and curing, the properties of the concrete, and loading conditions. In addition, shrinkage and creep often compound the degree of complexity to the extent that engineers are unable to pinpoint the precise causes of cracks or accurately predict the behavior of the cracked concrete structural elements.

In general, concrete develops cracks because of one or more of the following reasons: abnormal setting of cement paste, heat of hydration and expansion of cement paste (Lea 1971; Soroka 1979); alkali aggregate reactions, poor gradation of aggregate (ACI Committee 221 1961); inadequate mixing, concrete construct defects (Powers 1968; ACI Committee 302 1971; ACI Committee 304 1972; ACI Committee 308 1971); overloading or abnormal loading, and other factors. Traditionally, assessment of cracks in concrete structures was done by experienced engineers only; however, expert systems have been developed to conserve time, make expertise more widely available, and simplify decision making. Similar expert systems have been successfully used in medicine (Shortliffe 1976; Adlassnig 1982; Binaghi 1990), mineral exploration (Duda and Reboh 1984), structural analysis (Bennet and Engelmore 1979; Adeli 1988; Adeli and Balasubramanyam 1988), construction material selection (Clifton and Oltikar 1987), and building repair technology (Kalyanasundaram et al. 1990; Wang et al. 1991).

The causes of cracking in concrete are complicated and interrelated, and the characteristics of cracks are difficult to describe precisely. These characteristics include how long after casting cracks develop, the depth of cracks, whether cracks are regular or irregular, the types of concrete members that develop cracks, crack patterns, and crack locations. Fuzzy set theory (Zadeh 1965) has been used to describe similar characteristics successfully in the field of medical diagnostic systems and in pile type selection (Mishido et al. 1990). Pattern recognition has been applied in linguistic pattern search techniques (Sanchez et al. 1982), character recognition (Chatterjii 1982), texture classification (Hajnal and Koczy 1982), and earthquake engineering (Fu et al. 1982; Ishizuka et al. 1982; Watada et al. 1984).

Cause-and-effect diagrams have been employed in construction management to classify the relationships between defects and their causes. These diagrams and fuzzy pattern recognition can be combined to identify fuzzy relationships between the cause of cracking and the characteristics cracks exhibit. In this paper we propose a two-level system for compiling data on the causes of cracks; the method can be extended to more levels if necessary.

MATHEMATICAL MODELING

An attempt was made to explore the feasibility of using fuzzy pattern recognition in the investigation of concrete structures exhibiting cracks. The concept of fuzzy set theory and pattern recognition may be new to readers in the field of concrete, so this section briefly describes some important terminology used in this paper.

1. Membership function. In fuzzy sets, an object's membership in a set may be whole, partial, or nonexistent. The degree of membership is expressed as its membership function, which is defined as follows: If X is a collection of objects denoted by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$. $\mu_{\tilde{A}}(x)$ is called the membership function of object x in $\tilde{A}$. The membership function is a real number $0 \le \mu_{\tilde{A}}(x) \le 1$.
2. Fuzzy vector. A fuzzy set defined by a finite universal set $X = \{x_1, x_2, \ldots, x_n\}$ can be represented by a vector $\tilde{A} = (a_1, a_2, \ldots, a_n)$, where $a_i = \mu_{\tilde{A}}(x_i)$, $i = 1, 2, \ldots, n$.
3. Fuzzy relation. Fuzzy relationship is an important concept in fuzzy set theory. A relationship is an association between elements, which is also called a mapping because it associates elements from one domain with those in another domain. Let $X, Y \subseteq U$ be universal sets. Then, $\tilde{R} = \{((x, y), \mu_{\tilde{R}}(x, y)) | (x, y) \subseteq X \times Y\}$ is a fuzzy relation of $X \times Y$. A fuzzy relation is a fuzzy subset in the Cartesian product universe. A convenient way of representing a relationship is by means of a matrix. The Cartesian product of two crisp sets X and Y, denoted by $X \times Y$, is the crisp set of all ordered pairs such that the first element in each pair is a member of X and the second element is a member of Y. Such a set can be represented as $X \times Y = \{(x, y) | x \in X, y \in Y\}$. Zadeh and other

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Note. Discussion open until September 1, 1998. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on November 27, 1995. This paper is part of the ***Journal of Computing in Civil Engineering***, Vol. 12, No. 2, April, 1998. ©ASCE, ISSN 0887-3801/98/0002-0111–0119/$4.00 + $.50 per page. Paper No. 12102.

J. Comput. Civ. Eng. 1998.12:111-119.

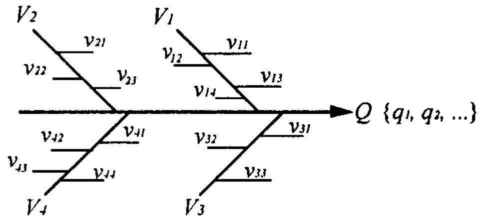

FIG. 1. Cause-and-Effect Diagram

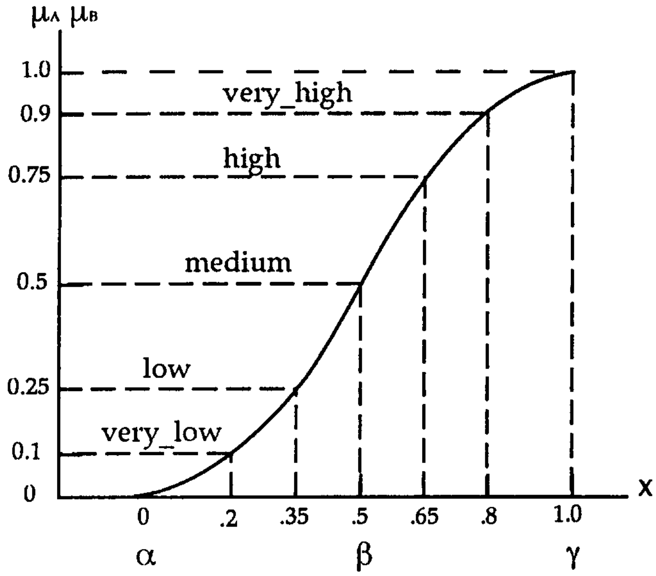

FIG. 2. Membership Functions of Linguistic Variables Sets A and B

authors have suggested additional definitions for fuzzy set operations, such as algebraic product and weighted Hamming distance.

4. Algebraic product. The algebraic product of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is defined as $\tilde{C} = \tilde{A}\cdot\tilde{B}$; then, $\tilde{C} = \{(x, \mu_{\tilde{C}}(x)) | x \in X\}$, where $\mu_{\tilde{C}}(x) = \mu_{\tilde{A}}(x)\cdot\mu_{\tilde{B}}(x)$.
5. Weighted Hamming distance. Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy vectors on $X = \{x_1, x_2, \ldots, x_n\}$. The weighted Hamming distance is defined as $d_w(\tilde{\mathbf{A}}, \tilde{\mathbf{B}}) = \Sigma_{i=1}^{n} W(x_i)\cdot[\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i)]$, $i = 1, 2, \ldots, n$, where $W(x_i)$ is the value of the weight for x_i. If $\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i) \geq 0$, it is a positive distance from $\tilde{A}$ to $\tilde{B}$. If $\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i) < 0$, it is a negative distance from $\tilde{\mathbf{A}}$ to $\tilde{\mathbf{B}}$.

Zadeh defines standard piecewise quadratic functions (Zadeh 1981) as

$$f_1(x; \alpha, \beta, \gamma) = S(x; \alpha, \beta, \gamma) \tag{1}$$

$$f_2(x; \alpha, \beta) = \begin{cases} f_1(x; \beta - \alpha, \beta - \alpha/2, \beta); & x \leq \beta \\ 1 - f_1(x; \beta, \beta + \alpha/2, \beta + \alpha); & x > \beta \end{cases} \tag{2}$$

$$f_3(x; \alpha, \beta, \gamma) = 1 - f_1(x; \alpha, \beta, \gamma) \tag{3}$$

where $S(x; \alpha, \beta, \gamma)$ is an S-function that is often used in fuzzy sets as a membership function and in this paper is defined as follows:

$$\mu_{\tilde{A}}(x) = S(x; \alpha, \beta, \gamma) = \begin{cases} 0; & x \leq \alpha \\ 2\left(\dfrac{x - \alpha}{\gamma - \alpha}\right)^2; & \alpha \leq x \leq \beta \\ 1 - 2\left(\dfrac{x - \gamma}{\gamma - \alpha}\right)^2; & \beta \leq x \leq \gamma \\ 1; & x \geq \gamma \end{cases} \tag{4}$$

where $\alpha = 0$; $\beta = 0.5$; and $\gamma = 1.0$.

Fuzzy sets and linguistic variables can be used to quantify concepts used in natural language, which can then be manipulated. A linguistic variable must have a valid syntax and semantics, which can be specified by fuzzy sets or rules. A syntactic rule defines the well-formed expressions in $T(L)$. The term set $T(L)$ of a linguistic variable, L, is the set of values it may take. For example, T(Age) = {very_young, young, medium, old, very_old}, where each of these values may itself be a linguistic variable that can take on values that are fuzzy sets. The membership function could be defined as the S function $\mu_{old}(x) = S(x; 60, 70, 80)$.

A cause-and-effect diagram is shown in Fig. 1; the cause parameters can be divided into several groups according to their properties. We call these groups the primary-level cause parameters and denote them by $V = \{V_1, V_2, \ldots, V_n\}$. Each primary-level cause parameter contains several subparameters, which are known as secondary-level cause parameters and expressed as $V_i = \{v_{i1}, v_{i2}, \ldots, v_{im}\}$ (the number of levels can be extended if necessary). $Q = \{q_1, q_2, \ldots, q_r\}$ is the crisp universal set of all characteristics.

Linguistic variables are used to describe the degree of relationship between a cause V_i and a characteristic q_k, which is defined as a set A = {very_low, low, medium, high, very_high}. Then, the fuzzy set is defined as $\tilde{A}(x)$, $x = \{0, 0.1, 0.2, \ldots, 1.0\}$, and the membership function is defined as $\mu_{\tilde{A}}(x) = S(x; 0, 0.5, 1.0)$, $x \in X$, where X is the relation space. Linguistic variables are also used to describe the degree to which it is confirmed that a characteristic q_k is exhibited, which is defined as a set B = {very_low, low, medium, high, very_high}. The fuzzy set is then $\tilde{B}(x)$, $x = \{0, 0.1, 0.2, \ldots, 1.0\}$, and the membership function is defined as $\mu_{\tilde{B}}(x) = S(x; 0, 0.5, 1.0)$, $x \in Y$, where Y is the confirmation space.

In this paper, the membership functions of the element sets A and B can be chosen from among the following equations:

$$\mu_{very\_low}(x) = S(x; 0, 0.5, 1.0) = 0.1 \tag{5}$$

where $x = 0.2$.

$$\mu_{low}(x) = S(x; 0, 0.5, 1.0) = 0.25 \tag{6}$$

where $x = 0.35$.

$$\mu_{medium}(x) = S(x; 0, 0.5, 1.0) = 0.5 \tag{7}$$

where $x = 0.5$.

$$\mu_{high}(x) = S(x; 0, 0.5, 1.0) = 0.75 \tag{8}$$

where $x = 0.65$.

$$\mu_{very\_high}(x) = S(x; 0, 0.5, 1.0) = 0.9 \tag{9}$$

where $x = 0.8$.

The membership functions are illustrated in Fig. 2.

For the primary level, we can define a fuzzy relation $\tilde{\mathbf{R}}^{(1)}$ on the set $Q \times V$ in which membership function $\mu_{\tilde{R}^{(1)}}(q_i, V_j)$, $(q_i \in Q, V_j \in V)$ indicates the degree of relationship between characteristic q_i and cause V_j. This relation can be expressed in matrix form:

$$\tilde{\mathbf{R}}^{(1)} = \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \\ \cdot \\ q_r \end{array} \begin{array}{c} \begin{array}{ccccc} V_1 & V_2 & V_3 & \cdot & V_n \end{array} \\ \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} & \cdot & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdot & \cdot & \cdot \\ \mu_{31} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \mu_{r1} & \cdot & \cdot & \cdot & \mu_{rn} \end{bmatrix} \end{array} \tag{10}$$

where $\mu_{ij} = \mu_{\tilde{A}}(q_i, V_j)$; $i = 1, 2, \ldots, r$; $j = 1, 2, \ldots, n$.

For the secondary level, using the cause parameters for each V_j, define a fuzzy relation $\tilde{\mathbf{R}}_j^{(2)}$ on the set $Q \times V_j$ in which the membership function $\mu_{\tilde{R}_j^{(2)}}(q_s, v_{ji})$, $(q_s \in Q, v_{ji} \in V_j)$ indicates

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the degree of relationship between characteristic q_s and cause $v_{jt} \cdot \tilde{\mathbf{R}}_j^{(2)}$ can also be written in matrix form:

$$\tilde{\mathbf{R}}_j^{(2)} = \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \\ \cdot \\ q_h \end{array} \begin{array}{c} \begin{array}{ccccc} v_{j1} & v_{j2} & v_{j3} & \cdot & v_{jm} \end{array} \\ \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \cdot & \gamma_{1m} \\ \gamma_{21} & \gamma_{22} & \cdot & \cdot & \cdot \\ \gamma_{31} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \gamma_{h1} & \cdot & \cdot & \cdot & \gamma_{hm} \end{bmatrix} \end{array} \quad (11)$$

where $\gamma_{st} = \mu_{\tilde{A}}(q_s, v_{jt})$; $s = 1, 2, \ldots, h$; $t = 1, 2, \ldots, m$; and $j = 1, 2, \ldots, n$.

An observational fuzzy vector $\tilde{\mathbf{P}}$ can be defined on a set $Q = \{q_1, q_2, \ldots, q_r\}$ to indicate the degree to which it is confirmed that a particular crack characteristic is exhibited. $\tilde{\mathbf{P}}$ can be expressed as $\tilde{\mathbf{P}} = (p_1, p_2, \ldots, p_r)$, where $p_i = \mu_{\tilde{B}}(x)$, $i = 1, 2, \ldots, r$. Let $\tilde{\mathbf{P}}^{(1)}$ indicate the degree of confirmation for the primary level, and let $\tilde{\mathbf{P}}^{(2)}$ indicate the degree of confirmation for the secondary-level.

We define a matrix $\mathbf{H} = (h_{ij})_{r\times r}$, whose component h_{ij} is called the importance factor. If $h_{ij} = k$, this means crack characteristic q_i is k times more important than crack characteristic q_j for the cause of cracks, and $h_{ji} = 1/k$. Then the weighting vector $\mathbf{W}$ is defined as $\mathbf{W} = (\Sigma_{j=1}^r h_{1j}, \Sigma_{j=1}^r h_{2j}, \ldots, \Sigma_{j=1}^r h_{rj}) = (w_1, w_2, \ldots, w_r)$ and the sum of the components of this vector is unity, $\Sigma_{i=1}^r w_i = 1$. $w_i > 0$ are weights that express the relative importance of the crack characteristics set Q. Let $\mathbf{W}^{(1)}$ indicate the primary-level weighting vector and $\mathbf{W}^{(2)}$ indicate the secondary-level weighting vector.

For the primary level, cause parameters define a fuzzy vector $\tilde{\mathbf{V}}_i$ on $Q^{(1)} = \{q_1, q_2, \ldots, q_s\}$ as a fuzzy pattern, which is represented by a vector $\tilde{\mathbf{V}}_i = (\alpha_1, \alpha_2, \ldots, \alpha_s)$, $i = 1, 2, \ldots, n$, where $\alpha_j = \mu_{\tilde{V}_i}(q_j)$, $j = 1, 2, \ldots, s$. We then perform pattern comparison for each pair of fuzzy patterns $\tilde{\mathbf{V}}_i$ and $\tilde{\mathbf{V}}_j$ using the weighted Hamming distance $d_w(\tilde{\mathbf{V}}_i, \tilde{\mathbf{V}}_j) = \Sigma_k^s w_k \cdot [\mu_{\tilde{V}_i}(q_k) - \mu_{\tilde{V}_j}(q_k)]$. If $d_w(\tilde{\mathbf{V}}_i, \tilde{\mathbf{V}}_j) > 0$, the fuzzy pattern $\tilde{\mathbf{V}}_i$ is selected. If $d_w(\tilde{\mathbf{V}}_i, \tilde{\mathbf{V}}_j) = 0$, the fuzzy pattern $\tilde{\mathbf{V}}_i$ and $\tilde{\mathbf{V}}_j$ are selected. If $d_w(\tilde{\mathbf{V}}_i, \tilde{\mathbf{V}}_j) < 0$, the fuzzy pattern $\tilde{\mathbf{V}}_j$ is selected. In each step of the process one pattern is screened out. Eventually, only one fuzzy pattern is left; this pattern is identified as the cause on the primary level.

From the observational fuzzy vector $\tilde{\mathbf{P}}^{(1)}$, the last selected fuzzy pattern $\tilde{\mathbf{V}}_i$, and the weighted vector $\mathbf{W}^{(1)}$, the degree of confirmation $C_i^{(1)}$ for the fuzzy pattern $\tilde{\mathbf{V}}_i$ can be computed by the following formula: $C_i^{(1)} = \Sigma_{j=1}^s (W^{(1)} \cdot \tilde{P}^{(1)}) \cdot \tilde{\mathbf{V}}_i$, $i = 1, 2, \ldots, n$.

After $\tilde{\mathbf{V}}_i$ is selected, a fuzzy vector $\tilde{\mathbf{v}}_{ik}$ on $Q^{(2)} = \{q_1, q_2, \ldots, q_p\}$ is defined as a fuzzy pattern on the secondary level. This vector can be expressed as follows:

$$\tilde{\mathbf{v}}_{ik} = (\gamma_1, \gamma_2, \ldots, \gamma_p), \; i = 1, 2, \ldots, n, \; k = 1, 2, \ldots, m.$$

where $\gamma_j = \mu_{\tilde{v}_{ik}}(q_j)$, $j = 1, 2, \ldots, p$.

For the secondary level, using γ_j, $\tilde{\mathbf{v}}_{ik}$, $\tilde{\mathbf{P}}^{(2)}$, $\mathbf{W}^{(2)}$, and $C_{ik}^{(2)}$ instead of α_j, $\tilde{\mathbf{V}}_i$, $\tilde{\mathbf{P}}^{(1)}$, $\mathbf{W}^{(1)}$, and $C_i^{(1)}$ respectively, we duplicate the procedures performed for the primary level to obtain the fuzzy pattern $\tilde{\mathbf{v}}_{il}$ and the degree of confirmation. If necessary, the procedure can be repeated for more levels.

CRACK MODELS

According to actual investigations and suggestions reported in engineering publications (Lerch 1957; ACI Committee 224 1972; Beaufait and Hoadley 1973; Price 1974; ACI 1974) the primary causes of cracks in reinforced concrete structural elements can be classified into four primary-level parameters. These can be expressed as $V = \{V_1, V_2, V_3, V_4\}$, where V_1 represents causes related to the quality of the concrete material, V_2 represents causes related to the procedure used to construct the concrete, V_3 represents causes related to environmental factors, and V_4 represents causes related to the applied loads. Each primary-level cause parameter and its subcauses are shown in Fig. 3 and in Tables 1–5.

Cracks can be described on the basis of seven characteristics: how soon after casting they develop, their depth, their regularity, whether they appear only in a concrete member or throughout the overall structure, the type of member in which they appear, their patterns, and their locations. The first four of these characteristics will be referred to as primary-level characteristics and the other as secondary-level characteristics. The primary-level characteristics, $Q_1 = \{q_1, q_2, q_3, q_4\}$, are shown in Table 6, and the secondary-level characteristics, $Q_2 = \{q_5, q_6, q_7\}$, are shown in Table 7. The fuzzy relation matrix $\tilde{\mathbf{R}}$ for each level is shown in Tables 8–12. All the data and characteristics of cracks can be extended properly by experts or experienced engineering.

APPLICATIONS

Three examples were used to verify the applicability of the model. The data used in these three examples are listed in Tables 8–12.

Example 1

In a reinforced concrete structure, fine cracks occurred on the slab surface three days after casting. The cracks are random in nature and have no regularity.

The degree of confirmation for each of the crack characteristics was as follows:

Crack time (q_1): very_high $\therefore\ p_1 = \mu_{very\_high}(x) = 0.90$
Crack depth (q_2): high $\therefore\ p_2 = \mu_{high}(x) = 0.75$
Crack regularity (q_3): very_high $\therefore\ p_3 = \mu_{very\_high}(x) = 0.90$
Crack range (q_4): very_high $\therefore\ p_4 = \mu_{very\_high}(x) = 0.90$
Crack member (q_5): very_high $\therefore\ p_5 = \mu_{very\_high}(x) = 0.90$
Crack pattern (q_6): very_high $\therefore\ p_6 = \mu_{very\_high}(x) = 0.90$
Crack location (q_7): high $\therefore\ p_7 = \mu_{high}(x) = 0.75$.

According to these crack characteristics ($q_1 = q_{12}$, $q_2 = q_{21}$, $q_3 = q_{32}$, $q_4 = q_{41}$), four fuzzy patterns for the primary level were found in Table 8, and the primary-level observational fuzzy vectors $\tilde{\mathbf{P}}^{(1)}$ were expressed as follows:

$\tilde{\mathbf{V}}_i = (q_{12} \quad q_{21} \quad q_{32} \quad q_{41})$ $i = 1, 2, 3, 4$.
$\tilde{\mathbf{V}}_1 = (.50\ .90\ .90\ .90)$, $\tilde{\mathbf{V}}_2 = (.75\ .50\ .50\ .90)$, $\tilde{\mathbf{V}}_3 = (.25\ .50\ .50\ .90)$,
$\tilde{\mathbf{V}}_4 = (.25\ .10\ .10\ .90)$, and $\tilde{\mathbf{P}}^{(1)} = (p_1, p_2\ p_3, p_4) = (.90\ .75\ .90\ .90)$.

For the primary-level characteristics, the importance factors assigned were

$$\mathbf{H} = \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \\ q_4 \end{array} \begin{array}{c} \begin{array}{cccc} q_1 & q_2 & q_3 & q_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1.5 & 1.5 \\ 1 & 1 & 1.5 & 1.5 \\ 0.67 & 0.67 & 1 & 1 \\ 0.67 & 0.67 & 1 & 1 \end{bmatrix} \end{array}$$

Therefore, $\mathbf{W}^{(1)} = (5\ 5\ 3.34\ 3.34)$, the weighting vector was normalized as $\mathbf{W}^{(1)} = (.3\ .3\ .2\ .2)$.

The weighted Hamming distance was computed by the following process:

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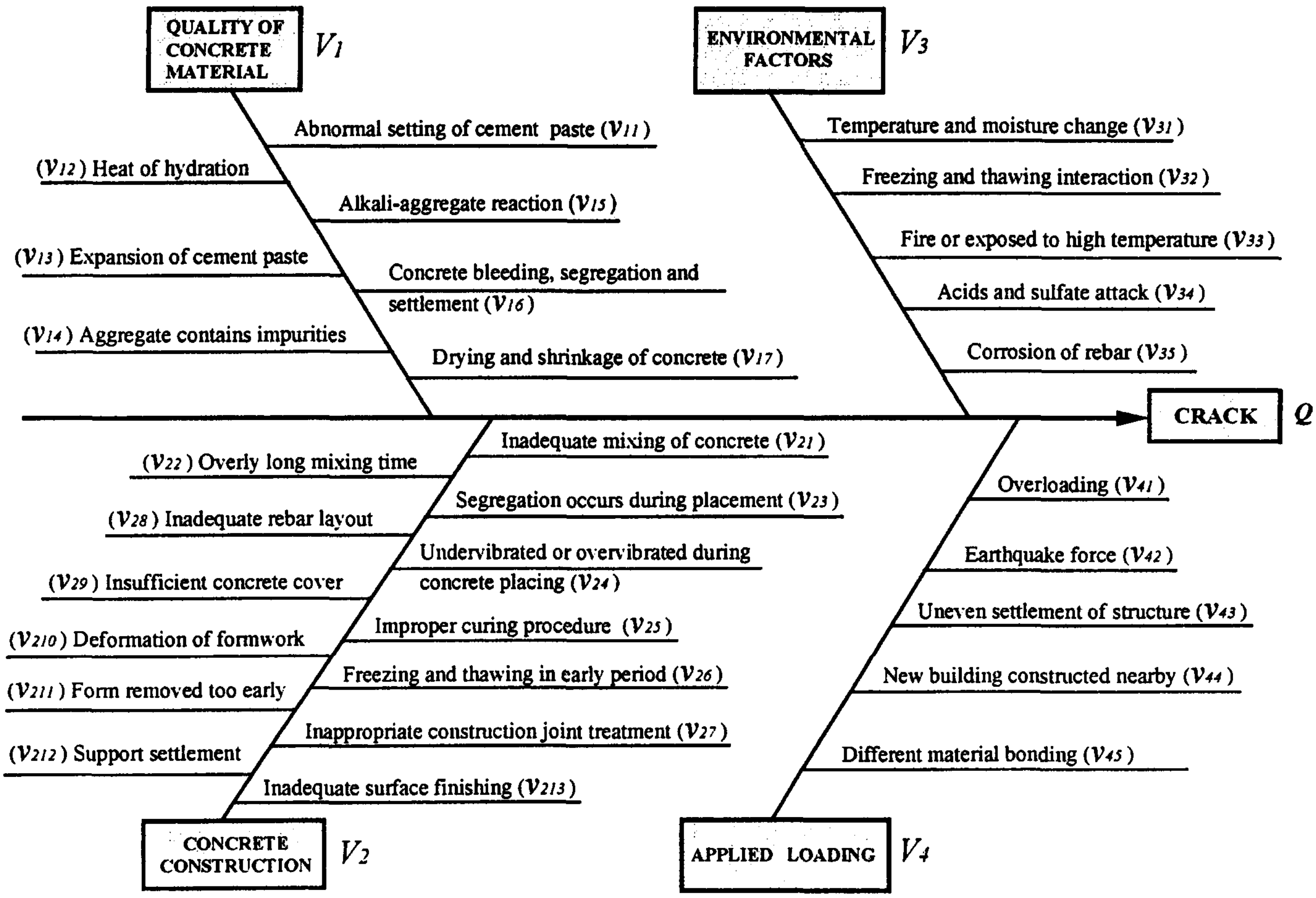

FIG. 3. Cause-and-Effect Diagram of Cracks

TABLE 1. Causes of Crack of Primary Level

Cause (1)	Characteristic (2)
V_1	Quality of concrete material
V_2	Concrete construction procedure
V_3	Environmental factors
V_4	Applied loads

TABLE 2. Quality of Concrete Material (V_1)

Cause (1)	Characteristic (2)
v_{11}	Abnormal setting of cement paste
v_{12}	Heat of hydration
v_{13}	Expansion of cement paste
v_{14}	Aggregate contains impurities
v_{15}	Alkali-aggregate reaction
v_{16}	Concrete bleeding, segregation, and settlement
v_{17}	Drying and shrinkage of concrete

$d_w(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_2) = .3\cdot(.50-.75) + .3\cdot(.9-.5) + .2\cdot(.9-.5) + .2\cdot(.9-.9) = 0.125$

$d_w(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_2) = 0.125 > 0$; $\tilde{\mathbf{V}}_1$ is selected.

$d_w(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_3) = .3\cdot(.50-.25) + .3\cdot(.9-.5) + .2\cdot(.9-.5) + .2\cdot(.9-.9) = 0.275$

$d_w(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_3) = 0.275 > 0$; $\tilde{\mathbf{V}}_1$ is selected.

$d_w(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_4) = .3\cdot(.50-.25) + .3\cdot(.9-.1) + .2\cdot(.9-.1) + .2\cdot(.9-.9) = 0.475$

$d_w(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_4) = 0.475 > 0$; $\tilde{\mathbf{V}}_1$ is selected.

$$C_1^{(1)} = \sum_{j=1}^{4} (\mathbf{W}^{(1)}\cdot\tilde{\mathbf{P}}^{(1)})\cdot\tilde{\mathbf{V}}_1 = 0.662$$

TABLE 3. Concrete Construction Procedure (V_2)

Cause (1)	Characteristic (2)
v_{21}	Inadequate mixing of concrete
v_{22}	Overly long mixing time
v_{23}	Segregation occurs during placement
v_{24}	Undervibrated or overvibrated during concrete placing
v_{25}	Improper curing procedure
v_{26}	Freezing and thawing in early period
v_{27}	Inappropriate construction joint treatment
v_{28}	Inadequate rebar layout
v_{29}	Insufficient concrete cover
v_{210}	Deformation of formwork
v_{211}	Form removed too early
v_{212}	Support settlement
v_{213}	Inadequate surface finishing

TABLE 4. Environmental Factors (V_3)

Cause (1)	Characteristic (2)
v_{31}	Temperature and moisture change
v_{32}	Freezing and thawing interaction
v_{33}	Fire or exposed to high temperature
v_{34}	Acids and sulfate attack
v_{35}	Corrosion of rebar

The primary-level crack cause is related to the quality of the concrete material (fuzzy pattern $\tilde{\mathbf{V}}_1$), and the degree of confirmation is 66.2%.

For the secondary level there are seven fuzzy patterns related to the quality of the concrete material. The fuzzy vector

TABLE 5. Applied Loads (V_4)

Cause (1)	Characteristic (2)
v_{41}	Overloading
v_{42}	Earthquake force
v_{43}	Uneven settlement of structure
v_{44}	New building constructed nearby
v_{45}	Different material bonding

TABLE 6. Primary-Level Characteristic Data Q_1

Characteristic (1)	Value (2)
Time (q_1)	q_{11}: One hour–one day
	q_{12}: One day–28 days
	q_{13}: More than 28 days
Depth (q_2)	q_{21}: Shallow and fine on surface
	q_{22}: Deep and wide
Regularity (q_3)	q_{31}: Regular
	q_{32}: Random
Range (q_4)	q_{41}: Member (beam, column, slab, wall)
	q_{42}: Overall structure

TABLE 7. Secondary-Level Characteristic Data Q_2

Characteristic (1)	Value (2)
Member (q_5)	q_{51}: Beam
	q_{52}: Column
	q_{53}: Slab
	q_{54}: Wall
	q_{55}: Overall structure
Pattern (q_6)	q_{61}: Longitudinal
	q_{62}: Transverse
	q_{63}: Diagonal
	q_{64}: X shape
	q_{65}: /\ shape
	q_{66}: \/ shape
	q_{67}: + shape
	q_{68}: Turtle-back shape
	q_{69}: Random shape
	q_{610}: Honeycombing
	q_{611}: Spall
	q_{612}: Spiral shape
	q_{613}: Corrosion of rebar
Location (q_7)	q_{71}: End parts
	q_{72}: Central parts
	q_{73}: Corner parts
	q_{74}: Member surface
	q_{75} Opening hole
	q_{76} Joint of members

was found from Table 9 ($q_5 = q_{53}$, $q_6 = q_{69}$, $q_7 = q_{74}$), the observational fuzzy vectors $\tilde{\mathbf{P}}^{(2)}$ were expressed as follows:

$\tilde{\mathbf{v}}_{1j} = (q_{53} \quad q_{69} \quad q_{74}) \quad j = 1, 2, \ldots, 7.$
$\tilde{\mathbf{v}}_{11} = (.90\ .90\ .90)$, $\tilde{\mathbf{v}}_{12} = (.10\ .10\ .25)$, $\tilde{\mathbf{v}}_{13} = (.75\ .10\ .90)$, $\tilde{\mathbf{v}}_{14} = (.90\ .25\ .90)$,
$\tilde{\mathbf{v}}_{15} = (.50\ .25\ .90)$, $\tilde{\mathbf{v}}_{16} = (.75\ .10\ .90)$, $\tilde{\mathbf{v}}_{17} = (.90\ .75\ .90)$,
$\tilde{\mathbf{P}}^{(2)} = (p_5\ p_6\ p_7) = (.90\ .90\ .75)$.

The importance factors assigned were

$$\mathbf{H} = \begin{array}{c} \\ q_5 \\ q_6 \\ q_7 \end{array} \begin{array}{c} \begin{array}{ccc} q_5 & q_6 & q_7 \end{array} \\ \begin{bmatrix} 1 & 0.67 & 1.5 \\ 1.5 & 1 & 2 \\ 0.67 & 0.5 & 1 \end{bmatrix} \end{array}, \quad \mathbf{W}^{(2)} = (3.17\ 4.5\ 2.17)$$

Therefore, weighted vector $\mathbf{W}^{(2)} = (.30\ .45\ .25)$.

$d_w(\tilde{\mathbf{v}}_{11}, \tilde{\mathbf{v}}_{12}) = .3\cdot(.9-.1) + .45\cdot(.9-.1) + .25\cdot(.9-.25) = 0.7625 > 0$ $\tilde{\mathbf{v}}_{11}$ is selected.
$d_w(\tilde{\mathbf{v}}_{11}, \tilde{\mathbf{v}}_{13}) = 0.405$; $\tilde{\mathbf{v}}_{11}$ is selected.
$d_w(\tilde{\mathbf{v}}_{11}, \tilde{\mathbf{v}}_{14}) = 0.293$; $\tilde{\mathbf{v}}_{11}$ is selected.
$d_w(\tilde{\mathbf{v}}_{11}, \tilde{\mathbf{v}}_{15}) = 0.413$; $\tilde{\mathbf{v}}_{11}$ is selected.
$d_w(\tilde{\mathbf{v}}_{11}, \tilde{\mathbf{v}}_{16}) = 0.405$; $\tilde{\mathbf{v}}_{11}$ is selected.
$d_w(\tilde{\mathbf{v}}_{11}, \tilde{\mathbf{v}}_{17}) = 0.068$; $\tilde{\mathbf{v}}_{11}$ is selected.

Fuzzy pattern $\tilde{\mathbf{v}}_{11}$ is selected, but fuzzy pattern $\tilde{\mathbf{v}}_{17}$ is near $\tilde{\mathbf{v}}_{11}$; therefore, $\tilde{\mathbf{v}}_{17}$ is also selected:

$$C_{11}^{(2)} = \sum_{j=1}^{3} (\mathbf{W}^{(2)} \cdot \tilde{\mathbf{P}}^{(2)}) \cdot \tilde{\mathbf{v}}_{11} = 0.776$$

$$C_{17}^{(2)} = \sum_{j=1}^{3} (\mathbf{W}^{(2)} \cdot \tilde{\mathbf{P}}^{(2)}) \cdot \tilde{\mathbf{v}}_{17} = 0.716$$

Thus, the cause related to the quality of the concrete material involves two issues:

1. Abnormal setting of cement paste (fuzzy pattern $\tilde{\mathbf{v}}_{11}$), with a degree of confirmation at 77.6%.
2. Drying and shrinkage of concrete (fuzzy pattern $\tilde{\mathbf{v}}_{17}$), with a degree of confirmation of 71.6%.

Example 2

In a reinforced concrete structure, cracks were found in wall surfaces one year after casting. The cracks were very deep and formed a regular pattern, like an X shape, near the center of the walls.

The degree of confirmation for each of the crack characteristics was as follows:

Crack time (q_1): very_high $\therefore p_1 = \mu_{very\_high}(x) = 0.90$
Crack depth (q_2): very_high $\therefore p_2 = \mu_{very\_high}(x) = 0.90$
Crack regularity (q_1): high $\therefore p_3 = \mu_{high}(x) = 0.75$
Crack range (q_4): high $\therefore p_4 = \mu_{high}(x) = 0.75$
Crack member (q_5): very_high $\therefore p_5 = \mu_{very\_high}(x) = 0.90$
Crack pattern (q_6): very_high $\therefore p_6 = \mu_{very\_high}(x) = 0.90$
Crack location (q_7): high $\therefore p_7 = \mu_{high}(x) = 0.75$.

TABLE 8. Primary-Level: Causes-Characteristics Matrices

$$q_1 \Rightarrow \begin{array}{c} \\ q_{11} \\ q_{12} \\ q_{13} \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} .9 & .75 & .1 & .1 \\ .5 & .75 & .25 & .25 \\ .75 & .25 & .9 & .9 \end{bmatrix} \end{array}$$

$$q_2 \Rightarrow \begin{array}{c} \\ q_{21} \\ q_{22} \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} .9 & .5 & .5 & .1 \\ .1 & .5 & .75 & .9 \end{bmatrix} \end{array}$$

$$q_3 \Rightarrow \begin{array}{c} \\ q_{31} \\ q_{32} \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} .1 & .5 & .5 & .9 \\ .9 & .5 & .5 & .1 \end{bmatrix} \end{array}$$

$$q_4 \Rightarrow \begin{array}{c} \\ q_{41} \\ q_{42} \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} .9 & .9 & .9 & .9 \\ .1 & .1 & .9 & .9 \end{bmatrix} \end{array}$$

TABLE 9. Secondary-Level: Quality of Concrete Material Matrices

$$
\begin{array}{rc}
 & \begin{array}{ccccccc} v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} & v_{17} \end{array} \\
\begin{array}{r} q_{51} \\ q_{52} \\ q_5 \Rightarrow q_{53} \\ q_{54} \\ q_{55} \end{array} &
\begin{bmatrix}
.25 & .75 & .5 & .75 & .75 & .9 & .1 \\
.25 & .1 & .25 & .5 & .9 & .75 & .1 \\
.9 & .1 & .75 & .9 & .5 & .75 & .9 \\
.75 & .5 & .75 & .75 & .5 & .75 & .9 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1
\end{bmatrix}
\end{array}
$$

$$
\begin{array}{rc}
 & \begin{array}{ccccccc} v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} & v_{17} \end{array} \\
\begin{array}{r} q_{61} \\ q_{62} \\ q_{63} \\ q_{64} \\ q_{65} \\ q_{66} \\ q_6 \Rightarrow q_{67} \\ q_{68} \\ q_{69} \\ q_{610} \\ q_{611} \\ q_{612} \\ q_{613} \end{array} &
\begin{bmatrix}
.1 & .5 & .1 & .1 & .1 & .75 & .9 \\
.1 & .5 & .1 & .1 & .1 & .1 & .9 \\
.1 & .1 & .1 & .1 & .1 & .1 & .75 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .75 \\
.1 & .1 & .1 & .1 & .1 & .1 & .75 \\
.5 & .25 & .5 & .25 & .1 & .75 & .1 \\
.25 & .1 & .75 & .9 & .5 & .1 & .75 \\
.9 & .1 & .1 & .25 & .25 & .1 & .75 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.1 & .1 & .75 & .25 & .9 & .1 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1
\end{bmatrix}
\end{array}
$$

$$
\begin{array}{rc}
 & \begin{array}{ccccccc} v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} & v_{17} \end{array} \\
\begin{array}{r} q_{71} \\ q_{72} \\ q_{73} \\ q_7 \Rightarrow q_{74} \\ q_{75} \\ q_{76} \end{array} &
\begin{bmatrix}
.1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .9 \\
.9 & .25 & .9 & .9 & .9 & .9 & .9 \\
.1 & .1 & .1 & .1 & .1 & .1 & .9 \\
.1 & .1 & .1 & .1 & .1 & .1 & .9
\end{bmatrix}
\end{array}
$$

According to the crack data above, the method and process used in Example 1, four fuzzy patterns for the primary level were found in Table 8, and the primary-level observational fuzzy vectors $\tilde{\mathbf{P}}^{(1)}$ were expressed as follows:

$\tilde{\mathbf{V}}_i = (q_{13}\ q_{22}\ q_{31}\ q_{41})$ $i = 1, 2, 3, 4.$
$\tilde{\mathbf{V}}_1 = (.75\ .10\ .10\ .90)$, $\tilde{\mathbf{V}}_2 = (.25\ .50\ .50\ .90)$, $\tilde{\mathbf{V}}_3 = (.90\ .75\ .50\ .90)$, $\tilde{\mathbf{V}}_4 = (.90\ .90\ .90\ .90)$, $\tilde{\mathbf{P}}^{(1)} = (.90\ .90\ .75\ .75)$, and $\mathbf{W}^{(1)} = (.3\ .3\ .2\ .2)$.

The weighted Hamming distance was computed as follows:

$d_w(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_2) = .3\cdot(.75-.25) + .3\cdot(.10-.50) + .2\cdot(.10-.50) + .2\cdot(.90-.90) = -0.05$
$d_w(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_2) = -0.05 < 0$; $\tilde{\mathbf{V}}_2$ is selected.
$d_w(\tilde{\mathbf{V}}_2, \tilde{\mathbf{V}}_3) = .3\cdot(.25-.90) + .3\cdot(.50-.75) + .2\cdot(.50-.50) + .2\cdot(.90-.90) = -0.27$
$d_w(\tilde{\mathbf{V}}_2, \tilde{\mathbf{V}}_3) = -0.27 < 0$; $\tilde{\mathbf{V}}_3$ is selected.
$d_w(\tilde{\mathbf{V}}_3, \tilde{\mathbf{V}}_4) = .3\cdot(.90-.90) + .3\cdot(.75-.90) + .2\cdot(.50-.90) + .2\cdot(.90-.90) = -0.125$
$d_w(\tilde{\mathbf{V}}_3, \tilde{\mathbf{V}}_4) = -0.125 < 0$; $\tilde{\mathbf{V}}_4$ is selected.

The degree of confirmation for fuzzy pattern $\tilde{\mathbf{V}}_4$ is

$$C_4^{(1)} = \sum_{j=1}^{4} (\mathbf{W}^{(1)}\cdot\tilde{\mathbf{P}}^{(1)})\cdot\tilde{\mathbf{V}}_4 = 0.756.$$

Therefore, the cause of cracking is related to applied loads (fuzzy pattern $\tilde{\mathbf{V}}_4$), and the degree of confirmation is 75.6%.

On the secondary level there are five fuzzy patterns related to the applied loads. The fuzzy vector was formulated from Table 12, and $\tilde{\mathbf{P}}^{(2)}$ are as follows:

$\tilde{\mathbf{v}}_{4j} = (q_{54}\ q_{64}\ q_{72})$ $j = 1, 2, \ldots, 5.$
$\tilde{\mathbf{v}}_{41} = (.50\ .50\ .90)$, $\tilde{\mathbf{v}}_{42} = (.90\ .90\ .90)$, $\tilde{\mathbf{v}}_{43} = (.25\ .50\ .25)$
$\tilde{\mathbf{v}}_{44} = (.50\ .50\ .25)$, $\tilde{\mathbf{v}}_{45} = (.90\ .10\ .10)$,
$\tilde{\mathbf{P}}^{(2)} = (.90\ .90\ .75)$.

Weighted vector: $\mathbf{W}^{(2)} = (.30\ .45\ .25)$.

$d_w(\tilde{\mathbf{v}}_{41}, \tilde{\mathbf{v}}_{42}) = .3\cdot(.50-.90) + .45\cdot(.50-.90) + .25\cdot(.90-.90) = -0.3$
$d_w(\tilde{\mathbf{v}}_{41}, \tilde{\mathbf{v}}_{42}) = -0.3 < 0$; $\tilde{\mathbf{v}}_{42}$ is selected.
$d_w(\tilde{\mathbf{v}}_{42}, \tilde{\mathbf{v}}_{43}) = 0.538 > 0$; $\tilde{\mathbf{v}}_{42}$ is selected.
$d_w(\tilde{\mathbf{v}}_{42}, \tilde{\mathbf{v}}_{44}) = 0.463 > 0$; $\tilde{\mathbf{v}}_{42}$ is selected.
$d_w(\tilde{\mathbf{v}}_{42}, \tilde{\mathbf{v}}_{45}) = 0.56 > 0$; $\tilde{\mathbf{v}}_{42}$ is selected.

and

$$C_{42}^{(2)} = \sum_{j=1}^{3} (\mathbf{W}^{(2)}\cdot\tilde{\mathbf{P}}^{(2)})\cdot\tilde{\mathbf{v}}_{42} = 0.776.$$

Hence the specific cause of these loading-related cracks was earthquake force (fuzzy pattern $\tilde{\mathbf{v}}_{42}$), with a degree of confirmation of 77.6%.

Example 3

In a reinforced concrete column, honeycombing occurred on the member surface seven days after casting. The cracks were deep and had no regularity.

The degree of confirmation for each of the crack characteristics was as follows:

Crack time (q_1): very_high $\therefore$ $p_1 = \mu_{very\_high}(x) = 0.90$
Crack depth (q_2): very_high $\therefore$ $p_2 = \mu_{very\_high}(x) = 0.90$
Crack regularity (q_3): high $\therefore$ $p_3 = \mu_{high}(x) = 0.75$
Crack range (q_4): very_high $\therefore$ $p_4 = \mu_{very\_high}(x) = 0.90$
Crack member (q_5): very_high $\therefore$ $p_5 = \mu_{very\_high}(x) = 0.90$
Crack pattern (q_6): high $\therefore$ $p_6 = \mu_{high}(x) = 0.75$
Crack location (q_7): high $\therefore$ $p_7 = \mu_{high}(x) = 0.75.$

According to the crack data above, the method and process used in Example 1, four fuzzy patterns for the primary level were found in Table 8, and the primary-level observational fuzzy vectors $\mathbf{P}^{(1)}$ were expressed as follows.

$\tilde{\mathbf{V}}_i = (q_{12}\ q_{22}\ q_{32}\ q_{41})$ $i = 1, 2, 3, 4.$
$\tilde{\mathbf{V}}_1 = (.50\ .10\ .90\ .90)$, $\tilde{\mathbf{V}}_2 = (.75\ .50\ .50\ .90)$, $\tilde{\mathbf{V}}_3 = (.25\ .75\ .50\ .90)$, $\tilde{\mathbf{V}}_4 = (.25\ .90\ .10\ .90)$, $\tilde{\mathbf{P}}^{(1)} = (.90\ .90\ .75\ .90)$, and $\mathbf{W}^{(1)} = (.3\ .3\ .2\ .2)$.

The weighted Hamming distance was computed as follows:

$d_w(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_2) = .3\cdot(.50-.75) + .3\cdot(.10-.50) + .2\cdot(.90-.50) + .2\cdot(.90-.90) = -0.115$
$d_w(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_2) = -0.115 < 0$; $\tilde{\mathbf{V}}_2$ is selected.
$d_w(\tilde{\mathbf{V}}_2, \tilde{\mathbf{V}}_3) = .3\cdot(.75-.25) + .3\cdot(.50-.75) + .2\cdot(.50-.50) + .2\cdot(.90-.90) = 0.075$
$d_w(\tilde{\mathbf{V}}_2, \tilde{\mathbf{V}}_3) = 0.075 > 0$; $\tilde{\mathbf{V}}_2$ is selected.

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TABLE 10. Secondary-Level: Concrete Construction Procedure Matrices

$$
q_5 \Rightarrow \begin{array}{c} \\ q_{51} \\ q_{52} \\ q_{53} \\ q_{54} \\ q_{55} \end{array}
\begin{array}{c}
\begin{array}{ccccccccccccc} v_{21} & v_{22} & v_{23} & v_{24} & v_{25} & v_{26} & v_{27} & v_{28} & v_{29} & v_{210} & v_{211} & v_{212} & v_{213} \end{array} \\
\begin{bmatrix}
.9 & .9 & .5 & .75 & .25 & .5 & .75 & .75 & .5 & .9 & .9 & .9 & .75 \\
.9 & .9 & .9 & .9 & .1 & .1 & .1 & .5 & .75 & .75 & .75 & .25 & .25 \\
.9 & .9 & .25 & .75 & .9 & .9 & .1 & .9 & .9 & .9 & .9 & .9 & .9 \\
.9 & .9 & .9 & .9 & .1 & .1 & .9 & .25 & .75 & .25 & .5 & .5 & .9 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1
\end{bmatrix}
\end{array}
$$

$$
q_6 \Rightarrow \begin{array}{c} \\ q_{61} \\ q_{62} \\ q_{63} \\ q_{64} \\ q_{65} \\ q_{66} \\ q_{67} \\ q_{68} \\ q_{69} \\ q_{610} \\ q_{611} \\ q_{612} \\ q_{613} \end{array}
\begin{array}{c}
\begin{array}{ccccccccccccc} v_{21} & v_{22} & v_{23} & v_{24} & v_{25} & v_{26} & v_{27} & v_{28} & v_{29} & v_{210} & v_{211} & v_{212} & v_{213} \end{array} \\
\begin{bmatrix}
.5 & .9 & .5 & .5 & .5 & .9 & .9 & .1 & .9 & .9 & .9 & .1 & .1 \\
.5 & .9 & .5 & .5 & .5 & .9 & .9 & .75 & .9 & .9 & .9 & .9 & .1 \\
.1 & .1 & .9 & .5 & .25 & .1 & .9 & .5 & .1 & .9 & .9 & .9 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.9 & .9 & .1 & .1 & .9 & .9 & .1 & .5 & .75 & .1 & .75 & .1 & .9 \\
.75 & .5 & .1 & .5 & .9 & .5 & .1 & .5 & .75 & .1 & .5 & .1 & .75 \\
.5 & .5 & .75 & .9 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .75 & .1 & .1 & .1 & .1 \\
.1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\
.1 & .1 & .1 & .1 & .75 & .1 & .1 & .1 & .9 & .1 & .1 & .1 & .1
\end{bmatrix}
\end{array}
$$

$$
q_7 \Rightarrow \begin{array}{c} \\ q_{71} \\ q_{72} \\ q_{73} \\ q_{74} \\ q_{75} \\ q_{76} \end{array}
\begin{array}{c}
\begin{array}{ccccccccccccc} v_{21} & v_{22} & v_{23} & v_{24} & v_{25} & v_{26} & v_{27} & v_{28} & v_{29} & v_{210} & v_{211} & v_{212} & v_{213} \end{array} \\
\begin{bmatrix}
.5 & .5 & .75 & .9 & .1 & .5 & .5 & .5 & .1 & .5 & .75 & .25 & .1 \\
.5 & .75 & .9 & .9 & .1 & .5 & .9 & .5 & .9 & .9 & .75 & .5 & .1 \\
.5 & .5 & .5 & .9 & .1 & .5 & .5 & .75 & .1 & .5 & .5 & .25 & .1 \\
.5 & .5 & .75 & .9 & .9 & .75 & .75 & .1 & .5 & .5 & .75 & .75 & .9 \\
.5 & .5 & .5 & .9 & .1 & .5 & .75 & .9 & .1 & .1 & .1 & .1 & .1 \\
.5 & .5 & .5 & .9 & .1 & .75 & .9 & .9 & .1 & .1 & .1 & .1 & .1
\end{bmatrix}
\end{array}
$$

$d_w(\tilde{\mathbf{V}}_2, \tilde{\mathbf{V}}_4) = .3\cdot(.75-.25) + .3\cdot(.50-.90) + .2\cdot(.50-.10) + .2\cdot(.90-.90) = 0.11$

$d_w(\tilde{\mathbf{V}}_2, \tilde{\mathbf{V}}_4) = 0.11 > 0$; $\tilde{\mathbf{V}}_2$ is selected.

The degree of confirmation for fuzzy pattern $\tilde{\mathbf{V}}_2$ is

$$C_2^{(1)} = \sum_{j=1}^{4} (\mathbf{W}^{(1)}\cdot\tilde{\mathbf{P}}^{(1)})\cdot\tilde{\mathbf{V}}_2 = 0.575.$$

Therefore, the cause of cracking is related to the concrete construction procedure (fuzzy pattern $\tilde{\mathbf{V}}_2$), and the degree of confirmation is 57.5%.

On the secondary level there are 13 fuzzy patterns related to the concrete construction procedure. The fuzzy vector was formulated from Table 10 and $\tilde{\mathbf{P}}^{(2)}$ as follows:

$\tilde{\mathbf{v}}_{2j} = (q_{52}\ q_{610}\ q_{74})\quad j = 1, 2, \ldots, 13.$

$\tilde{\mathbf{v}}_{21} = (.90\ \ .50\ \ .50)$, $\tilde{\mathbf{v}}_{22} = (.90\ \ .50\ \ .50)$, $\tilde{\mathbf{v}}_{23} = (.90\ \ .75\ \ .75)$

$\tilde{\mathbf{v}}_{24} = (.90\ \ .90\ \ .90)$, $\tilde{\mathbf{v}}_{25} = (.10\ \ .10\ \ .90)$, $\tilde{\mathbf{v}}_{26} = (.10\ \ .10\ \ .75)$

$\tilde{\mathbf{v}}_{27} = (.10\ \ .10\ \ .75)$, $\tilde{\mathbf{v}}_{28} = (.50\ \ .10\ \ .10)$, $\tilde{\mathbf{v}}_{29} = (.75\ \ .10\ \ .50)$

$\tilde{\mathbf{v}}_{210} = (.75\ \ .10\ \ .50)$, $\tilde{\mathbf{v}}_{211} = (.75\ \ .10\ \ .75)$, $\tilde{\mathbf{v}}_{212} = (.25\ \ .10\ \ .75)$

$\tilde{\mathbf{v}}_{213} = (.25\ \ .10\ \ .90)$, $\tilde{\mathbf{P}}^{(2)} = (.90\ \ .75\ \ .75)$, $\mathbf{W}^{(2)} = (.30\ \ .45\ \ .25)$.

$d_w(\tilde{\mathbf{v}}_{21}, \tilde{\mathbf{v}}_{22}) = .3\cdot(.90-.90) + .45\cdot(.50-.50) + .25\cdot(.50-.50) = 0$

$d_w(\tilde{\mathbf{v}}_{21}, \tilde{\mathbf{v}}_{22}) = 0$; $\tilde{\mathbf{v}}_{21}$ or $\tilde{\mathbf{v}}_{22}$ is selected.

$d_w(\tilde{\mathbf{v}}_{21}, \tilde{\mathbf{v}}_{23}) = -0.175 < 0$; $\tilde{\mathbf{v}}_{23}$ is selected.

$d_w(\tilde{\mathbf{v}}_{23}, \tilde{\mathbf{v}}_{24}) = -0.105 < 0$; $\tilde{\mathbf{v}}_{24}$ is selected.

$d_w(\tilde{\mathbf{v}}_{24}, \tilde{\mathbf{v}}_{25}) = 0.6 > 0$; $\tilde{\mathbf{v}}_{24}$ is selected.

$d_w(\tilde{\mathbf{v}}_{24}, \tilde{\mathbf{v}}_{26}) = 0.6375 > 0$; $\tilde{\mathbf{v}}_{24}$ is selected.

$d_w(\tilde{\mathbf{v}}_{24}, \tilde{\mathbf{v}}_{27}) = 0.6375 > 0$; $\tilde{\mathbf{v}}_{24}$ is selected.

$d_w(\tilde{\mathbf{v}}_{24}, \tilde{\mathbf{v}}_{28}) = 0.68 > 0$; $\tilde{\mathbf{v}}_{24}$ is selected.

$d_w(\tilde{\mathbf{v}}_{24}, \tilde{\mathbf{v}}_{29}) = 0.505 > 0$; $\tilde{\mathbf{v}}_{24}$ is selected.

$d_w(\tilde{\mathbf{v}}_{24}, \tilde{\mathbf{v}}_{210}) = 0.505 > 0$; $\tilde{\mathbf{v}}_{24}$ is selected.

$d_w(\tilde{\mathbf{v}}_{24}, \tilde{\mathbf{v}}_{211}) = 0.4425 > 0$; $\tilde{\mathbf{v}}_{24}$ is selected.

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$d_w(\tilde{\mathbf{v}}_{24}, \tilde{\mathbf{v}}_{212}) = 0.5925 > 0$; $\tilde{\mathbf{v}}_{24}$ is selected.
$d_w(\tilde{\mathbf{v}}_{24}, \tilde{\mathbf{v}}_{213}) = 0.555 > 0$; $\tilde{\mathbf{v}}_{24}$ is selected.

and

$$C_{24}^{(2)} = \sum_{j=1}^{3} (\mathbf{W}^{(2)} \cdot \tilde{\mathbf{P}}^{(2)}) \cdot \tilde{\mathbf{v}}_{24} = 0.716.$$

Therefore, the crack cause related to concrete construction was undervibrated or overvibrated during concrete placing (fuzzy pattern $\tilde{\mathbf{v}}_{24}$), with a degree of confirmation of 71.6%.

CONCLUSION

A fuzzy pattern recognition model based on cause-and-effect diagramming and fuzzy pattern recognition has been developed and applied to diagnose cracks in reinforced concrete structures. The following conclusions can be drawn from this paper:

TABLE 11. Secondary-Level: Environmental Factors Matrices

		v_{31}	v_{32}	v_{33}	v_{34}	v_{35}
	q_{51}	.5	.5	.9	.5	.9
	q_{52}	.5	.5	.9	.25	.9
$q_5 \Rightarrow$	q_{53}	.5	.75	.9	.9	.9
	q_{54}	.9	.9	.9	.9	.9
	q_{55}	.9	.75	.25	.25	.25

		v_{31}	v_{32}	v_{33}	v_{34}	v_{35}
	q_{61}	.5	.1	.5	.25	.5
	q_{62}	.5	.1	.5	.25	.5
	q_{63}	.1	.1	.1	.1	.1
	q_{64}	.9	.1	.1	.1	.1
	q_{65}	.9	.1	.1	.1	.1
	q_{66}	.9	.1	.1	.1	.1
$q_6 \Rightarrow$	q_{67}	.1	.5	.9	.5	.25
	q_{68}	.1	.75	.75	.5	.1
	q_{69}	.1	.75	.1	.75	.1
	q_{610}	.1	.1	.1	.1	.1
	q_{611}	.1	.75	.5	.9	.75
	q_{612}	.1	.1	.1	.1	.1
	q_{613}	.1	.1	.1	.1	.9

		v_{31}	v_{32}	v_{33}	v_{34}	v_{35}
	q_{71}	.25	.1	.1	.1	.1
	q_{72}	.25	.1	.1	.1	.1
	q_{73}	.9	.25	.1	.1	.1
$q_7 \Rightarrow$	q_{74}	.75	.9	.9	.9	.9
	q_{75}	.9	.1	.1	.1	.1
	q_{76}	.5	.25	.1	.1	.1

TABLE 12. Secondary-Level: Applied Loads Matrices

		v_{41}	v_{42}	v_{43}	v_{44}	v_{45}
	q_{51}	.9	.5	.75	.75	.1
	q_{52}	.75	.9	.25	.25	.1
$q_5 \Rightarrow$	q_{53}	.75	.5	.5	.5	.1
	q_{54}	.5	.9	.25	.5	.9
	q_{55}	.5	.75	.9	.9	.1

		v_{41}	v_{42}	v_{43}	v_{44}	v_{45}
	q_{61}	.1	.1	.1	.1	.9
	q_{62}	.9	.1	.1	.1	.5
	q_{63}	.9	.75	.75	.5	.1
	q_{64}	.5	.9	.5	.5	.1
	q_{65}	.1	.1	.9	.9	.1
	q_{66}	.1	.1	.9	.9	.1
$q_6 \Rightarrow$	q_{67}	.1	.1	.1	.1	.1
	q_{68}	.1	.1	.1	.1	.1
	q_{69}	.1	.1	.1	.1	.1
	q_{610}	.1	.1	.1	.1	.1
	q_{611}	.1	.1	.1	.1	.5
	q_{612}	.75	.5	.25	.25	.1
	q_{613}	.1	.1	.1	.1	.1

		v_{41}	v_{42}	v_{43}	v_{44}	v_{45}
	q_{71}	.9	.9	.75	.75	.1
	q_{72}	.9	.9	.25	.25	.1
	q_{73}	.5	.1	.75	.75	.1
$q_7 \Rightarrow$	q_{74}	.5	.1	.75	.75	.1
	q_{75}	.75	.75	.5	.5	.1
	q_{76}	.75	.75	.5	.5	.9

1. Cause-and-effect diagrams can be effectively used to establish a diagnostic model, particularly because they clearly depict the relationships among the causes of cracks and the characteristics of the cracks.
2. Fuzzy sets and fuzzy pattern recognition enable us to deal effectively with the ambiguity in diagnosing the causes of cracks. This ambiguity is almost impossible to solve using traditional mathematical models.
3. By combining fuzzy pattern recognition and cause-and-effect diagrams, one can narrow down the possible causes of crack formation.
4. If the data base, the weighted vector, and Hamming distance formula are valid, which depends on the data collected and the experience of the base designer, the proposed model produces reliable diagnostic results. The model offers an effective tool for diagnosing cracks in concrete structures and may be useful for professionals in the field of concrete engineering.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

$C_i^{(1)}$ = degree of confirmation of fuzzy pattern $\tilde{V}_i$ for primary level;
$C_{ik}^{(2)}$ = degree of confirmation of fuzzy pattern $\tilde{v}_{ik}$ for secondary level;
$d_w(\tilde{V}_i, \tilde{V}_j)$ = weighted Hamming distance between fuzzy pattern $\tilde{V}_i$ and $\tilde{V}_j$;
$\tilde{P}^{(1)}$ = observational fuzzy vector for primary level;
$\tilde{P}^{(2)}$ = observational fuzzy vector for secondary level;
q_i = ith characteristic of crack;
$\tilde{R}^{(1)}$ = fuzzy relation matrix of characteristic and cause for primary level;
$\tilde{R}_j^{(2)}$ = fuzzy relation matrix of characteristic and cause for secondary level;
$\tilde{V}_i$ = ith fuzzy pattern on primary level;
$\tilde{v}_{lm}$ = mth fuzzy pattern on secondary level;
$W^{(1)}$ = weighting vector of primary level; and
$W^{(2)}$ = weighting vector of secondary level.

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